

**UNIVERSIDAD NACIONAL DE EDUCACIÓN A DISTANCIA**

ESCUELA INTERNACIONAL DE DOCTORADO  
PROGRAMA DE DOCTORADO EN TECNOLOGÍAS INDUSTRIALES

**DEVELOPMENT OF A DYNAMIC RISK ASSESSMENT  
AND CONTROL METHODOLOGY FOR INDUSTRIAL  
ACCIDENTS**

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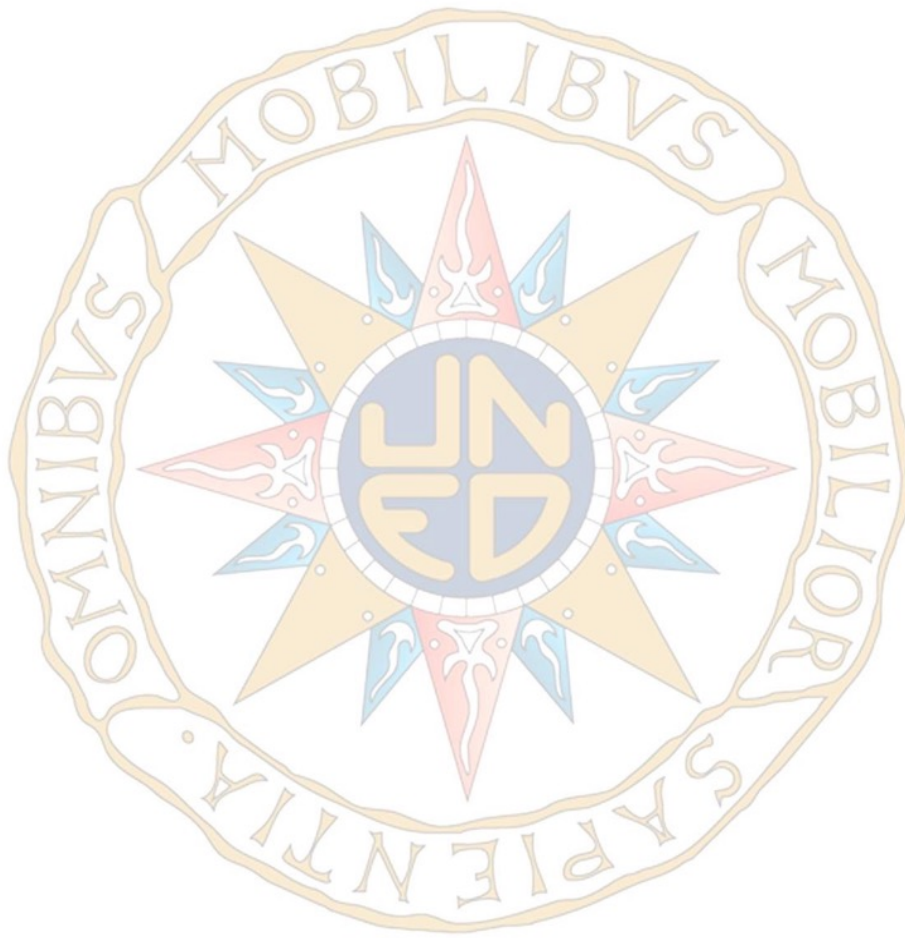
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# Resumen

Los tipos de industrias más afectados por accidentes, en la Unión Europea dentro del período 1979-2019, corresponden a los sectores químico-farmacéutico y petroquímico por causas que abarcan las incidencias en operaciones, los fallos en los equipos, explosiones e incendios y errores humanos en trabajos de producción y mantenimiento. A esta situación le sigue una accidentabilidad laboral que fué de 2 accidentes fatales por 100,000 personas empleadas en el 2015. Para poder presentar una herramienta que permita gestionar las diferentes situaciones que puedan estar afectadas por riesgos tanto en los procesos industriales como en el trabajo, primero se realiza una revisión de las herramientas existentes teniendo en cuenta tres características: prevención, simultaneidad e inmediatez; como resultado, se presenta una nueva metodología dinámica denominada Control Estadístico de Riesgos (SRC) basada en la utilización de la inferencia Bayesiana, cuadros de control y análisis aplicando cadenas ocultas de Markov, para que a partir de las causas iniciales de incidencias y los fallos en las barreras de seguridad, sea posible detectar una situación fuera de límites con la suficiente antelación como para permitir acciones correctivas con el fin de reducir el riesgo antes de que ocurra un accidente. Se desarrollan varios casos aplicando diferentes modelos de inferencia Bayesiana. La metodología cubre riesgos industriales, accidentes laborales, fuga de material inflamable con posible efecto dominó y desviaciones en coste y tiempo. Se han utilizado un total de 12 modelos de inferencia con los que se realizan los análisis de las observaciones recopiladas a partir de las causas iniciales de riesgo y fallos en las barreras de seguridad, utilizando , cuando no hay solución analítica, una herramienta de muestreo basada en el algoritmo de Metrópolis-Hastings. Los valores recogidos se presentan en tablas o gráficos de control que visualizan si la situación está fuera de los límites. Los resultados muestran que la metodología ofrece un procedimiento formal, para obtener una valoración de las probabilidades de riesgo y para prevenir o mitigar accidentes y riesgos laborales en escenarios de fabricación y procesos industriales, advirtiéndolo de su existencia con el fin de poder actuar de antemano, corrigiendo sus causas y ofreciendo una visión del proceso analizado de la manera más simple y práctica y a la vez respondiendo a las tres características de prevención (P), simultaneidad (S) e inmediatez (I).

**Palabras Clave**— *Riesgo, Bayesiano, Evaluación, Control, Accidente laboral, Coste, Efecto dominó.*



# Abstract

The most affected types of industries affected by accidents in the European Union in the period of years 1979-2019 are chemical-pharmaceutical and petrochemical-refining with causes covering operations, failure in equipment, explosion and fire and human errors. This situation is followed by an occupational accidentability rate of 2 fatal accidents by 100,000 persons employed in 2015. To manage situations that affect risks in industrial processes and occupational risk at work, a review of existing tools is first carried out taking into account three prevention, simultaneity and immediacy characteristics. As a result, a new dynamic methodology called Statistical Risk Control (SRC) based on Bayesian inference, control charts and analysis applying hidden Markov chains for the initiating causes and safety barriers is presented. The objective is to detect the situation outside the limits early enough to allow corrective actions to reduce the risk before an accident occurs under the concept of immediacy. Several cases are developed testing different Bayesian inference models. The methodology covers industrial risks, occupational accidents, loss of containment with possible domino effect and deviations in cost-time. A total of 12 inference models have been tested performing the analysis of collected observations of initiating causes of risk and safety barriers failure using a Metropolis-Hastings sampling when it is needed. Collected values are presented in tables or control charts visualizing when the situation is out of limits. The results show that the methodology offers a formal procedure for to have a determination of failure probabilities that can prevent or mitigate accidents and occupational hazards in manufacturing scenarios and industrial processes warning of the existence of a risk to act in advance correcting causes and offering a complete vision in the simplest and most practical way possible and responding to the three characteristics of prevention (P), simultaneity (S) and immediacy (I).

**Keywords**— *Risk, Bayesian, Assessment, Control, Occupational accident, Cost, Domino effect*





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# Symbols, abbreviations and acronyms

AcciMap	Map of Accident
AC	Administration Cost control
ACF	Auto Correlation Function
AE	Accident Event
AEB	Accident Evolution and Barrier Function
AGE	Ageing
AIChE	American Institute of Chemical Engineers
AMF	Actuation through Manifold Failure
AMIS	Administration Control Mistake
AR	Acceptance Rate
ARR	Accounting Rate of Return
ba	Basic Event
bar	Pressure unit equivalent to 10000 Pa
BLEVE	Boiling Liquid Expanding Vapor Explosion
BN	Bayesian Network
BORA	Barrier and Operational Risk Analysis
BOW-TIE	Integration of FTA and ETA
C	Distance of control limits, in standard deviation units, from the center line
CCPS	Center for Chemical Process Safety
CD	Communication Documentation
CI	Confidence Interval
CJS	Contract Job Security
CMDF	Corrosion monitoring deficiency-failure
CMIS	Communication Mistake
CO	Continuous Operation
COMAH	Control of Major Hazards
COOLSF	Cooling System Failure
CPC	Common Performance Conditions
CPR	Commissie voor de Preventie van Rampen. Commission for Disaster Prevention
CREAM	Cognitive Reliability and Error Analysis Method

CSTR	Continuous Stirred Tank Reactor
DEF	Deviation from a procedure
DRA	Dynamic Risk Assessment
DRAn	Dynamic Risk Analysis
DREAM	Driving Reliability and Error Analysis Method
DSBd	Degree of Safety Barriers Definition
DyPASI	Dynamic Procedure for Atypical Scenarios Identification
EAG	Earnings
EBM	Energy Barrier Model
EDEF	Engineering Maintenance Deficiency
EDW	Emotional Demands at Work
EEC	European Economic Community
ELCF	Emergency Level Control Failure
EM	Engineering Maintenance
EMHPWCF	Emergency High Pressure Water Curtain Failure
EMIS	Engineering Maintenance Mistake
EMPF	Emergency Power Failure
EMSHUTF	Emergency Shutdown Failure
EN	European Norm
EPC	Error Promoting Condition
EPCF	Emergency Pressure Control Failure
ETA	Event Tree Analysis
EU	European Union
FIFO	First In First Out
FMEA	Failure Mode Effect Analysis
FMECA	Failure Mode Effect and Criticality Analysis
FP	Flash Point
FRAM	Functional Resonance Accident Model
FTA	Fault Tree Analysis
FTS	Failed test
GFCF	General Foam Curtain Failure
GLCF	General Liquid Containment Failure
GSPKYF	General Sprinkler-Spray Failure
ha	Human factor
HAZID	Hazard Identification
HAZOP	Hazard and Operability
HEART	Human Error and Assessment Technique
HR	Human Resource
HRMIS	Human Resource Mistake



HRDEF	Human Resource Deficiency
HSC	Human and Social Changes
HSI	Human System Interface
I	Amount of heat radiation. Immediacy
IB	Installation Behavior
IBP	Initial Boiling Point
ic	Initiating Cause
IEC	International Electrotechnical Commission. Immediate Error after Intervention
IFRS	International Financial Reporting Standard
IMD	Investment Modifications
INF	Information
IPL	Independent Protection Layer
IPMA	International Project Management Association
IRR	Internal Rate of Return
IS	Intermediate Safety. Preventive Barrier
ISO	International Standard Organization
JBD	Job Design
JSC	Job Self Control
JSU	Job Supervision
kg	Kilogram
kJ	Kilo Joule
kPa	Kilo Pascal
kW	Kilo-Watt
l	Liter
LCI	Lack in Communication and Information
LCL	Low Control Limit
LCLLF	Liquid Collector Failure
LCMF	Liquid Containment Failure
LC50	Lethal Concentration. Concentration in air which causes the death of 50% of a group of test animals.
LD50	Lethal Dose. Amount of a material, given all at once, which causes the death of 50% of a group of test animals.
LFCF	Local Foam Curtain Failure
LHPWCF	Local High Pressure Water Curtain Failure
LMIS	Logistics Mistake
LNG	Liquefied Natural Gas
LO	Logistics
LOC	Loss of Containment

LOPA	Layer of Protection Analysis
LPG	Liquefied Petroleum Gas
m	Meter
cm	Centimeter
MAPP	Major Accident Prevention Policy
MARS	Major Accident Reporting System
MAT	Malicious Acts derived from War or Terrorism
MATLAB	Numerical Computing System with its own programming language
MDEF	Marketing Deficiency
MDF	Medium Density Fiber
mg	Milligram
MH	Metropolis Hastings sampler
MK	Marketing
MW	Mega-Watt
mm	Milimeter
MMIS	Marketing Mistake
MORT	Management Oversight and Risk Tree
MTO	Man Technology and Organization
NISHW	Spanish-National Institute of Safety and Health at Work
NORSOK	Norsk Sokkels Konkuranseposisjon. Norwegian standards for competitive position
NPV	Net Present Value
OP	Operations
OPC	Operator Characteristics
OPE	Operator Environment. Over Pressure
OREDA	Offshore and Onshore Reliability Data
ORG	Oganization
OSHA	Occupational Safety and Health Administration
p	Parameter.
P	Prevention
p <sub>s,f</sub>	Probability of success or failure
Pa	Pascal
PACF	Partial Autocorrelation Function
PBP	Pay Back Period
PDEF	Planning Scheduling Deficiency
PHA	Preliminary Hazard Analysis
PHE	Physical Environment
PHPAM	Process Hazard Prevention Accident Models
PMBOK	Project Management Body of Knowledge

PMIS	Planning Scheduling Mistake
PN	Planning
ppm	Part per million
pot	Potential Causes
PRF	Pumping Recycle Failure
PRINCE	Projects In Controlled Environments
PRO	Prospects
PWL	Poor Work Life balance
QDEF	Quality Deficiency
QMIS	Quality Mistake
QNAC	Quality Not Acceptance
QR	Quality-Reliability
QRA	Quantitative Risk Assessment
RA	Risk Assessment. Run Away
RCD	Concatenation. Domino effect
RDI	Research, Development, Innovation
RDIDEF	Research, Development, Innovation Deficiency
RDIMIS	Research, Development, Innovation Mistake
RIF	Risk Influencing Factor
RRA	Reliability Assessment
s	Second - unit of time
S	Simultaneity
SCADA	System Control and Data Acquisition
SCAT	Systematic Cause Analysis Technique
SCDF	System Control with Deficiency-Failure
SD	Shut Down
SDEL	Supply - Distribution Delay
SEN	Social Environment
seqobs	Sequence observed
SF <sub>n</sub>	Mitigative Barrier (n)
SF <sub>nm</sub>	Mitigative barrier (n) sub-function (m)
SF <sub>ISnm</sub>	Preventive barrier (n) sub-function (m)
SF <sub>ISg</sub>	General sub-function
SHE	Safety, Health, Environment
SHECH	Safety, Health, Environment Changes in Regulations
SHEDEF	Safety, Health, Environment Deficiency
SHEMIS	Safety, Health, Environment Mistake
SHESAN	Safety, Health, Environment Sanction
SHIPP	System Hazard Identification Prediction and Prevention

SHUTF	Shutdown failure
SHW	Safety and Health at Work
SKD	Skills and Discretion
SOL	Safety through Organizational Learning
SPKF	Local sprinkler failure
SPW55	Days of Speed of Wind over or equal to 55 km/h
SRC	Statistical Risk Control
SSC	Safety Self Control
SSU	Safety Supervision
STAMP	Systems Theoretic Accident Model and Processes
STEP	Sequential Time Events Plotting
TFT	Technical design failure
THERP	Technique for Human Error Rate Prediction
TNT	Trinitrotoluene
TOF	Toxicity-Operation-Flammability
UCL	Upper Control Limit
UNT	Unplanned New Technology
V	Volume
VCE	Vapor Cloud Explosion
VOC	Volatile Organic Compound
WBA	Why Because Analysis
WCF	Welding Control Failure
WIT	Work Intensification
WKD	Workplace Design
WKI	Work Intensity
WTQ	Working Time Quality
XCS	Extreme Climatic Situation

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# Chapter 1

## Introduction.

Industrial and occupational accidents are produced by the use of materials that can be toxic, flammable and explosive; for the mistakes in human actuations; and for the equipment failure despite the controls and preventive maintenance interventions.

Examining, for the European Union, the accident rate by types of industry in the years 1979-2019, [1], it is found that the most affected industrial activities are: chemical-pharmaceutical (36%); petrochemical-refining (22%); wholesale, retail, storage and distribution (including Liquefied Petroleum Gas (LPG)) (10%); metals processing (10%); pesticides, fungicides and fertilizers (6%); plastic and rubber (4%); food and beverages (3%); agriculture (1%); waste treatment (3%); power generation and distribution (4%) and paper (2%), Figure 1.

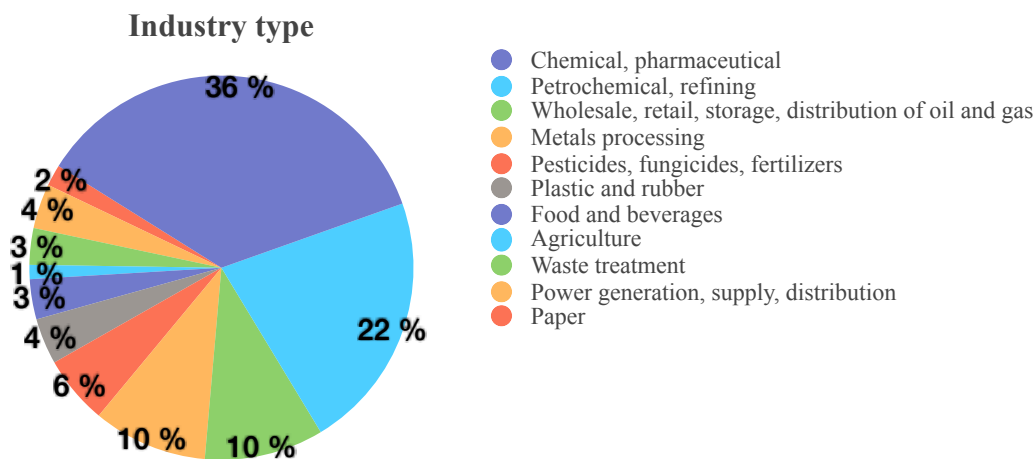


Figure 1.- Rate of accidents by type of industry, years 1979-2019 (source: [1]).

Analyzing the causes of accidents in the offshore oil and natural gas extraction sector [2], in the years 2013-2017, a sector which groups a set of operations in which chemical and petrochemical processes of extraction, treatment, conditioning and transport of toxic and flammable materials are used, it is obtained that the main causes of accident are generated from: production (64%); lifting operations (11%); failure in equipment or their damage (8%); explosion and fire (7%); human errors (4%); work-over and maintenance (3%); leakages (2%) and weather (storm, lightning) (0.15%), Figure 2.

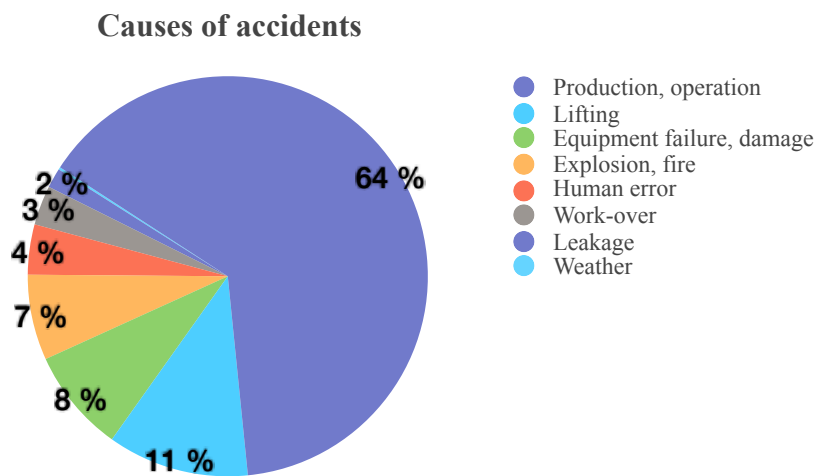


Figure 2.- Causes of accidents in the oil&gas offshore sector, years 2013-2017 (source: [2]).

The rate of occupational accidents in the European Union , according to the Eurostat information [3] for the EU-28 region, shows a value of 2 fatal accidents by 100,000 persons employed in 2015 and being the most affected industrial activities: building (24%), manufacturing (19%), transportation and storage (19%), agriculture - fishing (15%), retail (9%), public administration (9%), water supply - waste management (3%) and mining (2%). The situation in Spain between years 2014-2018, is practically the same [4].

Also analyzing the main causes of occupational accidents in the Spanish country, for the same period 2014-2018, has been found in situations of human body movement involving lift, pushing-pulling and improper body turns under physical effort (33%), entrapment by machine elements, not coordinated less physical effort movements and sharp machine elements contact (22%), falls and slips (18%), loss of total or partial control of the machine (16%), support breakage and gliding (6%), leakage and spill (2%), aggressions (2%) and explosion - fire (1%) [4], Figure 3.

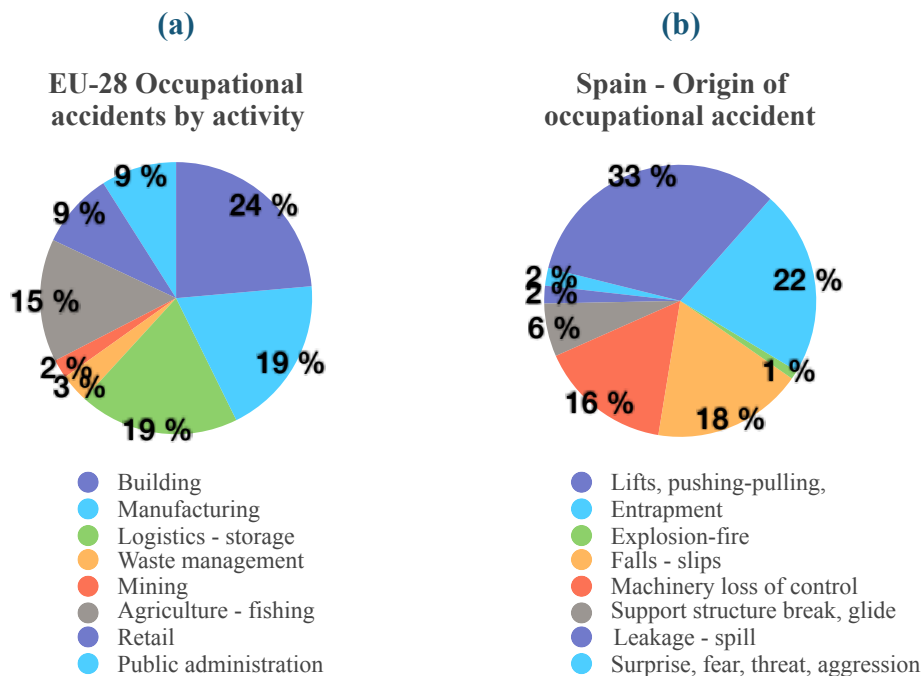


Figure 3.- (a) Accident rate by activity in the year 2015 in the EU-28 (source: [1]); (b) Description of Origin of occupational accidents in Spain for years 2014-2018 (source: [4]).

All of these causes have generated from the total of accidents a (99.1%) with minor consequences, with severe damage (0.8%) and fatal situations (0.1%).

## 1.1. Background.

This thesis has its origin in the experience and situations that its author has lived within the framework of professional projects and industrial works. And where, as a result of their realization, they were missing a procedure that could indicate, in parallel, through their own field or plant activity, when they could be close to an accident situation.

## 1.2. Research process.

With the idea of obtaining a formal procedure that can prevent or mitigate accidents and occupational hazards in manufacturing scenarios and industrial processes, warning of the existence of a risk well in advance to correct the situation that causes it, three characteristics are defined that the procedure must comply:

- Prevention (P): being the process of avoiding or mitigate the risks by reducing their probability of occurrence and their impacts in human and social; geographical and landscape; economical and infrastructures; environmental and ecosystem preservation; accident and safety (human, assets, production); perception and expectations.

- Simultaneity (S): being the capacity for to update the risk evolution according to the real time operation.
- Immediacy (I): being the capacity for to inform or infer of the existence of a risk with enough advance for making the needed corrections before the accident occurs.

To do this, it is first necessary to review what are the current tools related to risk assessment, verifying which are the most appropriate and to what degree they meet these three characteristics. The proposed methodology arises as a result of this review by completing the aspects and characteristics that other tools do not include.

### **1.3. Objective.**

After the revision of the existent methodologies the objective of this thesis is to develop and present a new dynamic tool called Statistical Risk Control (SRC) to manage and assess risk situations in industrial processes and occupational accidents, offering a complete view of the risk situation and the causes that generate it and warning of their existence enough in advance for correcting the situation and causes before it produces.

Once the general procedure has been established, its application is particularized on the control of possible accidents in industrial and work environments. Additionally, its application is examined in the control of projects extending the concept of risk in the deviations in time and cost as a deviation in objectives, and in industrial situations where the domino effect may occur.

### **1.4. Justification.**

There is a general need for to manage and assess the risk. Risk is a concept with many definitions for example in certain cases the risk arises from the occurrence of situations that occur with uncertainty [4], in others cases risk arises from the uncertainty in the objectives [5,6], or specifically risk is a measure of the lack of safety [7]. In the same consideration is the safety idea, that can simply be defined as a lack of accidents, or from an occupational point of view, how people are able to provide the required performance under expected and unexpected conditions [8], or describing safety as a numerical condition where the number of adverse outcomes is acceptably small [9] or from an analytical point of view defining it as the study of why things go wrong [10].

For to manage risk and control the major hazards in industrial and occupational scenarios the European Union has issued an initial preventive framework based on the Directive 2012/18/EU of 4 July 2012 on the control of major accident hazards involving dangerous substances also known as Seveso III, [11] which is the result of the initial



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Directive 82/501/EEC of 24 June 1982 on the major-accident hazards of certain industrial activities [12].

Additionally the risk and occupational safety at work is treated also in the European Union through the Directive 89/391/EEC of 12 June 1989 on the introduction of measures to encourage improvements in the safety and health of workers at work [13] addressed to employers to establish basic principles of prevention with evaluation of risks and their avoidance.

Also the European Union has established the Strategic Framework on Health and Safety at Work 2014-2020, with three key points focused on enhancing the health and safety, the prevention on work-diseases and aging of the workforce and additionally considering the growth of the new processes as green technologies, nano-materials and biotechnologies; to achieve this strategy seven key points have been defined, being among them the consideration of policy consolidation and coordination to simplify the international legislations and the need for focus on small enterprises; it is important the consideration for “Improving of the statistical data collection to have better evidence and developing monitoring tools”, [14]. But this key point has the aim of assess the quality of data on accidents at work transmitted by Member States and for to improve the availability and comparability of data on occupational diseases for creating a common data base on occupational exposures to allow the performing of occupational protection policies and performance indicators assessing their implementation.

Based on this European Strategic Framework the Spanish Council of Ministers approved the Occupational Health and Safety Strategy 2015-2020 [15] being a priority the improvement of occupational safety and health conditions and the psychosocial and organizational factors included in the EU.

In an analogous scheme the Spanish National Institute of Safety and Health at Work (NISHW), issued the actuation lines for 2018-2020, [16], with the goals of develop new methodologies of occupational risk assessment, to take in consideration the new work conditions and the health promoting by the detection of profesional diseases, ergonomics and limiting the chemical exposure.

The International Organization for Standardization has also issued the ISO 45001:2018 guideline with the aim to establish a framework for developing an occupational health and safety at work management system, [17].

The general guidelines that cover and establishes a risk management process and a risk assessment are the ISO 31000:2018. Risk Management Guidelines [18] and the ISO/IEC 31010:2019. Risk Management - Risk Assessment Techniques [19].

The ISO 31000:2018 and ISO/IEC 31010:2019 standards are based in the Deming cycle [20], consisting of a sequence of steps: “plan, do, check, act”; are the framework of the Risk Management and the Risk Assessment, (Figure 4).

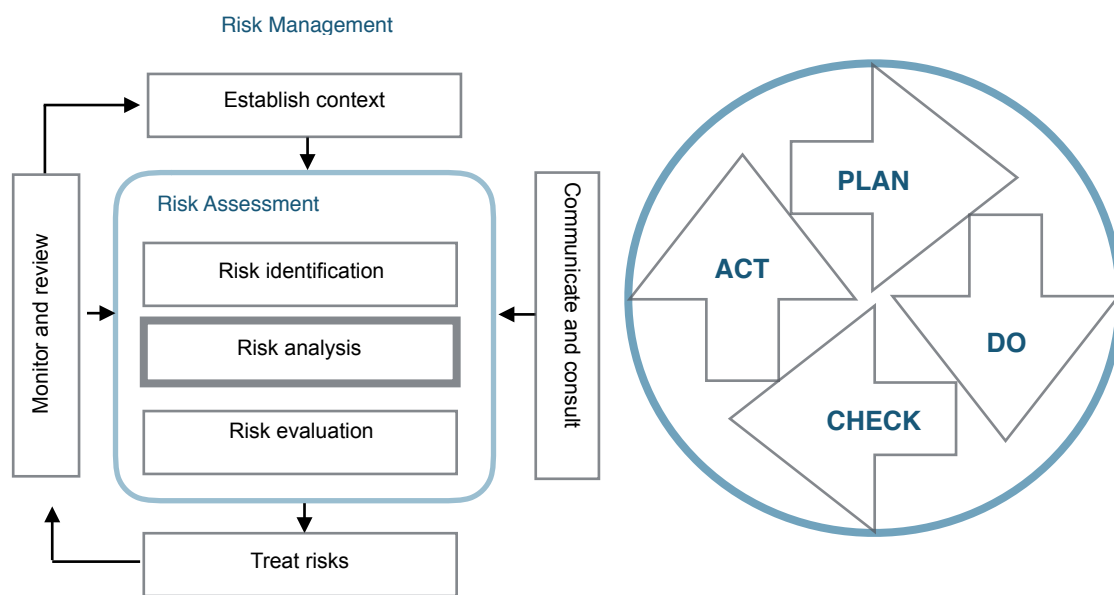


Figure 4.- Risk management and Risk Assessment process and Deming cycle equivalence (adapted from [18,19,20]).

The Quantitative Risk Assessment (QRA) is a formal and systematic risk analysis approach to quantifying the risks associated with the industrial and human processes. The Risk Assessment, is the general procedure that covers initially the risk identification process which can be performed based on historical data, through a panel of experts or using inductive cause-effect techniques; followed by the risk analysis through the application of: qualitative methods, indicating the levels of importance of the risk and its consequences; semiquantitative methods, by indicating numerical risk rating scales and their consequences; and quantitative methods, defining the probabilities of risk generation and its consequences, [18,19,21,22,23] and finally their evaluation that implies determining the importance and prioritizing from the point of view of risk-consequence or benefit-cost, [24].

In Spain the main occupational risk guidelines are supplied from the National Institute of Safety and Health at Work (NISHW), with the objective of analyze and study the

health and safety conditions in the workplace, as well as their improvement [25]. Additional but specific European regulations are applied to consider the maximum concentration of chemical agents at work [26] and for carcinogens and mutagens at work [27,28].

As previously indicated the European Strategic plan for health and safety at work presents, between the considerations of international coordination, the need for develop and improve the statistical techniques and monitoring tools related to the evaluation of occupational product exposures and the associated diseases.

But in these significant methodologies and guidelines there is no consideration about the need to have tools that allow not only to quantify and analyze the risk and its impact, but to be able to measure it and acting at the moment when the generation of a possible accident can be immediate.

### **1.5. Method.**

To achieve this objective, it is first necessary to perform a review for what are the current tools related to risk management, their field of application and its degree of fulfillment of the need for immediacy through the three defined characteristics. Based on this revision, the new Statistical Risk Control (SRC) methodology for to manage and assess the situations of risk, is presented. The development for industrial and occupational accidentes is performed and the application of different models is explored in order to verify its effectiveness using example cases. The analysis is extended to the deviations in time and cost and to the domino effect. Accordingly this work is organized as follows: in chapter (2) the revision of the state of the existing tools; in chapter (3) the development of the SRC methodology in general for industrial processes, occupational accidents and the special cases of project management and domino effect; in chapter (4) the presentation of cases of study and their results; in chapter (5) the discussion of results; and in chapter (6) the conclusions and further research work.



# Chapter 2

## Existing tools.

The analysis is performed in three main groups:

1. The first group is the corresponding to the standards, directives and regulations.
2. The second group covers methodologies and models differentiating the traditional and the modern approaches [29]. The traditional approach includes the sequential and the epidemiological models. The modern approach has five models: the systematic; cloud based; the fuzzy based, formal based and safety barrier based.
- 3.- The third group, which is encompassed in the modern methodologies, is specific for dynamic models.

### 2.1. Standards, Directives and Regulations.

#### 2.1.1. Directive 89/391/EEC.

Issued on 12 June 1989 is a framework directive for occupational accidents with the aim to establish the employers obligations for developing a prevention police oriented to the protection of safety and health, and for the prevention, assessment and elimination of risks and accident factors, and focused on workers and their representatives facilitating that they be informed, consulted, with a balanced participation in all aspects that affect safety and that they have adequate training [13]. It is applicable to industrial and public sectors of activity. This directive offers a definition of prevention on Article 3, “all the steps or measures taken or planned at all stages of work in the undertaking to prevent or reduce occupational risks”.

#### 2.1.2. ISO 45001:2018.

This standard is aimed at providing rules that allow the implementation of a management system of safety and health at work (SHW) [17], also based on the Deming cycle concept [20], but does not establish specific criteria for the performance of the (SHW) or for the design of a (SHW) management system.

### 2.1.3. National Institute of Safety and Health at Work (NISHW).

It is the Spanish governmental organization dependent from the Labour, Migrations and Social Security Office with the aim of analysis and study of health and safety conditions in the workplace, as well as the promotion and support to improve them. [16]

This organization issues a series of documents and guidelines for work safety in the areas of materials and chemicals storage, emergencies actuation, maintenance and prevention in industrial installations, workplace design, risks and explosion avoidance, electrical risks, fire safety and communication; covering risks in chemicals, biologicals, ergonomics, psychosocial and physical as radiation, vibrations and noise.

### 2.1.4. Law 31/1995 of prevention of occupational hazards.

It is the Spanish regulation defining the rights and duties for employers and workers in concordance with the Directive 89/391/EEC and requiring, in the 16th article, the establishment of an occupational risk prevention plan, including the risk assessment and preventive activity planning. [30]

### 2.1.5. Directives 98/24/EC and 2004/37/EC.

The 98/24/EC Directive of 7 April 1998 on the risks related to chemical agents at work and the 2004/37/EC of 29 April 2004 on the carcinogens and mutagens at work [26,27], are extensions of the 89/391/EEC Directive. The aim is to determine whether any hazardous chemical or mutagen agent is present at the workplace assessing any risk and performing the needed safety and health preventive and avoiding actions to reduce their concentration and presence at the limits of exposure established at Community and Member States. In the case of Spain the National Institute for Safety and Health at Work (NISHW) issues practically every year the Occupational exposure limits for chemical agents in Spain [31] and the Real Decreto 665/1997, of May 12, on the protection of workers against the risks related to exposure to carcinogens during work on the Annex III shows the exposure limit values for Benzene, Vinyl Chloride monomer and wood dust [32].

### 2.1.6. ISO 31000:2018 and ISO/IEC 31010:2019 standards.

As previously anticipated the ISO 31000:2018 and ISO/IEC 31010:2019 provide a set of principles being a framework for managing and assess the risks. It is based in the “Deming” cycle [20], considering a sequence of steps: “plan, do, check, act”. That concisely are: plan, what it is needed to do; do, execute the previous plan; check, if the plan has achieved the objectives and act, identifying improvement areas to apply in the

next cycle. In this guideline, the risk definition is based on uncertainty affecting the objectives that have been defined in the previous cycle. According to this cycle definition, and depending on the context and the set of circumstances that can occur, the deviation from the expected can be positive and then risk has a benefit or an opportunity for the organization or on the other hand it has a negative impact.

If the ISO 31000:2018 contains general principles of the Risk Management, the ISO/IEC 31010:2019 presents a detailed explanation about the Risk Assessment process and the applied tools, Figure 4.

The context defines the situation within the risks exists and it has to be considered the cultural, political, regulatory or legal and economical aspects whether at international, national, regional or local levels as external influences. Also considering the internal influences from the organization as information flows and decision processes, objectives, policies, standards and models adopted, governance, roles, values and culture. Also in the context a previous determination of consequences are performed.

The risk assessment comprises their: identification, analysis and evaluation and can be performed at different organizational levels.

The identification has the aim of analyze what might happen or what situations might exist into the form of possible causes and source of risk, as events or situations which could have an impact over the objectives. The risk identification methods can include: evidence based on check lists and reviews of historical data, inductive reasoning techniques and supporting methodologies as brainstorming or Delphi.

The analysis is performed to estimate the range of potential consequences that might arise from an event or situation with their associated probabilities, in order to measure the level of risk. The methods applied in this analysis can be qualitative or quantitative or a combination of both, in any case the obtained result is a ranking for the different levels of risk considered. This guideline considers that the level of risk is depending on the adequacy and effectiveness of existing controls based on proper documentation and assurance process in place. The event consequences is the consideration of nature and type of impact that can occur.

This analysis has three considered aspects:

- 1.- The consequence determines the nature and type of impact which could occur assuming that a particular event situation or circumstance has occurred and can be performed by using theoretical or empirical models to define the extension of the produced risk by affecting time, cost, life and environment, [29].

2.- The probability can be estimated from historical data to identify events or situations previously occurred in the past making an extrapolation to this probability for the future; or by using predictive techniques like Fault Tree Analysis (FTA) or Event Tree Analysis (ETA) or/and by using the experts information or by the application of the Delphi methodology, [23].

3.- The level of risk by performing a sensitivity analysis to determine the variation, significance and magnitude of risk to changes to with individual input parameters. The evaluation process allows to compare the estimated levels of risk obtained with a criteria previously defined, in order to determine the significance of the level and type of risk and it is presented in form of a risk-consequence graph.

The treatment of the risk has the purpose is to perform the corrective action according to the level of risk.

The actions of monitoring and reviewing are applied to take in consideration the variation in time of external factors previously defined in the context and by updating accordingly the risk assessment.

The communication and consult actions have the purpose to informatively extend into the organization the levels of risk and their originating events and situations together to with the corrective actions. [5,6]

#### 2.1.7. Directive 2012/18/EU and COMAH 2015.

The actual Directive 2012/18/EU, known as Seveso III directive with an important evolution from this accident, and their British homonym for the Control of Major Hazards (COMAH) state the minimum rules for the prevention of major accidents which involve dangerous substances, and the limitation of their consequences for human health and the environment. There is no raised any specific risk assessment methodology and the efforts are focused on considering the dangerous substances by taking care of their chemical, health and physical hazard properties; the Directives require the existence of an emergency plan with the notification of a major accident prevention policy (MAPP) into the establishment; the consideration of the possible domino effects caused by proximity and storage of hazardous and flammable materials and the need for to perform a safety report, as established in Annex II, demonstrating that: a MAPP and a safety management system have been implemented; that the possible major accident scenarios have been identified and the necessary measures have been taken; that the safety and reliability framework is present into the design, construction, operation, maintenance, storage, facilities, equipment and infrastructure



and that there is implemented an internal emergency plan and a mechanism for to inform the authorities and population. The safety report has an initial realization in function of the new or already in operation establishments, but, in any case, there is needed an update review of the safety report every 5 years. In Spain the based regulation is supported on the Real Decreto 840/2015 from September 21, which approves measures to control the risks inherent to serious accidents involving dangerous substances. [11,33,34]

#### 2.1.8. PMBOK and PRINCE2.

The Projects Management Institute and Prince2 are organizations oriented to the development of project management methodologies as PMBOK – Project Management Body of Knowledge and PRINCE2 – Projects In Controlled Environments. The PMBOK is a detailed framework of ten knowledge areas of project management: integration, scope, schedule, cost, quality, resource, communications, risk, procurement, and stakeholder; broken down into activities across five groups of processes considering the project life cycle: initiating, planning, executing, monitoring and controlling and closing; that are claimed to encompass the sum of knowledge generally recognized as good practice in the project management profession, and contemplated as an ANSI standard for project management [35,36,37]. A PRINCE2 project is driven by the project's business case, which describes the organization's justification, commitment and rationale for the deliverables or outcome. The business case is regularly reviewed during the project to ensure the business objectives, which often change during the lifecycle of the project, are still being met. PRINCE2 is designed to provide a common language across all the interested parties involved in a project.

Both methodologies set documentation which must be tailored to suit the occasion project. As previously indicated for the PMBOK, are considered five groups of processes, while for the PRINCE2, 8 groups are considered. However, it is possible to find equivalences between processes. The PMBOK process of initiating, is equivalent to the first two processes of PRINCE2, Starting up and Directing. The PMBOK planning process embodies the initiating and planning processes on PRINCE2. The executing PMBOK process will be equivalent to the controlling a stage and managing product delivery in PRINCE2. The monitoring and controlling from PMBOK are equivalent to the product delivery and directing in PRINCE2 and finally the closing process are equivalent in both methodologies.

The risk treatment for PMBOK and PRINCE2 in general follows the risk management steps as indicated by the ISO/IEC 31010, with the same cyclic structure: plan the risk

management, identify risks, performing a qualitative and quantitative analysis, plan and implement risk responses and monitor risks, (Figure 4). [37,38].

#### 2.1.9. Center for Chemical Process Safety (CCPS).

The Center for Chemical Process Safety is an organization dependent from the American Institute of Chemical Engineers (AIChE). The risk management procedures are mainly based on the ISO/IEC 31010:2019 approach and focused by the application of the Layer of Protection Analysis (LOPA) methodology. Conceptually, LOPA is used to understand how a process deviation can lead to a hazardous consequence if not interrupted by the successful operation of a safeguard called an independent protection layer (IPL). An IPL is a safeguard or safety barrier that can prevent a scenario from propagating to a consequence of concern without being adversely affected by either the initiating event or by the action (or inaction) of any other protection layer in the same scenario. [23,39,40,41]

#### 2.1.10. NORSOK.

The “Norsk Søkkel Konkuranseposisjon” (NORSOK) standards are developed by the Norwegian petroleum industry to ensure adequate safety with a balanced cost for petroleum industry developments and operations. The main guideline Z-013 covers the traditional (QRA) steps and within the ISO/IEC 31010:2019 guideline framework. [42,43,44]

#### 2.1.11. CPR18E.

The Netherlands advisory council of dangerous substances issued several publications from the old CPR (Commissie voor de Preventie van Rampen) still called CPR guidelines that are often used in the fields of occupational, transport and fire safety. This association offers a series of guidelines associated to one or grouped installations for analyzing the loss of containment events in chemical processes and the modeling of the associated flammable clouds, their dispersion and toxic effects. The CPR 18E Guidelines or “Purple Book” is applied for the quantitative risk assessment. [45]

#### 2.1.12. EN16991:2018.

European standards for chemical, power generation and manufacturing providing guidance for the inspection and risk evaluation in operations and maintenance. [46,47]

The Table 1 shows a summary and the compliance with the three characteristics.

Document	Application	P	S	I
89/391/EEC [13]	Occupational, framework - On the introduction of measures to encourage improvements in the safety and health of workers at work.	+	-	-
ISO 45001:2018 [17]	Implementation of a system of safety and health at work (SHW).	+	-	-
NISHW [16]	Spanish governmental organization of analysis and study for safety and health conditions in the workplace.	+	-	-
Law 31/1995 [30]	Law 31/1995, of November 8, on Prevention of Occupational Hazards. To establish an occupational risk prevention plan, risk assessment and preventive activity planning.	+	-	-
98/24/EC, 2004/37/EC [26,27,31,32]	Occupational - Chemicals and carcinogens concentration.	+	-	-
ISO/IEC 31010:2019 [5,6]	Risk management process, based on a iterative cycle. Risk assessment based on identification, analysis and evaluation. General application of (QRA).	+	-	-
2012/18EU COMAH 2015 [11,33,34]	European and British Control of Major Hazards for Seveso III Directive. Emergency plan with major accident prevention policy and information mechanism to authorities and population. A 5 years safety report.	+	-	-
PMBOK, PRINCE2 [35,36,37,38]	Documentation tailored for project management. Initiating, planning, executing, monitoring and controlling and closing.	+	-	-
CCPS [23,39,40,41]	Center for Chemical Process Safety. Layer of Protection Analysis (LOPA) methodology. A danger occurs if not interrupted by an independent protection layer (IPL).	+	-	-
NORSOK 2010 [42,43,44]	(Norsk Søkkel Konkuranseposisjon). Standards applied in the Norwegian petroleum industry, with the aim to follow the lifecycle of a project considering planning, execution and operation.	+	-	-
CPR18E [45]	Netherlands Commissie voor de Preventie van Rampen (CPR). Applied in hazardous installations and transport analyzing the loss of containment events and the modeling of the associated flammable clouds, their dispersion and toxic effects.	+	-	-
EN 16991:2018 [46,47]	European standards for chemical, power generation and manufacturing providing guidance for the inspection and risk evaluation in operations and maintenance.	+	-	-

Table 1.- Standards, Directives and Regulations group summary with their compliance to with the P: preventive; S: simultaneity and I: immediacy characteristics.

Taking in consideration that are standards or directives with the aim of avoiding or mitigate the risks by reducing their probability of occurrence and their impacts, all the components of this group cover the preventive characteristic indistinctly if it is applied for industrial processes or occupational accidents.

## **2.2. Methodologies and models. Traditional approach.**

### **2.2.1. Sequential models.**

Sequential models are representative of the Risk Assessment (RA) process regarding accidents as outcomes of a chain of discrete events or factors that take place in a temporal order. The events or factors, may be associated with the social environment, unsafe acts or conditions, breakdowns, incidents; all of them are related in the sense that if the first factor falls, a chain of failures can occur until reaching the last.

These models include, [23]: The Fault Tree Analysis (FTA) is a deductive graphical technique to quantify failure probability of human and technical systems. The Event Tree Analysis (ETA). Inductive and logic graphical technique for consequence analysis. It is used for human reliability assessment as part of THERP (Technique for Human Error Rate Prediction) a tool based on event-tree approach for evaluating human errors alone or in connection with equipment functioning, operational procedures and practices, or other system and human characteristics that influence system behavior; the aim of the tool is to demonstrate the effectiveness of protective systems, [48]. The BOW-TIE graphic is the integration of the FTA and ETA models to represent causes, designing or evaluating safe barriers, and conclude with the consequence events. The Failure Mode Effect Analysis (FMEA) it is a step-by-step analysis approach for identifying potential failures and then preventing them. The Failure modes, Effects and Criticality Analysis (FMECA), [49], is an upgrade of the previous model and it is used for preliminary hazard analysis (PHA) with the aim to identify potential failure or accident modes and how to avoid it. This analysis is generally previous to the performing of a risk assessment (RA). It is a process with six steps: definition of the scenario and scope of the analysis; constitution of a multidisciplinary team; identification or possible risks that can affect the process or the occupational activity; collect information; determination of the criticality and design avoidance measures and their implementation. The criticality is determined by associating an estimated gradation (defined in an interval with extreme low to high critical values) for every potential failure and their consequences and finally, developing and implementing corrective actions and monitoring measures to the highest potential failures. The Check list-What if, it a systematic and scenario imaging revision of equipment and installation to find malfunctions and compliance with a list of requirements or collect data in an orderly and systematic manner [49]. The Reliability Assessment (RRA), with the aim to quantify the probability of failure in a system, [50]. The Block Diagrams, is a graphical procedure describing the function of the system and showing the logical connections of components needed to fulfill a specified system function, [51].

The FTA and FMEA have been applied in cases study for occupational safety and health in risk analysis accidents for the textile industry, [52]. The FMECA is applied in a toxic exposure to contaminants in a drug industry work place, [53].

From this group the Bow-tie diagram is representative of the risk analysis of a system or process and for defining the preventive or proactive and mitigative or reactive barriers, [23], Figure 5.

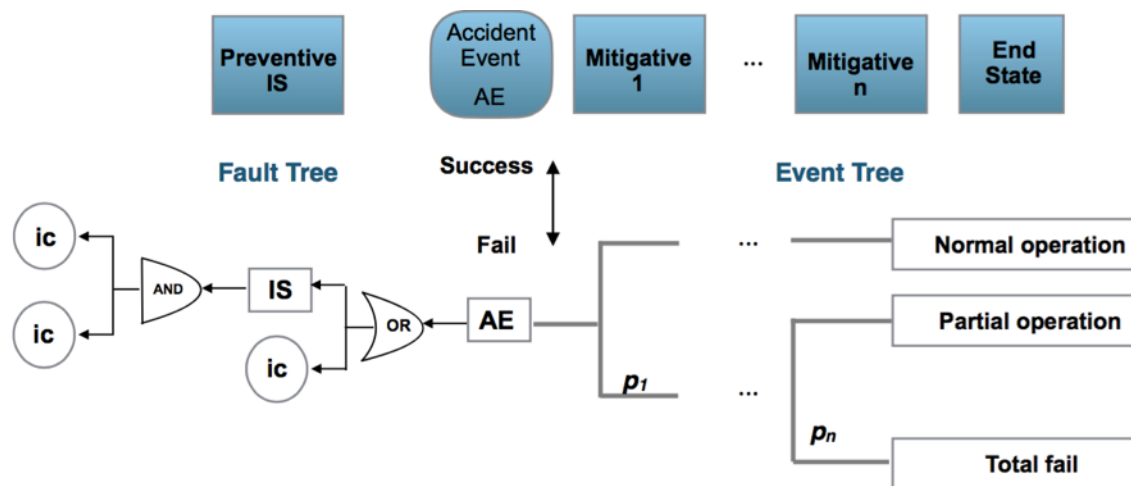


Figure 5.- Generic bow-tie. ic: initiating cause; IS: intermediate preventive safety barrier; AE: accident event; Mitigative: mitigative safety barrier;  $p_i$  : fail probability for the mitigative safety barrier “i”; End State: impact, consequence, (adapted from [23]).

The bow-tie is composed by the fault tree and the event tree, is a graphical presentation of which must be the causes that or by themselves or affecting the production of failures in preventive safety barriers can lead to the generation of an accident (AE) and if this occurs, what must be the safety barriers that avoid or mitigate its consequences. Mitigative safety barriers act in a chained manner, if a failure occurs in the first one, the second one is activated, and so on.

In occupational risk the Bow-tie is applied to analyze the chemicals exposure,[9], falls from height, crane activities, interaction with machines moving parts and situations of risk due to falling objects [54,55,56,57], for occupational accidents related with construction, operation and maintenance in onshore wind farms, [58].

The Table 2 shows a summary for the analyzed methods and the compliance with the three characteristics.

Models	Application	P	S	I
<b>Sequential</b>	Are representative of the Quantitative Risk Assessment (QRA) methodology regarding accidents as outcomes of a chain of discrete events or factors that take place in a temporal order. Analyzing causes and consequences of risk.			
FTA [23,52]	Fault Tree Analysis. Causes of risks. General application.	+	+/-	-
ETA [23]	Event Tree Analysis. Consequence analysis. General application.	+	+/-	-
THERP [48]	(Technique for Human Error Rate Prediction) a tool based on event-tree approach for evaluating human errors alone or in connection with equipment functioning, operational procedures and practices. General application.	+	+/-	-
BOW-TIE [23,54,55,56,57,58]	Graphic including FTA and ETA models to represent causes and consequence events. And what are the needed safety barriers. General application.	+	+/-	-
FMEA [49,52]	Failure Mode Effect Analysis. Step-by-step approach for identifying potential failures. General application.	+	+/-	-
FMECA [49,53]	Failure modes, Effects and Criticality Analysis. Upgrade of the FMEA. The criticality is determined classifying the degree of potential failures. General application.	+	+/-	-
Check list- What if [49]	Systematic revision to find malfunctions and compliance with a list of requirements. General application.	+	+/-	-
RRA [50]	Reliability Assessment. Quantification of the probability of failure in a system. General application.	+	+/-	-
Block Diagrams [51]	Graphical procedure describing the function of the system and showing the logical connections of components needed to fulfill a specified system function. General application.	+	+/-	-

Table 2.- Sequential models summary with their compliance to with the P: preventive; S: simultaneity and I: immediacy characteristics. General application means: industrial / occupational.

Following to with the sequential models, the HAZOP / HAZID methodologies are found, the Hazard and Operability study (HAZOP) is a structured and systematic examination of a complex planned or existing process or operation in order to identify and evaluate problems that may represent risks to personnel or equipment. The intention of performing a HAZOP is to review the design to pick up design and engineering issues that may otherwise not have been found. The technique is based on breaking the overall complex design of the process into a number of simpler sections called 'nodes' which are then individually reviewed. It is carried out by a suitably experienced multi-disciplinary team during a series of meetings. The HAZOP technique is qualitative, and methodically "brainstorms" aims to stimulate the imagination of participants to identify potential hazards and operability problems.

The HAZID concept is a hazard identification technique for early identification of hazards usually applied at the conceptual or detailed design stage, the study is carried out also by an experienced multi-discipline team using a structured approach based on a checklist of potential hazards [59]. The Energy Barrier model (EBM), [60,61], is based on the safety barrier management as “the coordinated activities to establish and maintain safety barriers so that they maintain their function at all times” and stating that an accident occur when hazards succeed to penetrate the safety barriers consisting in defects or deficiencies in their functional activity, Figure 6.

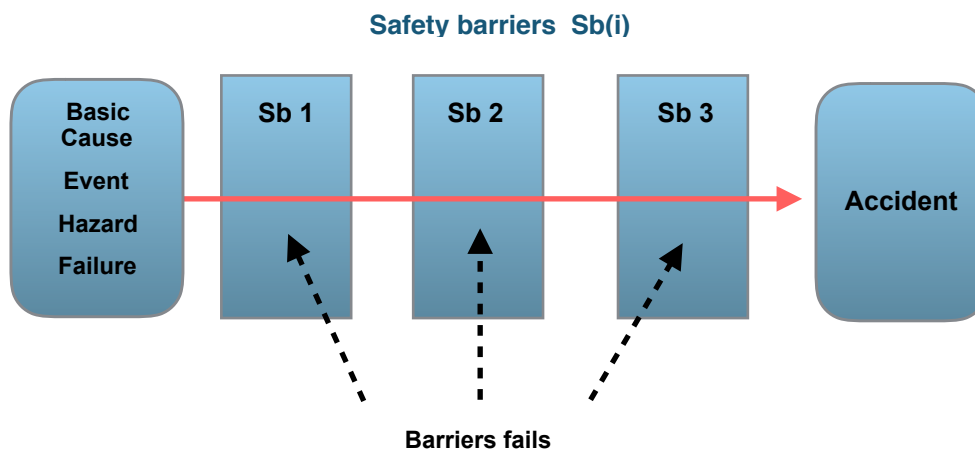


Figure 6.- Fail in all the safety barriers can allow for an accident (adapted from [61]).

The Management Oversight and Risk Tree (MORT), is a root cause analysis that identifies the set of multiple causes that together might create a potential accident. The analysis technique can be performed using tree techniques and check list methods, [62]; the Systematic Cause Analysis Technique (SCAT) is a causal analysis by using a poster-sized schematic which enables clear identification of the relevant corrective and preventive actions, [63]; the Sequential Time Events Plotting (STEP) a root cause analysis that identifies the set of multiple causes that together might create a potential occupational accident, [64]; the Man Technology and Organization (MTO) analysis method is also based on events and analyzing the safety barriers, the method applies checklists to identify root causes in occupational work affected by the organization, practice, management, procedures and deficiencies in: technology; communication; procedures, competence, and work environment, [65]; the Safety through Organizational Learning (SOL) is an event analysis in two steps: (1) the description of the actual event situation, and (2) the identification of contributing factors, [66].

The sequential models applying the quantitative risk assessment and adding specific tools are applied to manage occupational health and safety risks such as the use of fuzzy analytic hierarchy process with a case application to the mining industry [67] and two manufacturing plants [68]. A model for detecting and evaluating risks in process industries and a methodology for detect and evaluate emerging risks in industrial processes [69,70]. The application of a multi-objective evolutionary algorithm taking in consideration operators tasks, activities, the associated hazards and their safety with case studies for falling from a fixed scaffold [71, 72]. The use of Bayesian networks applied in offshore harsh environments including work conditions to prevent slips, trips and falls [73]. Risk assessment cases applied in the mining industry [74], in building for falls, scaffold, falling objects and contact with moving machine parts and vehicles [75] and in an aluminum process industry for operators on press extruders, forklifts, cranes, painters and production [76]. The use of block diagrams to define falling risks from a mobile ladder [77]. An event graph is applied to analyze chemicals exposure [78], falls from height, crane activities, interaction with machines moving parts and situations of risk due to falling objects [79,80,81,82], and for occupational accidents related with construction, operation and maintenance in onshore wind farms [83].

#### 2.2.2. Epidemiological models.

In this group the propagation of the events are modeled using the analogy of a disease spreading consisting in the investigation of the health conditions and determinants for have a disease distribution in human populations. Accidents are the result of manifest and latent events that take place under epidemic contexts. Under this idea the risk scenario is analyzed searching trends and determinants of the accidents. Cases are reported for helicopter and road accidents, [84,85].

The Table 3 shows a summary for the traditional analyzed methods and their compliance with the three characteristics.

From these two tables it is possible to see that sequential models have general application for assess risk in industrial production and occupational environments with specific application in occupational risk assessment using the failure modes FMEA and FMECA effects and criticality analysis, together with the fault tree, FTA and the BOW-TIE. But STEP and the MTO methodologies are the most specific in the occupational risk assessment.

According to the definitions, the sequential and epidemiological models offer not only prevention (P), also a degree of simultaneity (S), with possibility for update the risk evolution but with a delay caused by the own methodology being less based in real observational data and more in previous causal-effect analysis .



<b>Models</b>	<b>Application</b>	<b>P</b>	<b>S</b>	<b>I</b>
<b>Sequential</b>	(cont.)			
HAZOP/HAZID [59]	Technique for early identification of hazards usually applied in the design, the study is carried out by an experienced multi-discipline team using a checklist of potential hazards. General application.	+	+/-	-
EBM [60,61]	Energy Barrier Model defining a safety barrier management and considering that an accident occur when hazards succeed to penetrate the safety barriers deficiencies. General application.	+	+/-	-
MORT [62]	Management Oversight and Risk Tree. Root cause determination. General application.	+	+/-	-
SCAT [63]	Systematic Cause Analysis. Causal analysis using a poster schematic which enables the identification of relevant corrective and preventive actions. General application.	+	+/-	-
STEP [64]	Sequential Time Events Plotting. Identification of multiple causes in occupational accidents.	+	+/-	-
MTO [65]	Man Technology and Organization. Root causes in occupational work affected by the organization; practice; management; procedures and deficiencies in work environment.	+	+/-	-
SOL [66]	Safety through Organizational Learning. Event analysis in two steps: (1) description of the actual event situation, and (2) identification of contributing factors. Case for nuclear industry. General application.	+	+/-	-
<b>Epidemiological</b> [84,85]	Propagation of events is analogous to a disease spreading considering their distribution and determinants. Accidents are caused by latent events under epidemic context. Cases for helicopter and road accidents.	+	+/-	-

Table 3.- Traditional models summary with their compliance to with the P: preventive; S: simultaneity and I: immediacy characteristics. General application means: industrial / occupational.

## 2.3. Methodologies and models. Modern approach.

### 2.3.1. Systematic models.

Systematic Models are applying a general risk framework based on the Rasmussen's model, [86], that uses control theory concepts and considers that the social climate affects government policy and budgeting, regulatory associations, local area government planning and budgeting including the company and their organizational behavior, the company management, the staff and the work operational systems and their equipment and surroundings; for which their limitations and their interactions can get significantly preconditions for accidents, [87].

The most characteristic tools are the AcciMap, STAMP, CREAM, FRAM and AEB models.

The AcciMap is a cause event representation of the system interactions and focused on how to control the hazardous processes originated into of the organizational and socio - technical system. The method involves the construction of a multi-layered causal diagram in which the various causes of an accident are arranged between them according to their causal relations and layered according to Rasmussen’s classification, [88]. The Systems Theoretic Accident Model and Processes (STAMP), is based on the approach that systems are complex and dynamic subjected to external disturbances and can migrate toward the accident due to the physical, social and economic pressures together to with the inadequate control failures in safety barriers or due to safety-related constraints. The operating process actuation is supported by the human acting as controller and supporting the actions of the system development and system operations at top and from the physical process at bottom, (Figure 7). To facilitate the identification of control failures, a taxonomy is proposed including: inadequate control of actions; inadequate execution of control actions; and inadequate or missing feedback, [89,90,91].

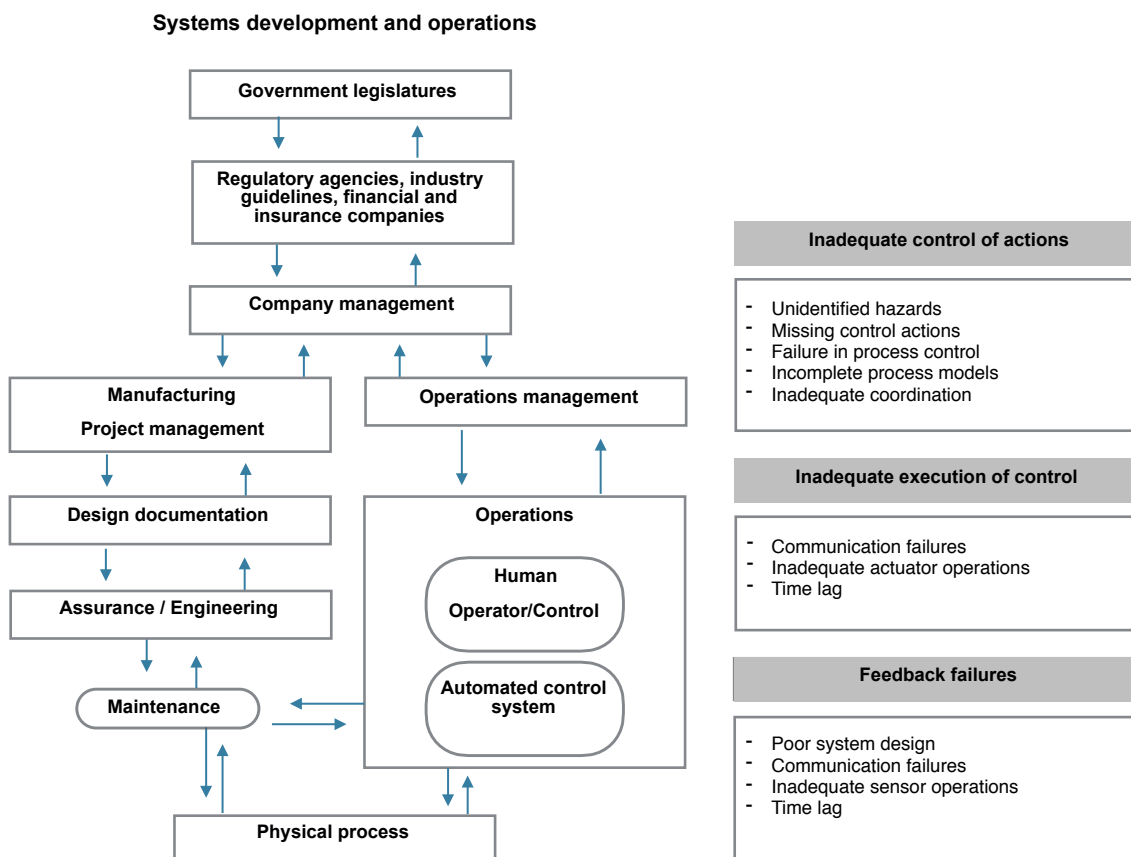


Figure 7.- STAMP’s generic complex system and control failure taxonomy. (adapted from 89).

The Cognitive Reliability and Error Analysis Method (CREAM) model characterizes the human performance to assess the consequences of the human errors on systems, [92,93]. As a variation, the Driving Reliability and Error Analysis Method (DREAM) model is established for driving accident assessment, [94]. The Functional Resonance Accident Model, (FRAM), [95], this systemic model states that as a result of the functional couplings appears resonance; the procedure needs to determine first what are the functional entities or basic processes in a given risk scenario and defining for each of them what are the inputs, outputs, resources (assets, procedures, energy, materials, manpower), supervision controls, preconditions and the time needed for to perform the function. The potential variability is determined as a dependence between functions considering what are the conditions (named performance conditions) needed for to perform the function (as example: availability of resources, training and experience, quality of communication, access to procedures, support of the organization, conditions of work, overpressure, collaboration quality) and rating these conditions as stable adequate or inadequate and unpredictable. The potential resonance can appear in unpredictable situations. The last step is to identify or define barriers for damping the potential resonances and monitor the new status for the affected conditions. The Accident Evolution and Barrier Function (AEB) describes the interaction between technical and human-organizational systems which may lead to an accident, the model gives equal weight to both these types of systems and needs interactive accident analysis by engineers and human accidents specialists. It can be used in predictive safety analyses or in post hoc incident analyses, [96].

### 2.3.2. Cloud based models.

Cloud models are applying the preliminary hazard analysis based on the FMECA tool by performing a cloud analysis of the critical hazards and establishing a hazard importance or grade with the scoring from the expert's knowledge [97].

This group is applicable on general industrial or occupational situations but CREAM and DREAM are focused on human behavior and actuation. FRAM model has applications on aircraft, maritime and industrial manufacturing hazards [95]. And the Cloud based has general applicability and a case for a gasification station has been described [97].

The Table 4 shows a summary for this systematic and cloudy based group and their compliance with the three characteristics.

Models	Application	P	S	I
<b>Systematic</b> [86,87]	General risk framework based on the Rasmussen's model using control theory concepts and considering that social climate is affected by government policy and budgeting, regulatory associations, organization, staff and the work operation systems for which their limitations and their interactions can get preconditions for accidents.			
AcciMap [88]	Cause event representation of the system interactions and how to control the hazardous processes originated into of the organizational and socio - technical system. General application.	+	+/-	-
STAMP [89,90,91]	Systems Theoretic Accident Model. Systems are subjected to external disturbances and can lead to accident due to the physical, social and economic pressures with inadequate control failures in safety barriers. Operation process is supported by human acting as controller and supporting the actions of the system. To identify failures a check is applied verifying: controls; execution of controls and missing feedback. General application.	+	+/-	-
CREAM [92,93]	Cognitive Reliability and Error Analysis Method. Human performance is modeled to asses the consequences of the human errors. Focused in human actuation.	+	+/-	-
DREAM [94]	Driving Reliability and Error Analysis Method. Stablished for driving accident assessment. Focused in human actuation.	+	+/-	-
FRAM [95]	Functional Resonance Accident Model. As a result of the functional couplings appears resonance. The functional or basic processes in a risk scenario are identified, defining for each of them what are the inputs needed; the outputs produced; the needed resources (equipment, procedures, energy, materials and manpower); the controls to supervise, the preconditions to be fulfilled to carry the process and the time. The resonance can appear in variability situations of dependence between processes. More focused in manufacturing and transportation.	+	+/-	-
AEB [96]	Accident Evolution and Barrier Function. Interaction between technical and human-organizational systems which may lead to an accident, the analysis needs work-team by engineers and human accidents specialists.	+	+/-	-
<b>Cloud based</b> [97]	Based on the FMECA . Performing a cloud analysis of the critical hazards by establishing a grade and the scoring from the expert's knowledge. General application.	+	+/-	-

Table 4.- Modern approach summary of the systematic and cloud based models with their compliance to with the P: preventive; S: simultaneity and I: immediacy characteristics. General application means: industrial / occupational.

### 2.3.3. Fuzzy based models.

Fuzzy models apply the fuzzy logical to define the human behavior in risk situations. Representative models of this group are the Human Error and Assessment Technique (HEART), [48], this fuzzy technique assumes that the reliability of any task performance may be modified by the influence of Error Promoting Conditions (EPCs). The procedure needs to previously identify the human operator tasks in a considered risk scenario; for every task a generic value of probability for human failure and what are the possible EPCs affecting are assigned by the assistance of a team of experts. The same team defines what are the strength affectation of every EPC into the generic task failure probability and this is transformed using fuzzy logic in an assessment factor that

modifies accordingly the generic probability of failure, [98]. An additional representative model of this group is the CREAM-BN a modern upgrade of the systemic CREAM model for determine the probability of the human failure accordingly to the human cognition and action, by using a generic control mode distribution integrated by four human behavior components as: strategic, tactical, opportunistic or scramble every one following a discrete distribution function associated to a failure probability. These human behavior components are affected by common performance conditions (CPCs) defined as: adequacy of the organization, working conditions, adequacy of man-machine interface and operational support, availability of procedures and plans, number of simultaneous goals, available time, time of day, adequacy of training and experience and collaboration quality. The probability of human failure due to their control mode behavior and their associated CPC's probabilities, is determined by scoring the probability distribution of every CPC using the fuzzy logic; a bayesian network is applied integrating all the scored CPC's, obtaining the human control modes risk probabilities. The tool can be applied to consolidate the behavior of crews, the workforce groups or for individual actuations. As expected, scramble and opportunistic human actuations allows for high failure probabilities, [99].

#### 2.3.4. Formal models.

Formal models apply the probabilistic approach for assess the scenarios of risk. Two basic tools are the Probabilistic model where the accident causation is assessed using probabilistic schemes and Bayesian networks to model the interaction between causes and effects. And the Why Because Analysis (WBA) model being also a probabilistic cause-effect relation supported by the use of Bayesian networks and stating that each component of the system is affected from the overall environment, [100].

#### 2.3.5. Safety barrier models.

Safety Barrier models. The representatives for this group are the Process Hazard Prevention Accident Models (PHPAM) and the System Hazard Identification Prediction and Prevention (SHIPP). Corresponding to the Process Hazard Prevention Accident Models (PHPAM) actually there are applications in the off-shore and oil-gas process industries for to analyze past accidents or preventing them. This model is founded on the assumption that accidents in these facilities are initiated by hydrocarbon release, which then propagates into accidents, to avoid this risk it is needed to establish the safety barriers into five groups of prevention functions: release, ignition, escalation, harm and loss; the new risk probabilities are evaluated after the barriers implementation, [101]. The System Hazard Identification Prediction and Prevention (SHIPP) model is an

upgrade of the previous prevention (PHPAM) model also it is applied also for to identify, evaluate, and model the accident process, and establishing the corresponding safety barriers but, as a difference of the previous model, adding to the quantitative evaluation of risk probability the possibility for to actualize the initial probability values in concordance to with the real data collected from the plant with the application of Bayesian inference, [102,103].

The Table 5 shows a summary for the fuzzy based, formal based and safety barrier models and their compliance with the three characteristics.

Models	Application	P	S	I
<b>Fuzzy based</b>	Application of fuzzy logical for define human behavior in risk situations.			
HEART [48,98]	Human Error and Assessment Technique. The reliability of any operator task performance may be modified by the influence of Error Promoting Conditions (EPCs). For every task a generic probability value for human failure and the (EPCs) affecting, and what are the strength of every (EPC) are defined by the assistance of a team of experts, then it is transformed using fuzzy logic in an assessment factor modifying the generic probability of failure. Human behavior in general applications.	+	+/-	-
CREAM-BN [99]	Upgrade of the systemic CREAM model. Human behavior has five components: strategic, tactical, opportunistic and scramble. And it is affected by common performance conditions (CPCs). Fuzzy logic is applied to every (CPC) and combined with human behavior applying a Bayesian network. Scramble and opportunistic human actuations allows for high failure probabilities. Human behavior and occupational accidents.	+	+/-	-
<b>Formal based</b>	Accident causation is approached using probabilistic schemes and Bayesian networks to model the interaction between causes and effects.			
WBA [100]	Why Because Analysis. Bayesian networks are applied considering that each component in a system is affected from the overall system environment. General application.	+	+/-	-
<b>Safety Barrier</b>				
PHPAM [101]	Process Hazard Prevention Accident Models. Accidents are initiated by hydrocarbon release and propagation, and it is needed to establish safety barriers into five groups of prevention: release, ignition, escalation, harm and loss. Risk probabilities are evaluated before and after barriers implementation. Chemical and petroleum industries.	+	+/-	-
SHIPP [102,103]	System Hazard Identification Prediction and Prevention. Update of the initial probability of risk according to the actual data collected and application of Bayesian inference. General application.	+	+	-

Table 5.- Modern approach summary of the fuzzy based, formal based and safety barrier models with their compliance to with the P: preventive; S: simultaneity and I: immediacy characteristics. General application means: industrial / occupational.

Cases of CREAM-BN application are developed for nuclear industry, aircraft transportation, manufacturing, retail and chemicals, [99]. Application cases for WBA model have been found in transportation and aircraft accidents, [100]. Applications of the Process Hazard Prevention Accident Model (PHPAM) are mainly in the off-shore

and oil-gas process industries for to analyze past accidents or preventing them, [101]; but the concept has general applicability.

Also from these previous two tables it is possible to see that modern models have general application for assess risk in general for industrial production and occupational environments. With specific applications of the CREAM and DREAM and their fuzzy based HEART and CREAM-BN models in human hazard situations.

Also according to the three characteristic definitions, the previous analyzed modern models offer not only prevention (P), also a degree of simultaneity (S), with the same possibility for update the risk evolution but with the observational delay because of the importance of environmental causal-effects, and their possible application on post accident analysis situations, and the less based on the real and continuous updating.

## **2.4. Methodologies and models. Dynamic models.**

This group covers a dynamic risk analysis concept using sequential models like Fault Tree Analysis (FTA), Event Tree Analysis (ETA) and the BOW-TIE graph approaches and performing a Bayesian inference analysis to update the failure probabilities from the information collected of the named accident precursors or precursor data. This group has five models; the Dynamic Risk Assessment (DRA) being the representative; the Dynamic Procedure for Atypical Scenarios Identification (DyPASI), the Dynamic Risk Analysis, the Risk Barometer methodology and the Dynamic Operational Risk Assessment, [104,105,106].

### **2.4.1. Dynamic Risk Assessment (DRA).**

The Dynamic Risk Assessment (DRA) is an extension of the Quantitative Risk Assessment (QRA) approach as presented on the ISO/IEC 31010:2019 guideline. The process needs to establish a prior function for the statistical parameter that models the risk probability. The precursors, events or causes that can lead to an accident are observed and formalized through the application of Bayesian inference to obtain the posterior function for the parameter that models the risk probability, [107,108,109, 110,111,112].

On Figure 8 is presented the schematic equivalences between Risk Assessment and (DRA) processes.

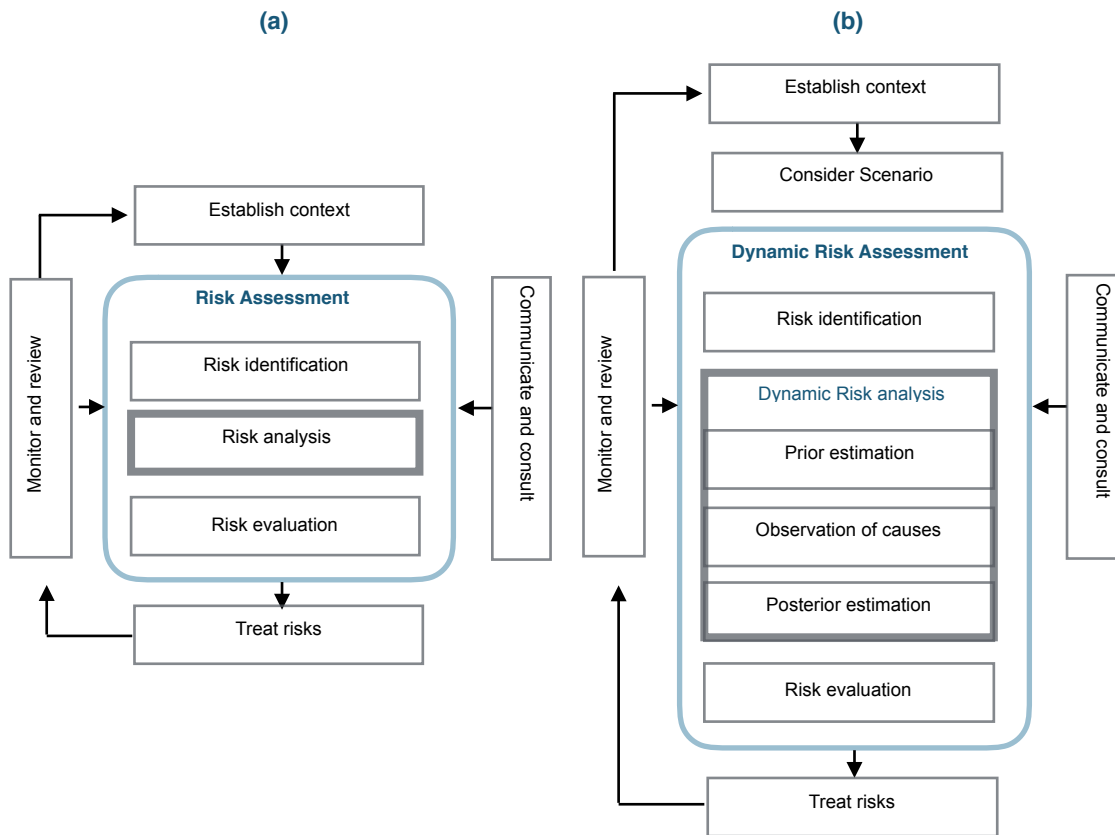


Figure 8.- Left (a) Risk Assessment compared to right (b) Dynamic Risk Assessment (DRA). (adapted from [18,19]).

The main highlights are;

- Risk identification being similar as presented in the ISO/IEC 31010:2019 guideline. The identification of potential risks are performed by the application of HAZOP/HAZID and FMEA, FMECA techniques, [113].
- Scenario consideration is similar to answer to the what if? question. Scenarios reflecting the ‘best case’, ‘worst case’ and ‘expected case’ may be used for quantifying the probability of potential consequences and obtain a sensitivity analysis.
- After identification of causes of risk, their paths and sequences through the safety barriers are defined using the (ETA) or (FTA) methods under a bow-tie graph and ending at the final states. Reliability data bases can be applied for human, equipment or barriers failure, or using experts judgment, [114,115].
- Observation of precursor data, events and situations from the workplace or process under analysis.



- Posterior estimation is performed using Bayesian inference through the expression;

$$f(p/Data) \propto g(Data/p) \cdot f(p) \quad (1)$$

Where  $p$  is the statistical parameter,  $f(p)$  is the prior statistical distribution for the parameter  $p$ ;  $g(Data/p)$  is corresponding to the observed precursor data and  $g(p/Data)$  is the posterior statistical distribution.

This strategy has been applied to a number of case studies in petrochemical industry including the case for a storage tank containing hazardous chemicals, a refinery, and the oil spill accidents; or additionally performing the inference using Bayesian Networks applied in offshore oil & gas accidents, [116,117,118,119,120,121].

#### 2.4.2. Dynamic Procedure for Atypical Scenarios Identification

The Dynamic Procedure for Atypical Scenarios Identification (DyPASI) have been developed for to perform an identification and assessment of the potential hazards based on information from atypical accident scenarios or situations, which are not captured by conventional HAZID/HAZOP techniques, and the needed safety functions are incorporated to the safety barriers, [122,123]. The main steps are:

- 1.- Identification of relevant accident scenarios and safety barriers. A bow-tie analysis is performed.
- 2.- Search for relevant information concerning undetected potential hazards for scenarios and safety barriers not considered in the previous bow-tie analysis.
- 3.- An assessment of risk is performed based on a traditional quantitative risk assessment (QRA) or based on a dynamic risk assessment (DRA) approaches, [124,125].
- 4.- The risk relevant scenarios and safety barriers functions, are analyzed using a cause-consequence scheme incorporating atypical situations.
- 5.- From the cause-consequence analysis arises the final definition and design of safety measures and safety barriers.

#### 2.4.3. Dynamic Risk Analysis

The Dynamic Risk Analysis (DRAn) is a part of the previous seen Dynamic Risk Assessment (DRA) a quantitative modern approach in which the frequency of accidents are updated by the application of the Bayesian inference, see Figure 9.

#### 2.4.4. Risk Barometer methodology

The Risk Barometer methodology is an iterative process with the aim to continuously monitor risk picture changes being based on an existing quantitative risk assessment (QRA) or a dynamic risk assessment (DRA) and on the Barrier and Operational Risk Analysis (BORA), [61,126,127,128,129,130]. The safety barriers are analyzed associating their performance to Risk Influencing Factors, (RIFs) based on human, operational, organizational and technical conditions. The main steps are:

- Development of the scenario risk model and corresponding safety barriers.
- Modeling the performance of safety barriers taking in consideration the attributes of: functionality, reliability/availability, response time, robustness and characteristics of the causes of possible failures.
- Quantitative Risk Assessment (QRA) or Dynamic Risk Assessment (DRA) to assign the probabilities/frequencies of risk. Or in default assignation of the average industrial failure values, based on data bases as Offshore and Onshore Reliability Data (OREDA), [131], or human error probability data [132].
- Development of risk influence diagrams. The aim is to incorporate the effect of the conditions for human, operational, organizational, and technical RIFs on the fail occurrences and safety barrier performance.
- Scoring of risk influencing factors (RIFs). The aim is to obtain for every safety barrier, a gradation of every RIF from best performed, medium performed to worst performed in a defined scale.
- Weighting of risk influencing factors (RIFs). The weights of the RIFs correspond to the relative difference in the frequency of occurrence of a barrier fail if the status (score) of the RIF is changed from the best scoring value, previously defined, to the worst. A low change in a RIF producing a high variation in the probability of risk and possible safety barrier fail, has higher weight or sensitivity. Usually the weighting of the RIFs is done by expert judgment. The basic method is:
  - Determination, for every safety barrier, of the most important RIF based on expert judgment.
  - Give this RIF a relative weight equal to 10.
  - Compare the importance of the other RIFs, affecting the safety barrier, and weight relative to the previous.

- Evaluate if the results are reasonable.
- The weights are normalized as the sum of the weights for the RIFs influencing a basic event should be equal to 1.

Another possibility is to perform, for the safety barrier  $j$ , a sensitivity analysis by determining the rate of change of the safety barrier reliability as a result of changes on the reliability of a single component, [51]. This component can be a risk influencing factor (RIF). Mathematically this represents the partial derivative for the reliability of the safety barrier  $j$  measured as change of risk  $R$  in a time interval  $t$  with respect to the reliability component  $q_j$  of the safety barrier.

$$IB(j/t) = \partial R(t) / \partial q_j(t) \quad (2)$$

This derivative can be linearized using a Taylor series; the change in risk  $R$  at time  $t$  as:

$$R(t) = R_0 + \sum IB(j/t) \Delta q_j(t) = R_0 + \Delta R(t) \quad (3)$$

Where  $R_0$  equals the risk value at time  $t=0$ , with parameter change  $\Delta q_j = q_{j,t} - q_{j,0}$  the difference between values of the parameter  $q$  at time  $t$  and 0 respectively.

- Then from this sensitivity analysis, a series of risk influencing factors (RIFs) are determined for every one of the most sensitive safety barriers.
- If there are more than one risk influencing factor (RIF) for the considered safety barrier  $j$  an aggregation of them based on a scored weighted importance for each risk influencing factor is performed to obtain an overall affecting value to the safety barrier probability of failure depending on the influencing factors changes.
- Finally the risk visualization of the probability of failure of the sensitive safety barriers according to the evolution of the considered risk influencing factors is performed by: (a) plotting the risk over time highlighting the risk trend or/and (b) performing an equivalent Risk Barometer diagram, consisting in a circular shape with an analog visualization to show changes in the safety barriers risk level.

On Figure 9 can be seen a schematic comparison between the DyPASI and Dynamic Risk Assessment (DRA) with the Risk Barometer methodologies.

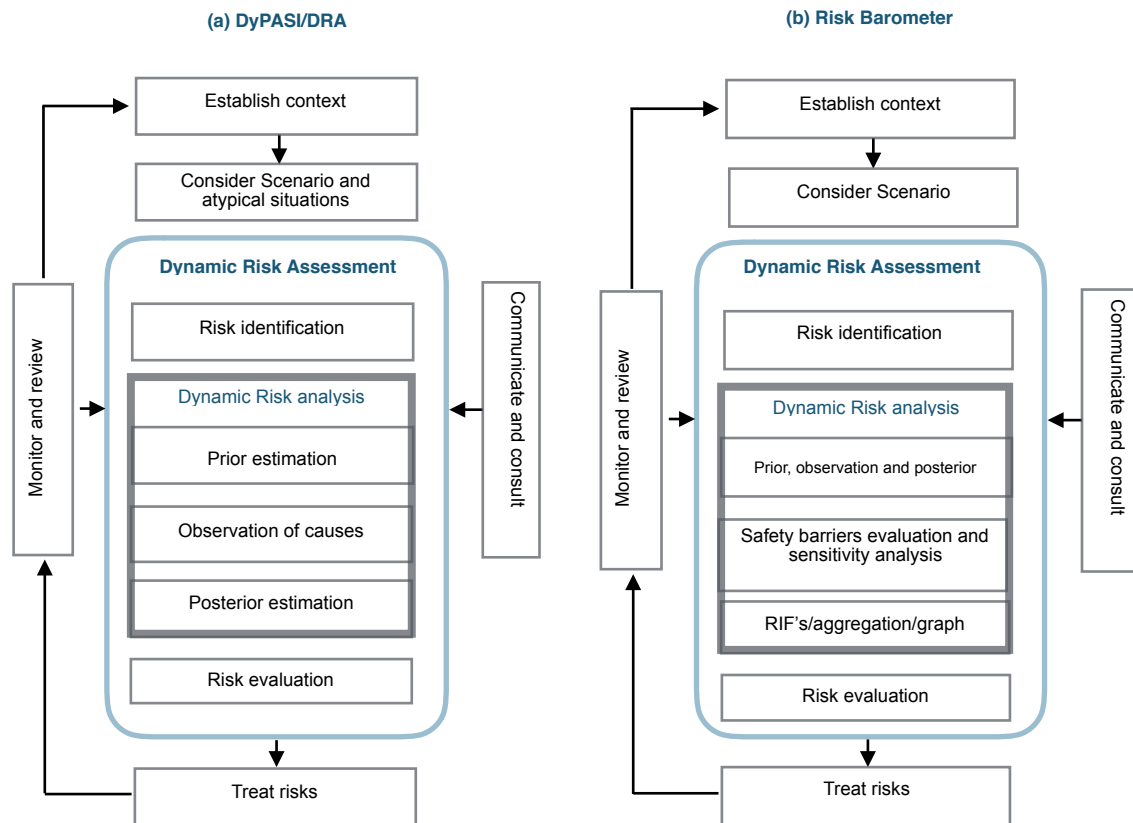


Figure 9.- Left (a) DyPASI /DRA methodologies compared to right (b) Risk Barometer methodology (adapted from [125]).

#### 2.4.5. Dynamic Operational Risk Assessment

The Dynamic Operational Risk Assessment is an extension of the Operational Risk Assessment approach, this last with the aim to follow the lifecycle of a project considering planning, execution and operation and including four types of decisions affecting risk as: strategic decisions with a long term horizon and evaluating alternatives in risks and cost/benefits, examples are the approval of major modifications, the change in maintenance strategy or the unexpected additional investment in a on going project; operational decisions are implemented in a short time, examples are the approval of works or initiate projects on a daily basis; instantaneous decisions are spontaneously made by sharp-end operators in a normal work, examples are the deviation in procedures, ignore or react to the deviations or avoid communication; and emergency decisions that are taken to manage and adapt to the hazardous situations and usually under pressure or stress situations, [43,44].

Dynamic Operational Risk Assessment has the greatest exponent in the risk analysis of a process following the traditional approach (QRA) and the dynamic approach using Markov and MonteCarlo chain simulations to analyze the incidence of events and causes in each component of a system-process and its behavior. The method simulates and analyzes visits in each of the four states, for every component of a system, in which they can be found: normal operation; abnormal not detected; abnormal detected and under repair, [133].

The Table 6 shows a summary for the dynamic models, and their compliance with the three characteristics.

Models	Application	P	S	I
<b>Dynamic</b>	Uses sequential models and the Bow-tie graph approach, performing a Bayesian inference analysis to update the failure probabilities from the information collected of the accident precursors.			
Dynamic Risk Assessment (DRA) [107,108,109]	A prior function for the statistical parameter that models the risk probability is defined. The precursors, events or causes that can lead to an accident are observed and formalized through the application of Bayesian inference to obtain the posterior function of the parameter.	+	+	+/-
DyPASI [122,123,124,125]	Dynamic Procedure for Atypical Scenarios Identification. Identification and assessment of the potential hazards based on information from atypical accident scenarios or situations, which are not captured by conventional HAZOP/HAZID techniques.	+	+	+/-
Dynamic Risk Analysis [124,125]	Analysis process as a step of the Dynamic Risk Assessment methodology being a quantitative modern approach in which the frequency of accidents are updated by the application of the Bayesian theory.	+	+	+/-
Risk Barometer [61,126,127,128,129,130]	Iterative process with the aim to continuously monitor risk picture changes being based on an existing Quantitative Risk Assessment (QRA) and on the Barrier and Operational Risk Analysis (BORA). The safety barriers are analyzed through influencing factors, named Risk Influencing Factors, (RIFs), that are correlated to with the (QRA) estimated probabilities, and their visualization in an equivalent barometer graph.	+	+	+/-
Dynamic Operational Risk Assessment [133]	Approach using Markov and MonteCarlo chain simulations applied to analyze the incidence of events and causes in each component of the process and its behavior. The method simulates and analyzes visits in each of the four states in which they can be found: normal operation; abnormal not detected; abnormal detected and under repair.	+	+	-

Table 6.- Summary of the dynamic models with their compliance to with the P: preventive; S: simultaneity and I: immediacy characteristics.

These models have been originated mainly from the need in the offshore, chemical and petrochemical industries. According to the three characteristic definitions, models offer prevention (P), and simultaneity (S), due to the own characteristic of update the probability of risk according to the observations, only the dynamic operational risk assessment shows the less compliance with this characteristic because there is no an

updating according to the observations that are matched with the simulated behavior of the system. These models show a certain degree of immediacy (I), being the Risk Barometer the one that comes closest to the immediacy characteristic definition, but there is a delay due to the scoring and weighting procedures applied to determine the most risk sensitive safety barriers.

# Chapter 3

## Statistical Risk Control (SRC) methodology.

The methodology is presented first developing the general method and considering their application for assess and control the risk in industrial environments covering process and manufacturing and project management considering risk in deviations for cost, time and investment. And second for to assess and control the risk in occupational accidents.

### 3.1. General application.

The methodology is compatible with the ISO guidelines using the Bayesian inference and hidden Monte Carlo - Markov chains [134], Figure 10.

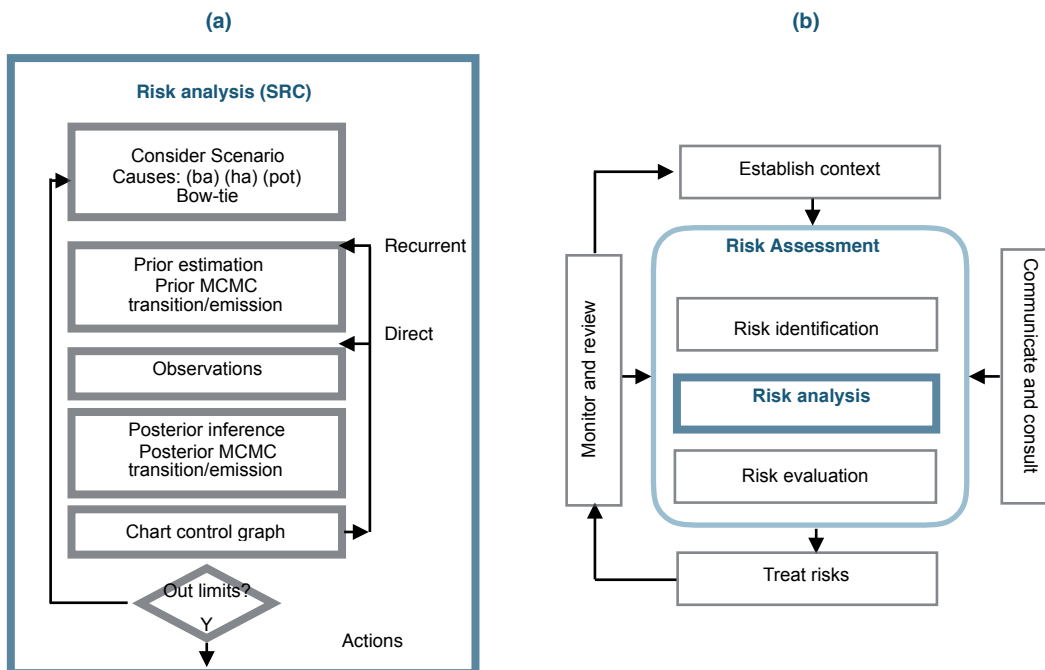


Figure 10.- (a) Statistical Risk Control (SRC) methodology ; (b) Position into the risk assessment scheme (adapted from [19]).

There is no literature in the use of the control charts for risk management but nevertheless they are applied in the earned value method in project management [135], in the environmental assessment [136] or for cost control and project duration [137].

Before describing the methodology, it is needed to expose the associated mathematical tools. In Annex A the Bayesian inference is presented with a description of the applied distributions. In Annex B the Metropolis-Hastings sampler is presented [138,139]. In Annex C the hidden time Markov Chains is also presented [140,141]. In Annex D the concept of control chart [142,143].

When the risk has been identified in a considered scenario, the following seven steps are applied:

- 1.- A bow-tie analysis is performed to provide a visual representation of the initiating causes (ic) classified as basic, human and potential that affect preventive and mitigative safety barriers and the consequences or final states when an accident occurs. Figure 11.
- 2.- The identification and definition of the initiating causes (ic's) which may be: basic events (ba) such as failures in control systems, equipment or processes; the human risk factors (ha) which are human errors and the potential causes (pot), which will be defined in the following subsection. The process is iterative between step 1 and step 2 until the causes and consequences have been clearly established.

In this scheme, the preventive or proactive safety barriers ( $IS_n$ ) have the function to avoid the accident event (AE) and have defined sub-functions ( $IS_{Snm}$ ) integrated by functional safety components of the main preventive safety barrier or procedures, automatism, alarm indications, actuators of a control system or the organizational culture itself. These functions can be affected by the defined initiating causes (ic) or by own failures. The ( $SF_{ISg}$ ) is a general sub-function safety barrier integrated by procedures, protocols, guidelines, automatism, control systems and operations management systems that guides the operation at each step of the process and does not let the next step be formalized if a number of conditions are not met. The mitigative or reactive safety barriers ( $SF_n$ ) have the function to avoid or mitigate the consequence of an accident event; these barriers have a chained action, if the first safety barrier fails the second is activated and so on, and also have defined sub-functions ( $SF_{nm}$ ) which can also be formed by functional safety components of the main mitigative or reactive safety barriers, or/and integrated also by procedures, automatism, alarm indications, actuators from a control system or the organizational culture itself. The safety barriers and their sub-functions are also affected by the initiating causes (ic) and by their own



failure. The final states represented are bounded at one end by the normal operation, if the first mitigative safety barrier acts correctly and the other by a total fail if the failure

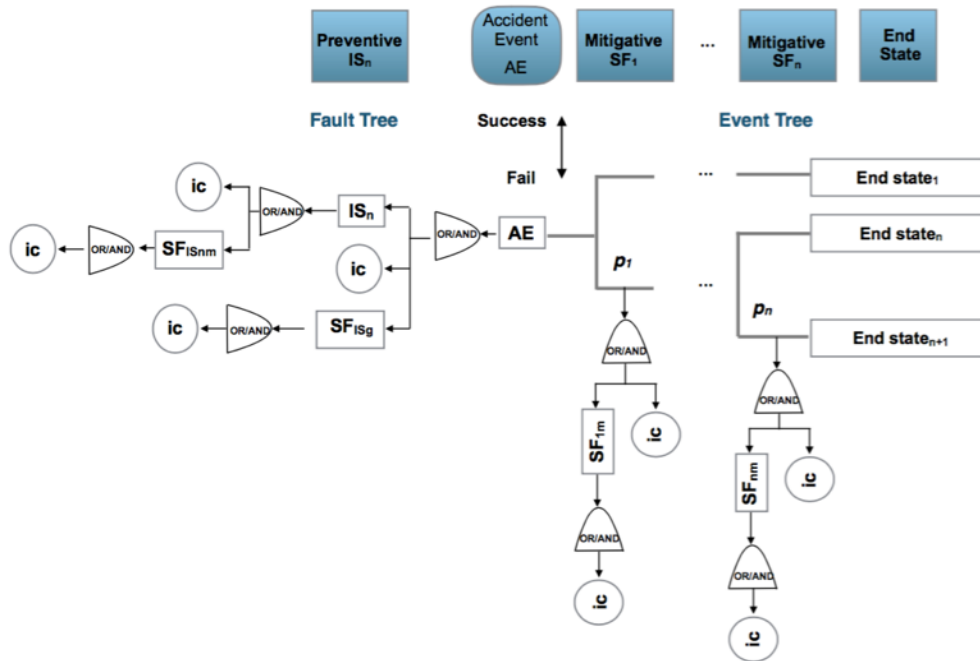


Figure 11.- General bow-tie of the SRC methodology. Safety barriers:  $IS_n$ : preventive barrier (n);  $SF_{ISnm}$ : preventive barrier (n) sub-function (m);  $SF_{ISg}$  general sub-function safety barrier;  $SF_n$ : mitigative barrier (n);  $SF_{nm}$ : mitigative barrier (n) sub-function (m); ic: initiating causes (ba, ha, pot);  $p_{s,f}$ : probability of success or fail in the mitigative safety barrier.

of all mitigative safety barriers occurs. The graph must be taken as a framework and can be adapted according to the processes, and the scenarios that are being analyzed.

3.- From the previous steps 1 and 2 the statistical parameter  $p$  that expresses the risk probability is also identified and the prior statistical distribution that reflects it can be established. Also can be defined the prior transition and emission matrices governing changes in the mitigative safety barriers.

4.- The observation of the initiating causes (ic) and end states are effectuated in a time interval.

5.- From the estimated prior  $f(p)$  and the observed initiating causes (ic), such as  $g(data/p)$  and applying equation (1), the posterior function for the statistical parameter ( $p$ ) can be obtained and if not there is analytical expression, the Metropolis - Hastings sampling method is applied to obtain the posterior distribution and its associated parameters,

[138,139]. Also corresponding to the hidden Markov chain, the prior transition and emission matrices are defined for the mitigative safety barriers and the posterior transition and emission matrices are obtained using the Baum-Welch algorithm, [140,141].

6.- Control chart presentation [142,143] to graph the evolution of the statistical parameter  $p$ , in a time interval. Chart determination has two modes:

a.- Direct: Uses the observed data up to the analyzed interval time, but with two possibilities: the mean established in the prior function that defines the statistical parameter ( $p$ ) is constant in every interval, and the standard deviation is determined using the observed data collected up to the analyzed interval, (Direct-Mean Prior) or by modifying the mean and the standard deviation also using the observed data collected up to the analyzed interval (Direct-Mean Posterior).

b.- Recurrent: Uses the observed data in every interval time, also with two possibilities: maintaining the mean posterior constant, and the standard deviation obtained in every interval is the new prior in the following interval (Recurrent-Mean Prior), or the mean and the standard deviation obtained in every interval are the prior values in the following interval (Recurrent-Mean Posterior).

Considering also the observed initiating causes (ic), and visualizing the bow-tie, there are two possibilities of analysis to include in the control chart:

a.- For the complete bow-tie scheme. Figure 12.

a.1.- Collecting the total of the initiating causes (ic) affecting all the preventive and mitigative safety barriers with their barrier sub-functions.

a.2.- Collecting only the first level for initiating causes (ic) and failures in preventive safety barriers functions and the first level for functions and sub-functions of the mitigative safety barriers with their corresponding initiating causes (ic).

b.- Observing the fault tree (FT) event tree (ET) and analyzing the response active (yes) or (no) for the preventive and mitigative safety barriers.

7.- Analysis applying a hidden Monte Carlo Markov Chain for the mitigative safety barriers, also with two possibilities. Figure 13.

a.- Analyzing the behavior of the mitigative safety barriers based on the end states. In this case a transition matrix is defined for the mitigative barriers and an emission matrix for the observed end states in function of the barriers transition.

b.- Analyzing the behavior of the end states based on the action of the mitigative barriers. In this case a transition matrix is defined for the end states and an emission matrix for the observed mitigative safety barriers in function of the end states.

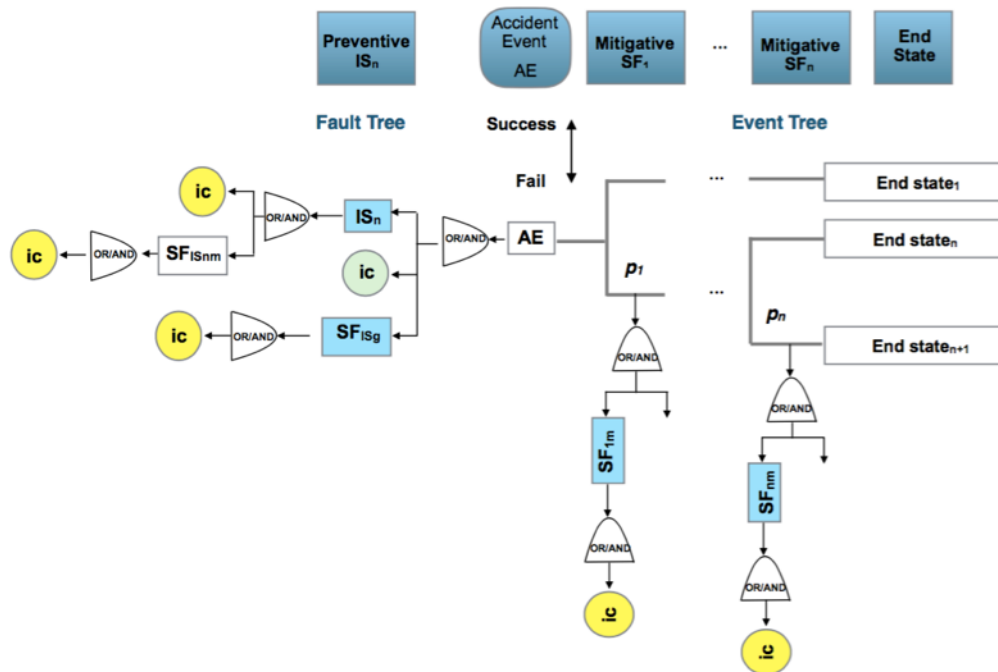


Figure 12.- Analysis modes. (a.1) collecting total of (ic's) of preventive and mitigative barriers and their sub-functions (highlighted green and yellow); (a.2) collecting at the first level of (ic's) and barriers sub-functions (highlighted green and blue).

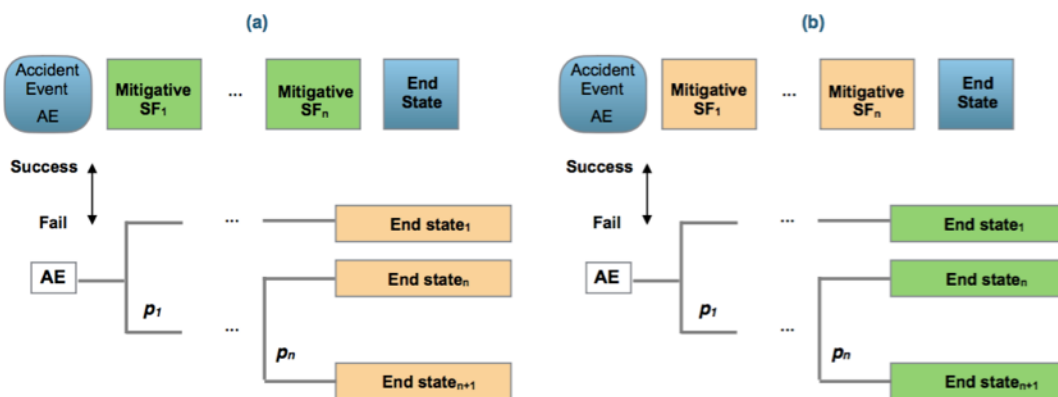


Figure 13.- Analysis modes of the hidden Markov chain. (a) defining transition probabilities for the mitigative barriers (highlighted green) and emission probabilities for the observed end states in function of the mitigative barriers (highlighted brown); (b) defining transition probabilities for the end states (highlighted green) and emission probabilities for the observed mitigative barriers states (highlighted brown).

The Baum Welch algorithm is applied, from the observations, and the posterior transition and emission matrices can be obtained using a direct or recurrent method.

### 3.2. Potential causes (pot).

Some authors consider [144] that when unexpected deviations arise due to causes that are difficult to predict because of their randomness, they classify it as a special risk [145,146]. From the point of view of the methodology (SRC), these causes that can lead to unexpected risk situations are called potential causes, and are summarized in Table 7.

Potential Cause (pot)		Attributes
Deviation from a procedure	(DEF)	Changes derived from operator decisions. Formal procedures that are established but not well applied.
Failed test	(FTS)	Test for integrity control, performance, safety operation, production.
Technical design failures	(TFT)	Incorrect design, big repair to adapt a mechanism or process.
Immediate errors after maintenance -safety - control	(IEC)	After reparation, audit or control a new intervention is needed.
Concatenation or domino effect	(RCD)	In operations, maintenance. Or in projects.
Unplanned operations or application of a new technology	(UNT)	In operations, maintenance. Or in projects.
Lack in communication and information	(LCI)	In operations, maintenance. Or in projects. Affecting procedures, messages, safety indications, rules of actuation, policies, Or defective in planning and scheduling.
Human and social changes	(HSC)	Government changes, economical variations, migrations or immigration of people, social unrest and violence.
Investment modifications (planned or not)	(IMD)	Affecting safety, with loss of social benefits, increasing work overtime and pressure. Poor cost control.
Extreme climatic situations	(XCS)	Despite historical weather and existent climate conditions. Planned work in rough weather and climate conditions.
Malicious acts derived from war or terrorism	(MAT)	Social upheavals, threats.

Table 7.- Potential causes (pot) and their attributes defined in the SRC methodology.

### 3.3. General risk assessment application in industrial processes.

The application of the Statistical Risk Control (SRC) methodology in scenarios belonging to manufacturing or chemical processes, is based on the previous general scheme that must be taken as a framework and must be adapted according to the processes and studied scenarios.

### 3.4. General risk assessment application in occupational accidents.

In accordance with the SRC methodology, a bow-tie is defined in each particular scenario. In the case of occupational accidents before being able to work with different

risk scenarios, it is necessary to analyze what human behavior is at work and what are the factors that affect it. Three views and organizations that cover the most critical have been considered. The first group compiled is general, [25,147,148]. The second group includes the factors considered critical of the 6th European Survey of Working Conditions [149], and the third group includes the emerging psychosocial risks related to occupational safety and health [150]. The aggregation of the three groups is presented in Table 8.

From (NISHW, 2018), (Baybutt 2013), (Kariuki and Löwe, 2007) [25,147,148]	From (Eurofound, 2017) [149]	From (EU-OSHA, 2009) [150]	Attributes
Organization (ORG)	Social environment (SEN)	Emotional demands at work (EDW)	Safety policy, culture, learning policy, management supervision and quality, social behavior and support, stress
Job design (JBD)	Working time quality (WTQ) Work intensity (WKI)	Contract - job security (CJS) Poor work - life balance (PWL) Work intensification (WIT)	Irregular scheduling, shifts and overtime. Low flexibility, atypical arrangements, quantitative and emotional demands, outsourcing, part-time, temporary situations, burn-out effects.
Operator environment (OPE)	Physical environment (PHE)		Ergonomics (noise, vibration, temperature, air), biological and chemical VOC's.
Operator characteristics (OPC)	Skills and discretion (SKD) Prospects (PRO)	Ageing (AGE)	Cognitive, concentration, Expertise-aptitudes, decision level, responsibility, steadiness, participation, skills, training support,
Human system interface (HSI)			Control design interface, displays, panels, actuators. Alarms.
Information (INF)			Training procedures, labeling, communication, labeling.
Workplace design (WKD)			Layout and configuration, accesibility.
	Earnings (EAG)		Expectations

Table 8.- Factors and attributes affecting human behavior at work.

These critical factors must be taken into account as possible causes of initiation (ic) of an occupational accident and must be present in all scenarios. However, it is considered that there are four situations that occur as a consequence and symptom of these critical factors. These are: failure in the self-control of work (JSC); failure to supervise work (JSU); failure in safety self-control (SSC) and failure in safety supervision (SSU). In addition, these four situations that are easily observable arise in automated process and manufacturing environments, where workers additionally perform control and supervision tasks. Also these four situations, by their control and supervision function, are associated with safety barriers. The general bow-tie for occupational accidents, is

equivalent to the general one presented above, with the differences in preventive barriers ( $IS_n$ ) integrated by the four safety barriers ( $IS_{JSC}$ ,  $IS_{JSU}$ ;  $IS_{SSC}$ ;  $IS_{SSU}$ ), Figure 14.

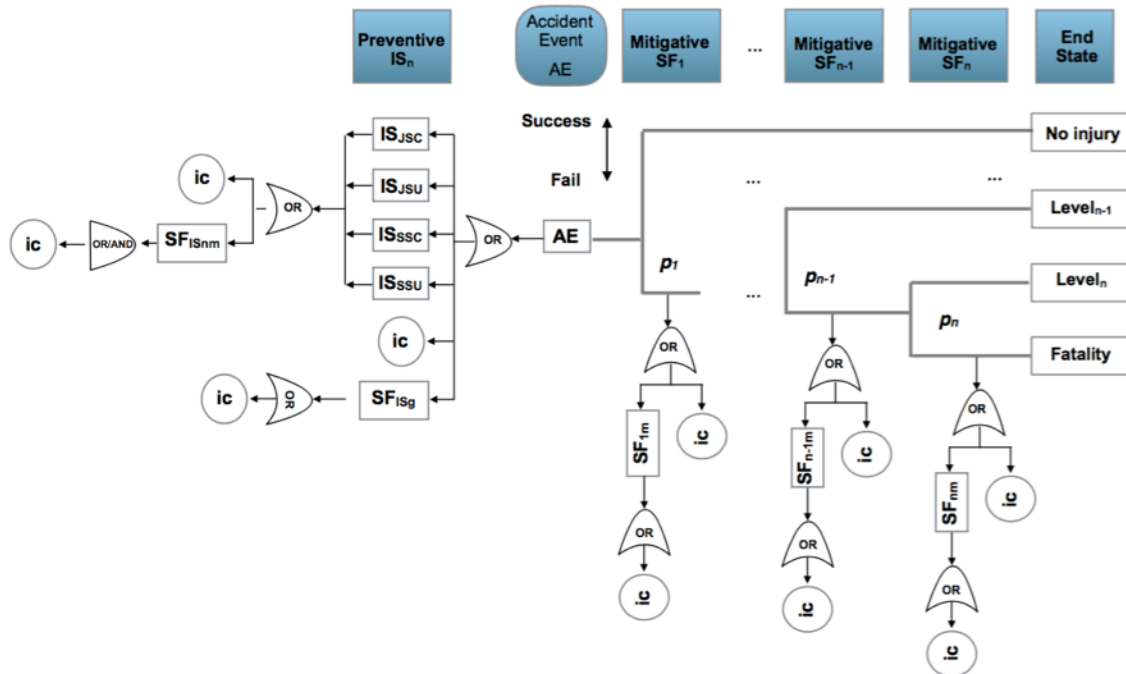


Figure 14.- General bow-tie for occupational accidents. Safety barriers:  $IS_n$ : preventive barrier (n) based on job self control (JSC); job supervision (JSU); safety self control (SSC) and safety supervision (SSU);  $SF_{ISnm}$ : preventive barrier (n) sub-function (m);  $SF_{ISg}$  general sub-function working parallel to the human actions;  $SF_n$ : mitigative barrier (n);  $SF_{nm}$ : mitigative barrier (n) sub-function (m); ic: initiating causes (ba, ha, pot);  $p_{s,f}$ : probability of success or fail of the mitigative safety barrier.

In this scheme, the ( $SF_{ISnm}$ ) are sub-functions of the four safety barriers, which can be formed by a procedure, an automatism, an alarm indication, an actuator of a control system or the organizational culture itself. The general safety barrier ( $SF_{ISg}$ ) works in parallel with the activity of the operators and may consist of automatic control systems, protections, alarms, actuators or an automated operations management that guides the operator at each step of the process and does not let the next step be formalized if a number of conditions are not met. Sub-function barrier ( $SF_{nm}$ ) covers the various functional components that integrate the mitigative safety barriers. The final states represented are bounded at one end by the absence of personal injuries, if the first mitigative safety barrier acts correctly and the other by a fatality if the failure of all mitigative safety barriers occurs. Also the graph must be taken as a framework and can be adapted according to the processes, the occupational works and the scenarios that are being analyzed.

### 3.5. General application in deviations of costs and time.

The elaboration of budgets for cost and time, can follow a top-down or a bottom-up scheme; both systems have advantages and disadvantages, from the point of view of risk management, the top-down scheme allows to examine the environmental characteristics and the variables that can affect the evolution of the project from a global perspective; the bottom-up scheme allows incorporating the detail, knowledge and experience of the people who work on a day-to-day basis; on the other hand, organizations oriented to the generation of value, usually adopt both criteria. In all cases, we must bear in mind that budgeting is an activity that has a high consumption of effort and resources, so the development of the additional information to be achieved and its control must be balanced, with the necessary effort to define it and manage it [151].

When analyzing the deviation in cost it is important that the accounting numbers in the project cost be available and integrated into the own company accounting system to gain homogeneity. The cost allocation is based on group of expense accounts of the Spanish national accounting plan based on the International Financial Reporting Standards (IFRS) [152], adopted by the European Union in 2002, in which the different groups of expenses are collected for each stage according to the Spanish RD 1514/2007, [153].

#### 3.5.1. A simple organizational model.

To be able to manage the risks in the deviations of cost and time of the projects, it is necessary to establish previously a basic organizational model, in which the projects and industrial processes should be developed; a first idea is suggested by Gómez-Senent et al. [154] when dealing with the IPMA competency model [155], and the strategy [156].

The proposed scheme is presented in the Figure 15. The functions depicted are the support of the main activity, ( activity<sub>j</sub> ), being physical and information the entries from previous activities ( in<sub>j-1</sub> ) and the result is also a physical and information out, ( out<sub>j</sub> ).

The functions are defined in Table 9. The information result is checked continuously and must be corrected if it shows deviations with respect to the programmed values.

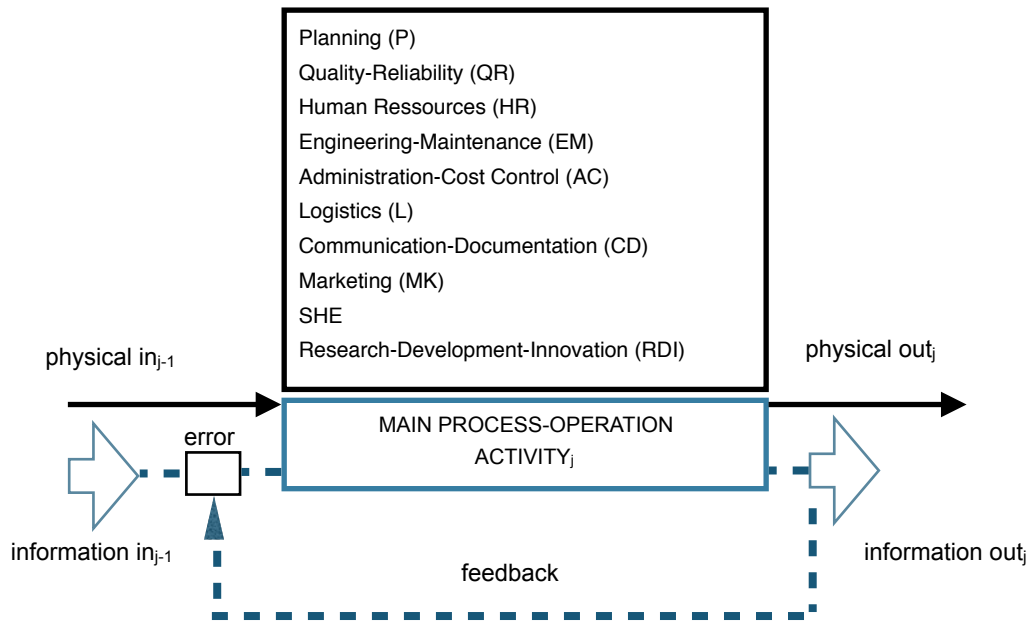


Figure 15.- A systemic basic scheme of organization in a project or industrial management units.

Being:

Main Process: main activity to be performed

Physical In/Out: Energy, materials, equipment, spares, people.

Information In/Out: Rules, policies, technical specifications, process parameters, material and energy balances, production, incidences, protocols, parameters, constraints, goals.

Thus, into the concept of risk management are involved not only the main activities also the functional support activities and the feedback error measured as the discrepancy between the observed or measured values and the planned or specified values, has to be constantly updated.



Function		Attributes
Planning	(PN)	Programming and control of the activities and resources in the different stages of the project or industrial process.
Quality - Reliability	(QR)	Issuer of the quality plan, control for variability and risk. Supervisor and guarantor that the activities, both the main ones and the support activities, are under control.
Human Resources	(HR)	Guarantor of the skills and conditions of people teams.
Engineering - Maintenance	(EM)	Product design. Design and installation of assets and equipment. Guarantor of the adequacy and availability of them to the needs of the project or industrial process.
Administration - Cost control	(AC)	General and cost accounting. Administration procedures and business information system.
Logistics	(LO)	Storage management and control of inventories, purchasing, planning and management of the supply and distribution chain.
Communication - Documentation	(CD)	Management of information related to manuals, regulations, plans, designs, specifications, records and work and quality documents.
Marketing	(MK)	Covers the establishment of all the activities needed for present to the customer target the product in price, place and promotion.
Safety - Health - Environment	(SHE)	Management of risk protection, safety regulations and preparation of protocols, procedures and worksheets revision. Safety, health and environmental prevention, risk analysis and impact evaluation.
Research - Development - Innovation	(RDI)	Search, characterization, creation and improvement of products, methods, protocols, processes and systematics.
Operations	(OP)	Activity and process realization according to the engineering design, quality specifications and with reliability and safety.

Table 9.- Functions and attributes of the systemic basic scheme of organization.

### 3.5.2. Deviations in cost and time. Previous concepts.

The cost distribution are made to the different work centers and to the activities of the projects. The method by which the work centers collect the costs is not relevant for this thesis work.

The planed income statement based on costs scheme is as presented on Figure 16. When elaborating the budget of costs, the following equality is true:

$$\text{Planned Income Statement for Costs} + \text{Planned Idle capacity} = 0 \quad (4)$$

The previous equation is valid since all the costs with the exception of the idle capacity, have to be applied to the product and service generating its cost of production. It is a also a consequence of a balance between the variable and fixed expenses and those that are attributed to the product or service as a result of the activities of the project or industrial process. If the idle capacity were zero, meaning that all work centers are at nominal occupation, all the expenses generated in the project or process, would be

<b>PLANNED INCOME STATEMENT based on COSTS on Date xx/xx/xxxx</b>	<b>COST</b>
Planned products and services from project activity	Standard costs and w.i.p. at standard cost
- Raw materials	Standard costs
- Components and supplies	Planned Purchasing mix
+ Stock variation ( $Ex_f - Ex_i$ )	
= PRODUCT OR SERVICE GROSS MARGIN for COSTS	
-VARIABLE COSTS	
External works	
Repair and conservation	
External and professional services	
Transportation	
Advertising	
Supplies	
Use and aging losses	
= PRODUCT OR SERVICE NET MARGIN for COSTS	
-FIXED COSTS	
R&D expenses	
Leases	
Insurances	
Financial services	
Tributes	
Personnel expenses	
Financial expenses	
Depreciations	
= INCOME STATEMENT for COSTS	
+ Planned Idle capacity	
	Planned costs
	Provisions and contingencies
	Planned costs
	Provisions and contingencies

Figure 16.- Planned income statement based on costs. Being: work in process w.i.p. Provisions and contingencies: applied for environmental actuations, long-term personnel expenses, tributes, assets restructuring, stock deterioration.

totally applied to the product or service obtaining their cost price and therefore the planned income statement based on costs would be 0. If, on the other hand, there is an inactive capacity other than zero, a part of the fixed costs cannot be applied to the product or service, which would increase its value. The product or service has a part of the costs applied, not all of them, the rest being the underutilization or idle capacity costs.

The idle capacity is the fixed cost, which remain unabsorbed due to underutilization of capacity, and it should be measured as it is equivalent to a risk that must be known. This concept will be clear with a simple example, be a project organization that have to generate 10 units per year of an item, suppose that this project unit has fixed costs for 200 monetary units and 100 monetary units of variable costs and suppose that its nominal capacity is precisely 10 units per year. When working at nominal capacity the resulting cost of the unit produced is:

$$C_{item} = \frac{\sum \text{fixed and variable costs}}{\sum \text{production}} = \frac{200 + 100}{10} = 30 \frac{\text{monetary units}}{1 \text{ produced unit}} \quad (5)$$

In Table 10 are presented the comparison between a planned income statement by costs at nominal capacity, and at 80% and 120% capacity.

Income statement based on costs	Planned at 100%	Planned at 80%	Planned at 120%	Units
Product from activity	10 units x 30 = 300	8 units x 30 = 240	12 units x 30 = 360	monetary units absorbed by production at standard cost
- Variable expenses	-100	-80	-120	monetary units
- Fixed expenses	-200	-200	-200	monetary units
Income cost of the activity	0	-40	40	monetary units
Idle capacity	0	40	-40	monetary units

Table 10.- Differences in planned idle costs due to under or over utilization.

If the planned realization would have been done considering 20% idle capacity, that is working at 80% capacity, then it is possible to recover 2 units affecting practically only the variable expenses and keeping practically fixed planned costs. But working at 120% capacity it is needed to pay overloads. If for example 10 monetary units have been contributed impacting on fixed costs, the actual income cost of the activity at 120% would be 30 and from expression (4) is; actual Income Statement for Costs + Actual Idle capacity = 30 -40 = -10 , that is the project or the activity has lost 10 monetary units in form of cost overruns, Table 11.

Income statement based on costs	Planned at 80%	Actual at 100%	Actual at 120%	Units
Product from activity	8 units x 30 = 240	10 units x 30 = 300	12 units x 30 = 360	monetary units absorbed by production at standard cost
- Variable expenses	-80	-100	-120	monetary units
- Fixed expenses	-200	-200	-210	monetary units
Income cost of the activity	-40	0	30	monetary units
Idle capacity	40	0	-40	monetary units

Table 11.- Differences in actual costs considering the idle capacity and cost overruns.

3.5.3. General SRC methodology applied for risk in cost and time deviation.

The application follows the general scheme as previously presented and requiring the realization of a bow-tie. Figure 17.

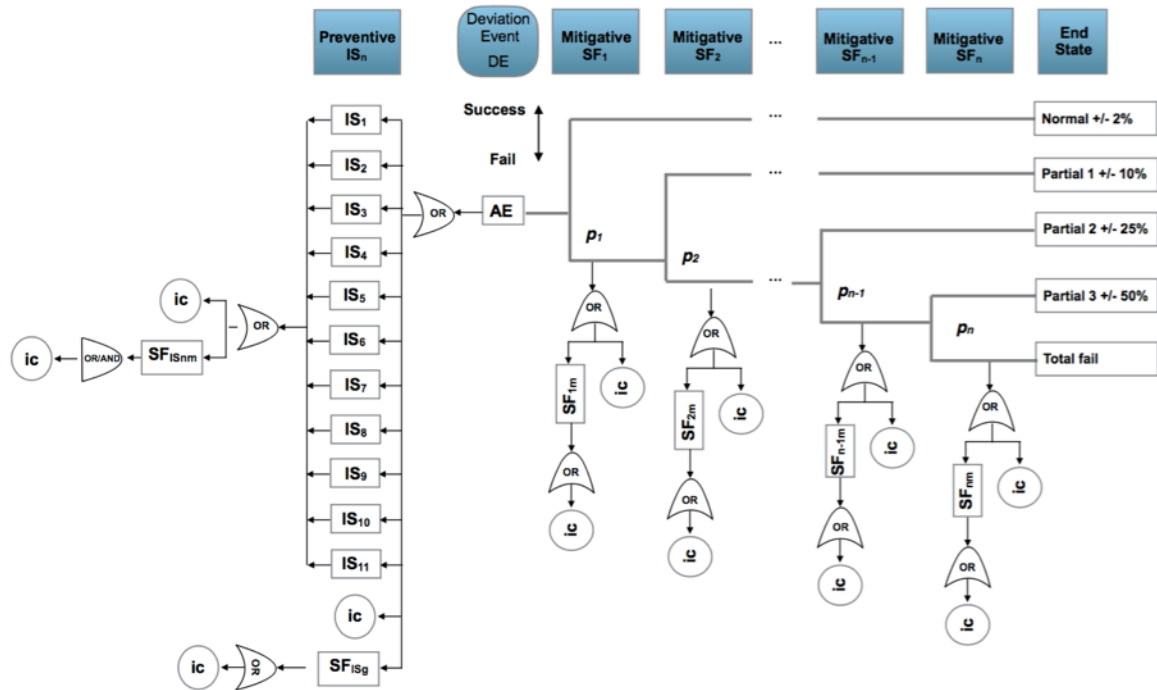


Figure 17.- General bow-tie for risk in cost and time deviation. Safety barriers:  $IS_n$ : preventive barrier (n);  $SF_{ISnm}$ : preventive or proactive barrier (n) sub-function (m);  $SF_{ISg}$  general parallel sub-function;  $SF_n$ : mitigative barrier (n);  $SF_{nm}$ : mitigative barrier (n) sub-function (m); ic: initiating causes (ba, ha, pot);  $p_{s,f}$ : probability of success or fail of the mitigative safety barrier.

Where (ic) are the defined initiating causes (ba, ha, pot).  $IS_n$  are the preventive safety barriers defined in Table 12. The ( $SF_{ISnm}$ ) are sub-functions of the preventive safety barriers, integrated by procedures, an automatic or manual control system or the organizational culture itself. The general safety barrier ( $SF_{ISg}$ ) works in parallel with the activity and may consist of automatic alarms or messages from information control systems or from organizational actuations. The general mitigative safety barriers ( $SF_n$ ) are defined in Table 13. The sub-function barrier ( $SF_{nm}$ ) covers the various functional components that integrate the mitigative safety barriers. The final states represented are bounded at one by targeted normal deviations, if the first mitigative safety barrier acts correctly and successive targeted partial deviations if the safety barriers fail consecutively. As previously stated the graph must be taken as a framework and can be adapted according to the scenarios that are being analyzed.

To perform the risk analysis the seven steps from the general method can be applied, making the observations considering the initiating causes (ic), and safety barriers sub-

functions fails, applying a hidden Monte Carlo Markov Chain for the mitigative safety barriers, but in this case an additional eighth step can be added as an additional possibility by analyzing the own deviations from planned in the cost and/or time situations.

<b>Preventive Safety barrier (IS<sub>n</sub>)</b>		<b>Attributes-Failures</b>
Planning	(IS <sub>1</sub> )	Planning - scheduling mistakes (PMIS). Planning - scheduling deficiencies (PDEF).
Quality - Reliability	(IS <sub>2</sub> )	Quality mistakes (QMIS). Quality deficiencies (QDEF). Quality not acceptances (QNAC).
Human Resources	(IS <sub>3</sub> )	Human Resources mistakes (HRMIS). Deficiencies (HRDEF).
Engineering - Maintenance	(IS <sub>4</sub> )	Engineering -maintenance mistakes (EMIS). Deficiencies (EDEF).
Administration - Cost control	(IS <sub>5</sub> )	Administration - control mistakes (AMIS).
Logistics	(IS <sub>6</sub> )	Logistics mistakes (LMIS). Supply-distribution delays (SDEL).
Communication - Documentation	(IS <sub>7</sub> )	Communication mistakes (CMIS).
Marketing	(IS <sub>8</sub> )	Marketing mistakes (MMIS). Marketing deficiencies (MDEF).
Safety - Health - Environment	(IS <sub>9</sub> )	SHE mistakes (SHEMIS). SHE deficiencies (SHEDEF). Changes in regulations (SHECH). Sanctions (SHESAN).
Research - Development - Innovation	(IS <sub>10</sub> )	Research development mistakes (RDIMIS). Deficiencies (RDIDEF).
Operations	(IS <sub>11</sub> )	Fail job self control (JSC). Fail safety self control (SSC). Fail job supervision (JSU). Fail safety supervision (SSU).

Table 12.- Preventive safety barriers (IS<sub>n</sub>) for cost and time deviation, attributes-failures.

<b>Mitigative Safety barrier (SF<sub>n</sub>)</b>		<b>Attributes-Failures</b>
Economic-Time controlling	(SF <sub>1</sub> )	(ic's) especially JSC, JSU failures. No available idle capacity. No possibility for reschedule.
Third's based penalties	(SF <sub>2</sub> )	(ic's) especially JSC, JSU failures. Undefined-incorrect agreement. Inability to reprogram with third parties. Not answer by third party.
Sharing responsibilities	(SF <sub>n-1</sub> )	(ic's) especially JSC, JSU failures. Partial capital funding not available due to bank credit failure. Undefined-incorrect agreement. Inability to share time with third parties. Not answered by counterparts.
Capital funding	(SF <sub>n</sub> )	(ic's) especially JSC, JSU failures. Capital funding not available due to bank credit failure. Undefined-incorrect agreement. Incorrect time schedule definition.

Table 13.- Mitigative safety barriers (SF<sub>n</sub>) for cost and time deviation, attributes-failures.

There is no previous scientific work considering risk of deviations in cost and time using chart control methodology. Some authors consider a risk indicator based on a weighted sum for to perform a comparison between the actual progress situation in front of the planned [157].

#### 3.5.4. General SRC methodology applied to risks in industrial investments.

As an extension of the analysis for risk of deviations in cost and time, also there is the situation of risk of deviations in industrial investments.

Traditional project valuation methods comprises methodologies as the Payback Period (PBP), the Accounting Rate of Return (ARR), the Net Present Value (NPV) and the Internal Rate of Return (IRR), [158]. Before analyze the possible options it is needed to introduce the income statement based on sales definition according to the accounting standards, [152,153], Figure 18.

The payback period (PBP) of a project represents the time it takes for an investment on that project to be recovered and it is determined by taking in consideration the number of years it takes until the initial investment for the project equals the sum of the forecasted cash flows (income plus depreciation). The advantage is their simplicity but possible problems after PBP period are ignored. Usually there is a metric application of 3 years. In this method it is possible to consider the time value money depreciation.

The accounting rate of return (ARR) method determines the ratio between the average project earnings after depreciation and taxes and the average investment value along the project life. The method is simple and can their accounting is based and available from the company accounting system. It is possible to take in consideration the time value money depreciation.

The net present value (NPV) is determined by deducting the present value of the capital investments from the present value of the economical cash flow revenues. The general expression is;

$$NPV = \sum_{t=1}^n \frac{C_t}{(1+r)^t} - \sum_{t=0}^n \frac{I_t}{(1+r)^t} \quad (6)$$

Where;

$n$  is the project time duration.

$t$  is the time period where a cash flow revenue  $C_t$  or a capital investment  $I_t$  occurs.

$C_t$  is the economical cash flow equal to;

$$C_t = \text{Project Income after taxes} + \text{depreciations} + \text{provisions and contingencies}$$

$r$  is the discount rate.

<b>PLANNED PROJECT INCOME STATEMENT based on SALES on Date xx/xx/xxxx</b>	<b>CONCEPT</b>
Planned products and services sales	Sales price
+ Stock variation (Ex-Ex)	Finished products and services and their W.I.P. (1)
	(2)
+ Own work done applicable to company assets	Purchasing mix
- Raw materials	
- Components and supplies	Expenses
= PRODUCT OR SERVICE GROSS MARGIN	Provisions and contingencies
-VARIABLE COSTS	
External works	
Repair and conservation	
External and professional services	
Transportation	
Advertising	
Supplies	
Use and aging losses	
= PRODUCT OR SERVICE MARGIN	Expenses
-FIXED COSTS	Provisions and contingencies
R&D expenses	
Leases	
Insurances	
Financial services	
Tributes	
Personnel expenses	
Financial expenses	
Depreciations	
= PROJECT INCOME STATEMENT NET MARGIN	
+ Other operative incomes and allowances	
- Other operative expenses and disposals	
= OPERATIVE PROJECT INCOME STATEMENT	
+/- Financial incomes and expenses	
= PROJECT INCOME before TAXES	
- Taxes	
= PROJECT INCOME STATEMENT after TAXES	Allowances and Assets disposal

Figure 18.- Planned income statement based on sales. Being: (1) work in process w.i.p.; (2) counterpart of the expenses made by the company for its fixed assets, using its equipment and its personnel, which are activated; also those carried out, on request, by other companies with research and development purposes, will also be accounted for. Provisions and contingencies: applied for environmental actuations, long-term personnel expenses, tributes, assets restructuring, commercial losses, stock deterioration.

The decision rule is to invest in any project with a positive NPV. And if different projects are compared with each other, then the prior is to invest in the project with highest positive NPV. The account for a risk associated is considered by the addition of a risk premium value added to the discount rate  $r$  leading to a risk-adjusted rate which is higher than the risk-free discount rate. The estimation of this risk premium value can be difficult, [158].

The internal rate of return (IRR) is closely related to the NPV method and is defined as the discount rate at which the NPV equals zero. The following equation has to be solved for (irr);

$$0 = \sum_{t=1}^n \frac{C_t}{(1 + irr)^t} - \sum_{t=0}^n \frac{I_t}{(1 + irr)^t} \tag{7}$$

The decision rule in this method, is to accept a project if its IRR is higher than the expected rate of return offered from other assets investment with the same amount of risk as the evaluated project.

Both methods (NPV) and (IRR) take into consideration the time value of the money and account for a certain amount of risk, but the NPV method measures the absolute contribution of a project to the company goal to maximize revenue, [158].

To take in consideration the risks and uncertainties that can allow for unexpected investment changes it is needed to take in consideration the causes and safety barrier fails that can affect an investment project along its life.

Because the evolution of the project cost affects directly to the NPV consecution, the reactive safety barriers are more related to this cost - time evolution and are the same as presented for previous deviation control of risk, Table 12 and Table 13. With general bow-tie, Figure 19.

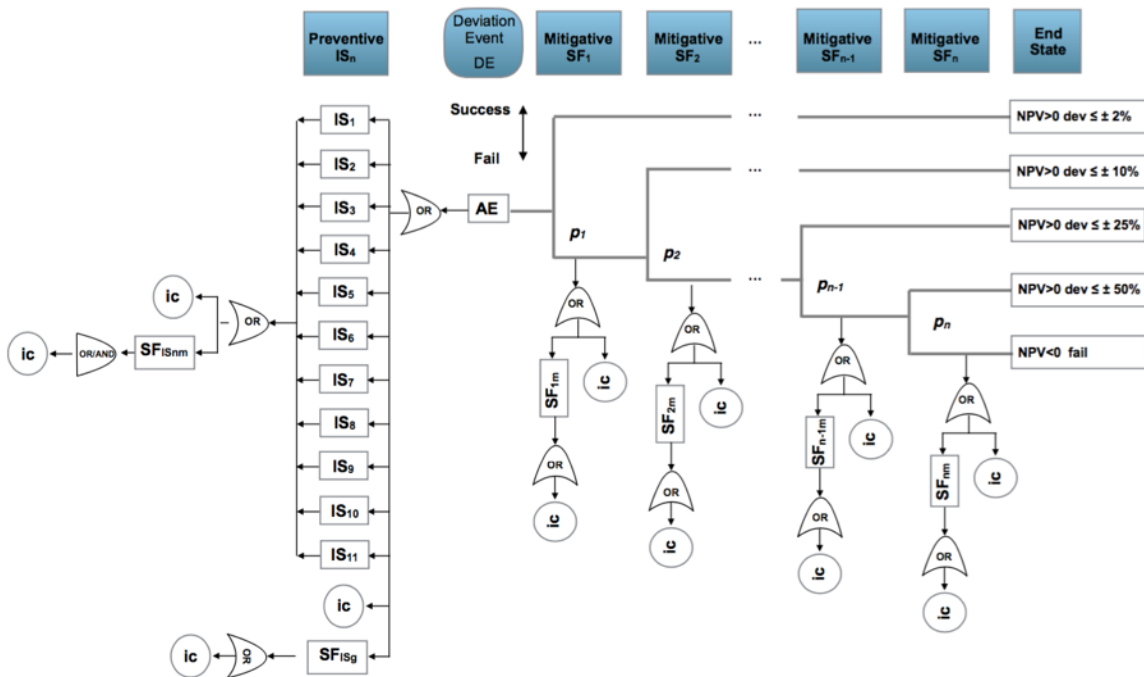


Figure 19.- General bow-tie for risk of deviations in industrial investment.



### 3.6. General application in LOC-domino effect risk analysis.

A review of some accidents involving the domino effect, as the case of France (1966), where eleven storage tanks were destroyed, covering a distance of 800 meters around the site. The Mexico accident (1984) involving a storage and distribution facility for Liquefied Petroleum Gas (LPG) from a pressure excess involving Flash and VCE together with pool fire explosions and affecting 15 tanks of 16,000 m<sup>3</sup>. The case of India (1997) where Flash and VCE and pool fire explosions affected the storage tanks at the refinery of the Hindustan Petroleum Corporation. Turkey (2014) from a LPG tanker. Louisiana (2015) the flash explosion and pool fire of a heat exchanger in a olefins plant. California (2015) the flash and vapor cloud explosions of a hydrofluoric acid unit used to increase the octane index in a refinery plant. Louisiana (2017), the explosion of a volatile liquids tank in a pulp mill plant after welding maintenance works in the above located pipe system. Shows the importance of this.

There is no generally accepted definition of what constitutes loss of containment (LOC) domino effect in the context of accidents in the chemical processing industry, although various authors have provided suggestions. An overview of some definitions identified in a review of the relevant documents is as follows.

Lees, [159], exposes two definitions: first as a factor to take into account of the hazards that can occur if leakage of a hazardous material and can lead to the escalation of the incident and second as an event whose consequence causes a separate event in a separate unit.

Bagster and Pitblado, [160], defined the domino effect as a loss of containment of a plant item which results from a serious incident on a nearby plant unit.

The AIChE-CCPS [23], defines the domino effect as an incident which starts in one item and may affect nearby items (e.g., vessels containing hazardous materials) by thermal, blast, or fragment impact causing an increase in consequence severity or in failure frequencies.

Delvosalle et al. [161], defined domino effect as a primary accident in a primary installation (this event might not be a major accident), inducing one (or more) secondary accident(s), concerning secondary installation(s). This (these) secondary accident(s) must be a major one(s) and must extend the damage caused by the primary accident. Therefore, the domino effects act in a chain, involving a number of installations.

Necci et al. [162], defined the domino effect as the existence of a primary accident scenario that usually propagates due to the failure of storage or process units with direct damage causing the loss of the structural integrity in secondary equipment being the more frequent cause of accident propagation. There are also indirect effects that are responsible of the accident propagation (e.g. loss of control of the installation due to control room damage or evacuation).

In Europe, the basic guidelines for preventing major accidents are stipulated in the Seveso III directive [11]; the article 8 specifies the term domino effect to denote the existence of establishments or groups of establishments where the likelihood and the possibility or consequences of a major accident may be increased because of the location and the proximity of such establishments, and their inventories of dangerous substances.

In terms of the regulations of the Committee on Control of Major Hazards HSE-COMAH, [33], defines a domino event as a loss of containment incident on a major hazard installation which has resulted either directly or indirectly from a loss of containment incident at an adjacent or nearby major hazard installation. More recently several authors have explored patterns of domino effect generation, Hou et al. [163], define as primary contributors the existence of errors in human actuations, equipment failure, management, environmental and external - malicious acts. Others perform an exhaustive search of patterns, [164], or analyze the propagation thresholds [165]. Fire is a major primary event in domino effects [166,167] and according to Khakzad et al. [168] a domino effect, also known as cascading event, is a sequence of events where an initial fire or explosion (primary event) causes damage to neighboring equipment or units and triggers other fires or explosions (secondary events), with overall consequences more severe than those of the primary event, [169]. The propagation (escalation) of a primary event to secondary events occurs by means of physical phenomena such as heat radiation, blast wave, or fragment projection. These are termed escalation vectors (e.g., magnitude of a tank fire's heat radiation) and the estimation of escalation probabilities (e.g., escalation probability of an atmospheric tank exposed to certain heat radiation) that some authors try to define using Bayesian networks [168].

From the previous analysis of the technical literature shows that all the accidental sequences where a relevant domino effect took place have the following common features:

1. A primary event;

2. The propagation, due to the physical effects of the primary event, that results in the damage of at least one secondary equipment item;
3. As consequence one or more secondary events appear (i.e. fire, explosion and toxic dispersion), involving the damaged equipment items.

Darbra, Palacios and Casal, [170], stated that the domino effect is an important aspect in risk analysis, as knowledge of the main hazards and features that can be used to identify additional safety measures, such as the minimum safe distances between certain types of equipment considering the domino effect the action of a primary event that propagates to nearby equipment, triggering one or more secondary events resulting in overall consequences more severe than those of the primary event.

In general the most analyzed scenario is the threshold and propagation in a tank farm, but domino effect can appear in other scenarios not only placed in process industries also in manufacturing environments as a consequence of the own activity that due to changes and modifications in the production process or in the maintenance performance, it may be located in spaces where flammable and toxic products are being used.

Several models were developed for the assessment of domino effects in industrial or manufacturing plants [171]: domino effect generated by fire and overpressure, and domino effect caused by projection of fragments.

### 3.6.1 Domino effect generated by fire and overpressure.

#### a.- First models

The more simple approach proposed for the assessment of damage to equipment caused by fires and explosions is to consider a threshold value for damage of 36 kPa. The probability of damage is zero if the physical effect is lower than the threshold value, and it is valued one if the physical effect is higher than the threshold value.

A damage probability function can be defined based on the distance from the center of the explosion:

$$F_d = \left(1 - \frac{r}{r_{th}}\right)^2 \quad ( 8 )$$

Where  $F_d$  is the damage probability,  $r$  is the distance from explosion center (m) and  $r_{th}$  is the distance from explosion center at which a threshold value of static overpressure is reached (36 kPa).

A quantitative study has been made describing possible approaches for quantifying the consequences of domino effects resulting from events giving rise to thermal radiation and from pressure equipment fragmentation, [172,173]. But a simplified model [174], assesses the damage probability of process equipment, caused by a blast overpressure defining a probit function to relate equipment damage to the peak static overpressure:

$$Y = a + b \cdot \ln(P^0) \quad (9)$$

Where  $Y$  is the probit function for equipment damage,  $P^0$  is the peak static overpressure (Pa),  $a$  and  $b$  are the probit coefficients ( $a = -23.8$  and  $b = 2.92$ ). See Figure 20.

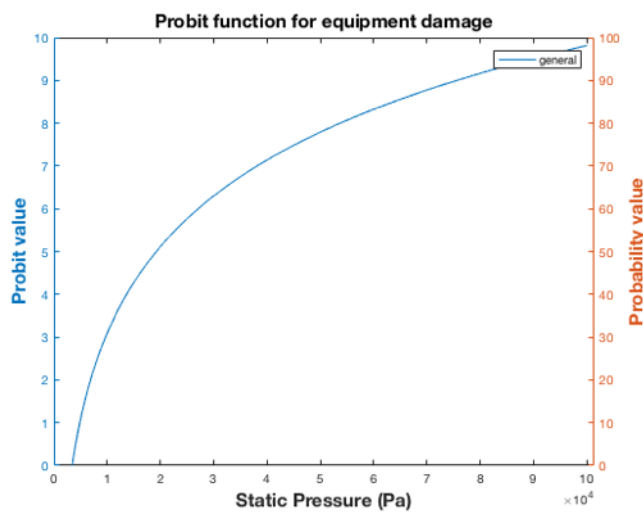


Figure 20.- Probability function for equipment damage in function of the static pressure [174].

#### b.- Advanced models

The probit approach was extended taking into account four categories of industrial equipments (atmospheric vessels, pressurized vessels, elongated vessels, and small equipments) [174]. The probit coefficients for overpressure damage probabilities for four equipment categories are represented in Table 14 and Figure 21.

Equipment category	a	B	Threshold
Atmospheric	-18.96	+2.44	22 kPa
Pressurized	-42.44	+4.33	16 kPa
Elongated	-28.07	+3.16	31 kPa
Small	-17.79	+2.18	37 kPa

Table 14.- Probit coefficients and probability distribution for damage probability in four categories of industrial equipment. (Values in kPa). Adapted from [174].

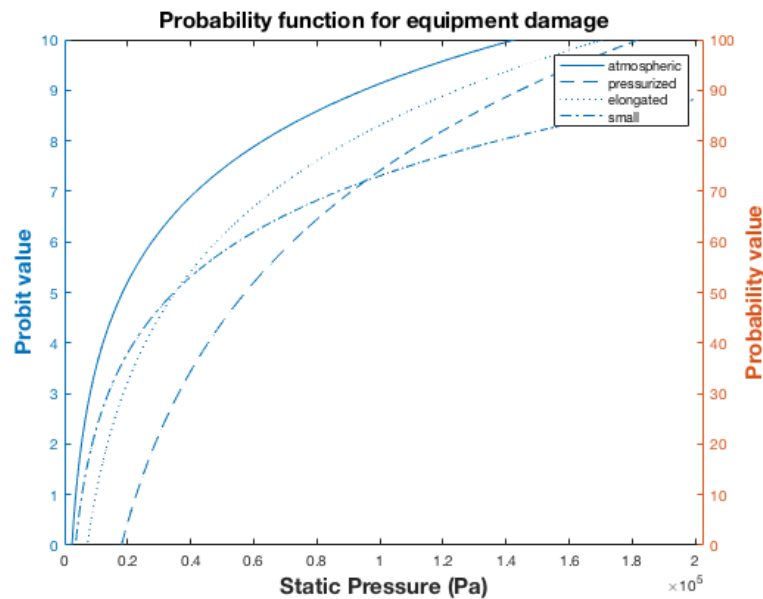


Figure 21.- Probit and probability function for equipment damage in function of the static pressure for atmospheric, pressurized, elongated and small equipment. Adapted from [174].

To improve these models, specific thresholds for domino effects were obtained for the different escalation vectors, taking into account the characteristics of different categories of industrial equipments.

Also the same probit model is applied to estimate the time to failure (ttf) of industrial equipments exposed to fire, the Table 15 presents the thresholds and probit models for two equipment categories.

Equipment category	Threshold	Correlation
Atmospheric	15 kW/m <sup>2</sup> t <sub>≥</sub> 10 min	Y=12.54-1.847×ln(ttf) ln(ttf)=-1.128×ln(I)-2.667×10 <sup>-5</sup> V+9.887
Pressurized	50 kW t <sub>≥</sub> 10 min	Y=12.54-1.847×ln(ttf) ln(ttf)=-0.947×ln(I)+8.835V <sup>0.032</sup>

Table 15.- Probit coefficients for time to failure (ttf) estimation due to fire. Y: probit function; ttf time to failure (s); V vessel volume (m<sup>3</sup>); I amount of heat radiation received (kW/m<sup>2</sup>). Adapted from [174]

### 3.6.2 Domino effect generated by projection of fragments.

In industrial plants, the projections generated by a primary explosion produced in tanks or equipment containing highly pressurized gas or liquids, with high internal energy, in

general lead to catastrophic failures due to the successive explosions that are generated by the projection of the fragments, creating secondary accidents, even tertiary, ... until the process stops. Two main accident scenarios are responsible for primary accidents resulting in the projection of fragments [171] the internal explosions due to confined deflagrations, and the Boiling Liquid Expanding Vapor Explosion (BLEVE).

Based on previous works, Djelosevic and Tepic and Sun et al. [172,173] developed a methodology to calculate the trajectories of fragments resulting from bursting spherical and cylindrical vessels containing gas at high pressure using the analytical solutions of the motion equations. Also, Nguyen et al. [175] proposed a probabilistic approach on the same subject; requiring three main steps:

1. Probabilistic modeling of the probability of occurrence of the primary explosion, where the fragments number, mass, velocity, departure angles, the geometric shape, dimensions and construction material properties are described with probability distributions.
2. Probabilistic modeling of the target where the number of projectiles, speed, angles, energy at the impact, construction materials, dimensions of the affected targets, and depths of penetration are described with probability distributions.
3. Risk assessment for the second scenario explosion (domino effect).

The application of momentum and energy equations for determining the effect caused by projectiles is outside the scope of this thesis.

### 3.6.3. General SRC methodology applied to scenarios of domino effect.

The explosion in a industrial process is preceded by the hydrocarbons release or gas release due to a loss of containment event (such as the collapse of a tank, a hole in a pipe, etc.) [176,177]. In quantitative risk analysis (QRA), the initiating events leading to loss of containment are normally described using generic hypotheses, as the described in the CPR 18E guidelines of the Loss of Containment (LOC) [45,178] and the Handbook of Scenarios for Assessing Major Chemical Accident Risks [179].

In Annex E, [45,174] are collected the Tables of LOC probabilities to apply, in general processes, transport and storage equipment, to assess the LOC in a process or manufacturing plant; with values considering the worst case.

### 3.6.4. General bow-tie.

A review of some accidents involving the domino effect, as the case of France (1966), where eleven storage tanks were destroyed, covering a distance of 800 meters around the site; the Mexico accident (1984) involving a storage and distribution facility for Liquefied Petroleum Gas (LPG) from a pressure excess involving flash and vapor cloud explosion (VCE) together with pool fire explosions and affecting 15 tanks of 16,000 m<sup>3</sup>; the case of India (1997) where flash, VCE and pool fire explosions affected the storage tanks at the refinery of the Hindustan Petroleum Corporation; in Louisiana (2015) with a flash explosion and pool fire of a heat exchanger in a olefins plant; also in Louisiana (2017) the explosion of volatile liquids tank in a pulp mill plant after welding maintenance works in the pipe system. All these accidents show that the production of flash-fire, vapor cloud explosion (VCE), jet fire, all alone or together with the pool fire are involved in the LOC event [177].

To model this evolution generic event trees have been proposed depending of the volatility [180]. In all cases, the initiation is dependent on the state and the amount of component released its flammability and the formation of vapor clouds. The presence of sources of ignition around the equipment together with the existence of safety barriers modifies the probability of having a direct ignition. After accidental release, if the initial ignition has not occurred, delayed ignition may occur depending on the formation of vapor clouds and the presence of ignition sources. The acceleration of the flame is produced by igniting the flammable mixture of the vapor, gas, or aerosol cloud, producing overpressure, [181,182].

In this thesis two generic event trees are defined for vapor-gas and liquid worst cases.

a.- Vapor - Gas. With preventive and mitigative safety barriers on Tables 16 and 17.

Preventive Safety barrier (IS <sub>n</sub> )		Attributes-Failures
Shutdown	(IS <sub>1</sub> )	Shutdown failure (SHUTF)
Sprinkler	(IS <sub>2</sub> )	Local Sprinkler failure (SPKF)
Emergency level control	(IS <sub>3</sub> )	Level control failure (ELCF)
Emergency pressure control	(IS <sub>4</sub> )	Pressure control failure (EPCF)
Corrosion monitoring	(IS <sub>5</sub> )	Deficiency - failure (CMDF)
Welding	(IS <sub>6</sub> )	Control failure (WCF)

Table 16.- Preventive safety barriers (IS<sub>n</sub>) attributes-failures in domino effect from vapor-gas.

Mitigative Safety barrier (SF <sub>n</sub> )		p <sub>f</sub>	Attributes-Failures
Complete Shutdown (avoid/delay ignition)	(SF <sub>1</sub> )	0.9	Emergency Shutdown failure (EMSHUTF) General Sprinkler/Spray failure (GSPKYF)
Palliative 1 (avoid/delay local flame/acceleration)	(SF <sub>2</sub> )	0.95	Local High pressure water curtain failure (LHPWCF) Local foam curtain failure (LFCF)
Palliative 2 (avoid/delay general flame/acceleration)	(SF <sub>3</sub> )	0.95	Emergency High pressure water curtain failure (EMHPWCF) General foam curtain failure (GFCF)

Table 17.- Reactive safety barriers (SF<sub>n</sub>) attributes-failures for vapor-gas domino effect and p<sub>f</sub> probability of safety barrier failure.

Corresponding to the bow-tie of the following Figure 22:

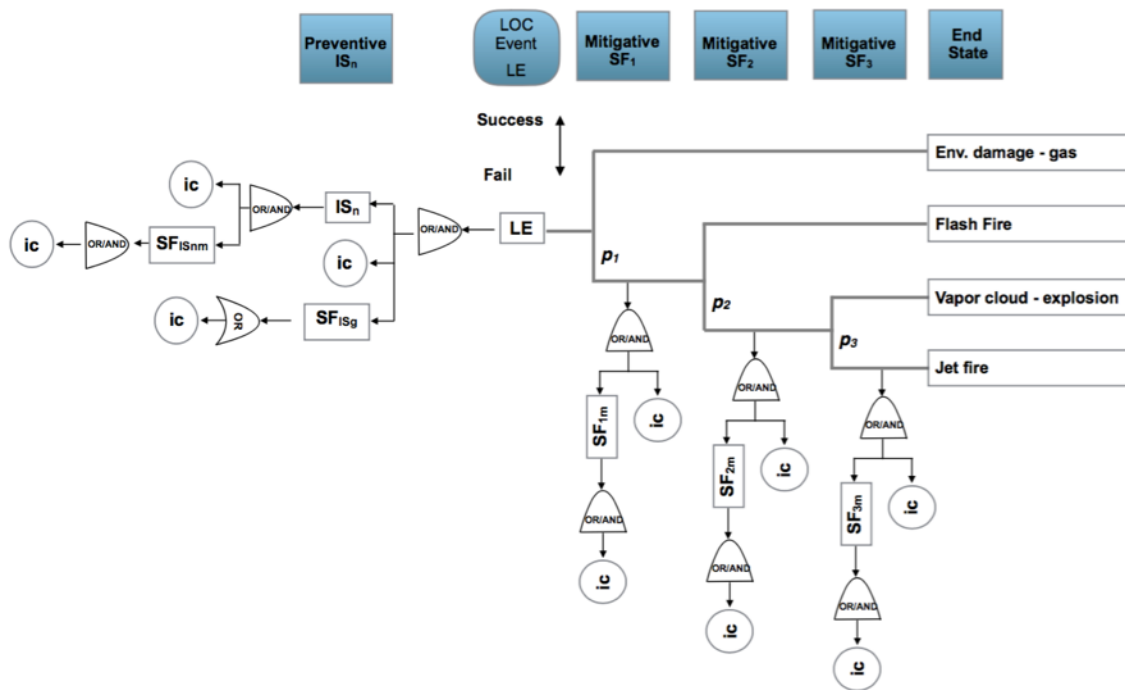


Figure 22.- General bow-tie in domino effect from vapor-gas loss of containment (LOC). Safety barriers: IS<sub>n</sub>: preventive barrier (n); SF<sub>ISnm</sub>: preventive or proactive barrier (n) sub-function (m); SF<sub>ISg</sub> general parallel sub-function; SF<sub>n</sub>: mitigative barrier (n); SF<sub>nm</sub>: mitigative barrier (n) sub-function (m); ic: initiating causes (ba, ha, pot); p<sub>1,2,3</sub>: probability of fail of the mitigative safety barriers.



b.- Liquid. With preventive and mitigative safety barriers on Tables 18 and 19. Corresponding to the bow-tie of Figure 23.

Preventive Safety barrier (IS <sub>n</sub> )		Attributes-Failures
Shutdown	(IS <sub>1</sub> )	Shutdown failure (SHUTF)
Containment	(IS <sub>2</sub> )	Liquid containment failure (LCMF) Liquid collector (LCLLF)
Sprinkler	(IS <sub>3</sub> )	Local Sprinkler failure (SPKF)
Emergency level control	(IS <sub>4</sub> )	Level control failure (ELCF)
Emergency pressure control	(IS <sub>5</sub> )	Pressure control failure (EPCF)
Corrosion monitoring	(IS <sub>6</sub> )	Deficiency - failure (CMDf)
Welding	(IS <sub>7</sub> )	Control failure (WCF)

Table 18.- Preventive safety barriers (IS<sub>n</sub>) attributes-failures in liquid domino effect.

Mitigative Safety barrier (SF <sub>n</sub> )		p <sub>f</sub>	Attributes-Failures
Complete Shutdown (avoid/delay ignition)	(SF <sub>1</sub> )	0.95	Emergency Shutdown failure (EMSHUTF) General Sprinkler/Spray failure (SPKYF) General liquid containment failure (GLCF)
Palliative 1 (avoid/delay local flame/ acceleration)	(SF <sub>2</sub> )	0.85	High pressure water curtain failure (HPWCF) Local foam curtain failure (LFCF)
Palliative 2 (avoid/delay general flame/ acceleration)	(SF <sub>3</sub> )	0.8	Emergency High pressure water curtain failure (EMHPWCF) General foam curtain failure (GFCF)

Table 19.- Reactive safety barriers (SF<sub>n</sub>) attributes-failures for liquid domino effect and p<sub>f</sub> the probability of safety barrier failure.

The probability of the various process equipment, mobile installations, devices or tanks to suffer a domino effect by being located at a certain distance from the element susceptible to suffering LOC can be modeled with the following relationship:

$$\text{Prob}DE_i = \text{ProbDamage} \cdot \left(1 - \frac{r_i}{r_{th,i}}\right)^2 \quad (10)$$

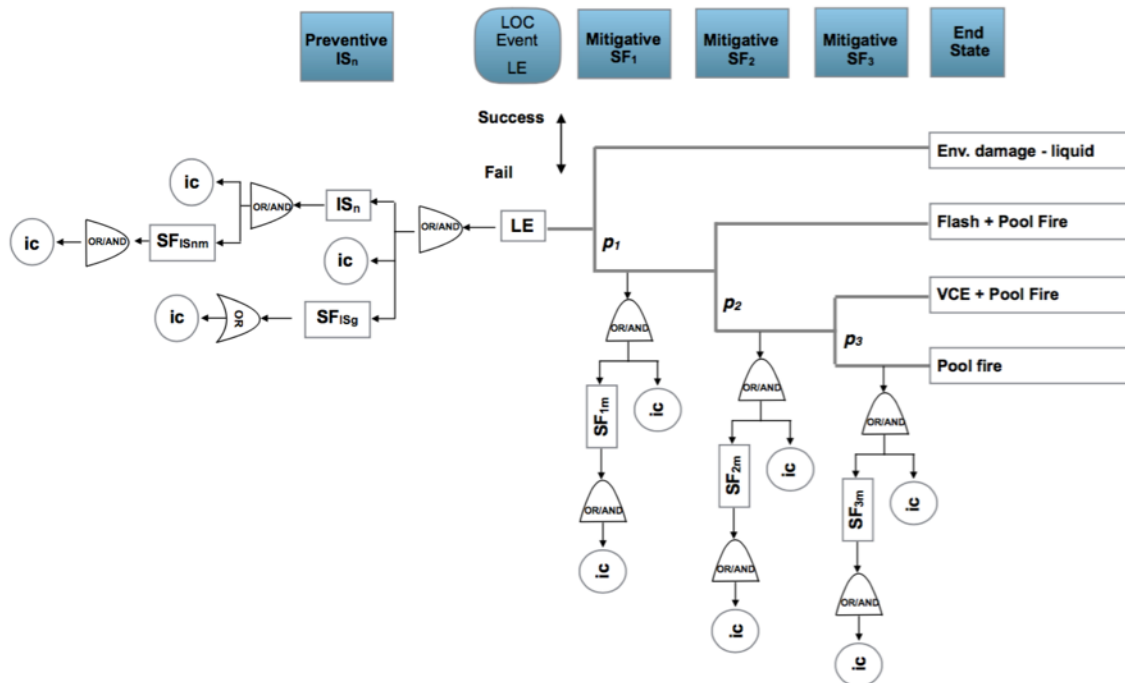


Figure 23.- General bow-tie in domino effect from liquid loss of containment (LOC). Safety barriers:  $IS_n$ : preventive barrier (n);  $SF_{ISnm}$ : preventive or proactive barrier (n) sub-function (m);  $SF_{ISg}$  general parallel sub-function;  $SF_n$ : mitigative barrier (n);  $SF_{nm}$ : mitigative barrier (n) sub-function (m); ic: initiating causes (ba, ha, pot);  $p_{1,2,3}$ : probability of fail of the mitigative safety barriers.

Where;

$ProbDE_i$  : for the equipment i, is the probability for having a Domino effect.

$ProbDamage$ : is the probability for the initiation of a LOC episode.

$\left(1 - \frac{r_i}{r_{th,i}}\right)$  : for the equipment i, is the probability for to get affected by proximity being

r the distance (m) from explosion center and  $r_{th}$  is the distance from the explosion center at which a threshold value of static overpressure is estimated to be reached (36 kPa) for light damage of buildings, [171].

As an alternative it is possible to apply, [160,183].

$$\text{Prob}DE_i = \text{ProbDamage} \cdot \left(1 - \frac{r_i}{r_{si}}\right)^2 \quad (11)$$

Where;

$r_{si}$  is the radius of zone damage, based on the expression:

$$r_{si} = K_1 G^{0.33} \left(1 + \left(\frac{3180}{G}\right)^2\right)^{-0.17} \quad (12)$$

With:

G equivalent charge of TNT weight (kg). The detonation of 1 kg of TNT with specific heat of explosion of 5000 kJ/kg and packed in 1 m tube, results in a power of 30·106 kW, that is the energy application in a time of 0.0002 s. And  $K_1$  coefficient according to the damage level, Table 20, being:

Damage level	Value of $K_1$
I: complete demolition	3.8
II: buildings, walls and roofs destroyed	9.6
III: moderate damages	28
IV: reparable damages	55

Table 20.- Values for  $K_1$  according to the damage levels.

The main objective of the application of the SRC methodology is to avoid the LOC event and mitigate if it is produced through the analysis of the initiating causes (ic) and the safety barriers.

In parallel because the possible scenario is depending of the geometry and layout of the elements of equipment, and affected by the toxicity, operation status and flammability conforming the affectation of the main product; and by the weather in terms of storm days, precipitation value and wind over 55 km/h, conforming the influence of the weather, and by the installation behavior in terms of values of main critical process variables (flow, pressure, temperature, speed, rate of quantity, time,...).

Taking into account these affectations, the general scheme of obtaining the final risk damage due to a LOC event, with a possible final state that depends on whether gas or liquid vapor has been emitted and at the distance  $r$  is;

1. Scenario consideration for gas-vapor or liquid.
2. Loss of containment estimation (LOC) and their corresponding end state according to the selected bow-tie.
3. Affection to elements  $i$  placed at distance  $r_i$  using equations (11) and (12). Obtaining a  $P_{LOC,distance}$  equal to the  $ProbDE_i$  probability.
4. Affection due to the Toxicity, Operation and Flammability (TOF) dimensions for which, in percentage, its relative importance should be assessed in the considered scenario. A ponderation in concordance with their values, considering the worst case for multi-components is performed. The result is a modified  $P_{LOC,distance,TOF}$  probability.
5. Affection due to the estimated storm days, precipitation value (mm) and wind speed over 55 km/h (SPW55) in days. Obtaining a  $f_{Weather,SPW55}$  factor.
6. Affection due to the installation behavior (IB) according to the main process variables affecting risk. Obtaining a  $f_{Behavior,IB}$  factor.

In step 4 a linear multi criteria analysis is performed. And for steps 5 and 6 the fuzzy logic is applied [184,185,186], (Annex F) in which an output factor that modifies the probability of risk is obtained.

And finally obtaining the damage probability value for the selected final state and distance.

$$P_{damage,distance} = P_{LOC,distance,TOF} \cdot f_{Weather,SPW55} \cdot f_{Behavior,IB} \quad (13)$$

### 3.7. Theoretical considerations.

In step 6 from the general SRC methodology and in all the previous cases: for industrial processes, occupational, deviations in cost and time, and in LOC events producing domino effect, when analyzing the initiating causes (ic) and the safety barriers the proposal for collecting the observations embracing the general process as a worst case obtaining a representative probability of risk for the overall installation has sense because observing the general bow-tie, of Figure 11, on the fault tree side, when the initiating causes (ic) affect the preventive safety barriers ( $IS_n$ ) and the general parallel sub-function ( $SF_{ISg}$ ), the probability of risk for every one is a function of;

If there are  $n$  preventive safety barriers; the risk for a safety barrier  $i$  is

$$P(IS_i) = f(ic, SF_{ISim}) = P(ic_i) + P(SF_{ISim}) \quad (14)$$

For the general parallel sub-function system;

$$P(SF_{ISg}) = f(ic_{sg}, SF_{ISg}) = P(ic_{sg}) + P(SF_{ISg}) \quad (15)$$

With  $n$  generic number of preventive barriers and  $m$  generic number of sub-functions.

With the hypothesis of independency of the safety barriers risk probabilities, the total probability for producing an accident event is;

$$P(AE) = \sum_{i=1}^n P(IS_i) + P(ic) + P(SF_{ISg}) \quad (16)$$

Showing that  $P(AE) = f(ic, IS_n, SF_{ISg})$

Observing the bow-tie from the event tree side, when an accident event has been produced the mitigative barriers take action leading to an end state.

For a mitigative safety  $j$  barrier;

$$P(SF_j) = f(ic, SF_{jn}) = P(ic_j) + P(SF_{jn}) \quad (14)$$

Being  $n$  the number of sub-functions of the mitigative  $j$  safety barrier.

If there are four end states, the probability of risk for the first end state is;

$$P(end\_state_1) = P(AE) \cdot (1 - P(SF_1))$$

For the second end state;

$$P(end\_state_2) = P(AE) \cdot P(SF_1) \cdot (1 - P(SF_2))$$

For the third end state;

$$P(end\_state_3) = P(AE) \cdot P(SF_1) \cdot P(SF_2) \cdot (1 - P(SF_3))$$

For the fourth end state;

$$P(end\_state_4) = P(AE) \cdot P(SF_1) \cdot P(SF_2) \cdot P(SF_3)$$

For obtaining an end state  $k$  ; if there are  $o$  end states is;

$$P(\text{end\_state}_k) = P(AE) \cdot \prod_1^{o-1} P(SF_k) \cdot (1 - P(SF_o))$$

And for the last state;

$$P(\text{end\_state}_o) = P(AE) \cdot \prod_1^o P(SF_k)$$

For obtaining any end state, that is the worst case situation for the overall installation is;

$$\begin{aligned} P(\text{overall\_risk}) &= \sum_1^{o-1} P(\text{end\_state}_k) + P(\text{end\_state}_o) = \\ &= \sum_1^{o-1} [P(AE) \cdot \prod_1^{o-1} P(SF_k) \cdot (1 - P(SF_o))] + P(AE) \cdot \prod_1^o P(SF_k) = \\ &= \sum_1^{o-1} \left[ \left( \sum_{i=1}^n P(IS_i) + P(ic) + P(SF_{ISg}) \right) \cdot \prod_1^{o-1} P(SF_k) \cdot (1 - P(SF_o)) \right] + \left( \sum_{i=1}^n P(IS_i) + P(ic) + P(SF_{ISg}) \right) \cdot \prod_1^o P(SF_k) \end{aligned}$$

Showing that;

$$P(\text{overall\_risk}) = f(ic, IS_n, SF_{ISg}, SF_n) \quad (15)$$

Then, the general realization of the observations, in an examination of the most unfavorable situation, which includes the initial causes (ic) and the failures in the preventive and mitigative safety barriers; it allows to obtain a representative probability of general risk of the installation or process being a practical treatment of the previous expression (15).

In addition, if necessary, the analysis can be performed only for either the fault tree or the event tree in order to obtain the probability failures of the safety barriers. Or it can focus on specific barriers in order to analyze the probability of risk and allow a sensitivity analysis with respect to obtaining which incidents affect their response.

# Chapter 4

## Results.

Several cases of the process and manufacturing environment that cover the SRC models applied in industrial, occupational, time-cost and domino-effect environments are presented below.

### 4.1. Industrial processes analysis. Separation unit

A generic separation unit placed in a oil and gas offshore extraction platform is used as example see Figure 24.

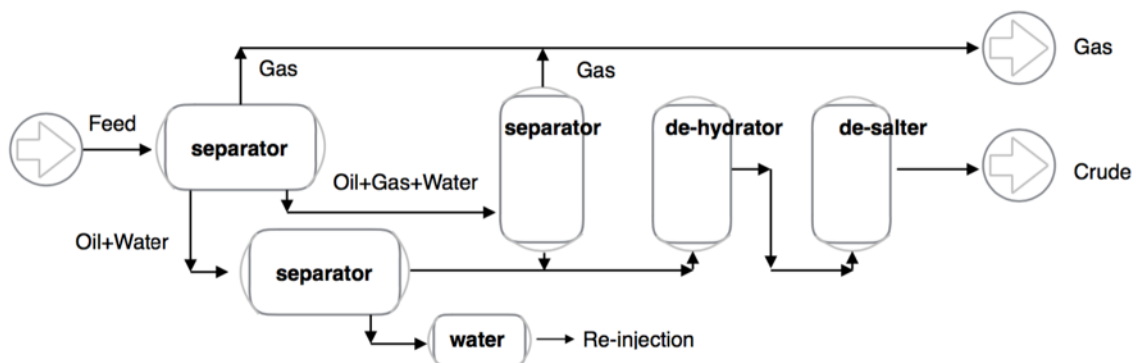


Figure 24.- Generic scheme for an oil/gas offshore separation unit.

The main function of the separation unit is to extract the water from the oil-gas mix obtained in a well production. Further separations of the oil and gas currents are performed in separator-flash units from which the obtained water current is re-injected, the gas current is exported to a compression and dehydration gas plant and the oil current is conditioned for export. The bow-tie for the separation unit is presented on Figure 25, and the event to avoid is the overpressure (OPE). The mitigative safety barriers have the function to protect the installation and reduce the risk of have a total failure with break and possible emission. Every fail in the response of a safety reactive barrier concatenates the response of the following. From the success or fail actuation of the barriers and their concatenation four end states are obtained.

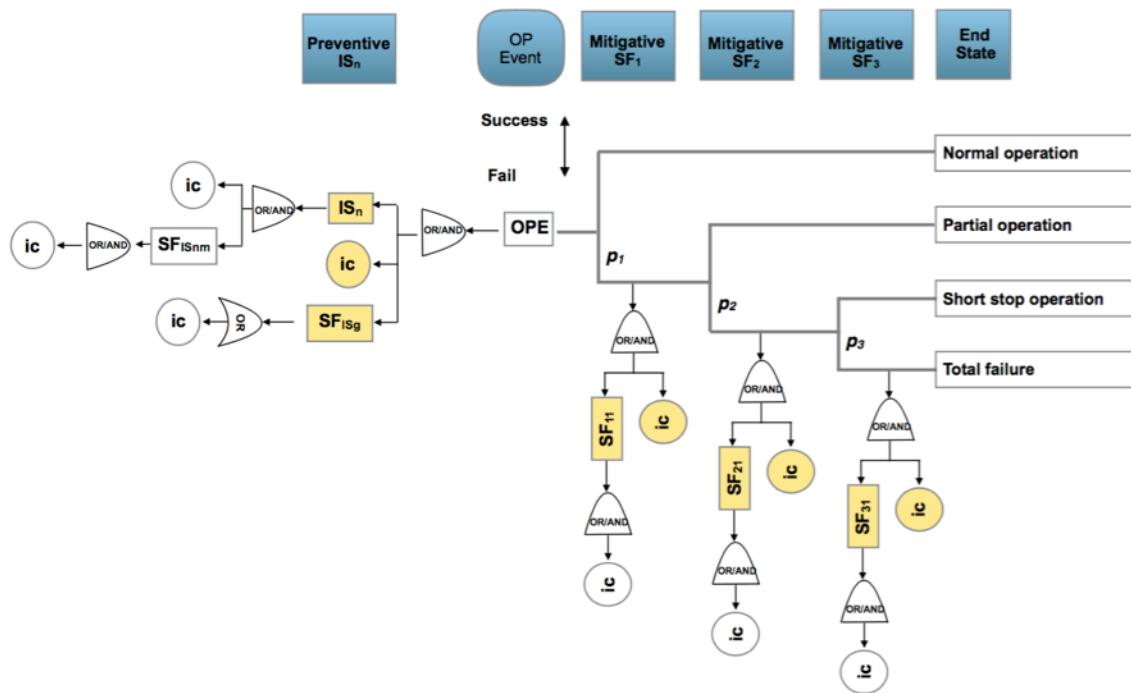


Figure 25.- Bow-tie for the generic oil/gas offshore separation unit.

The preventive safety barriers ( $IS_n$ ) are defined in the following Table 21.

Preventive Safety barrier ( $IS_n$ )	Attributes-Failures
Local shutdown-recycling	( $IS_1$ ) Shutdown failure (SHUTF) ic: ba power supply; ha:safety supervision fail (SSU), job supervision fail (JSU); pot
Emergency cooling system	( $IS_2$ ) Cooling system failure (COOLSF) ic: ba pumping station; ha:safety supervision fail (SSU), job supervision fail (JSU) ; pot
Emergency level control	( $IS_3$ ) Level control failure (ELCF) ic: ba sensor fail; ha: safety supervision fail (SSU), job supervision fail (JSU); pot
Emergency pressure control	( $IS_4$ ) Pressure control failure (EPCF) ic: ba sensor fail; ha: safety supervision fail (SSU), job supervision fail (JSU); pot
Control system	( $SF_{ISg}$ ) System control with deficiency - failure (SCDF) ic: ba: field equipment; ha: safety or job supervision fail (JSU)(SSU); pot

Table 21.- Preventive safety barriers, for the separation unit.

The most critical safety barriers with the most important initiating causes (ic) affecting them are described.

The mitigative safety barriers ( $SF_n$ ) are defined in the following Table 22.



Mitigative Safety barrier (SF <sub>n</sub> )		Attributes-Failures
Manifold - pressure correction	(SF <sub>1</sub> )	Actuation through manifold failure (AMF) ic: ba failure in valves; ha: safety or job supervision fail (SSU) (JSU); pot
Recycling	(SF <sub>2</sub> )	Pumping failure (PRF) ic: ba pump-valve failure; ha: safety or job supervision fail (SSU) (JSU); pot
Shutdown	(SF <sub>3</sub> )	Emergency power failure (EMPF) ic: connection-actuator failure; ha: safety or job supervision fail (SSU) (JSU); pot

Table 22.- Mitigative safety barriers, for the separation unit.

The analysis covers the general process and the bow-tie presents four final states: normal operation, partial operation, short stop operation and total failure. If an overpressure event (OPE) is generated the mitigative safety barriers (SF<sub>1</sub> SF<sub>2</sub> SF<sub>3</sub>) are activated. The final states represented are bounded at one end by a normal operation, if the first mitigative safety barrier (SF<sub>1</sub>) acts correctly, in case of failure the second barrier (SF<sub>2</sub>) acts ending with a partial operation if it works correctly, or in case of failure the third barrier (SF<sub>3</sub>) acts leading to a short shutdown in case of correct actuation, or on the contrary there a total failure of the process. The sub-functions for preventive (SF<sub>IS11</sub>, SF<sub>IS21</sub>, SF<sub>IS31</sub>, SF<sub>IS41</sub>, SF<sub>IS51</sub>) and mitigative (SF<sub>11</sub> SF<sub>21</sub> SF<sub>31</sub>) belong to the main function of each of the preventive and mitigative safety barriers and correspond to the functional components of each of them, and that include equipment, actuators, automatisms, procedures, alarms and active or passive protections. The probabilities of failure for the three mitigative safety barriers  $p_{1,2,3}$  are not known a priori.

The analysis is carried out at the first level, highlighted in yellow on the graph. The observations are made in the worst case taking the process in general, collecting data on the preventive safety barriers, the general safety barrier (SF<sub>ISg</sub>) and their corresponding initiating causes (ic); and on the other hand, from each of the functions and sub-functions of the mitigative barriers and their corresponding initiating causes, being the SF<sub>1</sub>-SF<sub>11</sub> and its (ic) the group for the general manifold action and pressure correction with the general valve actuators, positioners and passive and active protections; the SF<sub>2</sub>-SF<sub>21</sub> and its (ic's) the group of the recycling system with their general pump elements, actuators and passive and active protections, and the SF<sub>3</sub>-SF<sub>31</sub> and its (ic's) which group the general emergency shutdown and general actuators. The observations are done in a 10 time intervals in a day, covering all shifts, Table 23 shows three observed causes in intervals 4, 7 and 8.

Interval	1	2	3	4	5	6	7	8	9	10
Observations	-	-	-	1 (ba)	-	-	1 (ha)	1 (FTS)		

1 ba: valve failure affecting manifold SF<sub>11</sub>; 1 ha: fail in safety supervision (SSU) affecting IS<sub>2</sub> cooling; 1 pot failed test affecting SF<sub>3</sub> general emergency shutdown.

Table 23.- Initiating causes (ic) and safety barrier failures in 10 intervals.

Collecting observations and using equation (1)

$$f(p/Data) \propto g(Data/p) \cdot f(p) \quad (1)$$

Where  $p$  is the statistical parameter,  $f(p)$  is the prior statistical distribution for the parameter  $p$ ;  $g(Data/p)$  is corresponding to the observed precursor data and  $g(p/Data)$  is the posterior statistical distribution.

It is expected that can follow a Poisson, an exponential or a Weibull distributions, being the  $g(Data/p)$  in equation (1). The statistical parameter  $p$  is corresponding to the rate  $\lambda$  or frequency of events and the prior  $f(p)$  can be defined as a gamma or normal distributions.

#### 4.1.1. Poisson-gamma model.

With a Poisson-Gamma model the expression (1) with parameter  $p = \lambda$  is, (Annex A).

$$f(\lambda / Data) \propto g(Data / \lambda) f(\lambda) = \frac{\beta^\alpha \lambda^{\alpha-1} e^{-\beta\lambda}}{\Gamma(\alpha)} \cdot \prod_{i=1}^n \frac{e^{-\lambda} \lambda^{y_i}}{y_i!} \propto \lambda^{(\alpha+n\bar{y})-1} e^{-\lambda(\beta+n)} \quad (14)$$

Being the posterior a gamma distribution;

$$f(\lambda / Data) \propto \lambda^{(\alpha+n\bar{y})-1} e^{-\lambda(\beta+n)} \propto \lambda^{(\alpha+s)-1} e^{-\lambda(\beta+n_i)} \quad (15)$$

Where;

$$s = \sum Data_i = \sum y_i = n_i \bar{y} \quad (16)$$

Being  $n_i$  number of interval and  $s$  the sum of initiating causes (ic's) and safety barriers incidences as data  $y_i$  in the corresponding time interval  $i$ . The values  $\alpha$  and  $\beta$  are the parameters of the gamma distribution.

Modes recurrent and direct with mean prior and posterior will be applied.

a.- Recurrent method with mean prior.

A recurrent method with mean prior and equal to a desired value as a target, is applied. In this case the target is for have zero accidents, then the parameters of the gamma prior are  $\alpha=\beta=0.001$ . Working with  $\pm 1\sigma_{\text{post}}$  the posterior values are, Table 24;

Time interval	Prior		Posterior				Observed	LCL	MEAN	UCL
	alfa	beta	alfa	beta	lambda	sigma	lambda			
0	0.001	0.001	-	-	0	0	0	-	0	-
1	0.001	0.001	0.001	1	0	0.03	0	0	0	0.03
2	0.001	1	0.001	2	0	0.02	0	0	0	0.02
3	0.001	2	0.001	3	0	0.01	0	0	0	0.01
4	0.001	3	1	4	0.25	0.25	0.25	0	0	0.25
5	1	4	1	5	0.20	0.20	0	0	0	0.20
6	1	5	1	6	0.17	0.17	0	0	0	0.17
7	1	6	2	7	0.29	0.20	0.33	0	0	0.20
8	2	7	3	8	0.38	0.22	1	0	0	0.22
9	3	8	3	9	0.33	0.19	0	0	0	0.19
10	3	9	3	10	0.30	0.17	0	0	0	0.17

Table 24.- Separation unit. Poisson-gamma model. Observations using a recurrent method with mean prior. Out-of-limits highlighted blue.

With the following comments in every interval.

Interval 1.- With zero incidences, is,  $y_i=[0]$  and posterior density for  $\lambda$ ;  $gam(\alpha+s,\beta+Nt) = gam(0.001,1)$  with  $\lambda_{\text{post}}=0$  and  $\sigma_{\text{post}}=0.03$ .

Interval 2.- With zero incidences, is,  $y_i=[0]$  and posterior density for  $\lambda$ ;  $gam(\alpha+s,\beta+Nt) = gam(0.001,2)$  with  $\lambda_{\text{post}}=0$  and  $\sigma_{\text{post}}=0.02$ .

Interval 3.- With zero incidences, is,  $y_i=[0]$  and posterior density for  $\lambda$ ;  $gam(\alpha+s,\beta+Nt) = gam(0.001,3)$  with  $\lambda_{\text{post}}=0$  and  $\sigma_{\text{post}}=0.01$ .

Interval 4.- With one incidence in one valve affecting the SF<sub>11</sub> safety barrier sub-function, is in this case,  $y_i=[1]$  and posterior density for  $\lambda$ ;  $gam(\alpha+s,\beta+Nt) = gam(1,4)$  with  $\lambda_{\text{post}}=0.25$  and  $\sigma_{\text{post}}=0.25$ . Due to the no memory characteristic of the exponential, Poisson and Weibull distributions, the observed parameter value for 1 incidence in 4

time intervals is 0.25 that is coincident with the posterior. The value is on the upper control limit (UCL).

Interval 5.- With zero incidences, is,  $y_i=[0]$  and posterior density for  $\lambda$ ;  $gam(\alpha+s,\beta+Nt) = gam(1,5)$  with  $\lambda_{post}=0.20$  and  $\sigma_{post}=0.20$ . As a characteristic of the Bayesian inference, the posterior distribution has a soft reduction of parameter  $\lambda$  from 0.25 to 0.20.

Interval 6.- With zero incidences, is,  $y_i=[0]$  posterior density for  $\lambda$ ;  $gam(\alpha+s,\beta+Nt) = gam(1,6)$  with  $\lambda_{post}=0.17$  and  $\sigma_{post}=0.17$ . The Bayesian inference also responds softening the reduction of parameter  $\lambda$  from 0.20 to 0.17.

Interval 7.- With one incidence affecting a safety supervision (SSU) in the IS<sub>2</sub> cooling process, is  $y_i=[1]$  and posterior density for  $\lambda$ ;  $gam(\alpha+s,\beta+Nt) = gam(2,7)$  with  $\lambda_{post}=0.29$  and  $\sigma_{post}=0.20$ . The observed parameter value for 1 incidence in  $7-4=3$  time intervals is 0.33 that is practically coincident with the posterior. And showing an out-of-limits.

Interval 8.- With one incidence affecting a failed test in the emergency shutdown SF3 safety barrier, is,  $y_i=[1]$  and posterior density for  $\lambda$ ;  $gam(\alpha+s,\beta+Nt) = gam(3,8)$  with  $\lambda_{post}=0.38$  and  $\sigma_{post}=0.22$ . The observed parameter value for 1 incidence in  $8-7=1$  time intervals is 1. And also shows and out-of-limits.

Interval 9.- With zero incidences, is,  $y_i=[0]$  and posterior density for  $\lambda$ ;  $gam(\alpha+s,\beta+Nt) = gam(3,9)$  with  $\lambda_{post}=0.33$  and  $\sigma_{post}=0.19$ .

Interval 10.- With zero incidences, is,  $y_i=[0]$  and posterior density for  $\lambda$ ;  $gam(\alpha+s,\beta+Nt) = gam(3,10)$  with  $\lambda_{post}=0.30$  and  $\sigma_{post}=0.17$ .

Charts for observed values in intervals 4 and 7 are presented on Figure 26.

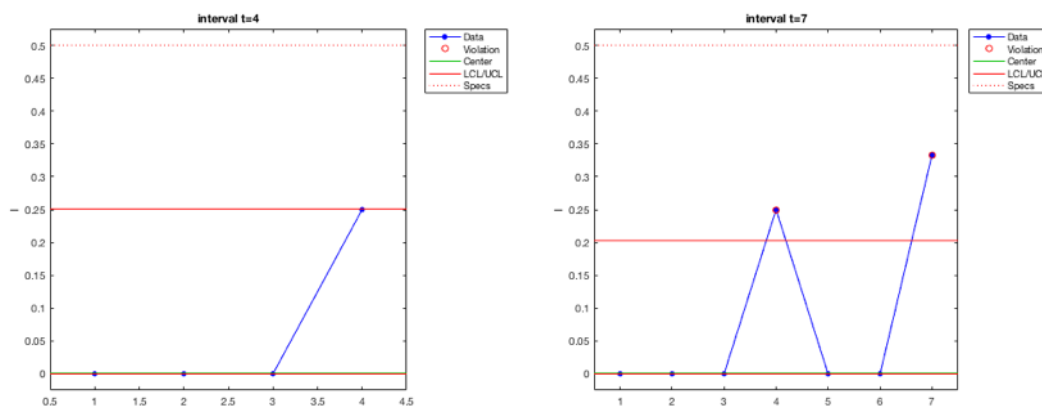


Figure 26.- Poisson-gamma model. Recurrent with mean prior method. Charts based on observed lambda evolution for intervals 4 and 7.

b.- Recurrent method with mean posterior.

A recurrent method with mean posterior is applied. With target for have zero accidents, parameters of the gamma prior  $\alpha=\beta=0.001$  and  $\pm 1\sigma_{post}$ . The posterior values are presented on Table 25; and charts for intervals 4 and 8 in Figure 27.

Time interval	Prior		Posterior				Observed	LCL	MEAN	UCL
	alfa	beta	alfa	beta	lambda	sigma	lambda			
0	0.001	0.001	-	-	0	0	0	-	0	-
1	0.001	0.001	0.001	1	0	0.03	0	0	0	0.03
2	0.001	1	0.001	2	0	0.02	0	0	0	0.02
3	0.001	2	0.001	3	0	0.01	0	0	0	0.01
4	0.001	3	1	4	0.25	0.25	0.25	0	0.25	0.50
5	1	4	1	5	0.20	0.20	0	0	0.20	0.40
6	1	5	1	6	0.17	0.17	0	0	0.17	0.33
7	1	6	2	7	0.29	0.20	0.33	0	0.29	0.49
8	2	7	3	8	0.38	0.22	1	0	0.38	0.59
9	3	8	3	9	0.33	0.19	0	0	0.33	0.53
10	3	9	3	10	0.30	0.17	0	0	0.30	0.47

Table 25.- Separation unit. Poisson-gamma model. Observations using a recurrent method with mean posterior. Out-of-limits highlighted blue.

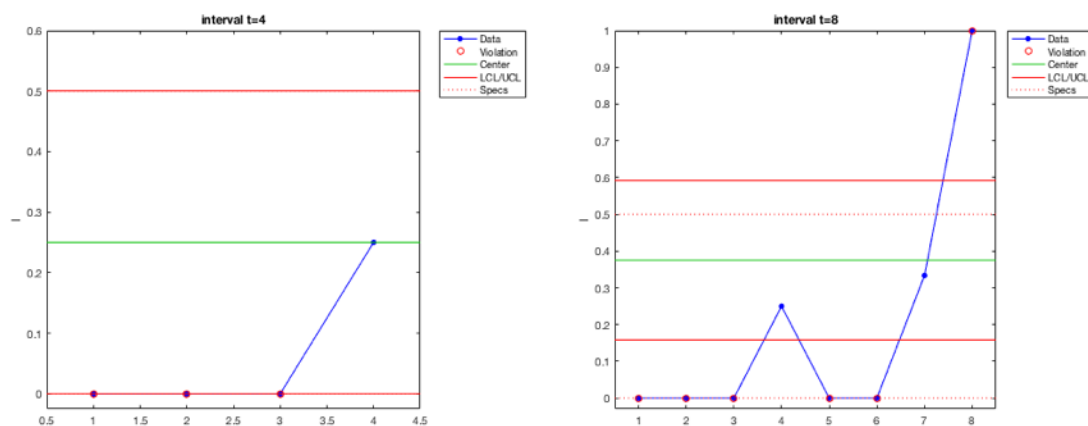


Figure 27.- Poisson-gamma model. Recurrent with mean posterior method. Charts based on observed lambda evolution for intervals 4 and 8.

c.- Direct method with mean prior.

A direct method with mean prior is applied. With the same target for have zero accidents, and gamma prior parameters  $\alpha=\beta=0.001$ . Also working with  $\pm 1\sigma_{\text{post}}$  the posterior values are presented on Table 26; and charts for intervals 7 and 8 in Figure 28.

Time interval	Prior		Posterior				Observed	LCL	MEAN	UCL
	alfa	beta	alfa	beta	lambda	sigma	lambda			
0	0.001	0.001	-	-	0	0	0	-	0	-
1	0.001	0.001	0.001	1	0	0.03	0	0	0	0.03
2	0.001	0.001	0.001	2	0	0.02	0	0	0	0.02
3	0.001	0.001	0.001	3	0	0.01	0	0	0	0.01
4	0.001	0.001	1	4	0.25	0.25	0.25	0	0	0.25
5	0.001	0.001	1	5	0.20	0.20	0	0	0	0.20
6	0.001	0.001	1	6	0.17	0.17	0	0	0	0.17
7	0.001	0.001	2	7	0.29	0.20	0.33	0	0	0.20
8	0.001	0.001	3	8	0.38	0.22	1	0	0	0.22
9	0.001	0.001	3	9	0.33	0.19	0	0	0	0.19
10	0.001	0.001	3	10	0.30	0.17	0	0	0	0.17

Table 26.- Separation unit. Poisson-gamma model. Observations using a direct method with mean prior. Out-of-limits highlighted blue.

With the following comments in every interval.

Interval 1.- With zero incidences, is,  $y_i=[0]$  and posterior density for  $\lambda$ ;  $gam(\alpha+s,\beta+Nt) = gam(0.001,1)$  with  $\lambda_{\text{post}}=0$  and  $\sigma_{\text{post}}=0.03$ .

Interval 2.- With zero incidences, is,  $y_i=[0 0]$  and posterior density for  $\lambda$ ;  $gam(\alpha+s,\beta+Nt) = gam(0.001,2)$  with  $\lambda_{\text{post}}=0$  and  $\sigma_{\text{post}}=0.02$ .

Interval 3.- With zero incidences, is,  $y_i=[0 0 0]$  and posterior density for  $\lambda$ ;  $gam(\alpha+s,\beta+Nt) = gam(0.001,3)$  with  $\lambda_{\text{post}}=0$  and  $\sigma_{\text{post}}=0.01$ .

Interval 4.- With one incidence in one valve affecting the SF<sub>11</sub> safety barrier sub-function, is in this case,  $y_i=[0 0 0 1]$  and posterior density for  $\lambda$ ;  $gam(\alpha+s,\beta+Nt) = gam(1,4)$  with  $\lambda_{\text{post}}=0.25$  and  $\sigma_{\text{post}}=0.25$ . Due to the no memory characteristic of the exponential, Poisson and Weibull distributions, the observed parameter value for 1

incidence in 4 time intervals is 0.25 that is coincident with the posterior. The value is on the upper control limit (UCL).

Interval 5.- With zero incidences, is,  $y_i=[0\ 0\ 0\ 1\ 0]$  and posterior density for  $\lambda$ ;  $gam(\alpha+s, \beta+Nt) = gam(1,5)$  with  $\lambda_{post}=0.20$  and  $\sigma_{post}=0.20$ . The parameter  $\lambda$  has a soft reduction from 0.25 to 0.20.

Interval 6.- With zero incidences, is,  $y_i=[0\ 0\ 0\ 1\ 0\ 0]$  posterior density for  $\lambda$ ;  $gam(\alpha+s, \beta+Nt) = gam(1,6)$  with  $\lambda_{post}=0.17$  and  $\sigma_{post}=0.17$ . The Bayesian inference also responds softening the reduction of parameter  $\lambda$  from 0.20 to 0.17.

Interval 7.- With one incidence affecting a safety supervision (SSU) in the IS<sub>2</sub> cooling process, is  $y_i=[0\ 0\ 0\ 1\ 0\ 0\ 1]$  and posterior density for  $\lambda$ ;  $gam(\alpha+s, \beta+Nt) = gam(2,7)$  with  $\lambda_{post}=0.29$  and  $\sigma_{post}=0.20$ . The observed parameter value for 1 incidence in 7-4=3 time intervals is 0.33 that is practically coincident with the posterior. And showing an out-of-limits.

Interval 8.- With one incidence affecting a failed test in the emergency shutdown SF3 safety barrier, is,  $y_i=[0\ 0\ 0\ 1\ 0\ 0\ 1\ 1]$  and posterior density for  $\lambda$ ;  $gam(\alpha+s, \beta+Nt) = gam(3,8)$  with  $\lambda_{post}=0.38$  and  $\sigma_{post}=0.22$ . The observed parameter value for 1 incidence in 8-7=1 time intervals is 1. And also shows and out-of-limits.

Interval 9.- With zero incidences, is,  $y_i=[0\ 0\ 0\ 1\ 0\ 0\ 1\ 1\ 0]$  and posterior density for  $\lambda$ ;  $gam(\alpha+s, \beta+Nt) = gam(3,9)$  with  $\lambda_{post}=0.33$  and  $\sigma_{post}=0.19$ .

Interval 10.- With zero incidences, is,  $y_i=[0\ 0\ 0\ 1\ 0\ 0\ 1\ 1\ 0\ 0]$  and posterior density for  $\lambda$ ;  $gam(\alpha+s, \beta+Nt) = gam(3,10)$  with  $\lambda_{post}=0.30$  and  $\sigma_{post}=0.17$ .

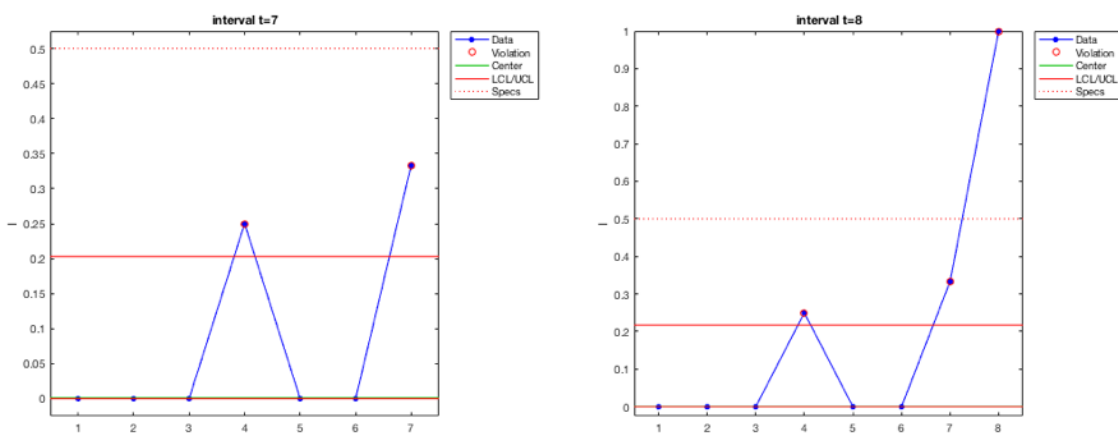


Figure 28.- Poisson-gamma model. Direct with mean prior method. Charts based on observed lambda evolution for intervals 7 and 8.

d.- Direct method with mean posterior.

A direct method with mean posterior is applied. With target for have zero accidents, parameters of the gamma prior  $\alpha=\beta=0.001$  and  $\pm 1\sigma_{post}$ . The posterior values are presented on Table 27; and charts for intervals 7 and 8 in Figure 29.

Time interval	Prior		Posterior				Observed	LCL	MEAN	UCL
	alfa	beta	alfa	beta	lambda	sigma	lambda			
0	0.001	0.001	-	-	0	0	0	-	0	-
1	0.001	0.001	0.001	1	0	0.03	0	0	0	0.03
2	0.001	0.001	0.001	2	0	0.02	0	0	0	0.02
3	0.001	0.001	0.001	3	0	0.01	0	0	0	0.01
4	0.001	0.001	1	4	0.25	0.25	0.25	0	0.25	0.50
5	0.001	0.001	1	5	0.20	0.20	0	0	0.20	0.40
6	0.001	0.001	1	6	0.17	0.17	0	0	0.17	0.33
7	0.001	0.001	2	7	0.29	0.20	0.33	0	0.29	0.49
8	0.001	0.001	3	8	0.38	0.22	1	0	0.38	0.59
9	0.001	0.001	3	9	0.33	0.19	0	0	0.33	0.53
10	0.001	0.001	3	10	0.30	0.17	0	0	0.30	0.47

Table 27.- Separation unit. Poisson-gamma model. Observations using a direct method with mean posterior. Out-of-limits highlighted blue.

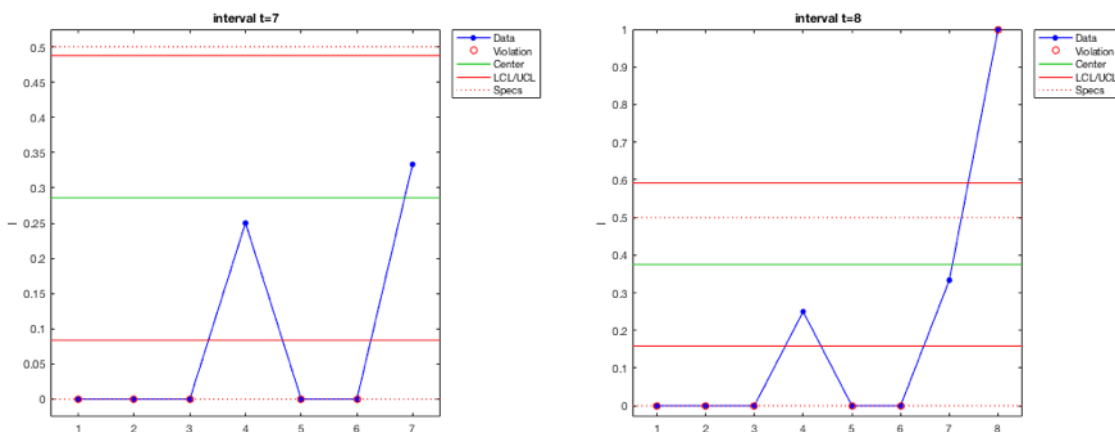


Figure 29.- Poisson-gamma model. Direct with mean posterior method. Charts based on observed lambda evolution for intervals 7 and 8.



#### 4.1.2. Exponential-gamma model.

With an exponential-gamma model the expression (1) with parameter  $p = \lambda$  is, (Annex A), is;

$$f(\lambda / Data) \propto g(Data / \lambda) f(\lambda) \propto y_i^{\alpha-1} e^{-y_i(\beta+\lambda)} \quad (17)$$

Being  $\alpha$  and  $\beta$  the parameters of the gamma distribution and  $y_i$  the data observed in the corresponding time interval  $i$ . Also modes recurrent and direct with mean prior and posterior will be applied.

a.- Recurrent method with mean prior.

A recurrent method with mean prior and equal to a target, being in this example changed to a less restrictive value of 1 accident in 20 intervals being equal to 0.05, with a sigma deviation also equal to 0.05 and with gamma prior parameters  $\alpha=0.5$  and  $\beta=10$ . Working with  $\pm 1\sigma_{\text{post}}$  and using the Metropolis-Hastings (MH) sampler, the Table 28 shows the observed values and the (MH) sampling in Figures 30, 31 and 32.

Time interval	Prior		Posterior					Observed		LCL	MEAN	UCL
	alfa	beta	alfa	beta	lambda	CI [5%-95%]	sigma	lambda				
0	0.5	10	-	-	0.05	-	0.05	0	-	0.05	-	
1	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10	
2	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10	
3	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10	
4	0.5	10	3.8	23	0.16	0.15-0.17	0.08	0.25	0	0.05	0.13	
5	3.8	23	3.8	23	0.16	0.15-0.17	0.08	0	0	0.05	0.13	
6	3.8	23	3.8	23	0.16	0.15-0.17	0.08	0	0	0.05	0.13	
7	3.8	23	6.9	34	0.20	0.19-0.21	0.08	0.33	0	0.05	0.13	
8	6.9	34	13.4	48	0.28	0.27-0.29	0.08	1	0	0.05	0.13	
9	13.4	48	13.4	48	0.28	0.27-0.29	0.08	0	0	0.05	0.13	
10	13.4	48	13.4	48	0.28	0.27-0.29	0.08	0	0	0.05	0.13	

Table 28.- Separation unit. Exponential-gamma model. Observations using a recurrent method with mean prior. Confidence interval (CI) [5%-95%]. Out-of-limits highlighted blue.

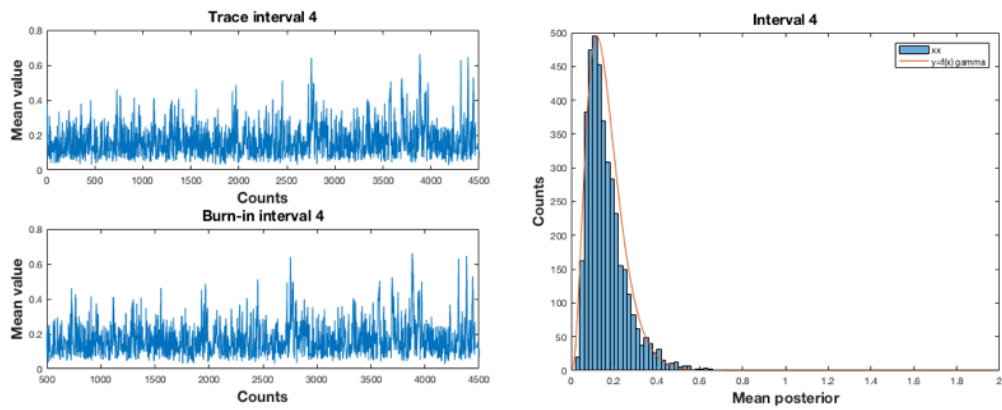


Figure 30.- Exponential-gamma. Interval 4. Sampling  $n=4500$ , burn=500; 10 cycles. AR= 48.9%.

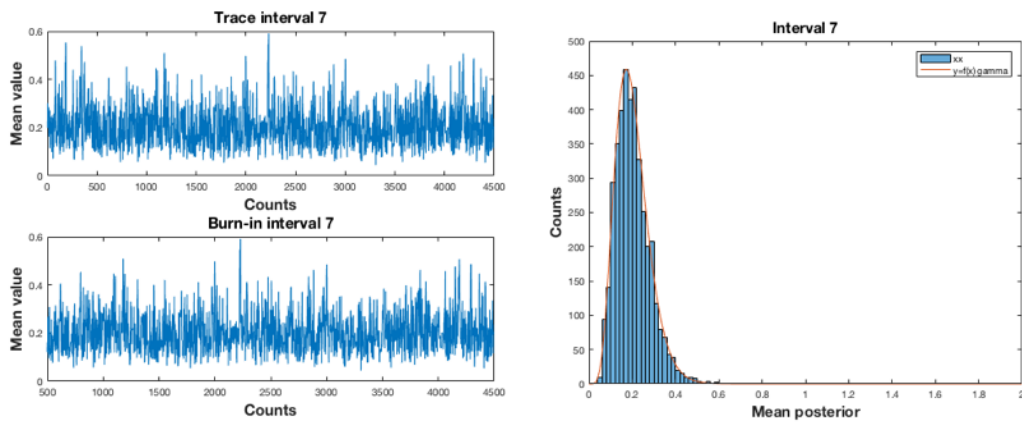


Figure 31.- Exponential-gamma. Interval 7. Sampling  $n=4500$ , burn=500; 10 cycles. AR= 51.9%.

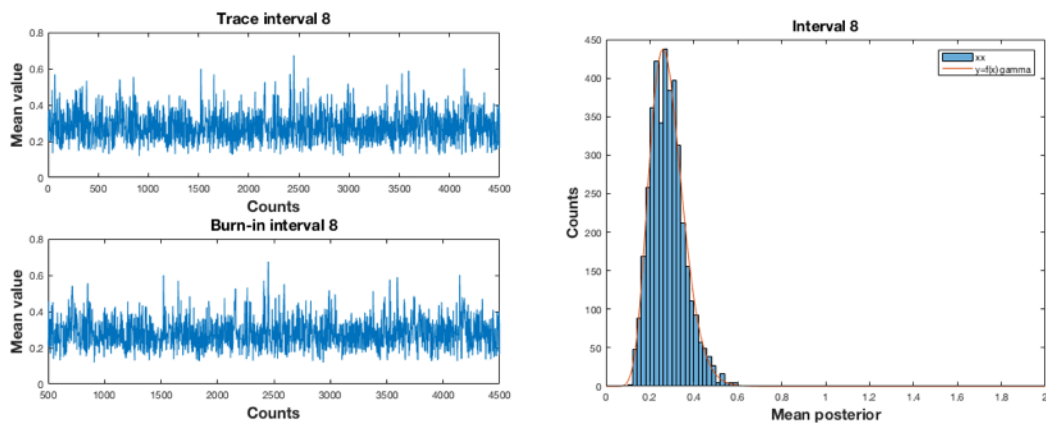


Figure 32.- Exponential-gamma. Interval 8. Sampling  $n=4500$ , burn=500; 10 cycles. AR= 52.7%.

b.- Recurrent method with mean posterior.

The posterior values are presented on Table 29.

Time interval	Prior		Posterior					Observed			
	alfa	beta	alfa	beta	lambda	CI [5%-95%]	sigma	lambda	LCL	MEAN	UCL
0	0.5	10	-	-	0.05	-	0.05	0	-	0.05	-
1	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
2	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
3	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
4	0.5	10	3.8	23	0.16	0.15-0.17	0.08	0.25	0	0.16	0.24
5	3.8	23	3.8	23	0.16	0.15-0.17	0.08	0	0	0.16	0.24
6	3.8	23	3.8	23	0.16	0.15-0.17	0.08	0	0	0.16	0.24
7	3.8	23	6.9	34	0.20	0.19-0.21	0.08	0.33	0	0.20	0.28
8	6.9	34	13.4	48	0.28	0.27-0.29	0.08	1	0	0.28	0.36
9	13.4	48	13.4	48	0.28	0.27-0.29	0.08	0	0	0.28	0.36
10	13.4	48	13.4	48	0.28	0.27-0.29	0.08	0	0	0.28	0.36

Table 29.- Separation unit. Exponential-gamma model. Observations using a recurrent method with mean posterior. Confidence interval (CI) [5%-95%]. Out-of-limits highlighted blue.

c.- Direct method with mean prior.

With a target value of 1 accident in 20 time intervals equal to 0.05, and a sigma deviation also equal to 0.05 and gamma prior parameters  $\alpha=0.5$  and  $\beta=10$ . Working with  $\pm 1\sigma_{\text{post}}$  and using the Metropolis-Hastings (MH) sampler the posterior values are presented on Table 30 and the (MH) sampling in Figures 33 and 34.

	Prior		Posterior					Observed			
Time interval	alfa	beta	alfa	beta	lambda	CI [5%-95%]	sigma	lambda	LCL	MEAN	UCL
0	0.5	10	-	-	0.05	-	0.05	0	-	0.05	-
1	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
2	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
3	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
4	0.5	10	3.8	23	0.16	0.15-0.17	0.08	0.25	0	0.05	0.13
5	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
6	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
7	0.5	10	4.3	23	0.18	0.17-0.19	0.09	0.33	0	0.05	0.14
8	0.5	10	7.2	23	0.32	0.31-0.33	0.12	1	0	0.05	0.17
9	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
10	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10

Table 30.- Separation unit. Exponential-gamma model. Observations using a direct method with mean prior. Confidence interval (CI) [5%-95%]. Out-of-limits highlighted blue.

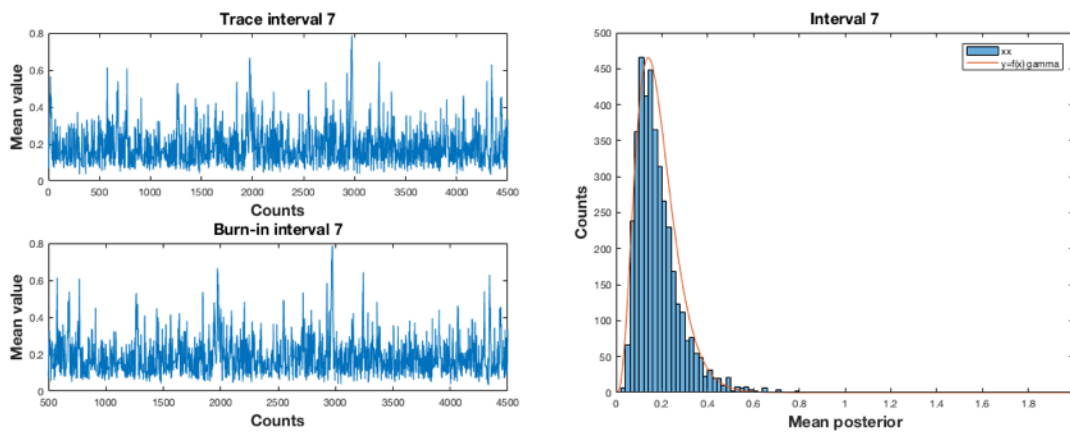


Figure 33.- Exponential-gamma. Interval 7. Sampling n=4500, burn=500; 10 cycles. AR= 52.4%.

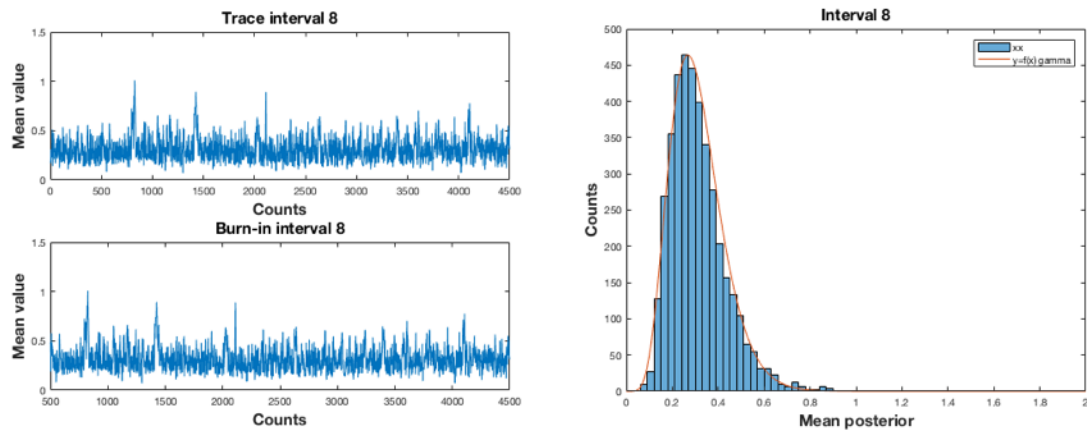


Figure 34.- Exponential-gamma. Interval 8. Sampling n=4500, burn=500; 10 cycles. AR= 65.3%.

d.- Direct method with mean posterior.

The posterior values are presented on Table 31.

Time interval	Prior		Posterior					Observed	LCL	MEAN	UCL
	alfa	beta	alfa	beta	lambda	CI [5%-95%]	sigma	lambda			
0	0.5	10	-	-	0.05	-	0.05	0	-	0.05	-
1	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
2	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
3	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
4	0.5	10	3.8	23	0.16	0.15-0.17	0.08	0.25	0	0.16	0.24
5	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
6	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
7	0.5	10	4.3	23	0.18	0.17-0.19	0.09	0.33	0	0.18	0.27
8	0.5	10	7.2	23	0.32	0.31-0.33	0.12	1	0	0.32	0.44
9	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
10	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10

Table 31.- Separation unit. Exponential-gamma model. Observations using a direct method with mean posterior. Confidence interval (CI) [5%-95%]. Out-of-limits highlighted blue.

#### 4.1.3. Weibull-gamma model.

With a Weibull-Gamma model the fatigue is considered in the analysis and the expression (1) with parameter  $p = \lambda$ , (Annex A), is;

$$f(\lambda / Data) \propto g(Data / \lambda) f(\lambda) \propto y_i^{\alpha-1} \cdot e^{-\beta y_i} \cdot \left(\frac{y_i}{\lambda}\right)^{c-1} e^{-\left(\frac{y_i}{\lambda}\right)^c} \quad (18)$$

Being  $\alpha$  and  $\beta$  the parameters of the gamma distribution and  $y_i$  the data observed in the corresponding time interval  $i$  and  $c$  is the fatigue parameter. When  $c=1$  the failure rate function is constant being equivalent to an exponential-gamma model. If  $c>1$  the failure rate function is increasing. If  $0<c<1$  the failure rate function is decreasing.

Also modes recurrent and direct with mean prior and posterior will be applied.

a.- Recurrent method with mean prior.

A recurrent method with mean prior and equal to the 0.05 target, with a sigma deviation also equal to 0.05 and gamma prior parameters  $\alpha=0.5$  and  $\beta=10$ . Working with  $\pm 1\sigma_{\text{post}}$  and using the Metropolis-Hastings (MH) sampler, the Table 32 shows the observed values and the (MH) sampling in Figures 35, 36 and 37.

Time interval	Prior		Posterior					Observed	LCL	MEAN	UCL
	alfa	beta	alfa	beta	lambda	CI [5%-95%]	sigma	lambda			
0	0.5	10	-	-	0.05	-	0.05	0	-	0.05	-
1	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
2	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
3	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
4	0.5	10	6.3	32.7	0.195	0.19-0.20	0.08	0.25	0	0.05	0.13
5	6.3	32.7	6.3	32.7	0.195	0.19-0.20	0.08	0	0	0.05	0.13
6	6.3	32.7	6.3	32.7	0.195	0.19-0.20	0.08	0	0	0.05	0.13
7	6.3	32.7	11.9	50.6	0.237	0.24-0.25	0.07	0.33	0	0.05	0.12
8	11.9	50.6	28.6	79.6	0.355	0.35-0.36	0.07	1	0	0.05	0.12
9	28.6	79.6	28.6	79.6	0.355	0.35-0.36	0.07	0	0	0.05	0.12
10	28.6	79.6	28.6	79.6	0.355	0.35-0.36	0.07	0	0	0.05	0.12

Table 32.- Separation unit. Weibull-gamma model. Observations using a recurrent method with mean prior. Confidence interval (CI) [5%-95%]. Out-of-limits highlighted blue.

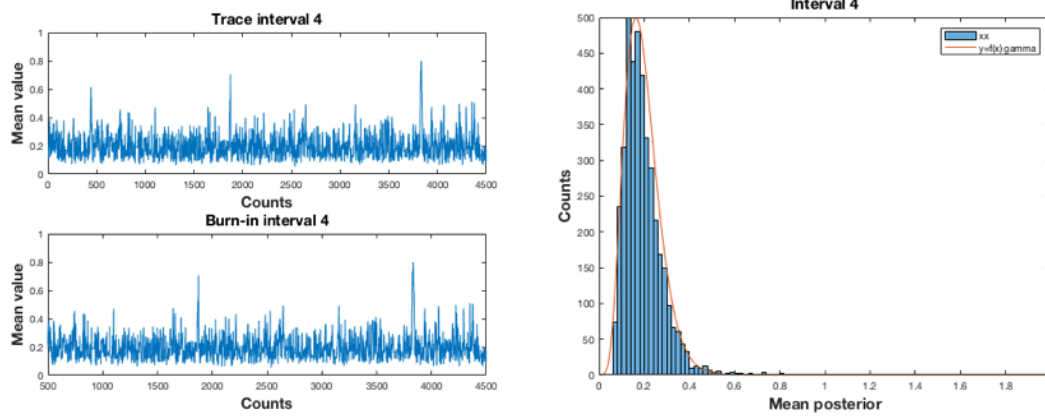


Figure 35.- Weibull-gamma. Interval 4. Sampling  $n=4500$ , burn=500; 10 cycles. AR= 49.2%.

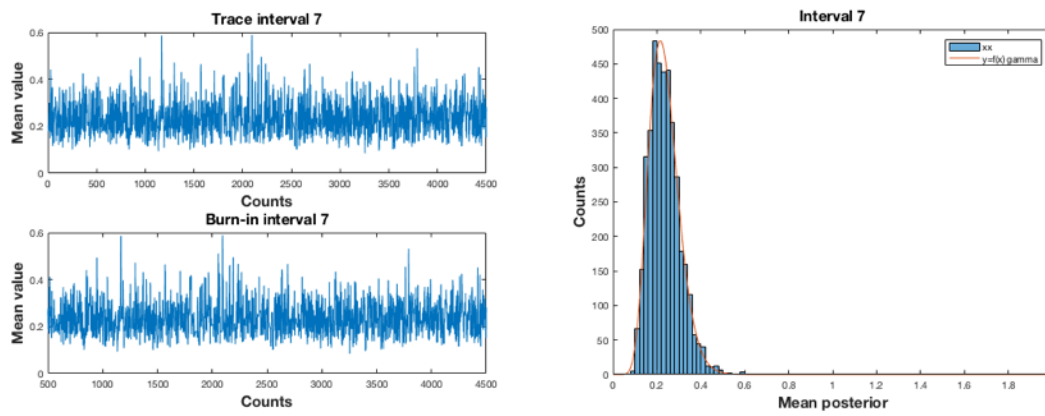


Figure 36.- Weibull-gamma. Interval 7. Sampling  $n=4500$ , burn=500; 10 cycles. AR= 47.7%.

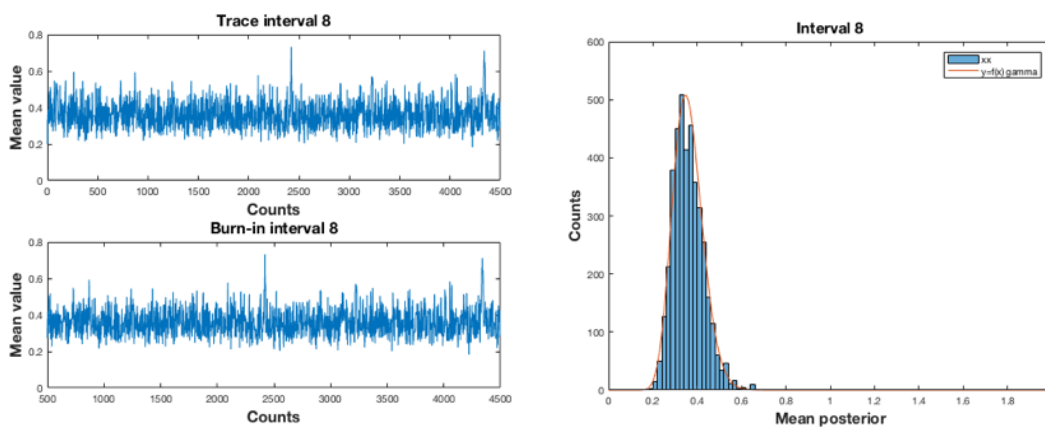


Figure 37.- Weibull-gamma. Interval 8. Sampling  $n=4500$ , burn=500; 10 cycles. AR= 47.6%.

b.- Recurrent method with mean posterior.

The posterior values are presented on Table 33.

Time interval	Prior		Posterior					Observed			
	alfa	beta	alfa	beta	lambda	CI [5%-95%]	sigma	lambda	LCL	MEAN	UCL
0	0.5	10	-	-	0.05	-	0.05	0	-	0.05	-
1	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
2	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
3	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
4	0.5	10	6.3	32.7	0.195	0.19-0.20	0.08	0.25	0	0.195	0.27
5	6.3	32.7	6.3	32.7	0.195	0.19-0.20	0.08	0	0	0.195	0.27
6	6.3	32.7	6.3	32.7	0.195	0.19-0.20	0.08	0	0	0.195	0.27
7	6.3	32.7	11.9	50.6	0.237	0.24-0.25	0.07	0.33	0	0.237	0.31
8	11.9	50.6	28.6	79.6	0.355	0.35-0.36	0.07	1	0	0.355	0.43
9	28.6	79.6	28.6	79.6	0.355	0.35-0.36	0.07	0	0	0.335	0.43
10	28.6	79.6	28.6	79.6	0.355	0.35-0.36	0.07	0	0	0.335	0.43

Table 33.- Separation unit. Weibull-gamma model. Observations using a recurrent method with mean posterior. Confidence interval (CI) [5%-95%]. Out-of-limits highlighted blue.

c.- Direct method with mean prior.

The Table 34 shows the observed values and the (MH) sampling in Figures 38 and 39.

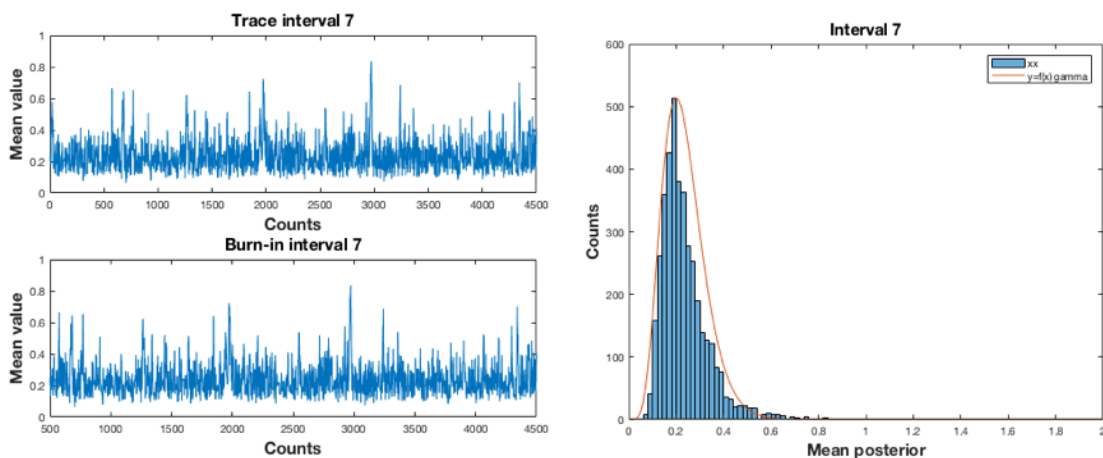


Figure 38.- Weibull-gamma. Interval 7. Sampling n=4500, burn=500; 10 cycles. AR= 53.2%.



Time interval	Prior		Posterior					Observed			
	alfa	beta	alfa	beta	lambda	CI [5%-95%]	sigma	lambda	LCL	MEAN	UCL
0	0.5	10	-	-	0.05	-	0.05	0	-	0.05	-
1	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
2	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
3	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
4	0.5	10	6.3	32.7	0.195	0.19-0.20	0.08	0.25	0	0.05	0.13
5	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
6	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
7	0.5	10	6.8	29.2	0.235	0.23-0.24	0.09	0.33	0	0.05	0.14
8	0.5	10	13.8	30.5	0.458	0.45-0.46	0.12	1	0	0.05	0.17
9	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
10	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10

Table 34.- Separation unit. Weibull-gamma model. Observations using a direct method with mean prior. Confidence interval (CI) [5%-95%]. Out-of-limits highlighted blue.

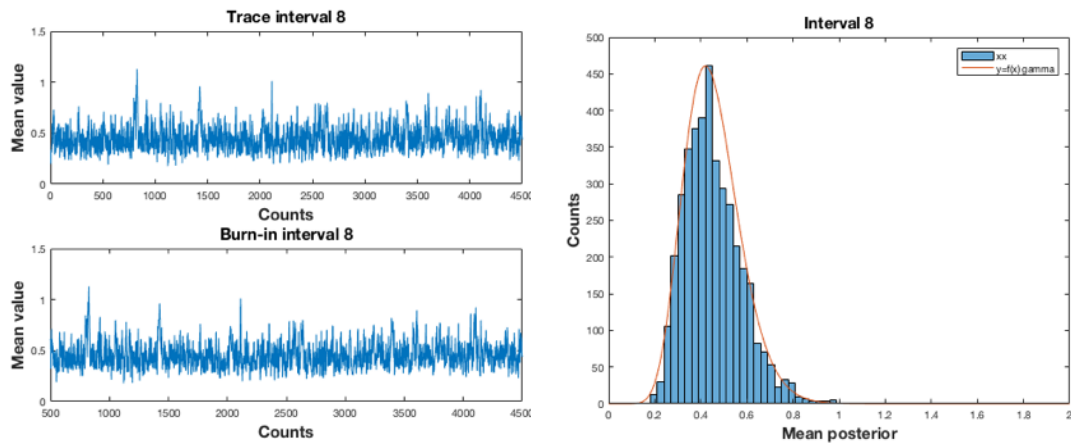


Figure 39.- Weibull-gamma. Interval 8. Sampling n=4500, burn=500; 10 cycles. AR= 68.1%.

d.- Direct method with mean posterior.

The Table 35 shows the observed values.

Time interval	Prior		Posterior					Observed			
	alfa	beta	alfa	beta	lambda	CI [5%-95%]	sigma	lambda	LCL	MEAN	UCL
0	0.5	10	-	-	0.05	-	0.05	0	-	0.05	-
1	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
2	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
3	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
4	0.5	10	6.3	32.7	0.195	0.19-0.20	0.08	0.25	0	0.195	0.28
5	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
6	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
7	0.5	10	6.8	29.2	0.235	0.23-0.24	0.09	0.33	0	0.235	0.33
8	0.5	10	13.8	30.5	0.458	0.45-0.46	0.12	1	0	0.458	0.58
9	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10
10	0.5	10	0.5	10	0.05	-	0.05	0	0	0.05	0.10

Table 35.- Separation unit. Weibull-gamma model. Observations using a direct method with mean posterior. Confidence interval (CI) [5%-95%]. Out-of-limits highlighted blue.

#### 4.1.4. Exponential-normal model.

With an exponential-normal model the expression (1) with parameter  $p = \lambda$  is, (Annex A), is;

$$f(\lambda / Data) \propto g(Data / \lambda) f(\lambda) \propto \lambda e^{-\lambda y_i} \cdot e^{-\frac{1}{2} \left( \frac{\lambda - \mu}{\sigma} \right)^2} \quad (19)$$

a.- Recurrent method with mean prior.

The Table 36 shows the observed values and the (MH) sampling in Figures 40, 41 and 42.

	Prior	Posterior			Observed			
Time interval	lambda	lambda	CI [5%-95%]	sigma	lambda	LCL	MEAN	UCL
0	0.05	0.05	-	0.05	0	-	0.05	-
1	0.05	0.05	-	0.05	0	0	0.05	0.10
2	0.05	0.05	-	0.05	0	0	0.05	0.10
3	0.05	0.05	-	0.05	0	0	0.05	0.10
4	0.05	0.159	0.158-0.161	0.06	0.25	0	0.05	0.11
5	0.159	0.159	0.158-0.161	0.06	0	0	0.05	0.11
6	0.159	0.159	0.158-0.161	0.06	0	0	0.05	0.11
7	0.159	0.242	0.239-0.247	0.09	0.33	0	0.05	0.14
8	0.242	0.562	0.552-0.573	0.20	1	0	0.05	0.25
9	0.562	0.562	0.552-0.573	0.20	0	0	0.05	0.25
10	0.562	0.562	0.552-0.573	0.20	0	0	0.05	0.25

Table 36.- Separation unit. Exponential-normal model. Observations using a recurrent method with mean prior. Confidence interval (CI) [5%-95%]. Out-of-limits highlighted blue.

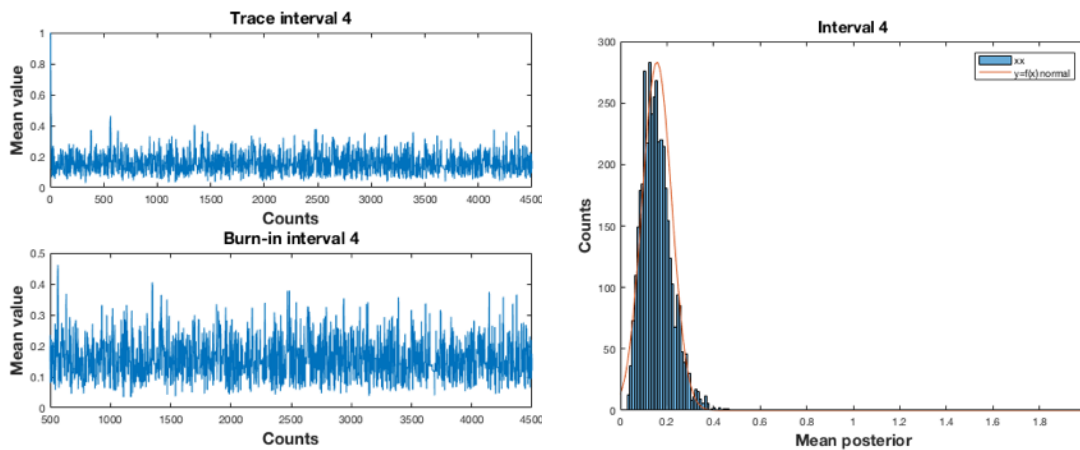


Figure 40.- Exponential-normal. Interval 4. Sampling n=4500, burn=500; 10 cycles. AR= 46.4%.

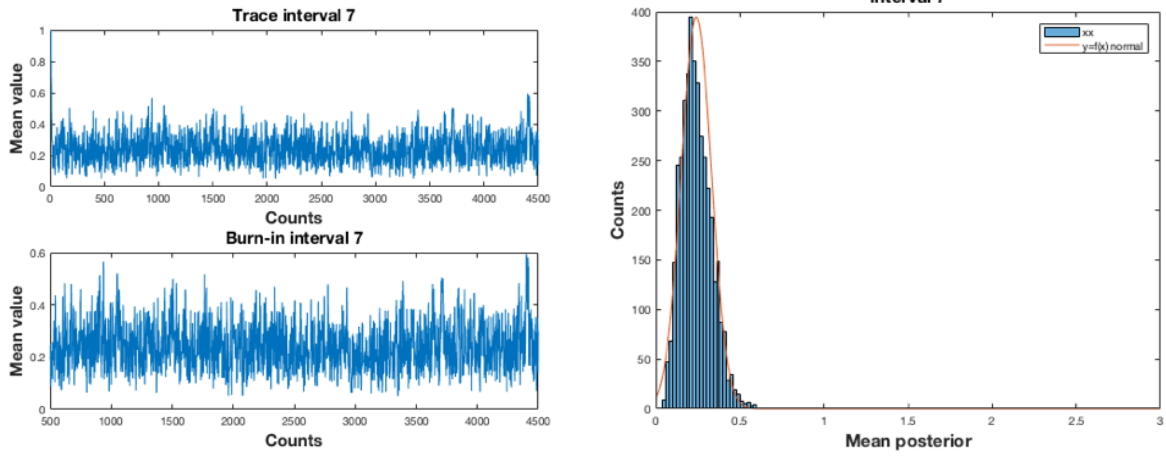


Figure 41.- Exponential-normal. Interval 7. Sampling  $n=4500$ , burn=500; 10 cycles. AR= 60.6%.

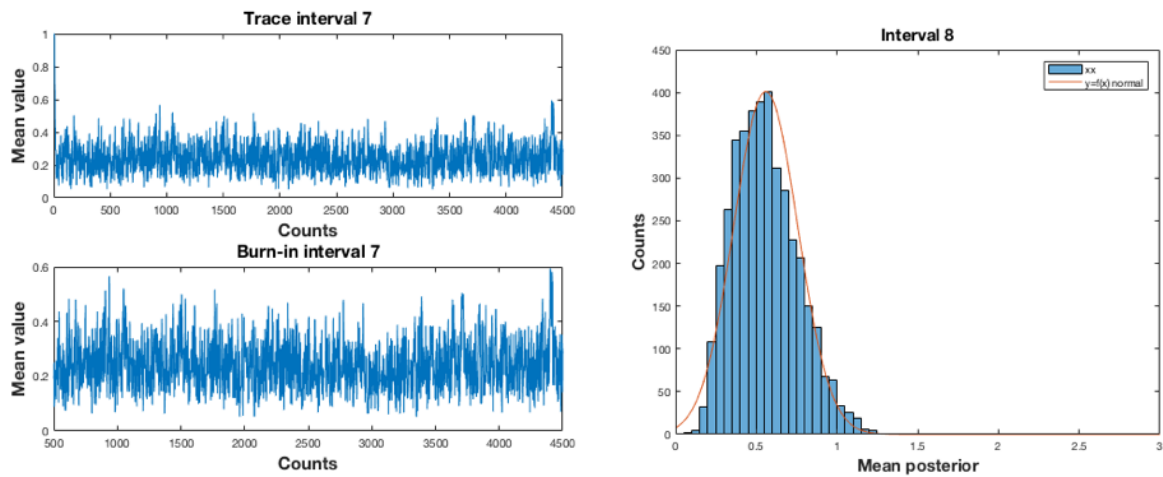


Figure 42.- Exponential-normal. Interval 8. Sampling  $n=4500$ , burn=500; 10 cycles. AR= 80.5%.

b.- Recurrent method with mean posterior.

The Table 37 shows the observed values.

	Prior	Posterior			Observed			
Time interval	lambda	lambda	CI [5%-95%]	sigma	lambda	LCL	MEAN	UCL
0	0.05	0.05	-	0.05	0	-	0.05	-
1	0.05	0.05	-	0.05	0	0	0.05	0.10
2	0.05	0.05	-	0.05	0	0	0.05	0.10
3	0.05	0.05	-	0.05	0	0	0.05	0.10
4	0.05	0.159	0.158-0.161	0.06	0.25	0	0.159	0.22
5	0.159	0.159	0.158-0.161	0.06	0	0	0.159	0.22
6	0.159	0.159	0.158-0.161	0.06	0	0	0.159	0.22
7	0.159	0.242	0.239-0.247	0.09	0.33	0	0.242	0.33
8	0.242	0.562	0.552-0.573	0.20	1	0	0.562	0.76
9	0.562	0.562	0.552-0.573	0.20	0	0	0.562	0.76
10	0.562	0.562	0.552-0.573	0.20	0	0	0.562	0.76

Table 37.- Separation unit. Exponential-normal model. Observations using a recurrent method with mean posterior. Confidence interval (CI) [5%-95%]. Out-of-limits highlighted blue.

c.- Direct method with mean prior.

The Table 38 shows the observed values and the (MH) sampling in Figures 43 and 44.

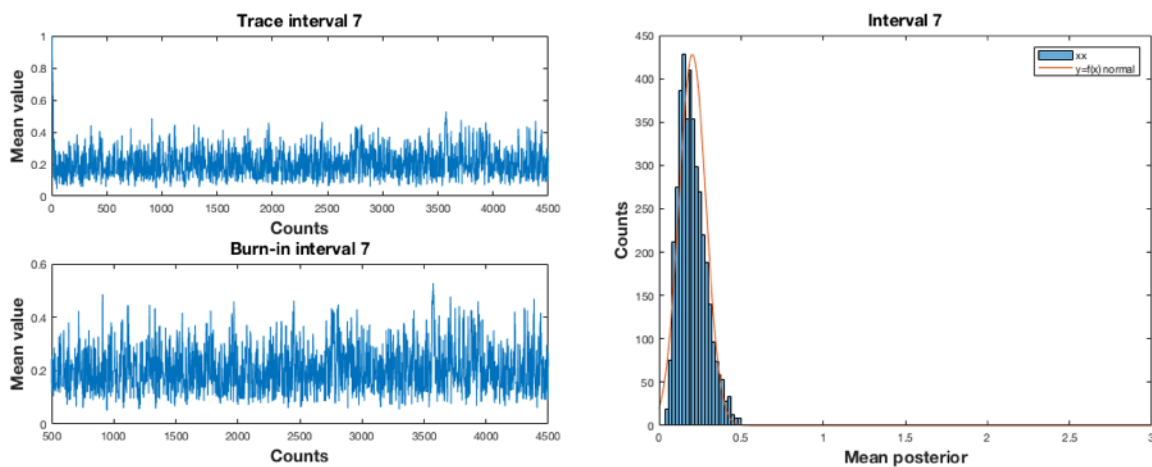


Figure 43.- Exponential-normal. Interval 7. Sampling n=4500, burn=500; 10 cycles. AR= 53.6%.

	Prior	Posterior			Observed			
Time interval	lambda	lambda	CI [5%-95%]	sigma	lambda	LCL	MEAN	UCL
0	0.05	0.05	-	0.05	0	-	0.05	-
1	0.05	0.05	-	0.05	0	0	0.05	0.10
2	0.05	0.05	-	0.05	0	0	0.05	0.10
3	0.05	0.05	-	0.05	0	0	0.05	0.10
4	0.05	0.159	0.158-0.161	0.06	0.25	0	0.05	0.11
5	0.05	0.05	-	0.05	0	0	0.05	0.10
6	0.05	0.05	-	0.05	0	0	0.05	0.10
7	0.05	0.199	0.197-0.201	0.08	0.33	0	0.05	0.13
8	0.05	0.497	0.489-0.505	0.19	1	0	0.05	0.24
9	0.05	0.05	-	0.05	0	0	0.05	0.10
10	0.05	0.05	-	0.05	0	0	0.05	0.10

Table 38.- Separation unit. Exponential-normal model. Observations using a direct method with mean prior. Confidence interval (CI) [5%-95%]. Out-of-limits highlighted blue.

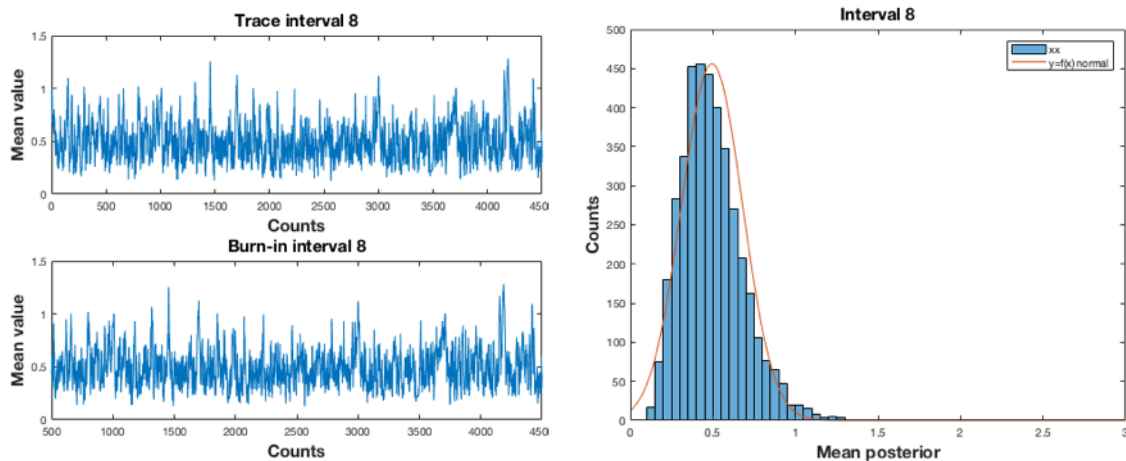


Figure 44.- Exponential-normal. Interval 8. Sampling n=4500, burn=500; 10 cycles. AR= 78.4%.

d.- Direct method with mean posterior.

The Table 39 shows the observed values.

Time interval	Prior	Posterior			Observed	LCL	MEAN	UCL
	lambda	lambda	CI [5%-95%]	sigma	lambda			
0	0.05	0.05	-	0.05	0	-	0.05	-
1	0.05	0.05	-	0.05	0	0	0.05	0.10
2	0.05	0.05	-	0.05	0	0	0.05	0.10
3	0.05	0.05	-	0.05	0	0	0.05	0.10
4	0.05	0.159	0.158-0.161	0.06	0.25	0	0.159	0.22
5	0.05	0.05	-	0.05	0	0	0.05	0.10
6	0.05	0.05	-	0.05	0	0	0.05	0.10
7	0.05	0.199	0.197-0.201	0.08	0.33	0	0.199	0.28
8	0.05	0.497	0.489-0.505	0.19	1	0	0.497	0.69
9	0.05	0.05	-	0.05	0	0	0.05	0.10
10	0.05	0.05	-	0.05	0	0	0.05	0.10

Table 39.- Separation unit. Exponential-normal model. Observations using a direct method with mean posterior. Confidence interval (CI) [5%-95%]. Out-of-limits highlighted blue.

4.1.5. Poisson-normal model.

With a Poisson-normal model the expression (1) with parameter  $p = \lambda$  is, (Annex A), is;

$$f(\lambda / Data) \propto g(Data / \lambda) f(\lambda) \propto \prod_{i=1}^n \frac{e^{-\lambda} \lambda^{y_i}}{y_i!} \cdot e^{-\frac{1}{2} \left( \frac{\lambda - \mu}{\sigma} \right)^2} \tag{20}$$

a.- Recurrent method with mean prior.

The Table 40 shows the observed values and the (MH) sampling in Figures 45, 46 and 47.

Time interval	Prior	Posterior			Observed	LCL	MEAN	UCL
	lambda	lambda	CI [5%-95%]	sigma	lambda			
0	0.05	0.05	-	0.05	0	-	0.05	-
1	0.05	0.05	-	0.05	0	0	0.05	0.10
2	0.05	0.05	-	0.05	0	0	0.05	0.10
3	0.05	0.05	-	0.05	0	0	0.05	0.10
4	0.05	0.154	0.152-0.155	0.08	0.25	0	0.05	0.13
5	0.154	0.154	0.152-0.155	0.08	0	0	0.05	0.13
6	0.154	0.154	0.152-0.155	0.08	0	0	0.05	0.13
7	0.154	0.240	0.238-0.242	0.11	0.33	0	0.05	0.16
8	0.240	0.468	0.459-0.477	0.22	1	0	0.05	0.27
9	0.468	0.468	0.459-0.477	0.22	0	0	0.05	0.27
10	0.468	0.468	0.459-0.477	0.22	0	0	0.05	0.27

Table 40.- Separation unit. Poisson-normal model. Observations using a recurrent method with mean prior. Confidence interval (CI) [5%-95%]. Out-of-limits highlighted blue.

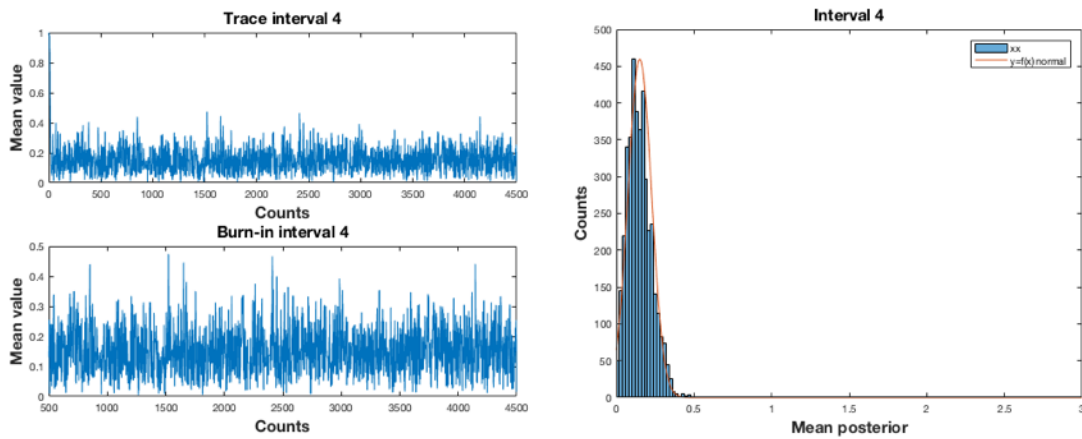


Figure 45.- Poisson-normal. Interval 4. Sampling n=4500, burn=500; 10 cycles. AR= 53.8%.



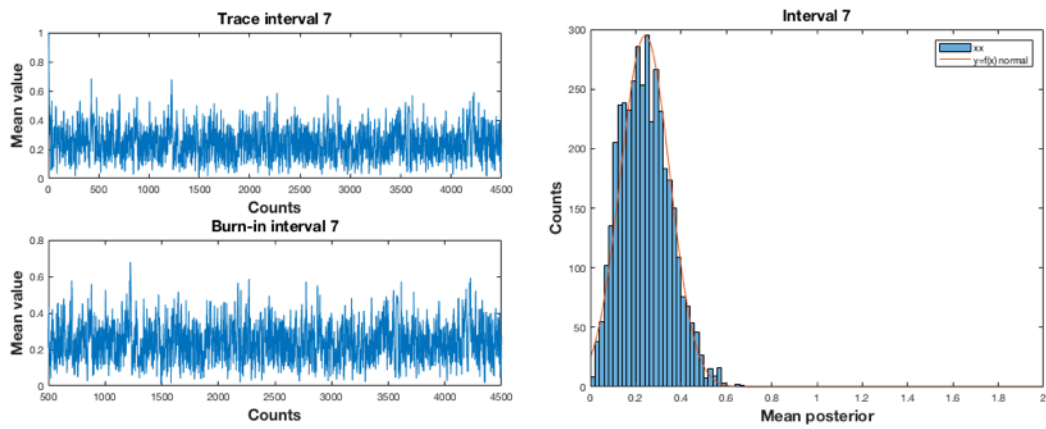


Figure 46.- Poisson-normal. Interval 7. Sampling  $n=4500$ , burn=500; 10 cycles. AR= 65.2%.

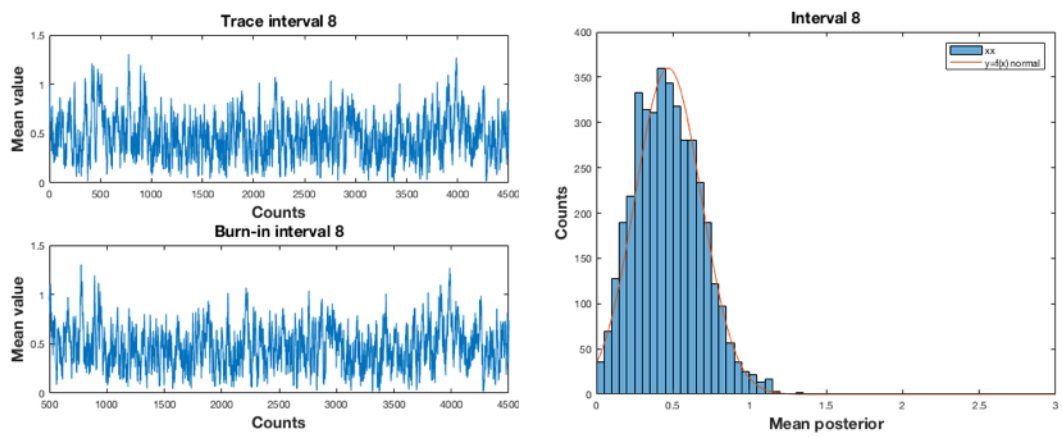


Figure 47.- Poisson-normal. Interval 8. Sampling  $n=4500$ , burn=500; 10 cycles. AR= 83.7%.

b.- Recurrent method with mean posterior.

The Table 41 shows the observed values

	Prior	Posterior			Observed			
Time interval	lambda	lambda	CI [5%-95%]	sigma	lambda	LCL	MEAN	UCL
0	0.05	0.05	-	0.05	0	-	0.05	-
1	0.05	0.05	-	0.05	0	0	0.05	0.10
2	0.05	0.05	-	0.05	0	0	0.05	0.10
3	0.05	0.05	-	0.05	0	0	0.05	0.10
4	0.05	0.154	0.152-0.155	0.08	0.25	0	0.154	0.23
5	0.154	0.154	0.152-0.155	0.08	0	0	0.154	0.23
6	0.154	0.154	0.152-0.155	0.08	0	0	0.154	0.23
7	0.154	0.240	0.238-0.242	0.11	0.33	0	0.240	0.35
8	0.240	0.468	0.459-0.477	0.22	1	0	0.468	0.69
9	0.468	0.468	0.459-0.477	0.22	0	0	0.468	0.69
10	0.468	0.468	0.459-0.477	0.22	0	0	0.468	0.69

Table 41.- Separation unit. Poisson-normal model. Observations using a recurrent method with mean posterior. Confidence interval (CI) [5%-95%]. Out-of-limits highlighted blue.

c.- Direct method with mean prior.

The Table 42 shows the observed values and the (MH) sampling in Figures 48 and 49.

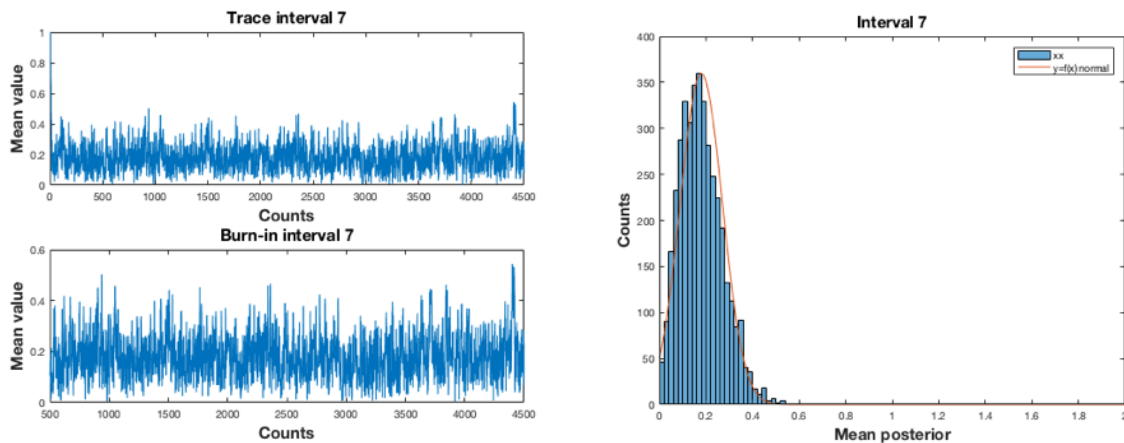


Figure 48.- Poisson-normal. Interval 7. Sampling n=4500, burn=500; 10 cycles. AR= 60.6%.

	Prior	Posterior			Observed			
Time interval	lambda	lambda	CI [5%-95%]	sigma	lambda	LCL	MEAN	UCL
0	0.05	0.05	-	0.05	0	-	0.05	-
1	0.05	0.05	-	0.05	0	0	0.05	0.10
2	0.05	0.05	-	0.05	0	0	0.05	0.10
3	0.05	0.05	-	0.05	0	0	0.05	0.10
4	0.05	0.153	0.151-0.155	0.07	0.25	0	0.05	0.12
5	0.05	0.05	-	0.05	0	0	0.05	0.10
6	0.05	0.05	-	0.05	0	0	0.05	0.10
7	0.05	0.181	0.178-0.185	0.09	0.33	0	0.05	0.14
8	0.05	0.387	0.376-0.399	0.19	1	0	0.05	0.24
9	0.05	0.05	-	0.05	0	0	0.05	0.10
10	0.05	0.05	-	0.05	0	0	0.05	0.10

Table 42.- Separation unit. Poisson-normal model. Observations using a direct method with mean prior. Confidence interval (CI) [5%-95%]. Out-of-limits highlighted blue.

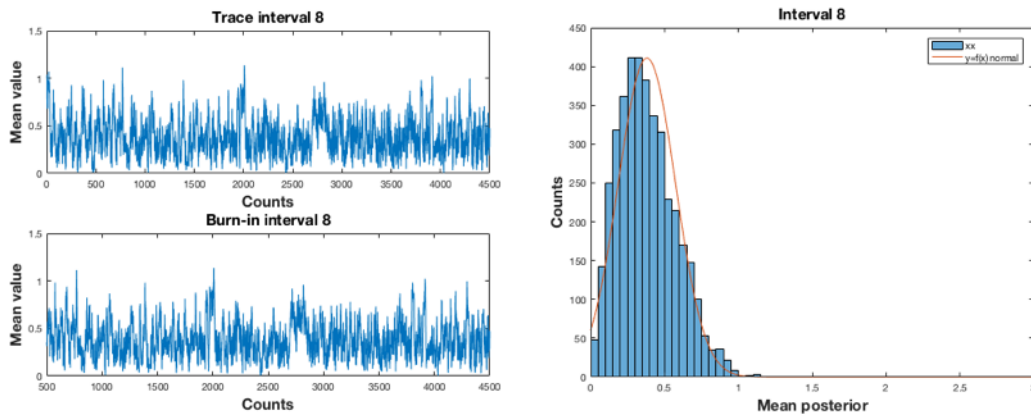


Figure 49.- Poisson-normal. Interval 8. Sampling n=4500, burn=500; 10 cycles. AR= 80.8%.

d.- Direct method with mean posterior.

The Table 43 shows the observed values.

	Prior	Posterior			Observed			
Time interval	lambda	lambda	CI [5%-95%]	sigma	lambda	LCL	MEAN	UCL
0	0.05	0.05	-	0.05	0	-	0.05	-
1	0.05	0.05	-	0.05	0	0	0.05	0.10
2	0.05	0.05	-	0.05	0	0	0.05	0.10
3	0.05	0.05	-	0.05	0	0	0.05	0.10
4	0.05	0.153	0.151-0.155	0.07	0.25	0	0.153	0.22
5	0.05	0.05	-	0.05	0	0	0.05	0.10
6	0.05	0.05	-	0.05	0	0	0.05	0.10
7	0.05	0.181	0.178-0.185	0.09	0.33	0	0.181	0.27
8	0.05	0.387	0.376-0.399	0.19	1	0	0.387	0.58
9	0.05	0.05	-	0.05	0	0	0.05	0.10
10	0.05	0.05	-	0.05	0	0	0.05	0.10

Table 43.- Separation unit. Poisson-normal model. Observations using a direct method with mean posterior. Confidence interval (CI) [5%-95%]. Out-of-limits highlighted blue.

The next step in the SRC methodology is the analysis of the safety barriers. This can be performed using a Poisson-gamma inference or applying a binomial-beta inference for the preventive and mitigative barriers or a hidden Markov chain analysis for the mitigative barriers.

The application of a Poisson-gamma inference is equivalent in methodology to the previously seen. The procedure is to observe, in this case, the eight safety barriers taking the inactive barriers as causes with  $\lambda$  rate.

For to apply a binomial-beta model it is needed to examine every one of the barriers, preventive or mitigative, independently of the others, to obtain the evolution of their probability of failure. For a selected barrier it is needed to perform groups of observations in a certain number of intervals; for example for the first mitigative safety barrier (SF<sub>1</sub>) the valve-manifold actuation, 10 intervals with 5 observations every one, are performed collecting if the safety barrier is not active. The observed data are summarized in the following Table 44.

Interval	1	2	3	4	5	6	7	8	9	10
1	A	A	A	A	A	A	A	A	A	A
2	A	A	A	F	A	A	F	A	A	A
3	A	A	A	A	A	A	F	A	A	A
4	A	A	A	A	A	A	A	F	A	A
5	A	A	A	A	A	A	A	A	A	A
Observed probability Failed/Collected	0/5	0/5	0/5	1/5	0/5	0/5	2/5	1/5	0/5	0/5

Table 44.- Separation unit. Collected data in the SF<sub>1</sub> safety barrier. Ten intervals with 5 collected observations.

#### 4.1.6. Binomial-beta model.

With a Binomial-beta model the expression (1) with parameter  $p = \theta$  equal to probability of failure is, (Annex A), is;

$$f(p / y_i) \propto f(p) f(y_i / p) = \frac{p^{\alpha-1} (1-p)^{\beta-1}}{B(\alpha, \beta)} \binom{n}{y_i} p^{y_i} (1-p)^{n-y_i} =$$

$$= p^{\alpha-1} (1-p)^{\beta-1} p^{y_i} (1-p)^{n-y_i} = p^{(\alpha+y_i)-1} (1-p)^{(\beta+n-y_i)-1}, \quad \text{for } 0 < p < 1 \quad (22)$$

And the posterior distribution of  $p$  is given by a beta distribution.

$$(p / y) \sim \text{Beta}(\alpha + y, \beta + n - y) \quad (23)$$

Being  $\alpha, \beta$  the parameters of the beta distribution,  $p$  the probability of failure,  $n$  the total observations in a group and  $y_i$  the not active barrier incidence counted in the group of observations.

An estimated probability of failure value  $p=0.2$  is applied with  $\sigma = 0.1$  as a prior.

Solving the equations;

$$0.2 = \frac{\alpha}{\alpha + \beta} \quad (24)$$

$$0.01 = \frac{\alpha \cdot \beta}{(\alpha + \beta)^2 + (\alpha + \beta + 1)} \quad (25)$$

Prior values of  $\alpha = 3$  and  $\beta = 12$  are obtained. The SRC methodology for safety barrier SF<sub>1</sub> applying control charts with  $\pm 1\sigma_{\text{post}}$  is presented in the following methods.

a.- Recurrent method with mean prior.

The Table 45 shows the observed values. With the following comments in every interval.

Interval 1.- With zero incidences, is,  $n=5$ ;  $y=0$  and posterior density for  $p$ ; is represented by  $\text{beta}(\alpha+y, \beta+n-y) = \text{beta}(3+0, 12+5-0) = \text{beta}(3, 17)$  with  $p_{\text{post}}=0.15$  and  $\sigma_{\text{post}}=0.08$ .

Interval 2.- With zero incidences, is,  $n=5$ ;  $y=0$  and posterior density for  $p$ ; represented by  $\text{beta}(\alpha+y, \beta+n-y) = \text{beta}(3+0, 17+5-0)$  with  $p_{\text{post}}=0.12$  and  $\sigma_{\text{post}}=0.06$ .

Interval 3.- With zero incidences, is,  $n=5$ ;  $y=0$  and posterior density for  $p$ ; represented by  $\text{beta}(\alpha+y, \beta+n-y) = \text{beta}(3, 27)$  with  $p_{\text{post}}=0.10$  and  $\sigma_{\text{post}}=0.05$ .

Interval 4.- With one no activity, is,  $n=5$ ;  $y=1$  and posterior density for  $p$ ; represented by  $\text{beta}(\alpha+y, \beta+n-y) = \text{beta}(3+1, 27+5-1) = \text{beta}(4, 31)$  with  $p_{\text{post}}=0.11$  and  $\sigma_{\text{post}}=0.05$ .

Interval 5.- With zero incidences, is,  $n=5$ ;  $y=0$  and posterior density for  $p$ ; represented by  $\text{beta}(\alpha+y, \beta+n-y) = \text{beta}(4, 36)$  with  $p_{\text{post}}=0.10$  and  $\sigma_{\text{post}}=0.05$ .

Interval 6.- With zero incidences, is,  $n=5$ ;  $y=0$  and posterior density for  $p$ ; represented by  $\text{beta}(\alpha+y, \beta+n-y) = \text{beta}(4, 41)$  with  $p_{\text{post}}=0.09$  and  $\sigma_{\text{post}}=0.04$ .

Interval 7.- With two no activities detected, is,  $n=5$ ;  $y=2$  and posterior density for  $p$ ; represented by  $\text{beta}(\alpha+y, \beta+n-y) = \text{beta}(6, 44)$  with  $p_{\text{post}}=0.12$  and  $\sigma_{\text{post}}=0.05$ .

Interval 8.- With one no activity detected, is,  $n=5$   $y=1$  and posterior density for  $p$ ; represented by  $\text{beta}(\alpha+y, \beta+n-y) = \text{beta}(7, 48)$  with posterior parameter  $p_{\text{post}}=0.13$  and  $\sigma_{\text{post}}=0.04$ .

Interval 9.- With zero incidences, is,  $n=5$ ;  $y=0$  and posterior density for  $p$ ; represented  $\text{gam}(\alpha+y, \beta+n-y) = \text{beta}(7, 53)$  with  $p_{\text{post}}=0.12$  and  $\sigma_{\text{post}}=0.04$ .

Interval 10.- With zero incidences, is,  $n=5$   $y=0$  and posterior density for  $p$ ; represented by  $\text{beta}(\alpha+y, \beta+n-y) = \text{beta}(7, 58)$  with  $p_{\text{post}}=0.11$  and  $\sigma_{\text{post}}=0.04$ .

Time interval	Prior		Posterior				Observed	LCL	MEAN	UCL
	alfa	beta	alfa+y	beta+n-y	p	sigma	p			
0	3	12	-	-	0.2	0.1	0	0.1	0.2	0.3
1	3	12	3	17	0.15	0.08	0	0.12	0.2	0.28
2	3	17	3	22	0.12	0.06	0	0.14	0.2	0.26
3	3	22	3	27	0.10	0.05	0	0.15	0.2	0.25
4	3	27	4	31	0.11	0.05	0.20	0.15	0.2	0.25
5	4	31	4	36	0.10	0.05	0	0.15	0.2	0.25
6	4	36	4	41	0.09	0.04	0	0.16	0.2	0.24
7	4	41	6	44	0.12	0.05	0.40	0.15	0.2	0.25
8	6	44	7	48	0.13	0.04	0.20	0.16	0.2	0.24
9	7	48	7	53	0.12	0.04	0	0.16	0.2	0.24
10	7	53	7	58	0.11	0.04	0	0.16	0.2	0.24

Table 45.- Separation unit. Binomial-beta model. Observations for SF<sub>1</sub> using a recurrent method with mean prior. Out-of-limits highlighted blue.

Charts for observed values in intervals 4 and 7 are presented on Figure 50.

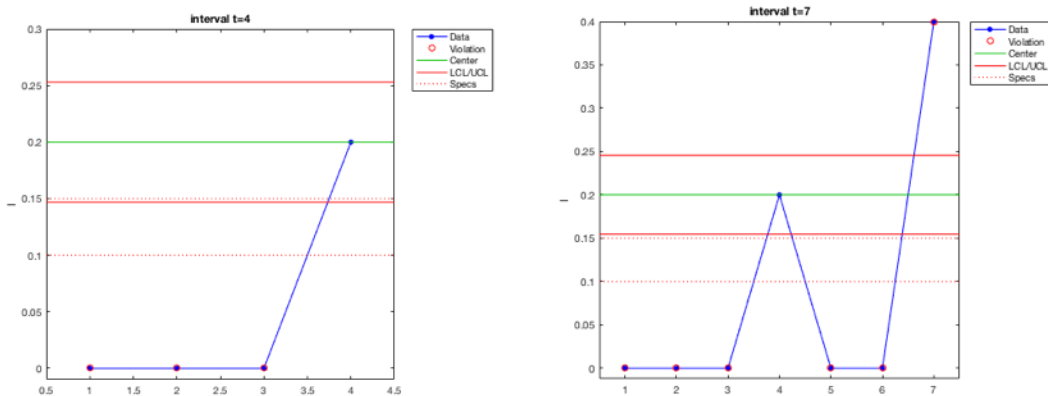


Figure 50.- Binomial-beta model. Recurrent with mean prior method. Charts based on observed failure probability evolution for intervals 4 and 7.

b.- Recurrent method with mean posterior.

The Table 46 shows the observed values.

Time interval	Prior		Posterior				Observed	LCL	MEAN	UCL
	alfa	beta	alfa+y	beta+n-y	p	sigma	p			
0	3	12	-	-	0.2	0.1	0	0.1	0.2	0.3
1	3	12	3	17	0.15	0.08	0	0.07	0.15	0.23
2	3	17	3	22	0.12	0.06	0	0.06	0.12	0.18
3	3	22	3	27	0.10	0.05	0	0.05	0.10	0.15
4	3	27	4	31	0.11	0.05	0.20	0.06	0.11	0.16
5	4	31	4	36	0.10	0.05	0	0.05	0.10	0.15
6	4	36	4	41	0.09	0.04	0	0.05	0.09	0.13
7	4	41	6	44	0.12	0.05	0.40	0.07	0.12	0.17
8	6	44	7	48	0.13	0.04	0.20	0.09	0.13	0.17
9	7	48	7	53	0.12	0.04	0	0.08	0.12	0.16
10	7	53	7	58	0.11	0.04	0	0.07	0.11	0.15

Table 46.- Separation unit. Binomial-beta model. Observations for SF<sub>1</sub> using a recurrent method with mean posterior. Out-of-limits highlighted blue.

c.- Direct method with mean prior.

With the same target probability of failure value  $p=0.2$  is applied with  $\sigma = 0.1$  as a prior and beta prior parameters  $\alpha=3$ ;  $\beta=12$ . Working with  $\pm 1\sigma_{\text{post}}$  the posterior values are presented on Table 47.

With the following comments in every interval.

Interval 1.- With zero incidences, is,  $n=5$ ;  $y=0$  and posterior density for  $p$ ; is represented by  $beta(\alpha+y, \beta+n-y) = beta(3+0, 12+5-0) = beta(3, 17)$  with  $p_{\text{post}}=0.15$  and  $\sigma_{\text{post}}=0.08$ .

Interval 2.- With zero incidences, is,  $n=10$ ;  $y=0$  and posterior density for  $p$ ; represented by  $beta(\alpha+y, \beta+n-y) = beta(3+0, 12+10-0) = beta(3, 22)$  with  $p_{\text{post}}=0.12$  and  $\sigma_{\text{post}}=0.06$ .

Interval 3.- With zero incidences, is,  $n=15$ ;  $y=0$  and posterior density for  $p$ ; represented by  $beta(\alpha+y, \beta+n-y) = beta(3, 27)$  with  $p_{\text{post}}=0.10$  and  $\sigma_{\text{post}}=0.05$ .

Interval 4.- With one no activity, is,  $n=20$ ;  $y=1$  and posterior density for  $p$ ; represented by  $beta(\alpha+y, \beta+n-y) = beta(3+1, 12+20-1) = beta(4, 31)$  with  $p_{\text{post}}=0.11$  and  $\sigma_{\text{post}}=0.05$ .



Interval 5.- With zero incidences, is,  $n=25$ ;  $y=0$  and posterior density for  $p$ ; represented by  $beta(\alpha+y, \beta+n-y) = beta(4,36)$  with  $p_{post}=0.10$  and  $\sigma_{post}=0.05$ .

Interval 6.- With zero incidences, is,  $n=30$ ;  $y=0$  and posterior density for  $p$ ; represented by  $beta(\alpha+y, \beta+n-y) = beta(4,41)$  with  $p_{post}=0.09$  and  $\sigma_{post}=0.04$ .

Interval 7.- With two no activities detected, is,  $n=35$ ;  $y=3$  and posterior density for  $p$ ; represented by  $beta(\alpha+y, \beta+n-y) = beta(6,44)$  with  $p_{post}=0.12$  and  $\sigma_{post}=0.05$ .

Interval 8.- With one no activity detected, is,  $n=40$   $y=4$  and posterior density for  $p$ ; represented by  $beta(\alpha+y, \beta+n-y) = beta(7,48)$  with posterior parameter  $p_{post}=0.13$  and  $\sigma_{post}=0.04$ .

Interval 9.- With zero incidences, is,  $n=45$ ;  $y=4$  and posterior density for  $p$ ; represented  $gam(\alpha+y, \beta+n-y) = beta(7,53)$  with  $p_{post}=0.12$  and  $\sigma_{post}=0.04$ .

Interval 10.- With zero incidences, is,  $n=50$   $y=4$  and posterior density for  $p$ ; represented by  $beta(\alpha+y, \beta+n-y) = beta(7,58)$  with  $p_{post}=0.11$  and  $\sigma_{post}=0.04$ .

Time interval	Prior		Posterior				Observed	LCL	MEAN	UCL
	alfa	beta	alfa+y	beta+n-y	p	sigma	p			
0	3	12	-	-	0.2	0.1	0	0.1	0.2	0.3
1	3	12	3	17	0.15	0.08	0	0.12	0.2	0.28
2	3	12	3	22	0.12	0.06	0	0.14	0.2	0.26
3	3	12	3	27	0.10	0.05	0	0.15	0.2	0.25
4	3	12	4	31	0.11	0.05	0.20	0.15	0.2	0.25
5	3	12	4	36	0.10	0.05	0	0.15	0.2	0.25
6	3	12	4	41	0.09	0.04	0	0.16	0.2	0.24
7	3	12	6	44	0.12	0.05	0.40	0.15	0.2	0.25
8	3	12	7	48	0.13	0.04	0.20	0.16	0.2	0.24
9	3	12	7	53	0.12	0.04	0	0.16	0.2	0.24
10	3	12	7	58	0.11	0.04	0	0.16	0.2	0.24

Table 47.- Separation unit. Binomial-beta model. Observations for  $SF_1$  using a direct method with mean prior. Out-of-limits highlighted blue.

And charts for intervals 7 and 8 in Figure 51.

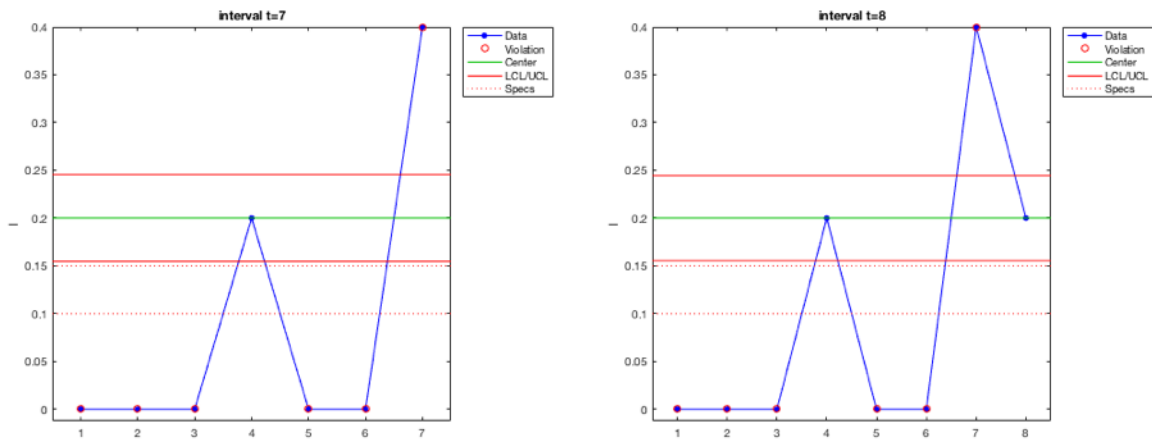


Figure 51.- Binomial-beta model. Direct with mean prior method. Charts based on observed failure probability evolution for intervals 7 and 8.

d.- Direct method with mean posterior.

The posterior values are presented on Table 48.

Time interval	Prior		Posterior				Observed	LCL	MEAN	UCL
	alfa	beta	alfa+y	beta+n-y	p	sigma	p			
0	3	12	-	-	0.2	0.1	0	0.1	0.2	0.3
1	3	12	3	17	0.15	0.08	0	0.07	0.15	0.23
2	3	12	3	22	0.12	0.06	0	0.06	0.12	0.18
3	3	12	3	27	0.10	0.05	0	0.05	0.10	0.15
4	3	12	4	31	0.11	0.05	0.20	0.06	0.11	0.16
5	3	12	4	36	0.10	0.05	0	0.05	0.10	0.15
6	3	12	4	41	0.09	0.04	0	0.05	0.09	0.13
7	3	12	6	44	0.12	0.05	0.40	0.07	0.12	0.17
8	3	12	7	48	0.13	0.04	0.20	0.09	0.13	0.17
9	3	12	7	53	0.12	0.04	0	0.08	0.12	0.16
10	3	12	7	58	0.11	0.04	0	0.07	0.11	0.15

Table 48.- Separation unit. Binomial-beta model. Observations for SF<sub>1</sub> using a direct method with mean prior. Out-of-limits highlighted blue.

And charts for intervals 7 and 8 in Figure 52.

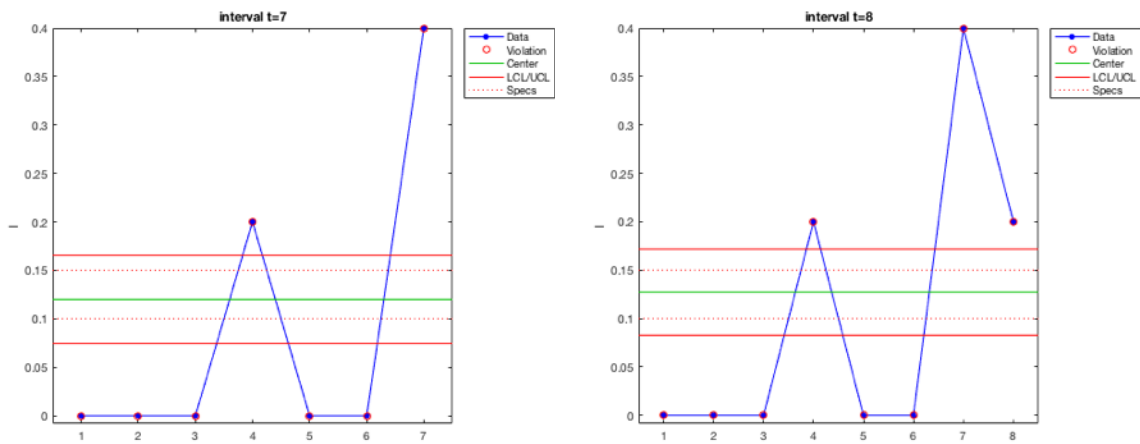


Figure 52.- Binomial-beta model. Direct with mean posterior method. Charts based on observed failure probability evolution for intervals 7 and 8.

#### 4.1.7. Poisson-gamma model

The procedure is analogous to the previous, it is needed to examine every one of the barriers, preventive or mitigative, independently of the others, to obtain the evolution of their incidences rate  $\lambda$ , also through groups of observations, when an incidence is produced for the analyzed barrier in a certain number of intervals, the observed data are summarized in the following Table 49.

Interval	1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	1	0	0	1	0	0	0
3	0	0	0	0	0	0	1	0	0	0
4	0	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	0	0	0	0
<b>Observed rate <math>\lambda</math></b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>0</b>	<b>2</b>	<b>1</b>	<b>0</b>	<b>0</b>

Table 49.- Separation unit. Collected data in the SF<sub>1</sub> safety barrier. Ten intervals with 5 collected observations of incidences.

The application of the Poisson-gamma inference model is the same as the previously seen.

#### 4.1.8. Analysis of the mitigative safety barriers observing the end states.

##### a.- Recurrent method

If “0” is defined as a barrier is “correct” and active and “1” is in “fail” the possible end states are defined in Table 50. Where “000” means that the first, second and third barriers are active and the end state is  $V_1 = \text{Normal operation}$ ; and the same logic with the rest.

Mitigative barrier situation	End state	Mitigative barrier situation	End state
000	Normal operation ( $V_1$ )	100	Partial operation ( $V_2$ )
001	Normal operation ( $V_1$ )	101	Partial operation ( $V_2$ )
010	Normal operation ( $V_1$ )	110	Short stop operation ( $V_3$ )
011	Normal operation ( $V_1$ )	111	Total failure ( $V_4$ )

Table 50.- End states in function of the mitigative safety barriers situation.

A prior transition matrix is defined for the three safety barriers. Being  $p_{11}$  the probability for the  $SF_1$  barrier to stay active in state 1;  $p_{12}$  the probability of transition from  $SF_1$  to  $SF_2$  because  $SF_1$  has failed;  $p_{13}$  is the probability of transition from  $SF_1$  to  $SF_3$  because  $SF_1$  and  $SF_2$  have failed, and so on. Additionally an emission matrix is defined to indicate the probabilities that a mitigative barrier  $SF_n$  is active based on the observed end states ( $V_1 V_2 V_3 V_4$ ).

$$Tran = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix} = \begin{pmatrix} 0.8 & 0.15 & 0.05 \\ 0.55 & 0.4 & 0.05 \\ 0.2 & 0.4 & 0.4 \end{pmatrix}$$

$$Emiss = \begin{pmatrix} & V_1 & V_2 & V_3 & V_4 \\ S_1 & 0.8 & 0.1 & 0.05 & 0.05 \\ S_2 & 0.4 & 0.2 & 0.2 & 0.2 \\ S_3 & 0.3 & 0.4 & 0.2 & 0.1 \end{pmatrix}$$

In concordance with the previous defined transition and emission matrices the expected use of the mitigative safety barriers and the end states are presented in the Figure 53.

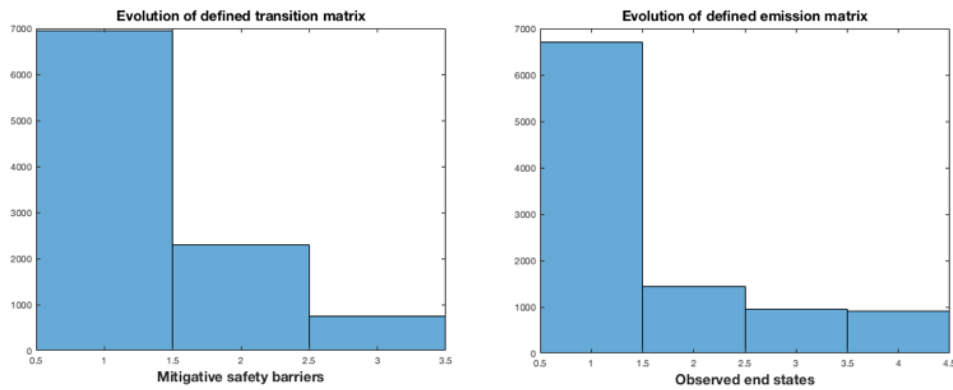


Figure 53.- Separation unit. Initial transitions of the safety barriers and emission states.

The observations are made by creating a group of ten and following a first in first out (FIFO) order. Each new one is added to the group and the oldest one disappears. The observed sequence is seqobs=[1 1 1 1 1 1 2 3 1 1] indicating that in the six first observations times normal operation has been sampled, one partial operation in the seventh, one short stop operation in the eighth, and normal operation in the next two observations. Performing a Baum-Welch algorithm (Annex C) the posterior transition and emission matrices are obtained from the observed sequence.

$$Transobs = \begin{pmatrix} 0.875 & 0 & 0.1250 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad Emisobs = \begin{pmatrix} V_1 & V_2 & V_3 & V_4 \\ S_1 & 1 & 0 & 0 & 0 \\ S_2 & 0 & 0 & 1 & 0 \\ S_3 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Being the relative occupancies of the safety barriers at steady state SF<sub>1</sub>=80%; SF<sub>2</sub>=10% and SF<sub>3</sub>=10%, Figure 54.

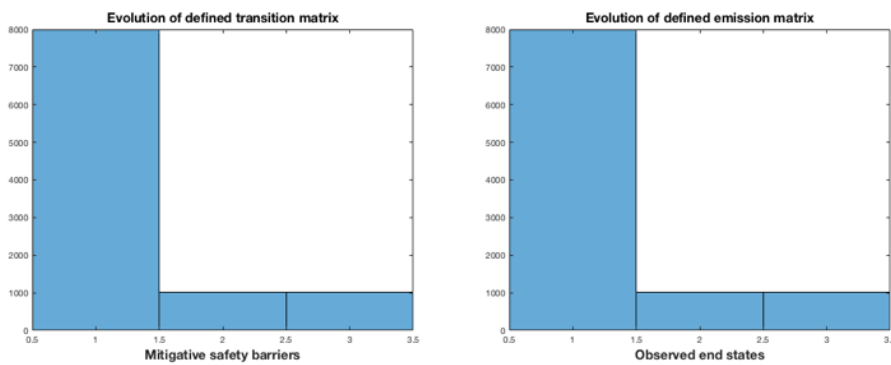


Figure 54.- Separation unit. Posterior transitions of the safety barriers and emission matrices according to the first sequence observation.

Thus in this 10 intervals the number of visits to every transition state for the three safety barriers are;

$$M(10) = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix} = \begin{pmatrix} 8.04 & 0.93 & 1.03 \\ 8.24 & 0.83 & 0.93 \\ 7.44 & 1.73 & 0.83 \end{pmatrix}$$

It is important also to know what will be the next passage, in observations intervals, from the SF1 and SF2 safety barriers to the most critical SF3 barrier. In this case is  $m_1=8$  and  $m_2=9$ , meaning that, according to the observations, is expected that from SF1 a transition to SF3 can be produced in 8 intervals, and from SF2 to SF3 in 9 intervals.

As a recurrent method the posterior transition and emission matrices (Transobs and Emisobs), can be the new prior in the next sampling, in this case has been observed a partial operation ( $V_2$ ); then seqobs=[1 1 1 1 1 2 3 1 1 2] and a new posterior for transition and emission matrices is obtained.

$$Transobs = \begin{pmatrix} 0.75 & 0 & 0.25 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad Emisobs = \begin{pmatrix} & V_1 & V_2 & V_3 & V_4 \\ S_1 & 1 & 0 & 0 & 0 \\ S_2 & 0 & 0 & 1 & 0 \\ S_3 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Being the relative occupancies of the safety barriers at steady state  $SF_1=66.6\%$ ;  $SF_2=16.7\%$  and  $SF_3=16.7\%$ , Figure 55.

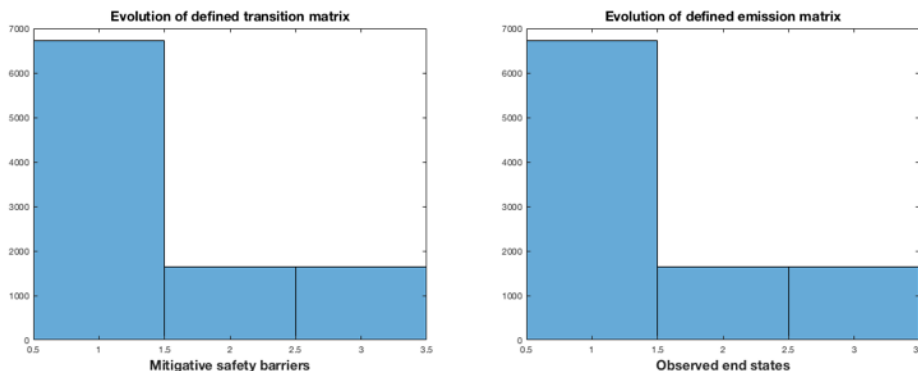


Figure 55.- Separation unit. Posterior transitions of the safety barriers and emission matrices, using a recurrent method, according to the second sequence observation.

And the number of visits to every transition state in these 10 intervals for the three safety barriers are;

$$M(10) = \begin{pmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{pmatrix} = \begin{pmatrix} 6.67 & 1.58 & 1.75 \\ 7.00 & 1.42 & 1.58 \\ 6.33 & 2.25 & 1.42 \end{pmatrix}$$

The next passage values for the third safety barrier are  $m_1=4$  and  $m_2=5$ , meaning that, according to the observations, is expected that from  $SF_1$  a transition to  $SF_3$  can be produced in 4 intervals, and from  $SF_2$  to  $SF_3$  in 5 intervals. The values are lower than the previous ones because the sampled observation has been for state  $V_2$ , a partial operation, which reduces the number of normal operation, state  $V_1$ , making more critical the possible use of the third barrier.

b.- Direct method

The same values are obtained using a direct method for the second observation.

4.1.9. Analysis of the end states observing the mitigative safety barriers.

a.- Recurrent method

The prior matrices are defined being the transition matrix for the four end states and the emission matrix indicating the probabilities to stay in an end state  $S_n$  being:  $S_1$ =Normal operation;  $S_2$ =Partial operation;  $S_3$ =Short stop operation and  $S_4$ =Total failure; in function of the observed three active mitigative barriers ( $V_1 V_2 V_3$ ).

A prior transition matrix is defined for the four end states. Being  $p_{11}$  the probability for the  $S_1$  state (normal operation) to stay active in this state 1;  $p_{12}$  the probability of transition from  $S_1$  (normal operation) to  $S_2$  (partial operation) because  $S_1$  has failed;  $p_{13}$  is the probability of transition from  $S_1$  (normal operation) to  $S_3$  (short stop operation) because  $S_1$  and  $S_2$  have failed,  $p_{14}$  is the probability of transition from  $S_1$  (normal operation) to  $S_4$  (total failure) because  $S_1$ ,  $S_2$  and  $S_3$  have failed, and so on. Additionally an emission matrix is defined to indicate the probabilities that an end state be active based on the observed safety barriers. ( $V_1 V_2 V_3$ ).

$$Tran = \begin{pmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{pmatrix} = \begin{pmatrix} 0.8 & 0.15 & 0.03 & 0.02 \\ 0.50 & 0.4 & 0.05 & 0.05 \\ 0.2 & 0.3 & 0.4 & 0.1 \\ 0.15 & 0.4 & 0.4 & 0.05 \end{pmatrix}$$

$$Emiss = \begin{pmatrix} & V_1 & V_2 & V_3 \\ S_1 & 0.7 & 0.2 & 0.1 \\ S_2 & 0.4 & 0.4 & 0.2 \\ S_3 & 0.3 & 0.3 & 0.4 \\ S_4 & 0.2 & 0.2 & 0.6 \end{pmatrix}$$

In concordance with the previous defined transition and emission matrices the expected use of the end states and the observed safety barrier are presented in the Figure 56.

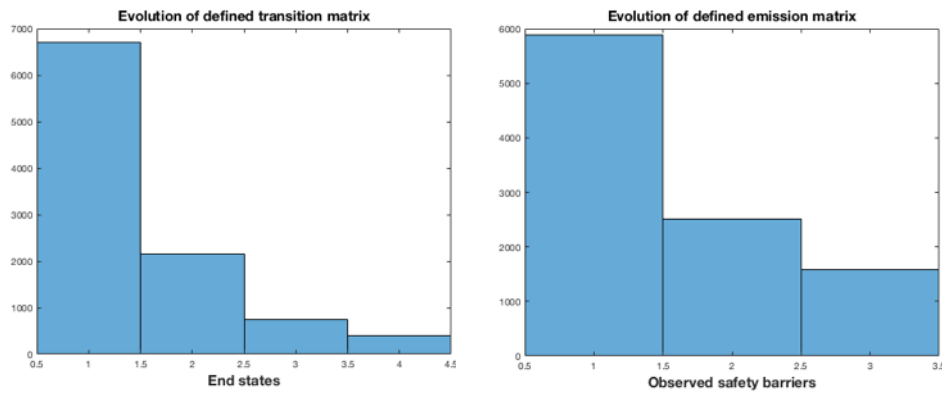


Figure 56.- Separation unit. Initial transitions of the end states and emission matrices.

The sampling is obtained by sequentially observing the barriers, if the first is active, then its value is 1, if it fails and the second barrier is active, then the value is 2 and so on; a group of ten observations is also maintained. The sampled sequence of the safety barriers is seqobs=[1 1 1 2 1 1 2 2 1 1]. And the posterior transition and emission matrices are.

$$Transobs = \begin{pmatrix} 0.5 & 0 & 0 & 0.5 \\ 0 & 0.33 & 0 & 0.67 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad Emissobs = \begin{pmatrix} & V_1 & V_2 & V_3 \\ S_1 & 1 & 0 & 0 \\ S_2 & 0 & 1 & 0 \\ S_3 & 1 & 0 & 0 \\ S_4 & 1 & 0 & 0 \end{pmatrix}$$

Being the relative occupancies  $S_1=0\%$ ,  $S_2=43\%$ ,  $S_3=29\%$  and  $S_4=28\%$  at steady state, Figure 57.



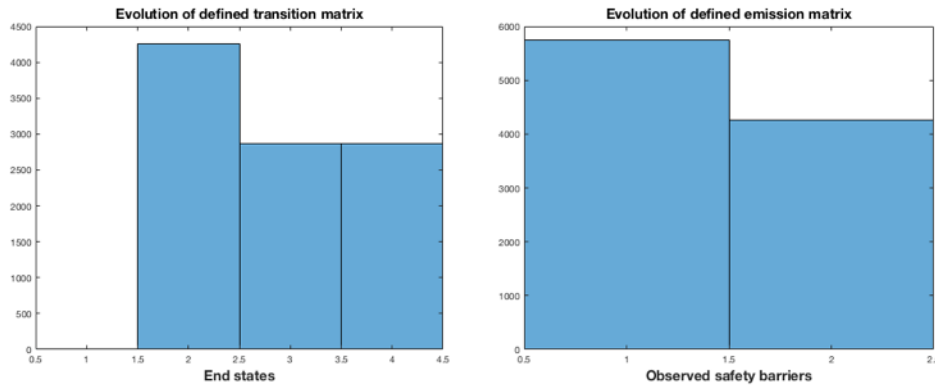


Figure 57.- Separation unit. Transitions of the end states and emission matrices according to the first sequence observation of safety barriers.

And the number of visits or transitions, in 10 intervals.

$$M(10) = \begin{pmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{pmatrix} = \begin{pmatrix} 0.9955 & 3.3950 & 2.6549 & 2.9545 \\ 0 & 4.1196 & 2.7840 & 3.0963 \\ 0 & 4.6397 & 2.5752 & 2.7851 \\ 0 & 4.1732 & 3.2504 & 2.5764 \end{pmatrix}$$

The next passage values for the fourth state, are  $m_1=2$ ,  $m_2=1.5$  and  $m_3=2.5$ .

Using a recurrent method the posterior transition and emission matrices (*Transobs* and *Emisobs*), can be the new prior in the next sampling, being  $seqobs=[1 \ 1 \ 2 \ 1 \ 1 \ 2 \ 2 \ 1 \ 1 \ 2]$  showing an increase of the use of the safety barrier  $SF_2$ , a new posterior for transition and emission matrices is obtained.

$$Transobs = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0.33 & 0 & 0.67 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad Emisobs = \begin{pmatrix} & V_1 & V_2 & V_3 \\ S_1 & 0 & 0 & 0 \\ S_2 & 0 & 1 & 0 \\ S_3 & 1 & 0 & 0 \\ S_4 & 1 & 0 & 0 \end{pmatrix}$$

Being the steady state occupations  $S_1=0\%$ ,  $S_2=43\%$ ,  $S_3=28\%$  and  $S_4=29\%$ , Figure 58.

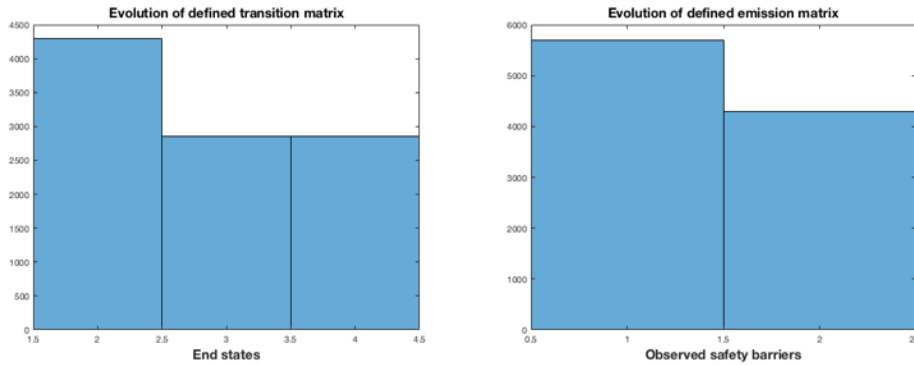


Figure 58.- Separation unit. Transitions of the end states and emission matrices, using a recurrent method, according to the second sequence observation of safety barriers.

And the number of visits or transitions, in 10 intervals.

$$M(10) = \begin{pmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{pmatrix} = \begin{pmatrix} 0 & 3.8619 & 2.8875 & 3.2506 \\ 0 & 4.1220 & 2.7832 & 3.0948 \\ 0 & 4.6422 & 2.5746 & 2.7832 \\ 0 & 4.1748 & 3.2506 & 2.5746 \end{pmatrix}$$

With next passage values for the fourth state;  $m_1=1$ ,  $m_2=1.5$  and  $m_3=2.5$ .

b.- Direct method

Applying a direct method the initially defined transition and emission matrices (Tran and Emiss), are the prior in the next sampling with  $seqobs=[1 \ 1 \ 2 \ 1 \ 1 \ 2 \ 2 \ 1 \ 1 \ 2]$ . And the posterior matrices are,

$$Transobs = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad Emisobs = \begin{pmatrix} V_1 & V_2 & V_3 \\ S_1 & 1 & 0 & 0 \\ S_2 & 0 & 1 & 0 \\ S_3 & 0.33 & 0.67 & 0 \\ S_4 & 1 & 0 & 0 \end{pmatrix}$$

Being the steady state occupations  $S_1=25\%$ ,  $S_2=25\%$ ,  $S_3=25\%$  and  $S_4=25\%$ , Figure 59.

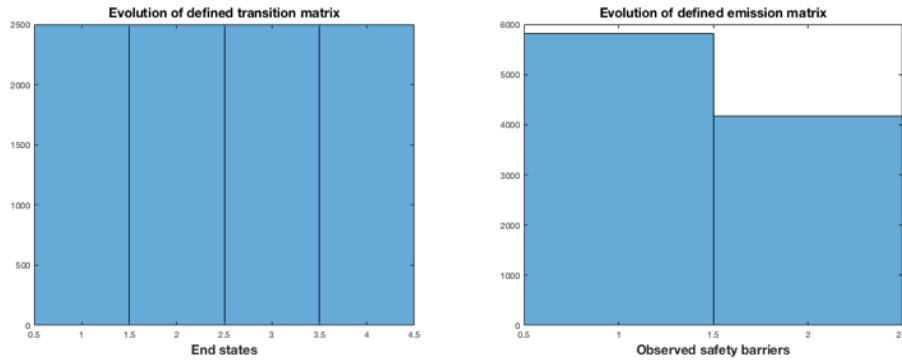


Figure 59.- Separation unit. Transitions of the end states and emission matrices, using a direct method, according to the second sequence observation of safety barriers.

And the number of visits or transitions, in 10 intervals.

$$M(10) = \begin{pmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{pmatrix} = \begin{pmatrix} 2 & 2 & 3 & 3 \\ 3 & 2 & 2 & 3 \\ 3 & 3 & 2 & 2 \\ 2 & 3 & 3 & 2 \end{pmatrix}$$

With next passage values for the fourth state;  $m_1=1$ ,  $m_2=2$  and  $m_3=3$ .

The difference with the recurrent and direct methods is due to the value of the transition and emission matrices that are used as prior, because the algorithm convergence is depending on the initial values [187, 188].

## 4.2. Occupational analysis. MDF process.

The occupational analysis can be performed using any of the Bayesian inference models described above since its application is based on the same scheme of analysis of initiating causes (ic), incidents in preventive and mitigative safety barriers and the analysis of the general safety barriers using Bayesian inference or the mitigative safety barriers and their end states applying hidden Markov chains.

Then, using the generic scheme of Figure 14 the same inference models following Poisson, exponential or Weibull distributions, with the statistical parameter  $p$  corresponding to the rate  $\lambda$  or frequency of events and defining gamma or normal distributions as the prior  $f(p)$  can be applied. Also the same Binomial-Beta model for the safety barriers analysis, in any of their methodologies, direct or recurrent with prior or posterior values. And the same consideration for the use of the hidden Markov chains.

But to expose the functionality of the method will affect a case of a manufacturing process.

The general process scheme is depicted in Figure 60. And its goal is to produce urea-melamine medium density fiber (MDF) board elements using as basic raw materials: paper, wood, melamine, urea, a resin (such as a polyamide or vinyl chloroacetate) and formaldehyde. The paper is subjected to a surface printing treatment continuing with the impregnation phase performed with melamine-formaldehyde. A drying and a cooling process are executed next in a single step if only the melamine-formaldehyde polymer is added or with one additional step if the urea-formaldehyde polymer is added, and with the same impregnation, drying and subsequent cooling steps. The process continues with the cutting and winding of the paper and its stacking. At the same time, the wood is splintered by subsequently drying the material at 180°C to reduce moisture. The dry material (8% moisture) is impregnated with the urea-formaldehyde solution and the resin. It follows a stage of forming and pressing at 200°C. The board thus obtained is subjected to a curing process, and is completed in a union-pressing stage of the board formed with the sheet of paper.

In plant there are 42 workers distributed in two shifts. The plant is highly automated with robots for handling, feeding, palletizing and control systems in every step. The finishing area is made up of panel sectioning machines composed of vertical bar and pressure sawing machines, as well as circular saws of one or several discs, in addition to a final sanding and calibration zone. Also in the work areas and in order to maintain the correct level of particles and VOC's emissions, there is a centralized air aspiration system with subsequent filtration and purification processes prior to its emission to the environment. Quality and safety policies are established. Workers wear personal protective equipment and there are periodic safety checks at the process plant.

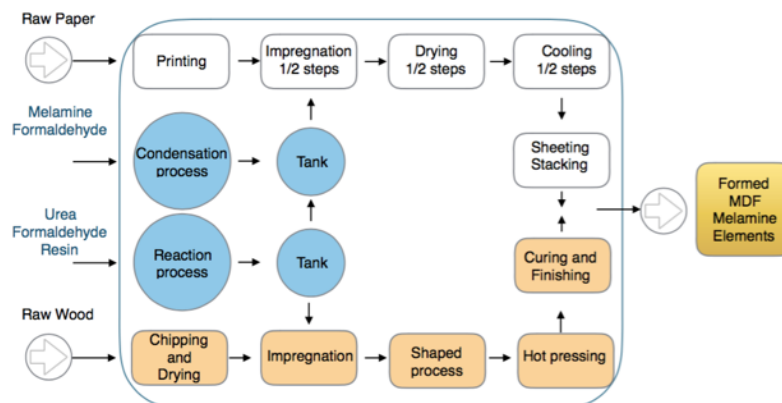


Figure 60.- General production scheme of a medium density fiber (MDF) urea-melamine plant.

The analysis covers the general plant and the bow tie is presented in Figure 61 with five final states: no injuries, minor, serious and fatal [189]. If an accident event (AE) is generated, the mitigating safety barriers (SF<sub>1</sub> SF<sub>2</sub> SF<sub>3</sub> SF<sub>4</sub>) are activated. The final states represented are bounded at one end by the absence of personal injuries, if the first mitigating safety barrier (SF<sub>1</sub>) acts correctly, in case of failure the second barrier (SF<sub>2</sub>) acts ending with a minor injury if it works correctly, in case of failure the third barrier (SF<sub>3</sub>) acts leading to a serious problem in case of correct operation, or in case of failure the fourth barrier activates (SF<sub>4</sub>) leading to a very serious if acts correctly of a fatality in case of failure. The sub-functions (SF<sub>11</sub> SF<sub>21</sub> SF<sub>31</sub> SF<sub>41</sub>) correspond to automatism, procedures, alarms and active or passive protections belonging to the main function of each of the mitigative safety barriers.

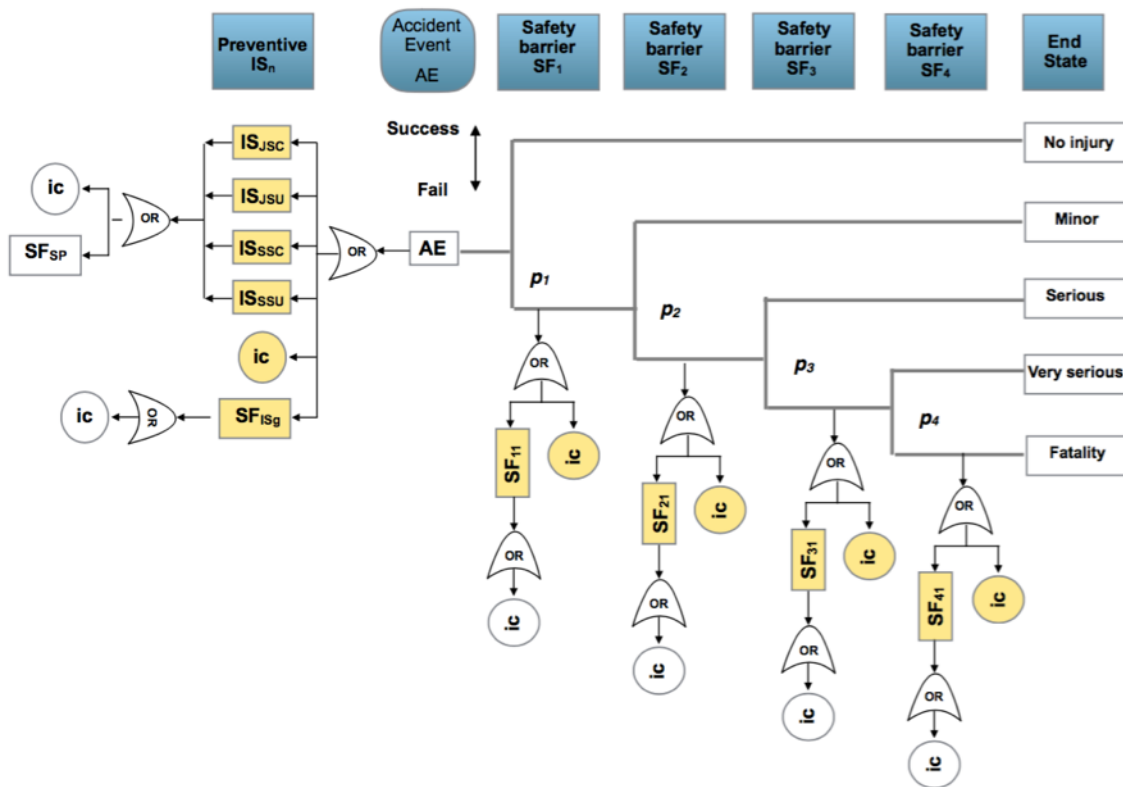


Figure 61.- Bow-tie for occupational accident performed at first level in the MDF process plant.

The preventive safety barriers (IS<sub>n</sub>) are defined in concordance to the general scheme from Figure 14 and their definition together to with the mitigative safety barriers are collected in the following Table 51.

The analysis is carried out at the first level, highlighted in yellow on the graph. The observations are made in the worst case taking the plant in general, which means that the observations are collected on the one hand for the preventive barriers, the general

safety barrier (SF<sub>ISg</sub>) and the initiating causes (ic) that collect all workers in one shift; and on the other hand, from each of the mitigative barriers. Being SF<sub>1</sub>-SF<sub>11</sub> and its (ic) which group the personal equipment protection at place and located in each workplace of the plant; the SF<sub>2</sub>-SF<sub>21</sub> and its (ic's) which group the work in place and local safety systems, fire extinction, alarms and the automated or manual shutdown of every workplace; the SF<sub>3</sub>-SF<sub>31</sub> and its (ic's) which group the general plant protections for shutdown, light, power and fire, and the SF<sub>4</sub>-SF<sub>41</sub> and its (ic's) which group the emergency interventions at own plant with rescue and first medical aid and the explosion-fire protections. The observations are done in a 10 time intervals in a day, covering all shifts.

Preventive Safety barrier (IS <sub>n</sub> )	Attributes-Failures
IS <sub>JSC</sub>	JSC Job Self Control
IS <sub>JSU</sub>	JSU Job Supervision
IS <sub>SSC</sub>	SSC Safety Self Control
IS <sub>SSU</sub>	SSU Safety Supervision
SF <sub>ISg</sub>	Automated actuators and control systems. Process operations control.
SF <sub>SP</sub>	Safety sub-functions: Organization (ORG), Job Design (JBD), Operator environment (OPE), Operator characteristics (OPC), Human system interface (HSI), Information (INF), Work place design (WKD).
SF <sub>1</sub> - SF <sub>11</sub>	Personal Protection Equipment. Equipment condition and state.
SF <sub>2</sub> - SF <sub>21</sub>	Work in place safety system. Automated local protections. Local fire system. Local shutdown.
SF <sub>3</sub> - SF <sub>31</sub>	General control safety system. Automated general protections. General fire system. General Shutdown.
SF <sub>4</sub> - SF <sub>41</sub>	Emergency intervention. First aid and medical. General rescue actuations. General passive-active for fire and explosion protections.

Table 51.- Preventive and mitigative safety barriers definition for the occupational analysis of the MDF process plant.

Table 52 shows the observed causes in different intervals in a working day.

Interval	1	2	3	4	5	6	7	8	9	10
Observations	-	-	1 IS <sub>JSC</sub>	1 SF <sub>1</sub> - SF <sub>11</sub>	-	1SF <sub>2</sub> - SF <sub>21</sub>	1 IS <sub>SSC</sub>	1 (FTS)		
1 JSC job self control failure, 1 personal equipment protection defect in impregnation, 1 automation failure in hot pressing, 1 SSC safety self control fail, 1 FTS failed test of the emergency power system.										

Table 52.- Initiating causes (ic) and safety barrier failures in 10 intervals.

4.2.1. Poisson-gamma model.

Applying this model with parameter  $p = \lambda$ , (Annex A) and with a recurrent with mean prior method and equal to a desired value as a target, is applied. In this case the target is for have zero accidents, then the parameters of the gamma prior are  $\alpha = \beta = 0.001$ . Working with  $\pm 1\sigma_{post}$  the posterior values are, Table 53.

Time interval	Prior		Posterior				Observed	LCL	MEAN	UCL
	alfa	beta	alfa	beta	lambda	sigma	lambda			
0	0.001	0.001	-	-	0	0	0	-	0	-
1	0.001	0.001	0.001	1	0	0.03	0	0	0	0.03
2	0.001	1	0.001	2	0	0.02	0	0	0	0.02
3	0.001	2	1	3	0.3	0.3	0.3	0	0	0.3
4	1	3	2	4	0.5	0.35	1	0	0	0.35
5	2	4	2	5	0.4	0.3	0	0	0	0.3
6	2	5	3	6	0.5	0.3	0.5	0	0	0.3
7	3	6	4	7	0.6	0.3	1	0	0	0.3
8	4	7	5	8	0.6	0.3	1	0	0	0.3
9	5	8	5	9	0.56	0.25	0	0	0	0.25
10	5	9	5	10	0.5	0.2	0	0	0	0.2

Table 53.- MDF plant. Poisson-gamma model. Observations using a recurrent method with mean prior. Out-of-limits highlighted blue.

And charts in intervals 6 and 7 on Figure 62.

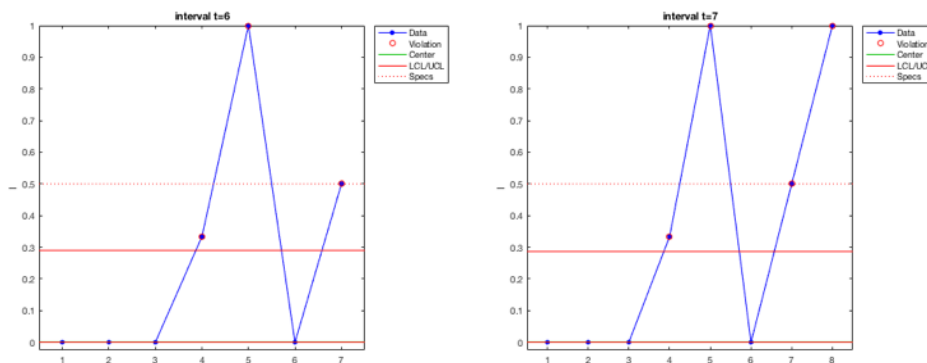


Figure 62.- Poisson-gamma model. Recurrent with mean prior method. Charts based on observed causes for intervals 6 and 7.

4.2.2. Analysis of the end states observing the mitigative safety barriers.

a.- Recurrent method

It is of interest to monitor the mitigative safety barriers activity in order to infer the evolution of the possible end states.

The prior matrices are defined being the transition matrix for the five end states and the emission matrix indicating the probabilities to stay in an end state  $S_n$  being:  $S_1$ =No injury;  $S_2$ =Minor injury;  $S_3$ =Serious injury;  $S_4$ =Very serious injury and  $S_5$ =Fatality; in function of the observed four active mitigative barriers ( $V_1 V_2 V_3 V_4$ ).

A prior transition matrix is defined for the five end states and additionally an emission matrix is defined to indicate the probabilities that an end state be active based on the observed safety barriers. ( $V_1 V_2 V_3 V_4$ ).

$$Tran = \begin{pmatrix} 0.8 & 0.10 & 0.05 & 0.03 & 0.02 \\ 0.5 & 0.3 & 0.1 & 0.05 & 0.05 \\ 0.2 & 0.3 & 0.3 & 0.1 & 0.1 \\ 0.15 & 0.3 & 0.4 & 0.1 & 0.05 \\ 0.10 & 0.2 & 0.3 & 0.35 & 0.05 \end{pmatrix} \quad Emiss = \begin{pmatrix} & V_1 & V_2 & V_3 & V_4 \\ S_1 & 0.6 & 0.2 & 0.15 & 0.05 \\ S_2 & 0.4 & 0.4 & 0.15 & 0.05 \\ S_3 & 0.3 & 0.3 & 0.25 & 0.15 \\ S_4 & 0.25 & 0.25 & 0.25 & 0.25 \\ S_5 & 0.20 & 0.20 & 0.25 & 0.35 \end{pmatrix}$$

In concordance with the previous defined transition and emission matrices the expected use of the end states and the observed safety barrier are presented in the Figure 63.

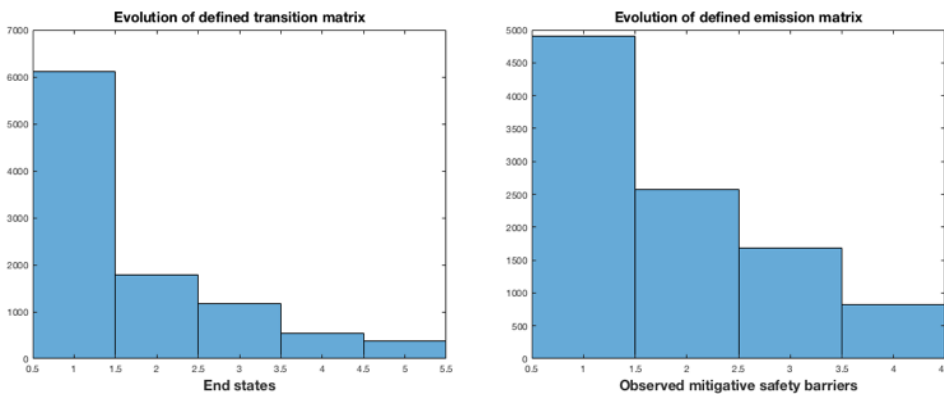


Figure 63.- MDF plant. Initial transitions of the end states and emission matrices.

As the previous case, the sampling is obtained by sequentially observing the barriers, if the first is active, then its value is 1, if it fails and the second barrier is active, then the



value is 2 and so on; a group of ten observations is also maintained. The sampled sequence of the safety barriers is  $seqobs=[1\ 1\ 1\ 1\ 1\ 1\ 2\ 3\ 1\ 1]$ . And the posterior transition and emission matrices are.

$$Transobs = \begin{pmatrix} 0.86 & 0 & 0 & 0.08 & 0.06 \\ 0.95 & 0.05 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad Emisobs = \begin{pmatrix} & V_1 & V_2 & V_3 & V_4 \\ S_1 & 1 & 0 & 0 & 0 \\ S_2 & 1 & 0 & 0 & 0 \\ S_3 & 0 & 0 & 1 & 0 \\ S_4 & 0 & 1 & 0 & 0 \\ S_5 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Being the relative occupancies  $S_1=69.65\%$ ,  $S_2=10.44\%$ ,  $S_3=9.95\%$ ,  $S_4=5.26\%$  and  $S_5=4.69\%$  at steady state, Figure 64.

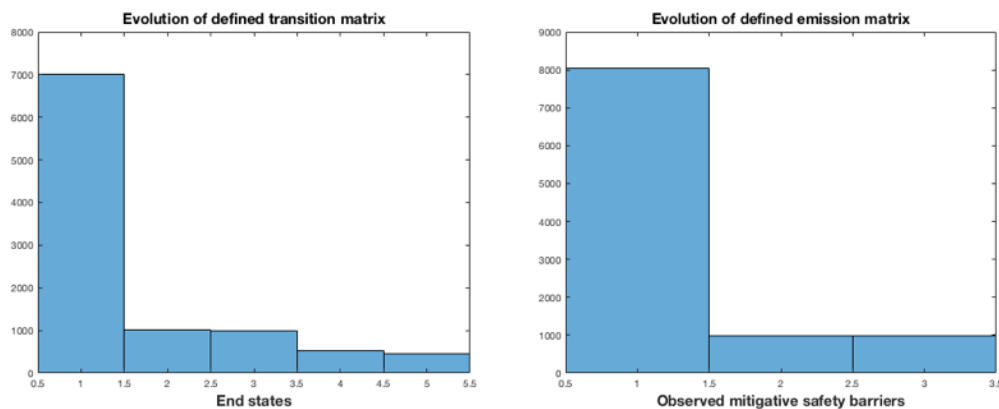


Figure 64.- MDF plant. Transitions of the end states and emission matrices according to the first sequence observation of safety barriers.

And number of visits or transitions, in 10 intervals.

$$M(10) = \begin{pmatrix} 7.0912 & 0.8953 & 0.9571 & 0.5581 & 0.4983 \\ 7.3602 & 0.8345 & 0.8530 & 0.5030 & 0.4492 \\ 6.6652 & 1.7285 & 0.7533 & 0.4506 & 0.4024 \\ 5.9709 & 1.6235 & 1.6522 & 0.3980 & 0.3554 \\ 5.9709 & 1.6235 & 1.6522 & 0.3980 & 0.3554 \end{pmatrix}$$

The next passage values for the fifth state, are  $m_1=18.25$ ,  $m_2=19.30$ ,  $m_3=20.30$  and  $m_4=21.30$ .

Using a recurrent method the posterior transition and emission matrices (Transobs and Emisobs), can be the new prior in the next sampling, being seqobs=[1 1 1 1 1 2 3 1 1 2] showing an increase of the use of the safety barrier SF<sub>2</sub>, a new posterior for transition and emission matrices is obtained.

$$\text{Transobs} = \begin{pmatrix} 0.31 & 0.31 & 0 & 0.20 & 0.18 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad \text{Emisobs} = \begin{pmatrix} & V_1 & V_2 & V_3 & V_4 \\ S_1 & 1 & 0 & 0 & 0 \\ S_2 & 1 & 0 & 0 & 0 \\ S_3 & 0 & 0 & 1 & 0 \\ S_4 & 0 & 1 & 0 & 0 \\ S_5 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Being the steady state occupations  $S_1=41\%$ ,  $S_2=28.2\%$ ,  $S_3=15.4\%$ ,  $S_4=8.1\%$  and  $S_5=7.3\%$ , Figure 65.

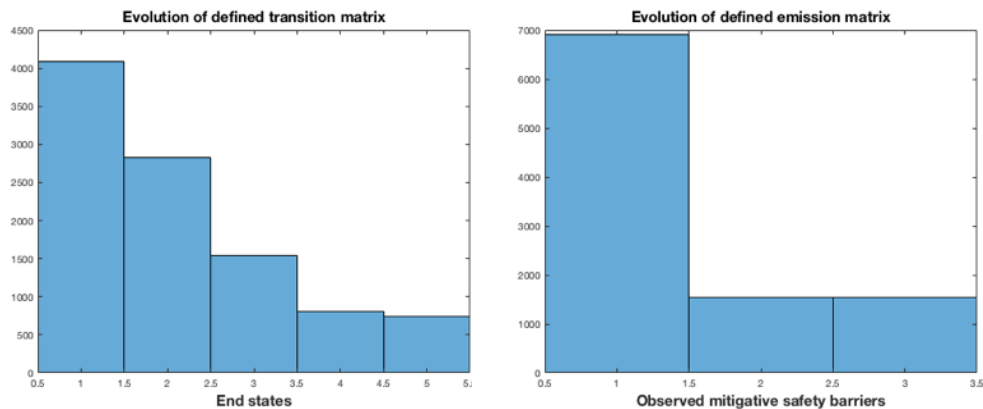


Figure 65.- MDF plant. Transitions of the end states and emission matrices, using a recurrent method, according to the second sequence observation of safety barriers.

And the number of visits or transitions, in 10 intervals.

$$M(10) = \begin{pmatrix} 3.9555 & 2.7958 & 1.5517 & 0.8965 & 0.8005 \\ 4.5223 & 2.5474 & 1.3786 & 0.8197 & 0.7320 \\ 4.1351 & 3.2347 & 1.2515 & 0.7283 & 0.6503 \\ 3.6739 & 2.9912 & 2.0834 & 0.6611 & 0.5904 \\ 3.6739 & 2.9912 & 2.0834 & 0.6611 & 0.5904 \end{pmatrix}$$

With next passage values for the fourth state;  $m_1=10.8$ ,  $m_2=11.8$ ,  $m_3=12.8$  and  $m_4=13.8$ .

b.- Direct method

Applying a direct method the initially defined transition and emission matrices (Tran and Emiss), are the prior in the next sampling with seqobs=[1 1 1 1 1 2 3 1 1 2]. And the posterior matrices are,

$$Transobs = \begin{pmatrix} 0.75 & 0.25 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad Emisobs = \begin{pmatrix} & V_1 & V_2 & V_3 & V_4 \\ S_1 & 1 & 0 & 0 & 0 \\ S_2 & 1 & 0 & 0 & 0 \\ S_3 & 0 & 0 & 1 & 0 \\ S_4 & 0 & 1 & 0 & 0 \\ S_5 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Being the steady state occupations  $S_1=0\%$ ,  $S_2=25\%$ ,  $S_3=25\%$ ,  $S_4=25\%$  and  $S_5=25\%$ , Figure 66.

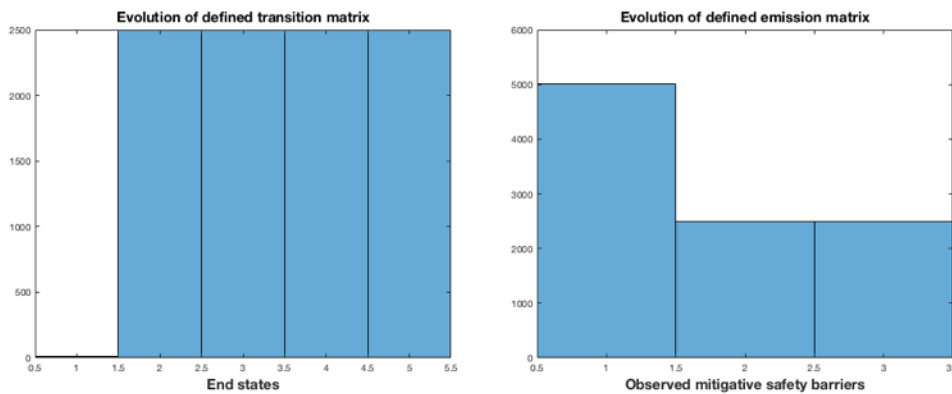


Figure 66.- MDF plant. Transitions of the end states and emission matrices, using a direct method, according to the second sequence observation of safety barriers.

And the number of visits or transitions, in 10 intervals.

$$M(10) = \begin{pmatrix} 2.8310 & 2.2032 & 1.4446 & 1.5835 & 1.9376 \\ 0 & 2.0000 & 2.0000 & 3.0000 & 3.0000 \\ 0 & 3.0000 & 2.0000 & 2.0000 & 3.0000 \\ 0 & 3.0000 & 3.0000 & 2.0000 & 2.0000 \\ 0 & 2.0000 & 3.0000 & 3.0000 & 2.0000 \end{pmatrix}$$

The next passage values for the fifth state, are  $m_1=5$ ,  $m_2=1$ ,  $m_3=2$  and  $m_4=3$ .

### 4.3. Cost-time deviations analysis. Cogeneration and compression units.

The same considerations set out in the previous section apply using the generic scheme of Figure 17 and therefore the same inference models and the same parameter  $p$  corresponding to the rate  $\lambda$  or frequency of events as regards the analysis of events apply. The same binomial-beta model is also applicable to carry out the analysis of safety barriers together with the use of hidden Markov chains.

But this is a case where the own deviations are applied as data collected and applied into the inference model to perform the SRC analysis.

The following case corresponds to an installation of energy cogeneration placed in a waste recovery plant.

This is a 24-7 uninterrupted power turbine unit of 1 MW producing electricity and steam. Figure 67.

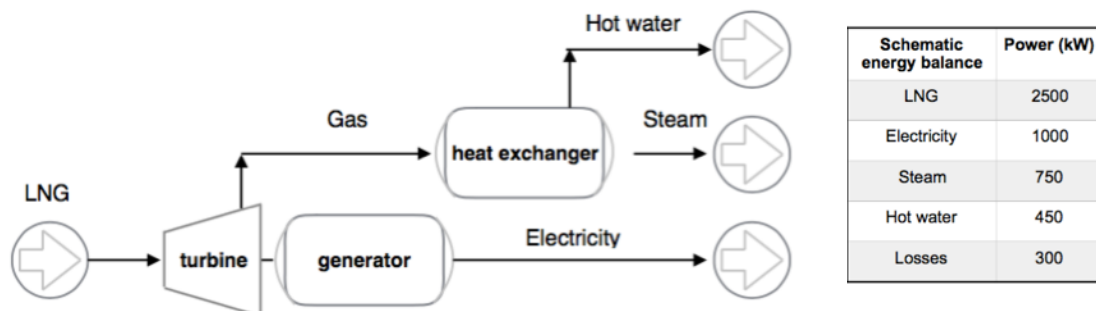


Figure 67.- Cogeneration unit

The project lasted 7 weeks. And the generic bow-tie diagram exposed in Figure 17 can be applied.

From Figure 16, the sum of raw materials + components + supplies + variable + fixed costs for the planned project planned and the actual costs, by period and their accumulated, are shown in the following Table 54.

Time interval / activity	Planned Costs (k€)	Accumulated planned costs (k€)	Actual Costs (k€)	Accumulated actual costs (k€)
1	68	68	69	69
2	74	142	73	142
3	82	224	85	227
4	110	334	115	342
5	115	449	125	467
6	93	542	90	557
7	86	628	79	636
Totals	628	-	636	-

Table 54.- Cogeneration unit project. Variable plus fixed costs evolution.

In order to evaluate whether it is possible to apply an inference model based on the data in a normal function, it is examined whether these correspond to this type of distribution, Figure 68.

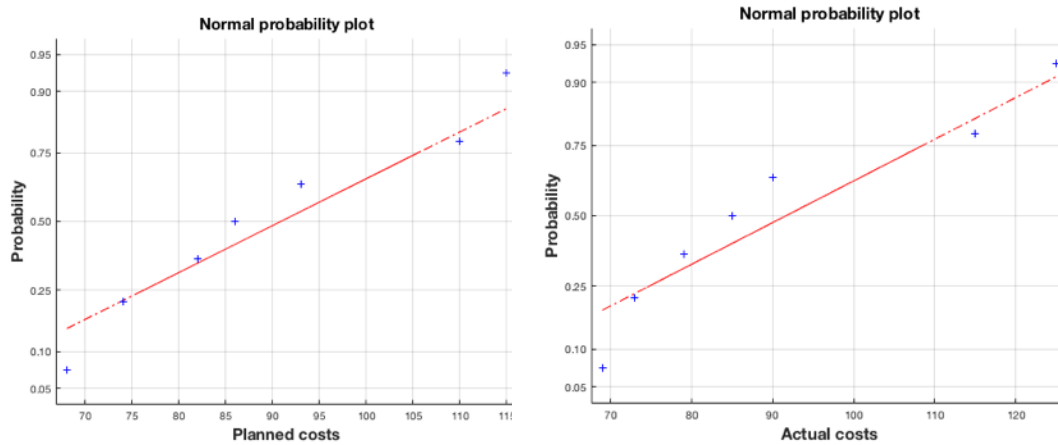


Figure 68.-Adjustment verification to a normal distribution for the planned and actual costs of the cogeneration project.

#### 4.3.1. Normal-normal model.

With a Normal-normal model the expression (1) with parameter  $p=\mu$ , (Annex A) offers a posterior normal distribution  $(\mu / y) \sim N(\mu_*, \sigma_*^2)$  with;

$$\mu_* = (1 - k)\mu_0 + k\bar{y} \tag{26}$$

$$\sigma_*^2 = k \frac{\sigma^2}{n} \quad (27)$$

$$\text{with } k = \frac{n}{n + \frac{\sigma^2}{\sigma_0^2}} \quad (28)$$

Where the data  $y$  follow a normal probability function, with a sample mean  $\bar{y} = \frac{(y_1 + \dots + y_n)}{n}$  and being  $\mu_0; \sigma_0^2$  the parameters of the prior normal distribution for  $\mu$  and  $\sigma^2$  the standard distribution of the  $n$  data.

Considering the planned costs as a prior with mean prior  $\mu_0 = 89.7$  and sigma prior  $\sigma_0 = 17.6$ . The SRC methodology for cost deviation applying control charts working with  $\pm 3\sigma_{post}$  has the following results.

a.- Recurrent method with mean prior.

The Table 55 shows the observed values. With the following comments in every interval.

Interval 1.- With value  $y_i = [69]$ , is,  $n=1$ ;  $\mu=69$ ;  $\sigma=0$ . Then from equations (27), (28) and (29) values of  $k=1$ ;  $\mu_{post} = 69$ ;  $\sigma_{post} = 0$  are obtained and with  $\pm 3\sigma_{post}$  the control limits are: LCL = 89.7; MEAN = 89.7 and UCL = 89.7.

Interval 2. With value  $y=73$ ; the data vector is  $y_i = [69 \ 73]$ , with  $n=2$ ;  $\mu=71$ ;  $\sigma=2$ . Values of  $k=1$ ;  $\mu_{post} = 71$ ;  $\sigma_{post} = 1.4$  are obtained, with control limits LCL = 85.5; MEAN = 89.7 and UCL = 93.9.

Interval 3. With value  $y=85$ ; the data vector is  $y_i = [69 \ 73 \ 85]$ , with  $n=3$ ;  $\mu=75.7$ ;  $\sigma=6.8$ . Values of  $k=0.1$ ;  $\mu_{post} = 71.5$ ;  $\sigma_{post} = 1.3$  are obtained, with control limits LCL = 85.7; MEAN = 89.7 and UCL = 93.7.

Interval 4. With value  $y=115$ , and data vector  $y_i = [69 \ 73 \ 85 \ 115]$ , with  $n=4$ ;  $\mu=85.5$ ;  $\sigma=18$ . Values of  $k=0.02$ ;  $\mu_{post} = 71.8$ ;  $\sigma_{post} = 1.3$  are obtained, with control limits LCL = 85.8; MEAN = 89.7 and UCL = 93.7.

Interval 5. With value  $y=125$ , and data vector  $y_i=[69\ 73\ 85\ 115\ 125]$ , with  $n=5$ ;  $\mu=93.5$ ;  $\sigma=22.6$ . Values of  $k=0.02$ ;  $\mu_{post}=72.2$ ;  $\sigma_{post}=1.3$  are obtained, with control limits LCL = 85.8; MEAN = 89.7 and UCL = 93.6.

Interval 6. With value  $y=90$ , and data vector  $y_i=[69\ 73\ 85\ 115\ 125\ 90]$ , with  $n=6$ ;  $\mu=92.8$ ;  $\sigma=20.6$ . Values of  $k=0.02$ ;  $\mu_{post}=72.7$ ;  $\sigma_{post}=1.3$  are obtained, with control limits LCL = 85.9; MEAN = 89.7 and UCL = 93.6.

Interval 7. With value  $y=79$ , and data vector  $y_i=[69\ 73\ 85\ 115\ 125\ 90\ 79]$ , with  $n=7$ ;  $\mu=90.9$ ;  $\sigma=19.7$ . Values of  $k=0.03$ ;  $\mu_{post}=73.2$ ;  $\sigma_{post}=1.3$  are obtained, with control limits LCL = 85.9; MEAN = 89.7 and UCL = 93.5.

Time interval	Normal Prior		Normal Posterior		Observed	LCL	MEAN	UCL
	mu	sigma	mu	sigma	y			
1	89.7	17.6	69	0	69	89.7	89.7	89.7
2	69	17.6	71	1.4	73	85.5	89.7	93.9
3	71	1.4	71.5	1.3	85	85.7	89.7	93.7
4	71.5	1.3	71.8	1.3	115	85.8	89.7	93.7
5	71.8	1.3	72.2	1.3	125	85.8	89.7	93.6
6	72.2	1.3	72.7	1.3	90	85.9	89.7	93.6
7	72.7	1.3	73.2	1.3	79	85.9	89.7	93.5

Table 55.- Cogeneration unit. Normal-normal model. Observations for cost deviation using a recurrent method with mean prior. Out-of-limits highlighted blue.

And charts for intervals 4 and 5 in Figure 69.

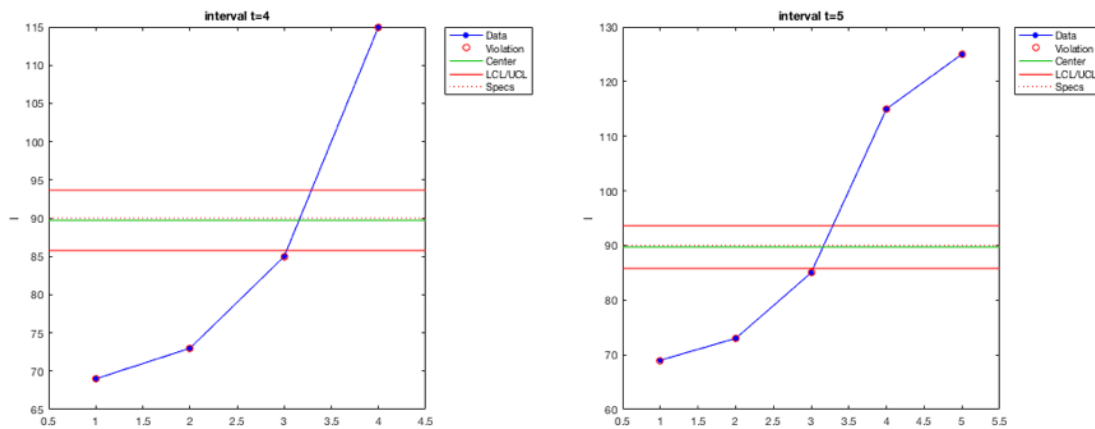


Figure 69.- Normal-normal model. Recurrent with mean prior method. Charts based on observed values in intervals 4 and 5.

b.- Recurrent method with mean posterior.

The Table 56 shows the observed values.

Time interval	Normal Prior		Normal Posterior		Observed	LCL	MEAN	UCL
	mu	sigma	mu	sigma	y			
1	89.7	17.6	69	0	69	69	69	69
2	69	17.6	71	1.4	73	66.8	71	75.2
3	71	1.4	71.5	1.3	85	67.5	71.5	75.5
4	71.5	1.3	71.8	1.3	115	67.9	71.8	75.8
5	71.8	1.3	72.2	1.3	125	68.3	72.2	76.1
6	72.2	1.3	72.7	1.3	90	68.8	72.7	76.5
7	72.7	1.3	73.2	1.3	79	69.4	73.2	77.0

Table 56.- Cogeneration unit. Normal-normal model. Observations for cost deviation using a recurrent method with mean posterior. Out-of-limits highlighted blue.

c.- Direct method with mean prior.

The Table 57 shows the observed values.



Time interval	Normal Prior		Normal Posterior		Observed	LCL	MEAN	UCL
	mu	sigma	mu	sigma	y			
1	89.7	17.6	69	0	69	89.7	89.7	89.7
2	89.7	17.6	71.1	1.4	73	85.5	89.7	93.9
3	89.7	17.6	76.3	3.8	85	78.2	89.7	101.2
4	89.7	17.6	86.4	8	115	65.7	89.7	113.8
5	89.7	17.6	92.5	8.8	125	63.5	89.7	115.9
6	89.7	17.6	92.3	7.6	90	66.9	89.7	112.5
7	89.7	17.6	90.7	6.9	79	69.1	89.7	110.3

Table 57.- Cogeneration unit. Normal-normal model. Observations for cost deviation using a direct method with mean prior. Out-of-limits highlighted blue.

d.- Direct method with mean posterior.

The Table 58 shows the observed values.

Time interval	Normal Prior		Normal Posterior		Observed	LCL	MEAN	UCL
	mu	sigma	mu	sigma	y			
1	89.7	17.6	69	0	69	69	69	69
2	89.7	17.6	71.1	1.4	73	66.9	71.1	75.3
3	89.7	17.6	76.3	3.8	85	64.8	76.3	87.8
4	89.7	17.6	86.4	8	115	62.3	86.4	110.4
5	89.7	17.6	92.5	8.8	125	66.2	92.5	118.7
6	89.7	17.6	92.3	7.6	90	69.5	92.3	115
7	89.7	17.6	90.7	6.9	79	70.1	90.7	111.3

Table 58.- Cogeneration unit. Normal-normal model. Observations for cost deviation using a direct method with mean posterior. Out-of-limits highlighted blue.

The possibility of applying a LogNormal distribution function to the planned and actual cost data can now be examined if they are adjusted after the examination of the probability graph, Figure 70.

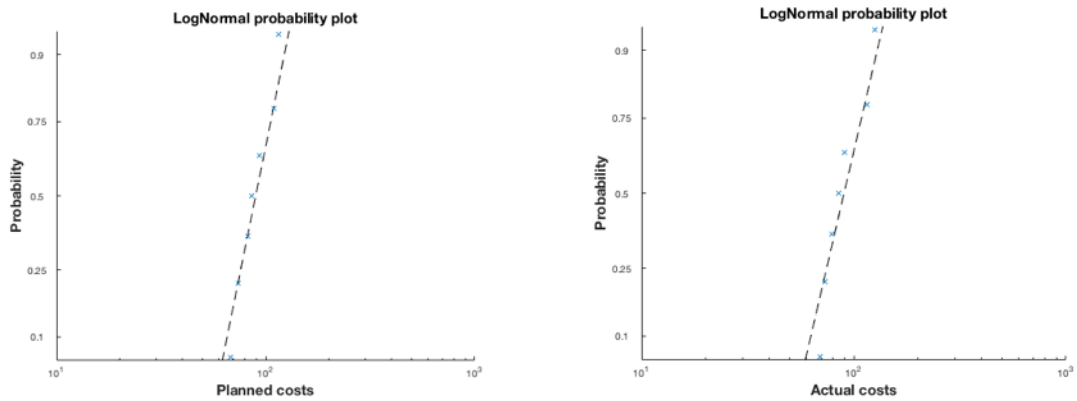


Figure 70.-Adjustment verification to a lognormal distribution for the planned and actual costs of the cogeneration project.

#### 4.3.2. Lognormal-lognormal model.

With a Lognormal-lognormal model the expression (1) with parameter  $p=\mu$  is, (Annex A).

$$f(\mu/y) \propto f(\mu)f(y/\mu) \propto \frac{1}{\mu} \exp\left\{-\frac{1}{2}\left(\frac{\mu-\mu_0}{\sigma_0}\right)^2\right\} \cdot \prod_{i=1}^n \frac{1}{y_i} \exp\left\{-\frac{1}{2}\left(\frac{\ln y_i - \mu}{\sigma}\right)^2\right\} \quad (29)$$

This expression needs to be solved using the Metropolis-Hastings sampler. Also considering the planned costs as a prior with mean prior  $\mu_0 = 89.7$  and sigma prior  $\sigma_0 = 17.6$ . The SRC methodology for cost deviation applying control charts working with  $\pm 3\sigma_{\text{post}}$  has the following results.

a.- Recurrent method with mean prior.

The Table 59 shows the observed values. An Figures 71 and 72 the Metropolis - Hastings values in intervals 4 and 5.

Time interval	LogNormal Prior		LogNormal Posterior			Observed	LCL	MEAN	UCL
	mu	sigma	mu	CI [5%-95%]	sigma	y			
1	89.7	17.6	79.4	79.1-79.8	10.5	69	58.2	89.7	121.2
2	79.4	10.5	76.9	76.6-77.3	8.3	73	64.8	89.7	114.6
3	76.9	8.3	76.4	76.3-76.7	6.0	85	71.7	89.7	107.7
4	76.4	6.0	77.7	77.5-77.8	5.8	115	72.3	89.7	107.1
5	77.7	5.8	78.9	78.8-79	5.8	125	72.3	89.7	107.1
6	78.9	5.8	80.2	80-80.4	5.5	90	73.2	89.7	106.2
7	80.2	5.5	81.2	81.1-81.4	5.3	79	73.8	89.7	105.6

Table 59.- Cogeneration unit. Lognormal-lognormal model. Observations for cost deviation using a recurrent method with mean prior. Out-of-limits highlighted blue.

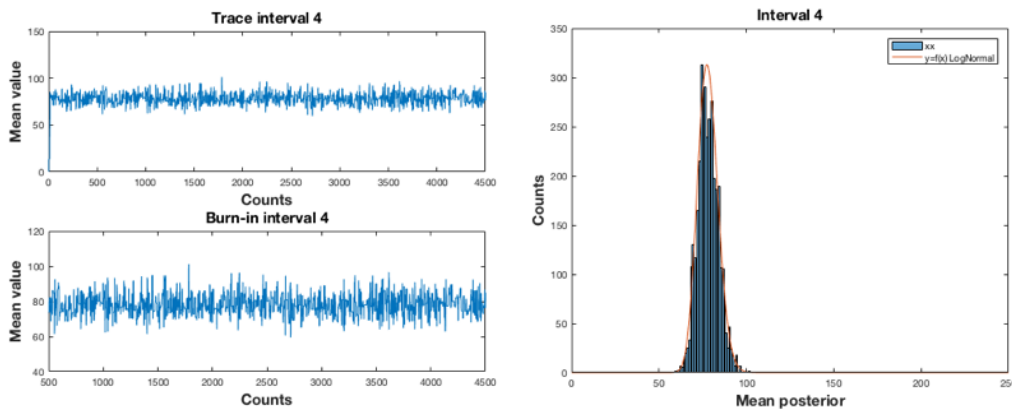


Figure 71.- Lognormal-lognormal. Interval 4. Sampling n=4500, burn=500; 10 cycles. AR= 25.8%.

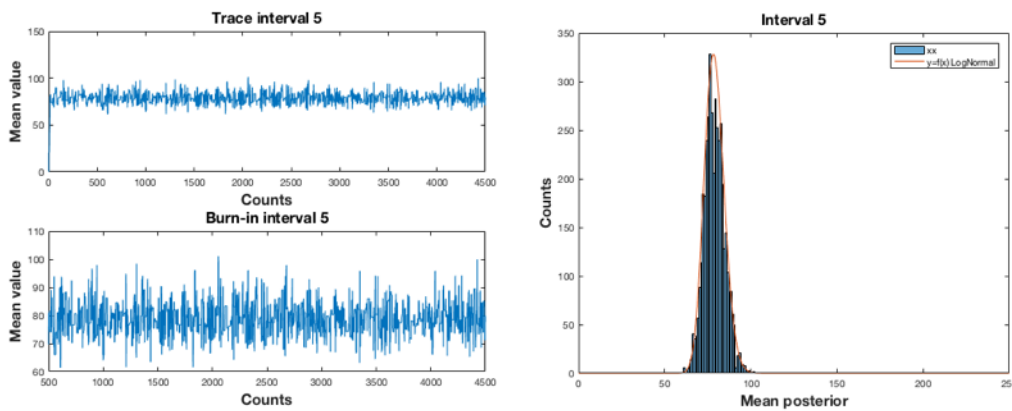


Figure 72.- Lognormal-lognormal. Interval 5. Sampling n=4500, burn=500; 10 cycles. AR= 25.7%.

b.- Recurrent method with mean posterior.

The Table 60 shows the observed values.

Time interval	LogNormal Prior		LogNormal Posterior			Observed	LCL	MEAN	UCL
	mu	sigma	mu	CI [5%-95%]	sigma	y			
1	89.7	17.6	79.4	79.1-79.8	10.5	69	47.9	79.4	110.9
2	79.4	10.5	76.9	76.6-77.3	8.3	73	52	76.9	101.8
3	76.9	8.3	76.4	76.3-76.7	6.0	85	58.4	76.4	94.4
4	76.4	6.0	77.7	77.5-77.8	5.8	115	60.3	77.7	95.1
5	77.7	5.8	78.9	78.8-79	5.8	125	61.5	78.9	96.3
6	78.9	5.8	80.2	80-80.4	5.5	90	63.7	80.2	96.7
7	80.2	5.5	81.2	81.1-81.4	5.3	79	65.3	81.2	97.1

Table 60.- Cogeneration unit. Lognormal-lognormal model. Observations for cost deviation using a recurrent method with mean posterior. Out-of-limits highlighted blue.

c.- Direct method with mean prior.

The Table 61 shows the observed values.

Time interval	LogNormal Prior		LogNormal Posterior			Observed	LCL	MEAN	UCL
	mu	sigma	mu	CI [5%-95%]	sigma	y			
1	89.7	17.6	79.4	79.1-79.8	10.5	69	58.2	89.7	121.2
2	89.7	17.6	80.3	80.1-80.6	11	73	56.7	89.7	122.7
3	89.7	17.6	79.1	78.9-79.3	7.7	85	66.6	89.7	112.8
4	89.7	17.6	88.9	88.5-89.3	13.7	115	48.6	89.7	130.8
5	89.7	17.6	92.3	91.9-92.6	14.4	125	46.5	89.7	132.9
6	89.7	17.6	92	91.6-92.4	14	90	46.5	89.7	131,7
7	89.7	17.6	91.4	91.2-91.7	14.1	79	47.4	89.7	132

Table 61.- Cogeneration unit. Lognormal-lognormal model. Observations for cost deviation using a direct method with mean prior. Out-of-limits highlighted blue.

The Figure 73 show the Metropolis-Hastings sampling execution for interval 7.

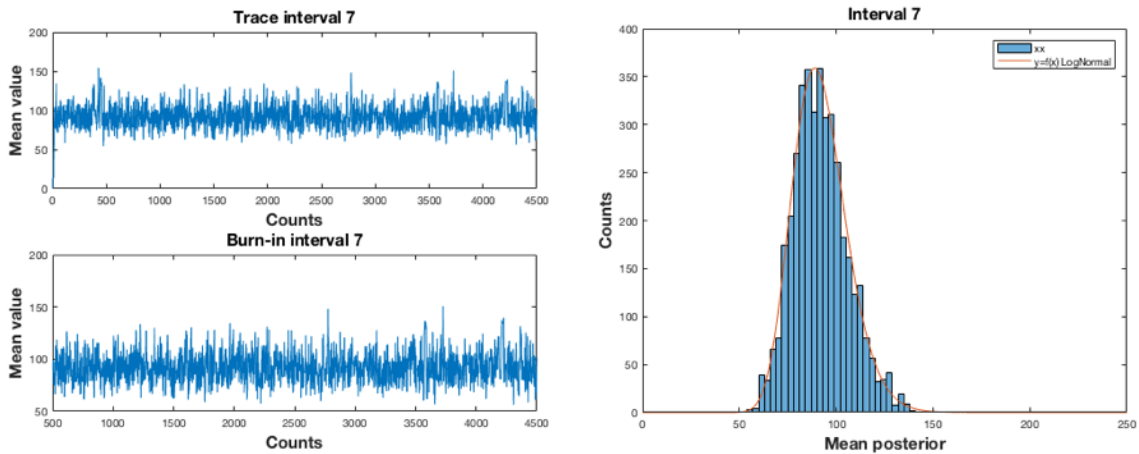


Figure 73.- Lognormal-lognormal. Interval 7. Sampling n=4500, burn=500; 10 cycles. AR= 53.4%.

d.- Direct method with mean posterior.

The Table 62 shows the observed values.

Time interval	LogNormal Prior		LogNormal Posterior			Observed	LCL	MEAN	UCL
	mu	sigma	mu	CI [5%-95%]	sigma	y			
1	89.7	17.6	79.4	79.1-79.8	10.5	69	47.9	79.4	110.9
2	89.7	17.6	80.3	80.1-80.6	11	73	47.3	80.3	113.3
3	89.7	17.6	79.1	78.9-79.3	7.7	85	56	79.1	102.2
4	89.7	17.6	88.9	88.5-89.3	13.7	115	47.8	88.9	130
5	89.7	17.6	92.3	91.9-92.6	14.4	125	49.1	92.3	135.5
6	89.7	17.6	92	91.6-92.4	14	90	50	92	134
7	89.7	17.6	91.4	91.2-91.7	14.1	79	49.1	91.4	133.7

Table 62.- Cogeneration unit. Lognormal-lognormal model. Observations for cost deviation using a direct method with mean posterior. Out-of-limits highlighted blue.

4.3.3. Lognormal-gamma model.

With a LogNormal-gamma model the expression (1) with parameter  $p=\mu$  is, (Annex A).

$$f(\mu / y) \propto f(\mu) f(y / \mu) \propto \mu^{\alpha-1} e^{-\frac{\mu}{\beta}} \cdot \prod_{i=1}^n \frac{1}{y_i} \exp \left\{ -\frac{1}{2} \left( \frac{\ln y_i - \mu}{\sigma} \right)^2 \right\} \quad (30)$$

This expression also needs to be solved using the Metropolis-Hastings sampler. With the same consideration of planned costs as a prior with mean prior  $\mu_0 = 89.7$  and sigma prior  $\sigma_0 = 17.6$ , the values for gamma are  $\alpha_0 = 26.1237$  and  $\beta_0 = 3.434$ . The SRC methodology for cost deviation applying control charts working with  $\pm 3\sigma_{\text{post}}$  has the following results.

a.- Recurrent method with mean prior.

The Table 63 shows the observed values.

Time interval	Gamma Prior		Gamma Posterior			Observed	LCL	MEAN	UCL
	mu	sigma	mu	CI [5%-95%]	sigma	y			
1	89.7	17.6	78.7	78.4-78.9	10.8	69	57.3	89.7	122.1
2	78.7	10.8	76.5	76.3-76.6	8.8	73	63.3	89.7	116.1
3	76.5	8.8	76.2	76.1-76.3	5.7	85	72.6	89.7	106.8
4	76.2	5.7	76.9	76.8-77	5.7	115	72.6	89.7	106.8
5	76.9	5.7	77.9	77.85-78	5.8	125	72.3	89.7	107.1
6	77.9	5.8	79.2	79.4-79.7	5.6	90	72.9	89.7	106.5
7	79.2	5.6	80.1	79.9-80.3	5.3	79	73.8	89.7	105.6

Table 63.- Cogeneration unit. Lognormal-gamma model. Observations for cost deviation using a recurrent method with mean prior. Out-of-limits highlighted blue.

The Figures 74 and 75 show the Metropolis-Hastings sampling execution for intervals 4 and 5.

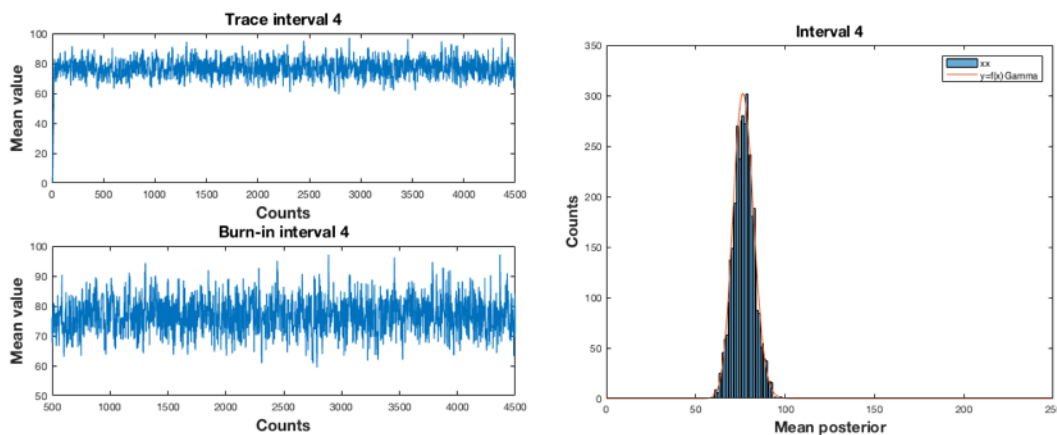


Figure 74.- Lognormal-gamma. Interval 4. Sampling n=4500, burn=500; 10 cycles. AR= 52.5%.

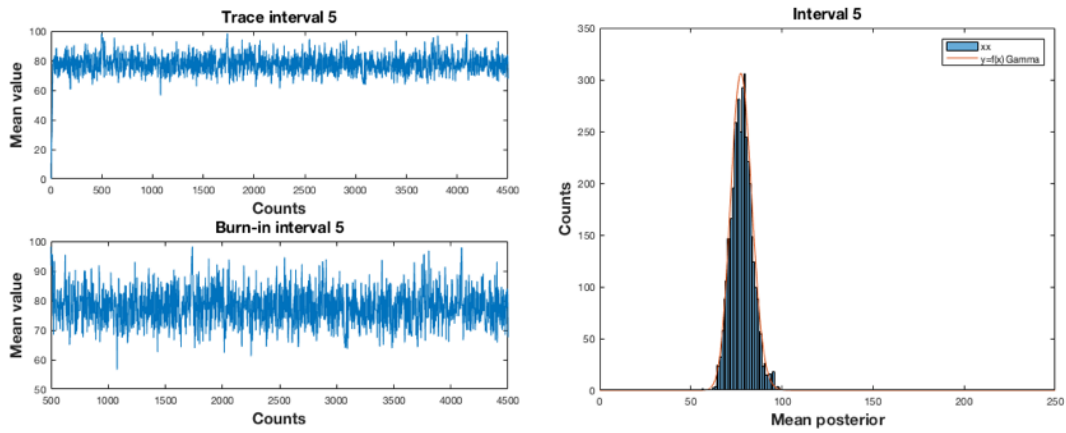


Figure 75.- Lognormal-gamma. Interval 5. Sampling n=4500, burn=500; 10 cycles. AR= 53.1%.

b.- Recurrent method with mean posterior.

The Table 64 shows the observed values.

Time interval	Gamma Prior		Gamma Posterior			Observed	LCL	MEAN	UCL
	mu	sigma	mu	CI [5%-95%]	sigma	y			
1	89.7	17.6	78.7	78.4-78.9	10.8	69	46.3	78.7	111.1
2	78.7	10.8	76.5	76.3-76.6	8.8	73	50.1	76.5	102.9
3	76.5	8.8	76.2	76.1-76.3	5.7	85	59.1	76.2	93.3
4	76.2	5.7	76.9	76.8-77	5.7	115	59.8	76.9	94
5	76.9	5.7	77.9	77.85-78	5.8	125	60.5	77.9	95.3
6	77.9	5.8	79.2	79.4-79.7	5.6	90	62.4	79.2	96
7	79.2	5.6	80.1	79.9-80.3	5.3	79	64.2	80.1	96

Table 64.- Cogeneration unit. Lognormal-gamma model. Observations for cost deviation using a recurrent method with mean posterior. Out-of-limits highlighted blue.

c.- Direct method with mean prior.

The Table 65 shows the observed values.

Time interval	Gamma Prior		Gamma Posterior			Observed	LCL	MEAN	UCL
	mu	sigma	mu	CI [5%-95%]	sigma	y			
1	89.7	17.6	78.7	78.5-79	10.9	69	57	89.7	122.4
2	89.7	17.6	79.8	79.4-80.2	11.3	73	55.8	89.7	123.6
3	89.7	17.6	78.7	78.5-78.9	7.6	85	66.9	89.7	112.5
4	89.7	17.6	88.2	87.9-88.5	13.3	115	49.8	89.7	129.6
5	89.7	17.6	91.5	90.9-92.1	14.1	125	47.4	89.7	132
6	89.7	17.6	91.1	90.4-91.8	14.2	90	47.1	89.7	132.3
7	89.7	17.6	90.3	89.8-90.9	13.3	79	49.8	89.7	129.6

Table 65.- Cogeneration unit. Lognormal-gamma model. Observations for cost deviation using a direct method with mean prior. Out-of-limits highlighted blue.

The Figure 76 show the Metropolis-Hastings sampling execution for interval 7.

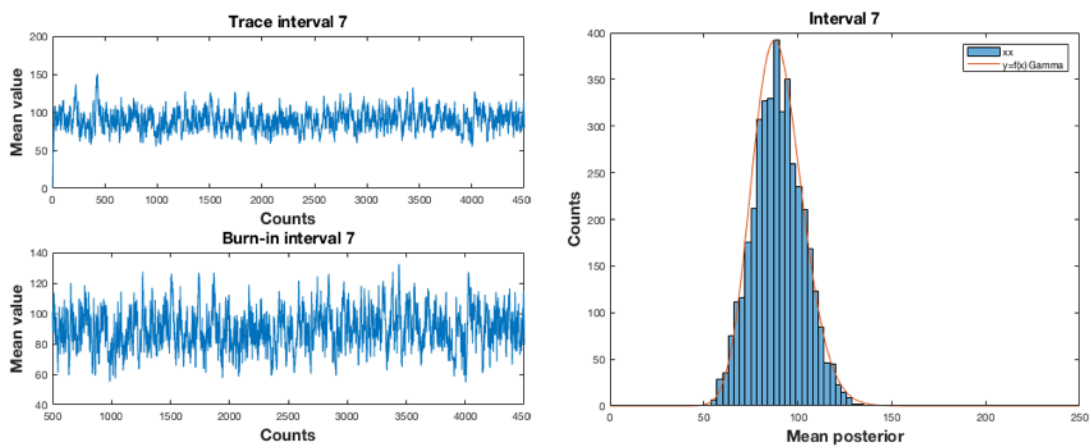


Figure 76.- Lognormal-gamma. Interval 7. Sampling n=4500, burn=500; 10 cycles. AR= 77.6%.

d.- Direct method with mean posterior.

The Table 66 shows the observed values.



Time interval	Gamma Prior		Gamma Posterior			Observed	LCL	MEAN	UCL
	mu	sigma	mu	CI [5%-95%]	sigma	y			
1	89.7	17.6	78.7	78.5-79	10.9	69	46	78.7	111.4
2	89.7	17.6	79.8	79.4-80.2	11.3	73	45.9	79.8	113.7
3	89.7	17.6	78.7	78.5-78.9	7.6	85	55.9	78.7	101.5
4	89.7	17.6	88.2	87.9-88.5	13.3	115	48.3	88.2	128.1
5	89.7	17.6	91.5	90.9-92.1	14.1	125	49.2	91.5	133.8
6	89.7	17.6	91.1	90.4-91.8	14.2	90	48.5	91.1	133.7
7	89.7	17.6	90.3	89.8-90.9	13.3	79	50.4	90.3	130.2

Table 66.- Cogeneration unit. Lognormal-gamma model. Observations for cost deviation using a direct method with mean posterior. Out-of-limits highlighted blue.

#### 4.3.4. Lognormal-uniform model.

With a LogNormal-uniform model the expression (1) with parameter  $p=\mu$  is, (Annex A).

$$f(\mu / y) \propto f(\mu) f(y / \mu) \propto \frac{1}{\mu} \cdot \prod_{i=1}^n \frac{1}{y_i} \exp \left\{ -\frac{1}{2} \left( \frac{\ln y_i - \mu}{\sigma} \right)^2 \right\} \quad (31)$$

This expression can be solved using the Metropolis-Hastings sampler. With this model of inference the SRC methodology considering the planned costs with mean prior  $\mu_0 = 89.7$  and sigma prior  $\sigma_0 = 17.6$  and working with  $\pm 1\sigma_{\text{post}}$  the methodology group the recurrent into only one and has not direct method.

##### a.- Mean prior

The Table 67 shows the observed values. Figures 77 and 78 show the sampling in the intervals 4 and 5.

	Posterior			Observed			
Time interval	mu	CI [5%-95%]	sigma	y	LCL	MEAN	UCL
1	70.4	70.2-70.6	14.2	69	75.5	89.7	103.9
2	72.3	71.9-72.6	14.3	73	75.4	89.7	104
3	76.1	75.9-76.3	8	85	81.7	89.7	97.7
4	88.3	87.9-88.7	20.6	115	69.1	89.7	110.3
5	96.9	95.9-98	26.2	125	63.5	89.7	115.9
6	95.3	94.6-96.1	23.2	90	66.5	89.7	112.9
7	92.9	91.8-94	21.6	79	68.1	89.7	111.3

Table 67.- Cogeneration unit. Lognormal-uniform model. Observations for cost deviation with mean prior. Out-of-limits highlighted blue.

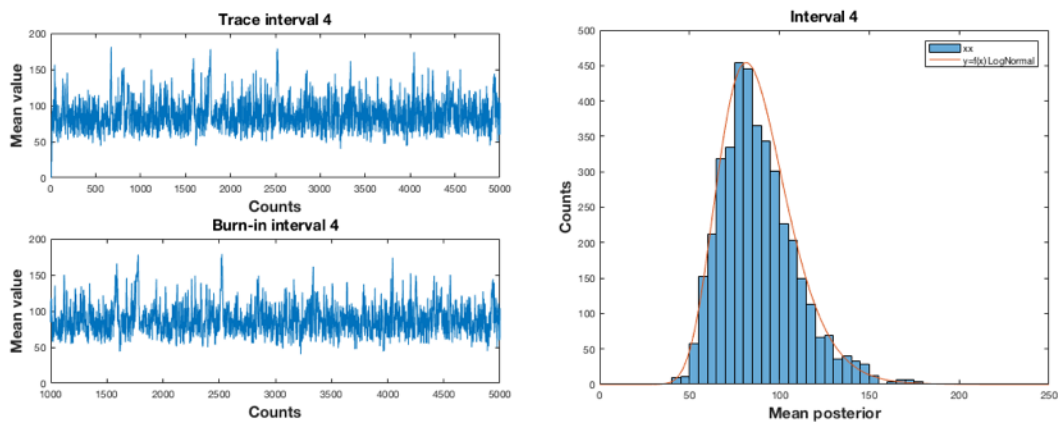


Figure 77.- Lognormal-uniform. Interval 4. Sampling n=4500, burn=500; 10 cycles. AR= 65.9%.

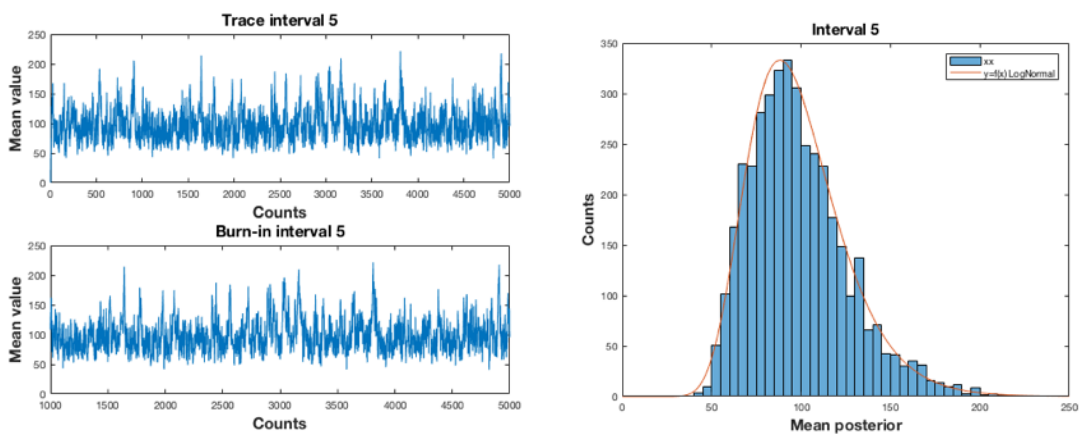


Figure 78.- Lognormal-uniform. Interval 5. Sampling n=4500, burn=500; 10 cycles. AR= 72.3%.

## b.- Mean posterior

The Table 68 shows the observed values.

Time interval	Posterior			Observed	LCL	MEAN	UCL
	$\mu$	CI [5%-95%]	$\sigma$	y			
1	70.4	70.2-70.6	14.2	69	56.2	70.4	84.6
2	72.3	71.9-72.6	14.3	73	58	72.3	86.6
3	76.1	75.9-76.3	8	85	68.1	76.1	84.1
4	88.3	87.9-88.7	20.6	115	67.7	88.3	108.9
5	96.9	95.9-98	26.2	125	70.7	96.9	123.1
6	95.3	94.6-96.1	23.2	90	72.1	95.3	118.5
7	92.9	91.8-94	21.6	79	71.3	92.9	114.5

Table 68.- Cogeneration unit. Lognormal-uniform model. Observations for cost deviation with mean posterior. Out-of-limits highlighted blue.

Until now the application of the SRC methodology has been carried out in a case of costs, now its application will be seen in a case of time deviation. The following case corresponds to an installation of a gas compression unit in an offshore platform. The gas stream from the separation unit goes to a recompression unit before their treatment, Figure 79.

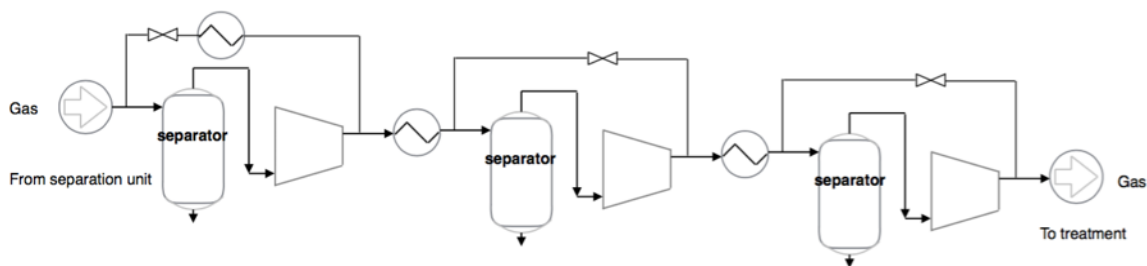


Figure 79.- General scheme of the recompression unit of gas in an offshore platform.

The objective of the project is to replace one of the compressors, the schedule in days of the activities and their actual duration, are shown in the Table 69.

Activity ID	1	2	3	4	5	6	7	8	9	10
PLAN	1	2	2	4	3	3	3	2	1	1
ACTUAL	1	3	3	3	2	4	4	2	1	1
A-P	0	1	1	-1	-1	1	1	0	0	0

Table 69.- Planned and actual realization of the compressor project substitution.

Although the extent of the data is reduced, it can be estimated if the distribution of planned and current days approximates a normal and lognormal distribution. For a normal distribution see Figure 80 and Figure 81 for a lognormal distribution.

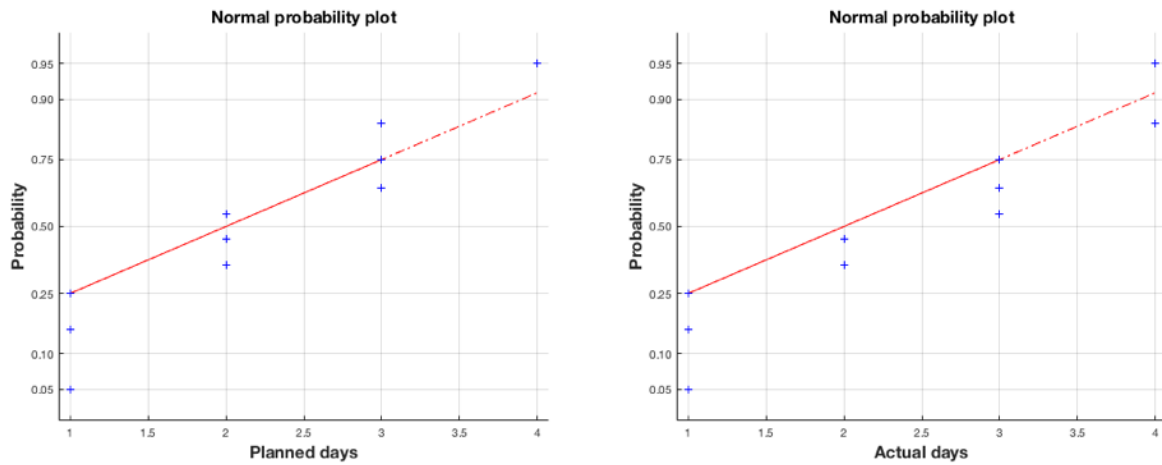


Figure 80.-Adjustment verification to a normal distribution for the planned and actual days of the compressor unit project.

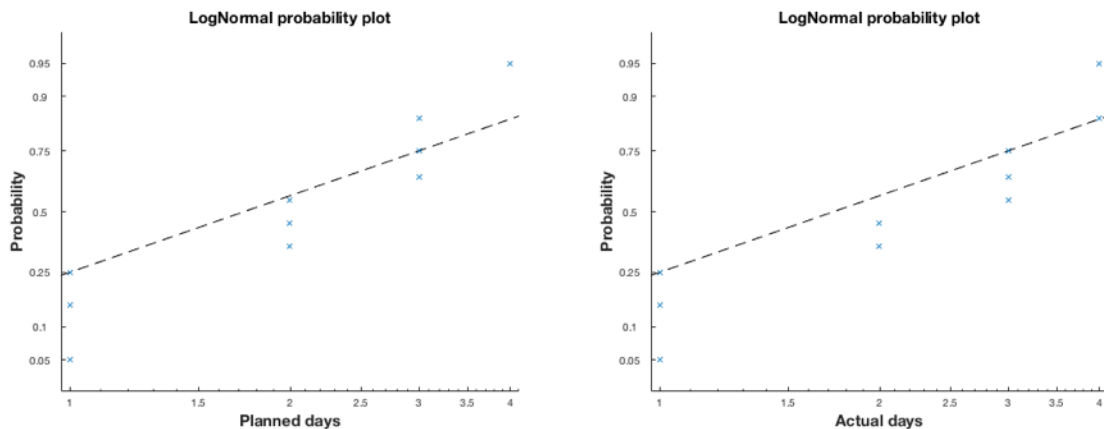


Figure 81.-Adjustment verification to a lognormal distribution for the planned and actual days of the compressor unit project.

It is not definitive, but it is quite approximate, the inference models seen above could be applied; the normal-normal, lognormal-lognormal, lognormal-gamma and lognormal-uniform.

#### 4.3.5. Normal-normal model.

For example applying a normal-normal model of inference working with the planned time as a prior with  $\mu_0 = 2.2$  and  $\sigma_0 = 1$ . Working with  $\pm 3\sigma_{\text{post}}$  the Table 70 shows the results for the direct method with mean posterior.

Time interval	Normal Prior		Normal Posterior		Observed	LCL	MEAN	UCL
	mu	sigma	mu	sigma	y			
1	2.2	1	1	0	1	1	1	1
2	2.2	1	2.1	0.6	3	0.3	2.1	3.8
3	2.2	1	2.3	0.5	3	0.9	2.3	3.7
4	2.2	1	2.5	0.4	3	1.3	2.5	3.7
5	2.2	1	2.4	0.3	2	1.4	2.4	3.4
6	2.2	1	2.6	0.4	4	1.5	2.6	3.7
7	2.2	1	2.8	0.4	4	1.7	2.8	3.8
8	2.2	1	2.7	0.3	2	1.7	2.7	3.7
9	2.2	1	2.5	0.3	1	1.5	2.5	3.5
10	2.2	1	2.4	0.3	1	1.4	2.4	3.4

Table 70.- Compression unit. Normal-normal model. Observations for days deviation using a direct method with mean posterior. Out-of-limits highlighted blue.

#### 4.3.6. Exponential-normal model.

As an additional possibility, the positive difference in days from Table 69 can be seen as number of incidences that are producing in different intervals with rate  $\lambda$ , if for example not more than 1 day of difference is admissible in 20 intervals, leads for the same rate 0.05 value as the previous examples. Applying an exponential-normal model with  $\pm 3\sigma_{\text{post}}$  the Table 71 shows the results for the recurrent with mean prior method.

	Prior	Posterior			Observed			
Time interval	lambda	lambda	CI [5%-95%]	sigma	lambda	LCL	MEAN	UCL
0	0.05	0.05	-	0.05	0	-	0.05	-
1	0.05	0.05	-	0.05	0	0	0.05	0.10
2	0.05	0.33	0.32-0.34	0.135	0.5	0	0.05	0.45
3	0.33	0.67	0.65-0.69	0.256	1	0	0.05	0.82
4	0.67	0.67	0.65-0.69	0.256	0	0	0.05	0.82
5	0.67	0.67	0.65-0.69	0.256	0	0	0.05	0.82
6	0.67	0.66	0.64-0.68	0.28	0.33	0	0.05	0.89
7	0.66	0.83	0.81-0.85	0.30	1	0	0.05	0.95
8	0.83	0.83	0.81-0.85	0.30	0	0	0.05	0.95
9	0.83	0.83	0.81-0.85	0.30	0	0	0.05	0.95
10	0.83	0.83	0.81-0.85	0.30	0	0	0.05	0.95

Table 71.- Compression unit. Exponential-normal model. Observations for days deviation using a recurrent method with mean prior. Confidence interval (CI) [5%-95%]. Out-of-limits highlighted blue.

#### 4.3.7. Normal-gamma I model.

With a Normal-gamma I model the expression (1) with parameter  $p=\sigma^2=1/\lambda$  is, (Annex A) allows to obtain a posterior  $f(\lambda/y) \sim \text{Gamma}(a,b)$  being a gamma distribution, where, the posterior parameters are,

$$a = \alpha + \frac{n}{2} \quad (32)$$

$$b = \beta + \frac{n}{2} s_{\mu}^2 \quad (33)$$

And

$$s_{\mu}^2 = \frac{1}{n} \sum_{i=1}^n (y_i - \mu)^2 \quad (34)$$

Being,  $\alpha, \beta$  the prior parameters of the gamma distribution,  $n$  the number of data from the vector  $(y_1, y_2, \dots, y_n)$ . The Plan is a Normal with population mean  $\mu_0 = \mu = 2.2$  and  $\sigma_0 = 1$ ; then the parameters for the prior distribution are calculated from

$$\sigma_0^2 = \frac{1}{\lambda} \rightarrow 1^2 = \frac{1}{\lambda} \rightarrow \lambda = 1 \quad (35)$$

This model examines de standard deviation as parameter, due to this only it is possible to apply the direct method. With  $\lambda_0=1$  and a variance of 0.5 defined the gamma prior parameters are  $\alpha=2$  and  $\beta=2$ , working with  $\pm 1/2\sigma_{\text{post}}$  the results are the following.

a.- Direct method with mean prior.

The Table 72 shows the observed values.

Time interval	Gamma Prior		Gamma Posterior		Observed	LCL	MEAN	UCL
	lambda	sigma	lambda	sigma	lambda			
1	1	0.7	0.92	0.58	-	0.71	1	1.29
2	1	0.7	0.99	0.57	1	0.72	1	1.28
3	1	0.7	1.04	0.56	1.13	0.72	1	1.28
4	1	0.7	1.09	0.54	1.33	0.73	1	1.27
5	1	0.7	1.22	0.57	1.56	0.71	1	1.29
6	1	0.7	0.94	0.42	1.13	0.79	1	1.21
7	1	0.7	0.79	0.34	1.02	0.83	1	1.17
8	1	0.7	0.86	0.35	1.07	0.82	1	1.18
9	1	0.7	0.85	0.33	0.9	0.83	1	1.17
10	1	0.7	0.83	0.32	0.8	0.84	1	1.16

Table 72.- Compression unit. Normal-gamma I model. Observations for days deviation using a direct method with mean prior. Out-of-limits highlighted blue.

b.- Direct method with mean posterior.

The Table 73 shows the observed values.

Time interval	Gamma Prior		Gamma Posterior		Observed	LCL	MEAN	UCL
	lambda	sigma	lambda	sigma	lambda			
1	1	0.7	0.92	0.58	-	0.63	0.92	1.21
2	1	0.7	0.99	0.57	1	0.70	0.99	1.27
3	1	0.7	1.04	0.56	1.13	0.76	1.04	1.32
4	1	0.7	1.09	0.54	1.33	0.82	1.09	1.36
5	1	0.7	1.22	0.57	1.56	0.93	1.22	1.50
6	1	0.7	0.94	0.42	1.13	0.73	0.94	1.15
7	1	0.7	0.79	0.34	1.02	0.62	0.79	0.96
8	1	0.7	0.86	0.35	1.07	0.69	0.86	1.04
9	1	0.7	0.85	0.33	0.9	0.68	0.85	1.01
10	1	0.7	0.83	0.32	0.8	0.68	0.83	0.99

Table 73.- Compression unit. Normal-gamma I model. Observations for days deviation using a direct method with mean posterior. Out-of-limits highlighted blue.

#### 4.4. LOC-domino effect analysis. Separation unit.

The separation unit is considered In this case the primary element that can be origin of the LOC event is a heat exchanger for LPG and oil. The scenario application is then for liquids and the bow-tie of application corresponds to the one in Figure 23. The SRC analysis follows the previously examined inference models considering the analysis of initiating causes (ic) and incidences in the safety barriers, the analysis of the safety barriers and the application of the hidden Markov chain analysis for the mitigative safety barriers.

But when maintenance works or equipment modifications are scheduled near to equipment that is likely to suffer a LOC event is necessary to consider when the situation of risks exceeds a certain limit and a special case of the application of the SRC methodology following the general scheme of obtaining the final risk damage due to a LOC event is applied. Remembering the general scheme is;

1. Scenario consideration for gas-vapor or liquid.
2. Loss of containment estimation (LOC) and their corresponding end state according to the selected bow-tie.





Applying equations (11) and (12), the  $P_{LOC,distance}$  probability is obtained. Then for an equivalent of 500 kg of TNT the radius of affectation  $r_{si}$  for a II level of damage, see Table 20, is;

$$r_{si} = K_i G^{0.33} \left( 1 + \left( \frac{3180}{G} \right)^2 \right)^{-0.17} = 9.6(500)^{0.33} \left( 1 + \left( \frac{3180}{500} \right)^2 \right)^{-0.17} = 40 \text{ (m)}$$

All the elements are inside the affectation with probabilities, Table 74.

Distance $r_i$ (m)	$P_{LOC,distance} \cdot 10^{-4}$
0	1.9000
15	0.7338
18	0.5659
20	0.4660
30	0.1121

Table 74.- Separation unit. Probabilities of damage due to LOC event in a heat exchanger.

These  $P_{LOC,distance}$  probabilities can be affected not only for the distance also for the exchanger operation status, and the worst value for flammability and toxicity of the components into the exchanger process. This situation defines three ponderations based on Toxicity (T), Operation (O) and Flammability (F).

The toxicity ponderation [190] is based on the following Table 75, where components are assigned to one of the five toxicity categories on the basis of LD50 (oral, dermal) or LC50 (inhalation) with the units of LD50 and LC50 are listed as follows:

- LD50: in mg/kg/bw. mg/kg bw/d stands for mg of substance per kg of body weight administered per day.
- LC50: in mg/L. mg/L is the estimated air concentration of a substance administered via inhalation route.

<b>Ponderation</b>	<b>Toxicity</b> (LD50 Oral and Dermal mg/kg Gas (ppm) (LC50 Vapour, dust and mist mg/l)
0	
1	Oral = 5000 mg/kg Dermal = 5000 mg/kg Gas > 20000 ppm Vapour > 20 mg/l Dust, mist > 5 mg/l
2	Oral = 2000 mg/kg Dermal = 2000 mg/kg Gas = 20000 ppm Vapour = 20 mg/l Dust, mist = 5 mg/l
3	Oral = 300 mg/kg Dermal = 1000 mg/kg Gas = 2500 ppm Vapour = 10 mg/l Dust, mist = 1 mg/l
4	Oral = 50 mg/kg Dermal = 200 mg/kg Gas = 500 ppm Vapour = 2 mg/l Dust, mist = 0.5 mg/l
5	Oral = 5 mg/kg Dermal = 50 mg/kg Gas = 100 ppm Vapour = 0.5 mg/l Dust, mist = 0.05 mg/l

Table 75.- Separation unit. Exchanger. Toxicity ponderation. Source [190].

The operation status ponderation is presented in the following Table 76.

<b>Ponderation</b>	<b>Status (process capability)</b>
0	No operation
1	25 %
2	50 %
3	75 %
4	100 %

Table 76.- Separation unit. Exchanger. Operation status ponderation.

The flammability ponderation, [45,190] is presented on the following Table 77.

Ponderation	Gases	Liquids Flash point (FP) Initial Boiling point (IBP)	Aerosols	Under pressure
0	-	-	-	-
1	Chemically unstable at a temperature higher than 20°C or a pressure greater than 101.3 kPa	-	In the foam test, the flame height is $\geq 20\text{cm}$ and the flame duration $\geq 2\text{s}$ ; or the flame height is $\geq 4\text{cm}$ and the flame duration $\geq 7\text{s}$ .	atmospheric pressure
2	Chemically unstable at 20°C and a standard pressure of 101.3 kPa	FP > 60°C and $\leq 93^\circ\text{C}$	Ignition occurs at a distance <75cm in an ignition test. However, the heat of combustion is >20kJ/g or ignition distance is $\geq 15\text{cm}$ or the space ignition test shows that the time equivalent $\leq 300\text{s}/\text{m}^3$ or the deflation density <300g/m <sup>3</sup> ;	Totally dissolved in a solvent
3	Ignite spontaneously in air at a temperature of 54°C or below	FP $\geq 23^\circ\text{C}$ and $\leq 60^\circ\text{C}$	Ignition occurs at a distance $\geq 75\text{cm}$ in an ignition test.	Partially dissolved in a solvent
4	At 20°C and a standard pressure of 101.3 kPa (14.7 psi), have a flammable range while mixed in air.	FP < 23°C IBP > 35°C	Concentration of the flammable components >85% and the heat of combustion is $\geq 30\text{ kJ/g}$ .	Partially liquid above -50°C
5	At 20°C and a standard pressure of 101.3 kPa (14.7 psi) that: (a) are ignitable when in a mixture of 13% or less by volume in air; or (b) have a flammable range with air of at least 12 percentage points regardless of the lower flammable limit.	FP < 23°C IBP $\leq 35^\circ\text{C}$	Concentration of the flammable components $\leq 1\%$ and the heat of combustion is < 20 kJ/	Gaseous at -50°C including all gases with critical temperature less than -50°C

Table 77.- Separation unit. Exchanger. Flammability ponderation. Sources [45,190].

The relative importance of every of these three dimensions is established in this scenario in: 25% Toxicity; 40% Operation and 35% Flammability. Their application allows for a corrected value of  $P_{\text{LOC,distance,TOF}}$  probability.

An important element that affects the probability of risk in situations of domino effect are the weather conditions in this case based on the storm days, precipitation value (mm) and wind speed over 55 km/h (SPW55) in days [191,192]. These values are obtained from the meteorological statistics. The Spain statistics, for precipitation, wind and storms in 2016, 2017 and 2018, [193] are presented in the following Table 78.

Month	1	2	3	4	5	6	7	8	9	10	11	12	total
Precip max (mm) 2016	17,48	21,69	55,86	18,91	14,78	6,39	4,96	3,85	15,12	22,38	31,25	21,35	234,03
Precip max (mm) 2017	18,68	18,67	21,78	13,64	13	11,32	11,61	14,95	8,74	13,96	16,65	20,25	183,24
Precip max (mm) 2018	21,72	23,89	21,40	20,85	17,09	16,96	8,97	10,41	15,51	32,60	26,21	10,14	225,73
Wind (km/h) 2016	11,63	15,11	13,29	13,25	13,03	12,76	11,82	12,01	10,70	8,64	9,97	8,38	10,86
Days wind 55	5,68	8,83	5,13	3,96	3,20	2,05	1,32	2,28	1,44	1,06	2,72	1,34	23,73
Days wind 91	0,20	0,42	0,15	0,07	0,05	0	0,04	0,02	0,02	0,04	0,07	0,09	0,09
Wind (km/h) 2017	10,15	12,78	12,47	12,56	12,35	13,09	13,07	12,03	10,94	9,04	9,09	11,95	10,65
Days wind 55	3,59	6,09	5,20	3,17	2,36	4,08	2,46	1,79	1,28	1,72	2,12	5,49	34
Days wind 91	0,12	0,67	0,11	0,11	0,05	0,08	0,02	0,02	0	0,07	0,11	0,35	1,5
Wind (km/h) 2018	11,63	11,56	16,61	14,01	12,42	12,04	12,55	11,62	10,43	10,84	11,09	9,19	11
Days wind 55	4,29	3,20	11,92	4,89	2,22	1,50	1,48	2,33	1,35	2,71	3,35	2,55	33,07
Days wind 91	0,17	0,05	0,70	0,16	0	0	0,02	0,03	0,02	0,06	0,08	0,06	0,93
Storm days 2016	0,23	0,29	0,50	0,67	1,51	1,12	1,68	0,99	1,24	0,98	0,92	0,27	10,11
Storm days 2017	0,48	0,52	0,72	0,46	2,19	2,85	1,57	2	0,51	0,55	0,61	0,28	12,57
Storm days 2018	0,52	0,31	0,92	1,31	3,80	2,70	2,06	2,42	3,29	1,61	0,48	0,08	19,11

Table 78.- Separation unit. Exchanger. Precipitation (mm), Wind days and Storm days for 2016, 2017 and 2018 years in the Spanish country. Source [193].

A probability of affected domino risk is dependent of the situation for precipitation, wind and storm, but there is no a lineal correlation, and then a fuzzy correlation is needed to obtain a ponderation  $f_{Weather,SPW55}$  factor.

The first step is to take the inputs and determine the degree to which they belong to each of the appropriate fuzzy sets via membership functions. In Fuzzy logic, the input is always a numerical value limited to the input variable interval, Annex E. In this case is between: 3.85 and 55.86 mm for precipitation value, but values can be higher in 24 hours; 1.06 and 11.92 days for wind greater or equal to 55 km/h; 0.08 and 3.80 days for storm and the output is a fuzzy degree always the interval between 1 and 2. The membership functions are presented in the following Figure 83 and the surfaces for the corresponding entries; Figure 84.

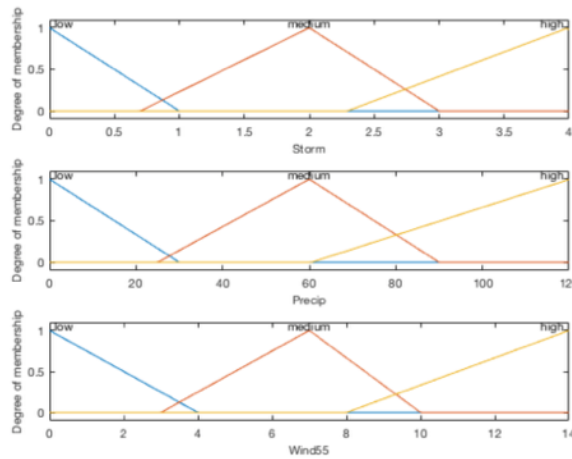


Figure 83.- Separation unit. Exchanger. Fuzzy membership functions for the three inputs: Storm (days), Precipitation (mm), Wind over 55 km/h (days).

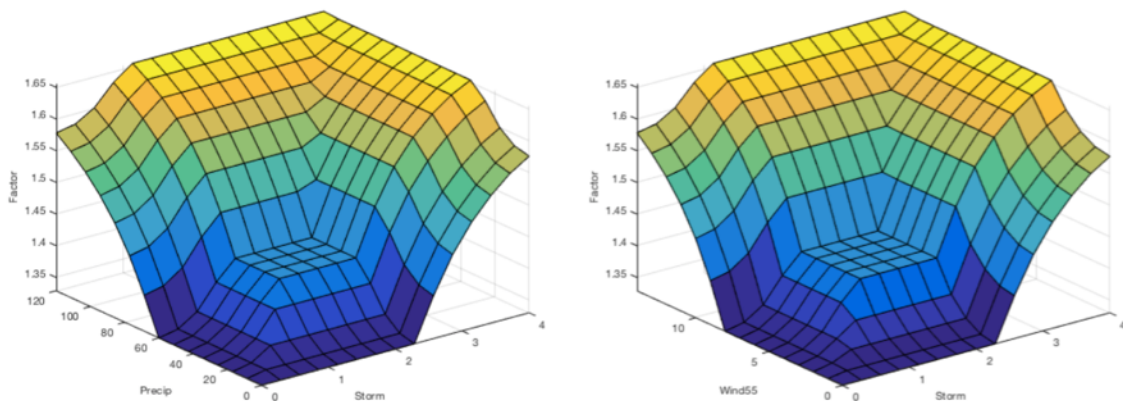


Figure 84.- Separation unit. Exchanger. Fuzzy surfaces for the three inputs: Storm (days), Precipitation (mm), Wind over 55 km/h (days) and the output ponderation factor.

The next affectation corresponds to the behavior of the installation, that is what is the situation of main streams, and pressures which by its characteristics, is understood to affect the risk of a LOC at the moment to perform the maintenance actuation or equipment modification. Data of the main flows and pressures are obtained from the system control and data acquisition (SCADA) system. From the data of currents and pressures affecting flash separators and exchangers, a main current affecting the exchanger with pressure input and the output exchanger pressure have been selected. Also a fuzzy logic is applied to determine the influence of these values for determine a  $f_{Behavior,IB}$  factor. The membership functions are presented in the following Figure 85 and the surfaces for the corresponding entries in Figure 86.

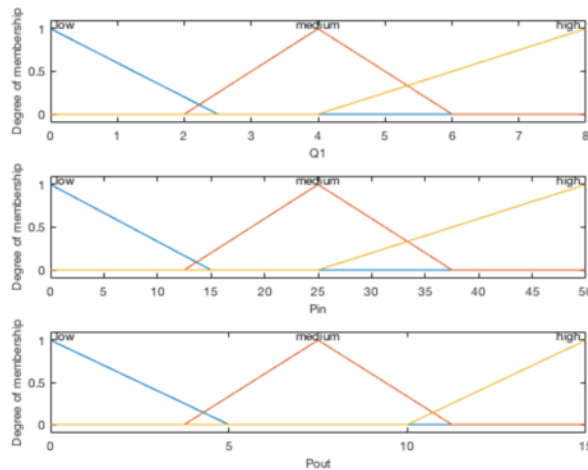


Figure 85.- Separation unit. Exchanger. Fuzzy membership functions for the three inputs: Q1 (kg/s), Pin (bar) and Pout (bar).

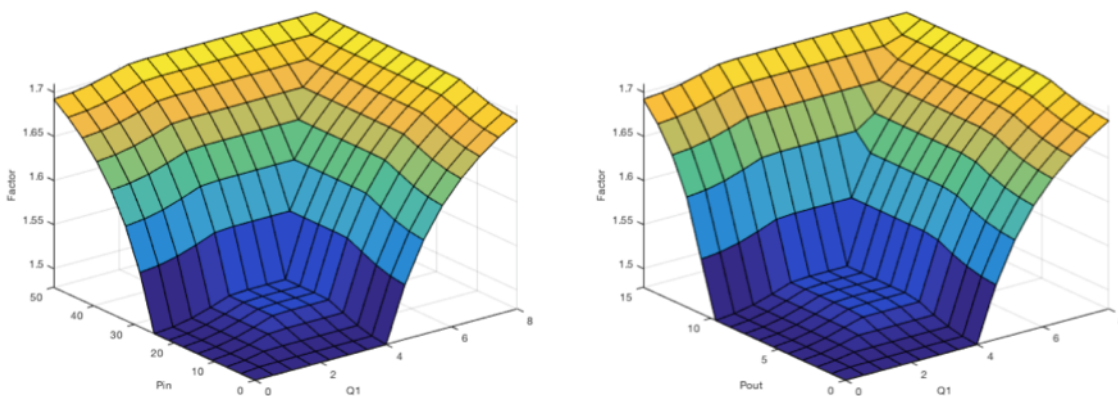


Figure 86.- Separation unit. Exchanger. Fuzzy surfaces for the three inputs: Q1 (kg/s), Pin (bar) and Pout (bar) and the output ponderation factor.

And finally obtaining the damage probability value for the selected final state and distance.

$$P_{\text{damage,distance}} = P_{\text{LOC,distance,TOF}} \cdot f_{\text{Weather,SPW55}} \cdot f_{\text{Behavior,IB}} \quad (13)$$

#### 4.4.1. Normal-gamma II model.

This value can be modeled through a normal-gamma II inference model, being a variation of the previously applied, where data follow a normal distribution with a mean  $\mu = P_{\text{damage,distance}}$  and this value following a gamma probability, (Annex A).

But before it is needed to define the probability damage value in the average conditions. That is to perform the calibration of the data for the considered location. Previously the value obtained for a LOC in a heat exchanger, from the bow-tie , Figure 82, selecting the worst end state case from producing primary flash explosion or/and a VCE plus a pool fire is  $1.9 \cdot 10^{-4}$  at distance of 0 m.

In normal operation at 0 m., the Toxicity, Operation and Flammability (TOF) conditions are: Toxicity = 2; Operation = 3 (75%) and Flammability = 3. With their relative importance previously established, is  $P_{\text{LOC,distance,TOF}} = 5.23 \cdot 10^{-4}$  , see Figure 87.

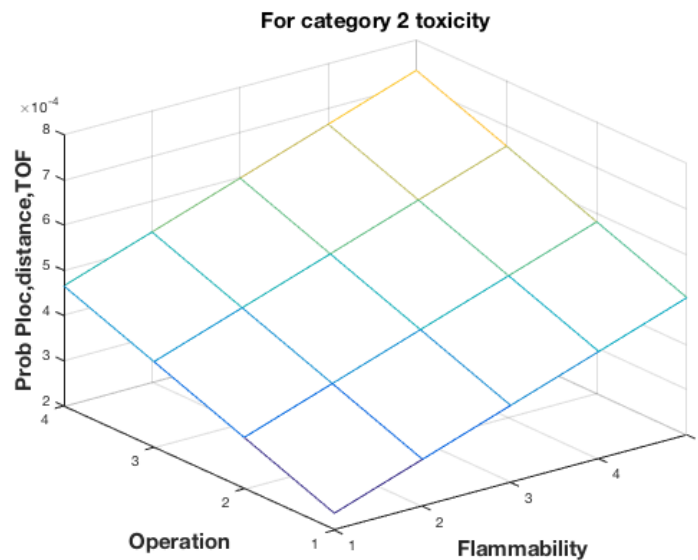


Figure 87.- Separation unit. Exchanger.  $P_{\text{LOC,distance,TOF}}$  probability in function of the toxicity equal to 2, operation and flammability.

And with middle conditions of storm (days), precipitation (mm) and wind55 (days) (SPW) of (1 20 1) a factor 1.2 is obtained , being the new probability is;

$$P^*_{\text{damage,distance}} = P_{\text{LOC,distance,TOF}} \cdot f_{\text{Weather,SPW55}} = 5.23 \cdot 10^{-4} \cdot 1.2 = 6.28 \cdot 10^{-4}$$



According to the installation behavior (IB), with a Q1 flow of 4 (kg/s) middle pressure of 25 (bar) an exit pressure of 7.5 (bar) is obtained a factor  $f_{\text{Behavior,IB}} = 1.5$  and the probability is then;

$$P_{\text{damage,distance}} = P_{\text{LOC,distance,TOF}} \cdot f_{\text{Weather,SPW55}} \cdot f_{\text{Behavior,IB}} = 5.23 \cdot 10^{-4} \cdot 1.2 \cdot 1.5 = 9.42 \cdot 10^{-4}.$$

This is the mean probability in middle circumstances for the plant location, in units of  $10^{-4}$  is Mean = 9.4 and sigma of 1 is considered enough variation; with control limits  $\pm 2\sigma_{\text{post}}$  and applying a Metropolis - Hastings procedure, for values taken in the worst case at 0 m, that is for the primary evolution, is;

Interval 1: With the same TOF conditions,  $P_{\text{LOC,distance,TOF}}=5.23$ ; 1 days of storm, 50 mm precipitation and 3.5 days wind are forecasted, being storm (days), precipitation (mm) and wind55 (days) (SPW), (1 50 3.5) with a factor is  $f_{\text{Weather,SPW55}}=1.4$ , and with a installation behavior (IB) change to Q3 =1 (kg/s);  $P_{\text{in}} = 5$  (bar) and  $P_{\text{out}} = 3.5$  (bar) due to maintenance a factor of the plant behavior  $f_{\text{Behavior,IB}}=1.3$ , then is;

$$P_{\text{damage,distance}} = P_{\text{LOC,distance,TOF}} \cdot f_{\text{Weather,SPW55}} \cdot f_{\text{Behavior,IB}} = 5.23 \cdot 1.35 \cdot 1.32 = 9.3.$$

Interval 2: With the same toxicity code 2, changes in TOF conditions to operation code 4 and flammability 4, being the  $P_{\text{LOC,distance,TOF}}=6.65$ ; and 1 days of storm, 30 mm precipitation and 2 days wind are forecasted, being storm (days), precipitation (mm) and wind55 (days) (SPW), (1 30 2) with a factor is  $f_{\text{Weather,SPW55}}=1.24$ , and with a installation behavior (IB) returning to Q3 =4 (kg/s);  $P_{\text{in}} = 25$  (bar) and  $P_{\text{out}} = 7.5$  (bar) with a factor of the plant behavior  $f_{\text{Behavior,IB}}=1.5$ , then is;

$$P_{\text{damage,distance}} = P_{\text{LOC,distance,TOF}} \cdot f_{\text{Weather,SPW55}} \cdot f_{\text{Behavior,IB}} = 6.65 \cdot 1.24 \cdot 1.5 = 12.4.$$

Interval 3: With the same previous values of TOF conditions, being the  $P_{\text{LOC,distance,TOF}}=6.65$ ; and 4 days of storm, 80 mm precipitation and 2 days wind are forecasted, being storm (days), precipitation (mm) and wind55 (days) (SPW), (4 80 2) with a factor is  $f_{\text{Weather,SPW55}}=1.63$ , and with the same installation behavior (IB) of Q3 =4 (kg/s);  $P_{\text{in}} = 25$  (bar) and  $P_{\text{out}} = 7.5$  (bar) with a factor of the plant behavior  $f_{\text{Behavior,IB}}=1.5$ , then is;

$$P_{\text{damage,distance}} = P_{\text{LOC,distance,TOF}} \cdot f_{\text{Weather,SPW55}} \cdot f_{\text{Behavior,IB}} = 6.65 \cdot 1.63 \cdot 1.5 = 16.3.$$

a.- Recurrent method with mean prior.

Values are presented on Table 79.

Time interval	Gamma Prior		Gamma Posterior			Observed	LCL	MEAN	UCL
	mu	sigma	mu	CI [5%-95%]	sigma	y			
1	9.4	1	9.32	9.31-9.33	0.4	9.3	8.5	9.4	10.3
2	9.32	0.4	9.80	9.78-9.81	0.4	12.4	8.5	9.4	10.3
3	9.8	0.4	10.07	10.06-10.09	0.4	16.3	8.5	9.4	10.3

Table 79.- Separation unit. Exchanger. Normal-gamma II model. Observations for  $P_{\text{damage, distance}}$  probability using a recurrent method with mean prior. Out-of-limits highlighted blue.

And the Metropolis-Hastings obtention in interval 2 on Figure 88.

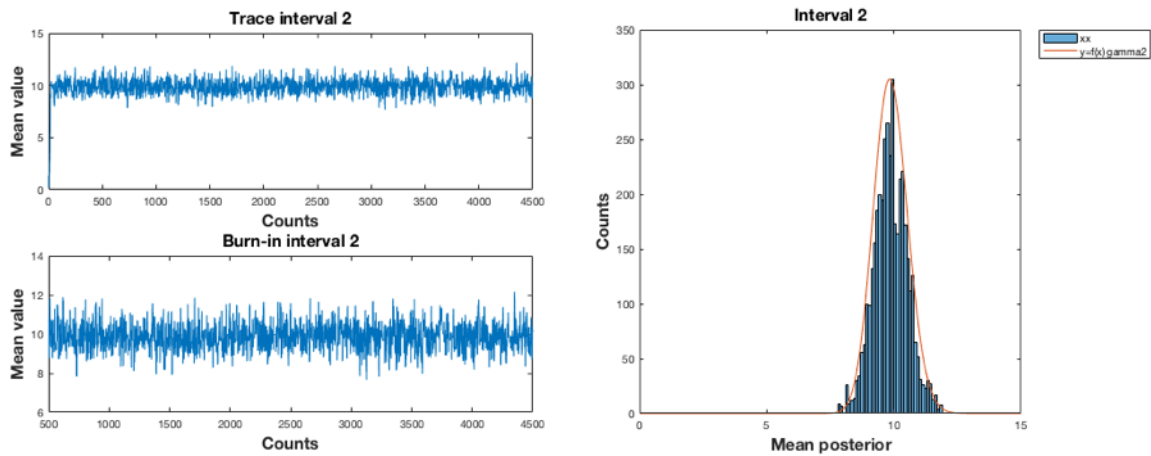


Figure 88.- Normal-gamma II. Interval 2. Sampling n=4500, burn=500; 10 cycles. AR= 47.0%.

b.- Recurrent method with mean posterior.

Values are presented on Table 80.

Time interval	Gamma Prior		Gamma Posterior			Observed	LCL	MEAN	UCL
	mu	sigma	mu	CI [5%-95%]	sigma	y			
1	9.4	1	9.32	9.31-9.33	0.4	9.3	8.5	9.32	10.1
2	9.32	0.4	9.80	9.78-9.81	0.4	12.4	9.0	9.80	10.6
3	9.8	0.4	10.07	10.06-10.09	0.4	16.3	9.3	10.07	10.9

Table 80.- Separation unit. Exchanger. Normal-gamma II model. Observations for  $P_{\text{damage,distance}}$  probability using a recurrent method with mean posterior. Out-of-limits highlighted blue.

c.- Direct method with mean prior.

The Table 81 shows the observed values.

Time interval	Gamma Prior		Gamma Posterior			Observed	LCL	MEAN	UCL
	mu	sigma	mu	CI [5%-95%]	sigma	y			
1	9.4	1	9.33	9.31-9.35	0.5	9.3	8.4	9.4	10.4
2	9.4	1	10.27	10.25-10.30	0.74	12.4	7.9	9.4	10.9
3	9.4	1	10.14	10.10-10.18	0.9	16.3	7.6	9.4	11.2

Table 81.- Separation unit. Exchanger. Normal-gamma II model. Observations for  $P_{\text{damage,distance}}$  probability using a direct method with mean prior. Out-of-limits highlighted blue.

And the Metropolis-Hastings obtention in interval 3 on Figure 89.

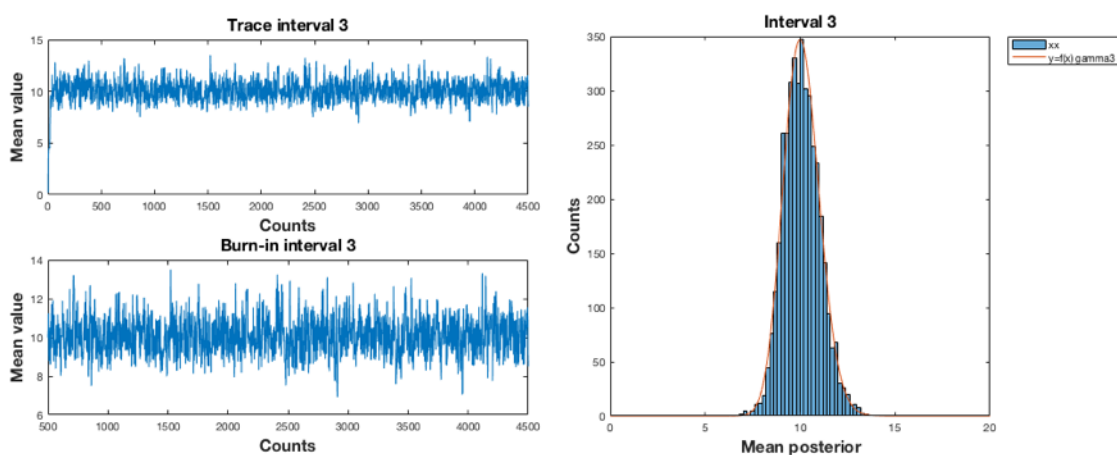


Figure 89.- Normal-gamma II. Interval 2. Sampling  $n=4500$ , burn=500; 10 cycles. AR= 61.9%.

d.- Direct method with mean posterior.

The Table 82 shows the observed values.

Time interval	Gamma Prior		Gamma Posterior			Observed	LCL	MEAN	UCL
	mu	sigma	mu	CI [5%-95%]	sigma	y			
1	9.4	1	9.33	9.31-9.35	0.5	9.3	8.33	9.33	10.33
2	9.4	1	10.27	10.25-10.30	0.74	12.4	8.79	10.27	11.75
3	9.4	1	10.14	10.10-10.18	0.9	16.3	8.34	10.14	11.94

Table 82.- Separation unit. Exchanger. Normal-gamma II model. Observations for  $P_{\text{damage, distance}}$  probability using a direct method with mean posterior. Out-of-limits highlighted blue.

Another possibility is to apply a LogNormal-gamma model of inference, (Annex A).

#### 4.4.2. Lognormal-gamma model.

The results are;

a.- Recurrent method with mean prior.

Values are presented on Table 83.

Time interval	Gamma Prior		Gamma Posterior			Observed	LCL	MEAN	UCL
	mu	sigma	mu	CI [5%-95%]	sigma	y			
1	9.4	1	9.37	9.35-9.40	0.7	9.3	8.0	9.4	10.8
2	9.37	0.7	9.59	9.58-9.60	0.7	12.4	8.0	9.4	10.8
3	9.59	0.7	9.74	9.72-9.75	0.6	16.3	8.2	9.4	10.6

Table 83.- Separation unit. Exchanger. Lognormal-gamma model. Observations for  $P_{\text{damage, distance}}$  probability using a recurrent method with mean prior. Out-of-limits highlighted blue.

b.- Recurrent method with mean posterior.

Values are presented on Table 84.

Time interval	Gamma Prior		Gamma Posterior			Observed	LCL	MEAN	UCL
	mu	sigma	mu	CI [5%-95%]	sigma	y			
1	9.4	1	9.37	9.35-9.40	0.7	9.3	8.0	9.37	10.8
2	9.37	0.7	9.59	9.58-9.60	0.7	12.4	8.2	9.59	11
3	9.59	0.7	9.74	9.72-9.75	0.6	16.3	8.5	9.74	10.9

Table 84.- Separation unit. Exchanger. Lognormal-gamma model. Observations for  $P_{\text{damage,distance}}$  probability using a recurrent method with mean posterior. Out-of-limits highlighted blue.

c.- Direct method with mean prior.

The Table 85 shows the observed values.

Time interval	Gamma Prior		Gamma Posterior			Observed	LCL	MEAN	UCL
	mu	sigma	mu	CI [5%-95%]	sigma	y			
1	9.4	1	9.35	9.33-9.38	0.7	9.3	8.0	9.4	10.8
2	9.4	1	9.69	9.66-9.72	0.86	12.4	7.7	9.4	11.1
3	9.4	1	9.75	9.72-9.78	0.9	16.3	7.6	9.4	11.2

Table 85.- Separation unit. Exchanger. Lognormal-gamma model. Observations for  $P_{\text{damage,distance}}$  probability using a direct method with mean prior. Out-of-limits highlighted blue.

d.- Direct method with mean posterior.

The Table 86 shows the observed values.

Time interval	Gamma Prior		Gamma Posterior			Observed	LCL	MEAN	UCL
	mu	sigma	mu	CI [5%-95%]	sigma	y			
1	9.4	1	9.35	9.33-9.38	0.7	9.3	8.0	9.35	10.8
2	9.4	1	9.69	9.66-9.72	0.86	12.4	7.97	9.69	11.4
3	9.4	1	9.75	9.72-9.78	0.9	16.3	7.95	9.75	11.6

Table 86.- Separation unit. Exchanger. Lognormal-gamma model. Observations for  $P_{\text{damage,distance}}$  probability using a direct method with mean posterior. Out-of-limits highlighted blue.

The analysis of the mitigative safety barriers through the application of the hidden Markov chain, can be performed for the mode of observing the state of the safety barriers to deduce the future transitions on the end states.

#### 4.4.3. Analysis of the end states observing the mitigative safety barriers.

From the prior probabilities of failure of the mitigative safety barriers of the general bow-tie for liquid LOC, Table 19, the following end probabilities are deduced, Table 87.

States	End State	Probabilities
state 1	Environmental damage liquid	0.050
state 2	Flash+Pool fire	0.143
state 3	VCE+Pool fire	0.162
state 4	Pool fire	0,646

Table 87.- Separation unit. Exchanger. Probabilities of end states after a LOC event.

##### a.- Recurrent method

The prior matrices are defined being the transition matrix for the four end sates and the emission matrix indicating the probabilities to stay in an end state  $S_n$  being:  $S_1$ =Environmental damage liquid;  $S_2$ =Flash+Pool fire;  $S_3$ =VCE+Pool fire and  $S_4$ =Pool fire; in function of the observed three active mitigative barriers ( $V_1 V_2 V_3$ ).

A prior transition matrix is defined for the four end states. Being  $p_{11}$  the probability for the  $S_1$  state (environmental damage liquid) to stay active in this state 1;  $p_{12}$  the probability of transition from  $S_1$  to  $S_2$  (Flash+Pool fire) because  $S_1$  has failed;  $p_{13}$  is the probability of transition from  $S_1$  to  $S_3$  (VCE+Pool fire) because  $S_1$  and  $S_2$  have failed,  $p_{14}$  is the probability of transition from  $S_1$  to  $S_4$  (Pool fire) because  $S_1$ ,  $S_2$  and  $S_3$  have failed, and so on. Additionally an emission matrix is defined to indicate the probabilities that an end state be active based on the observed safety barriers. ( $V_1 V_2 V_3$ ).

$$Tran = \begin{pmatrix} 0.05 & 0.14 & 0.16 & 0.65 \\ 0.15 & 0.10 & 0.10 & 0.65 \\ 0.05 & 0.05 & 0.20 & 0.70 \\ 0.05 & 0.15 & 0.15 & 0.65 \end{pmatrix} \quad Emiss = \begin{pmatrix} V_1 & V_2 & V_3 \\ S_1 & 0.75 & 0.15 & 0.05 \\ S_2 & 0.10 & 0.75 & 0.15 \\ S_3 & 0.10 & 0.15 & 0.75 \\ S_4 & 0.30 & 0.30 & 0.40 \end{pmatrix}$$

In concordance with the previous defined transition and emission matrices the expected use of the end states and the observed safety barrier are presented in Figure 90.

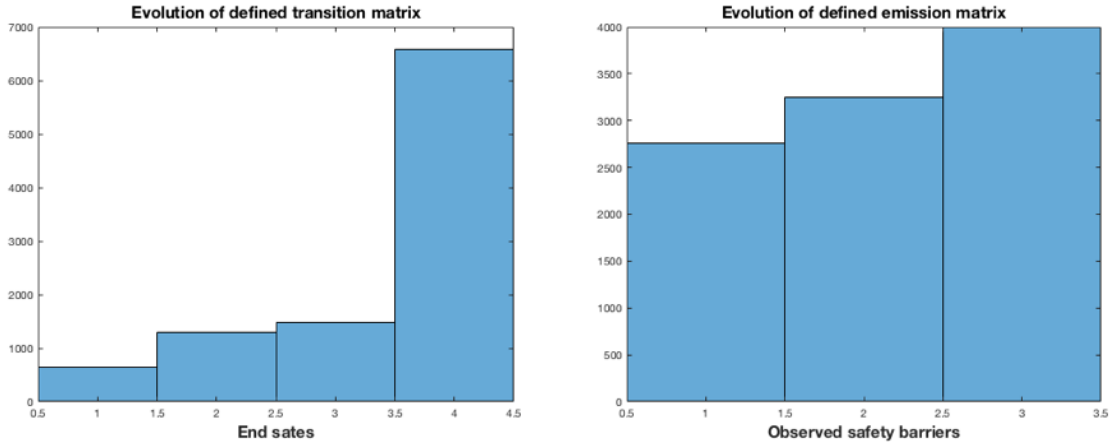


Figure 90.- LOC-domino effect. Initial transitions of the end states and emission matrices

As the previous case, the sampling is obtained by sequentially observing the barriers, if the first is active, then its value is 1, if it fails and the second barrier is active, then the value is 2 and so on; a group of ten observations is also maintained. The sampled sequence of the safety barriers is seqobs=[3 2 3 1 1 1 1 3 3]. And the posterior transition and emission matrices are.

$$\text{Transobs} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0.5 & 0 & 0.5 \\ 0.2 & 0 & 0 & 0.8 \end{pmatrix} \quad \text{Emisobs} = \begin{pmatrix} V_1 & V_2 & V_3 \\ S_1 & 0 & 0 & 1 \\ S_2 & 0 & 1 & 0 \\ S_3 & 0 & 0 & 1 \\ S_4 & 1 & 0 & 0 \end{pmatrix}$$

Being the relative occupancies  $S_1=11.1\%$ ,  $S_2=11.1\%$ ,  $S_3=22.2\%$  and  $S_4=55.6\%$  at steady state, Figure 91.

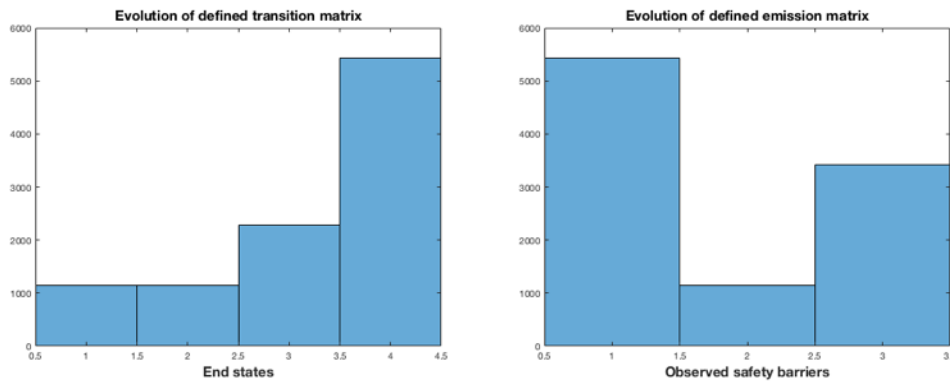


Figure 91.- LOC-domino effect. Transitions of the end states and emission matrices according to the first sequence observation of safety barriers.

And number of visits or transitions, in 10 intervals.

$$M(10) = \begin{pmatrix} 0.8391 & 1.3984 & 3.0075 & 4.7550 \\ 0.8391 & 1.3984 & 3.0075 & 4.7550 \\ 0.9510 & 1.5037 & 2.2376 & 5.3077 \\ 1.2839 & 0.8391 & 1.9020 & 5.9749 \end{pmatrix}$$

The next passage values in this case not for the last, but for the 3rd state, VCE+Pool fire, are  $m_1=1$ ,  $m_2=1$  and  $m_4=6$ .

Using a recurrent method the posterior transition and emission matrices (Transobs and Emisobs), can be the new prior in the next sampling, being seqobs=[2 3 1 1 1 1 1 3 3 1] showing an increase of the use of the first barrier SF<sub>1</sub>, a new posterior for transition and emission matrices is obtained.

$$Transobs = \begin{pmatrix} 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0.2 & 0 & 0 & 0.8 \end{pmatrix} \quad Emisobs = \begin{pmatrix} & V_1 & V_2 & V_3 \\ S_1 & 0 & 0 & 1 \\ S_2 & 0 & 1 & 0 \\ S_3 & 0 & 0 & 1 \\ S_4 & 1 & 0 & 0 \end{pmatrix}$$

Being the steady state occupations  $S_1=13.3\%$ ,  $S_2=6.7\%$ ,  $S_3=13.3\%$  and  $S_4=66.7\%$ , Figure 92.



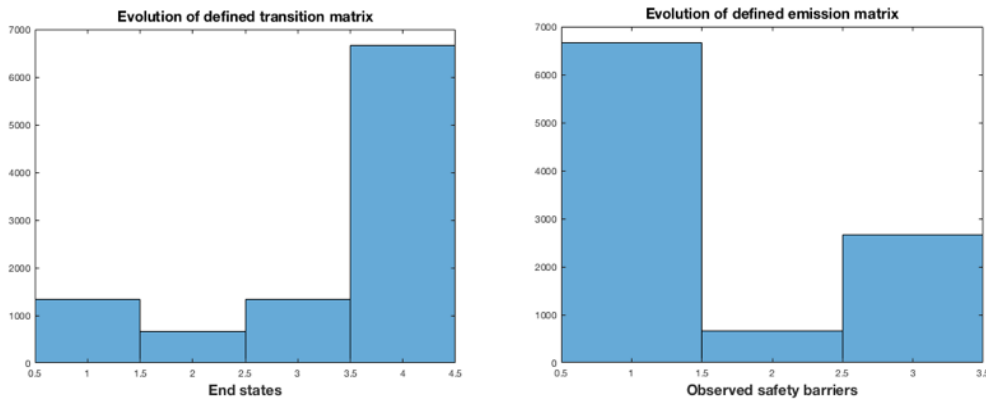


Figure 92.- LOC-domino effect. Transitions of the end states and emission matrices, using a recurrent method, according to the second sequence observation of safety barriers.

The number of visits or transitions, in 10 intervals.

$$M(10) = \begin{pmatrix} 1.0804 & 0.9736 & 1.8797 & 6.0663 \\ 1.1467 & 0.5070 & 1.9472 & 6.3991 \\ 1.2798 & 0.5734 & 1.0804 & 7.0664 \\ 1.4133 & 0.6399 & 1.2133 & 6.7335 \end{pmatrix}$$

And the next passage values for the 3rd state, VCE+Pool fire, are  $m_1=1.5$ ,  $m_2=1$  and  $m_4=6.5$ .

b.- Direct method

Using a direct method the initial transition and emission matrices (Tran and Emiss) be the prior in the next sampling, of seqobs=[2 3 1 1 1 1 1 3 3 1] and the new posterior for transition and emission matrices is obtained.

$$Transobs = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0.06 & 0.94 \end{pmatrix} \quad Emisobs = \begin{pmatrix} & V_1 & V_2 & V_3 \\ S_1 & 1 & 0 & 0 \\ S_2 & 0 & 1 & 0 \\ S_3 & 0 & 0 & 1 \\ S_4 & 0.79 & 0 & 0.21 \end{pmatrix}$$

Being the steady state occupations  $S_1=0\%$ ,  $S_2=0\%$ ,  $S_3=5\%$  and  $S_4=95\%$ , Figure 93.

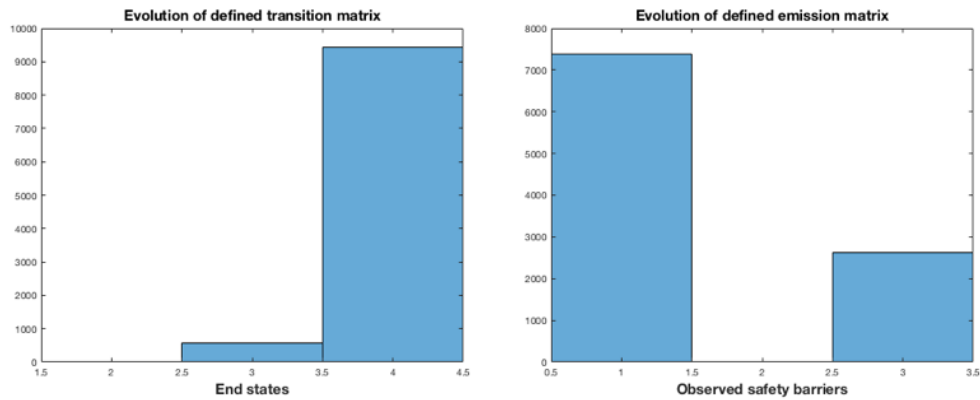


Figure 93.- LOC-domino effect. Transitions of the end states and emission matrices, using a direct method, according to the second sequence observation of safety barriers.

The number of visits or transitions, in 10 intervals.

$$M(10) = \begin{pmatrix} 0 & 1 & 1.3693 & 7.6307 \\ 0 & 0 & 1.4217 & 8.5783 \\ 0 & 0 & 0.4741 & 9.5259 \\ 0 & 0 & 0.5264 & 9.4736 \end{pmatrix}$$

And the next passage values for the 3rd state, VCE+Pool fire, are  $m_1=2$ ,  $m_2=1$  and  $m_4=18$ .

# Chapter 5

## Discussion of results.

### 5.1 Inference models.

An important number of inference models have been tested covering the recurrent or direct modes with mean prior or posterior in different environments, such as industrial, occupational process, cost-time deviation and loss of containment (LOC) producing domino effect and all of their MATLAB programs can be found in Annex H.

In the general application of the SRC methodology, with independency of the analyzed cases, the observations in which the risk parameter  $p$  is a frequency or ratio of risk for initiating causes and safety barriers incidences, requires the use of models based on Poisson, exponential and Weibull distributions being functions with the characteristic of no memory, and then every observation at each time interval begins as if it were new. The application of the gamma and normal distributions to characterize the risk parameter  $p$  as a ratio of risk, is adequate within the range of positive numbers, which this parameter can adopt, it should be kept in mind that gamma distribution is a special case of exponential and the exponential is related with the Poisson and Weibull distributions. For the general analysis of the preventive or mitigative safety barriers the application of the binomial-beta model inference is the most adequate because the analysis is for to determine if the safety barrier is active or not, this collected data follows a binomial distribution with a probability of failure expressed through a beta distribution as a prior.

These considerations also apply when analyzes are performed in the most specific presented cases of the general method such as industrial, occupational, cost-time and LOC events, because are developments of the general SRC methodology.

Additional inference models have been presented for the analysis of deviations in cost and time, in which the data is collected from the planned and actual values after examining their goodness of adjustment to normal or lognormal distributions. In the case of deviations in cost the normal and lognormal distributions have been tested for data and also the normal, lognormal, gamma and uniform for the parameter. And in the

analysis of time, the normal has been applied for data and Normal and Gamma I for prior. Additionally the Normal-gamma II have been tested in probability deviations in scenarios of LOC and further domino effect risk.

In the following Table 88 can be seen the summarized inference models tested in this work.

Inference models	Application	Inference models	Application
Poisson-gamma	Initiating causes-incidences rate	Normal-normal	Cost deviation Time deviation
Exponential-gamma	Initiating causes-incidences rate	Lognormal-Lognormal	Cost deviation
Weibull-gamma	Initiating causes-incidences rate	Lognormal-gamma	Cost deviation Probability deviation
Exponential-normal	Initiating causes-incidences rate	Lognormal-uniform	Cost deviation
Poisson-normal	Initiating causes-incidences rate	Normal-gamma I	Time deviation
Binomial-beta	Probability of failure	Normal-gamma II	Probability deviation

Table 88.- Summary of inference models used in this thesis and their application.

The posterior distribution because is a reflect of the evolution of the prior takes, with exception in the application of the Lognormal-uniform model, the same distribution function.

The use of a control chart first requires, in every situation, previous tests to adjust the control limits in in order to obtain the desired detection level.

Below are the most important aspects according to each case analyzed.

#### a.- Industrial processess

In the following Table 89 are compared the important parameters of every inference model as a result of their application on the analysis of the initiating causes and safety barriers incidences rate.

Inference models	1st interval of detection RM Prior	1st interval of detection RM Posterior	1st interval of detection DM Prior	1st interval of detection DM Posterior	Incidences and intervals	Sigma applied	Min. $\lambda$ variation [posterior: observed]	Max. $\lambda$ variation [posterior: observed]
Poisson-gamma	7	8	7	8	[1:4;1:7;1:8]	$\pm 1\sigma_{\text{post}}$	[0.25:0.25] all methods	[0.38:1] all methods
Exponential-gamma	4	4	4	4	[1:4;1:7;1:8]	$\pm 1\sigma_{\text{post}}$	[0.16:0.25] all methods	[0.28:1] all recurrent
Weibull-gamma	4	7	4	8	[1:4;1:7;1:8]	$\pm 1\sigma_{\text{post}}$	[0.20:0.25] all methods	[0.36:1] all recurrent
Exponential-normal	4	4	4	4	[1:4;1:7;1:8]	$\pm 1\sigma_{\text{post}}$	[0.16:0.25] [0.24:0.33] all recurrent	[0.50:1] all direct
Poisson-normal	4	4	4	4	[1:4;1:7;1:8]	$\pm 1\sigma_{\text{post}}$	[0.24:0.33] all recurrent	[0.39:1] all direct

Table 89.- Comparison of results for inference models applied in the analysis of the initiating causes and safety barriers incidences  $\lambda$  rate.

From these inference models, with a target for have not more than 1 incidence in 20 intervals and  $\pm 1\sigma_{\text{post}}$ , the exponential-gamma, exponential-normal and Poisson-normal show the first out-of-limits at the first incidence in interval 4, and this detection is maintained independently of the method applied, recurrent or direct. From the comparison of the minimal difference in  $\lambda$  value from the observation, all of them present this minimal difference at the low observed value 1/4 in the fourth interval, with exception of the exponential-normal model showing this also at the next interval, the seventh; with differences between the different models of  $\pm 20\%$ , and being the most nearest posterior value to the observed the one offered by the Poisson-gamma model. Likewise from the comparison of the maximal difference in  $\lambda$  value from the observation, all of them present this maximal difference at the last observation with a value of 1 in the eighth interval, in this case the model that approximates the value after the observed one is the exponential-normal showing 0.50 posterior to 1 observed, in direct method and 0.56 posterior to 1 observed in recurrent.

The Poisson-gamma model showed just at the upper limit the first observation with value 0.25, at low changes the Poisson-gamma respond better and with important changes shows better the exponential-normal.

Posterior values are inferred from the prior data and the observed data, see Figure 94 showing the evolution for the normal data in the exponential-normal model and for the Weibull data in the Weibull-gamma model, in intervals 4, 7 and 8 and using a recurrent method with mean prior. As a characteristic of the Bayesian inference, the posterior data and their subsequent distribution is a compromise between the prior and the observed distributions and it is adjusted according to the evolution of the observed data and the distribution function that represents them and the initial or prior considered distribution data.

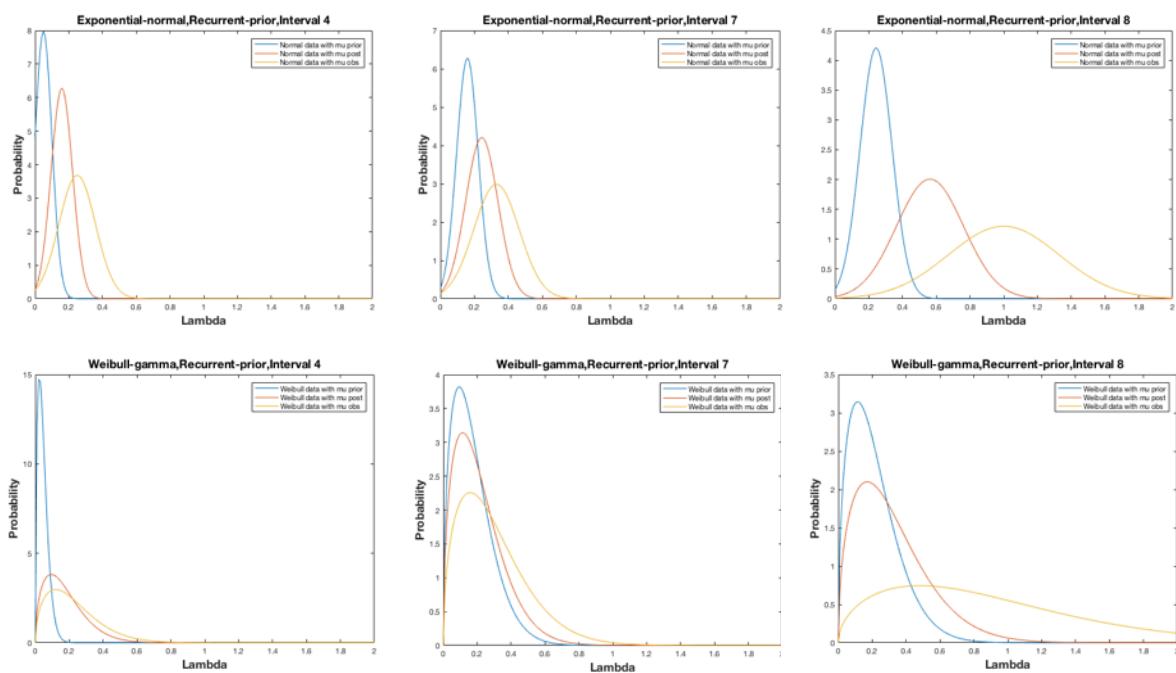


Figure 94.- Evolution of posterior distribution in two inference models: exponential-normal, from point of view of normal distribution and Weibull-gamma, from point of view of Weibull distribution in the intervals 4,7 and 8 with a recurrent method. Being blue:prior; orange: observed; red: posterior.

In the process of data collection when initiating causes occur, at the same time information is being obtained on the status of the process and the different elements of the equipment and the safety barriers, forcing a continuous review of its operation; for example, in the interval 4 due to a failure in a valve on the manifold SF<sub>11</sub> it is a situation that requires verifying the same type of valves; the same occurs when in the interval 7 there is a fail in safety supervision (SSU) affecting IS<sub>2</sub> cooling, then it is necessary to analyze the procedures and sub-functions that affect the safety design of the work, the environment of the operator, the characteristics of the operator, the interface of the system, procedures and information and design that affect this kind of workplaces. The last cause occurs in a failed test affecting SF<sub>3</sub> the general emergency shutdown requires

reviewing the causes, the status of the automatism and interlocks and the general procedure.

Considering the analysis of the safety barriers, the inference model of application is the binomial-beta, see Table 90.

Inference models	1st interval of detection RM Prior	1st interval of detection RM Posterior	1st interval of detection DM Prior	1st interval of detection DM Posterior	Incidences and intervals	Sigma applied	Min. <i>p</i> variation [posterior: observed]	Max. <i>p</i> variation [posterior: observed]
Binomial-beta	7	4	7	4	[1:4;2:7;1:8]	+/- 1σ <sub>post</sub>	[0.13:0.20] all methods	[0.12:0.40] all methods

Table 90.- Results for the binomial-beta model applied in the analysis of the probability *p* of failure of a safety barrier.

The analysis has a double purpose, making groups of observations distributed in different intervals, on the one hand it is wanted to determine the probability of failure of the safety, preventive or mitigating barrier, and on the other hand to control at what moment an out of control limits occurs .

This is a process, in which the collection of data to the extent possible is likely to be automated based on information from the control systems.

With a target value of  $p=0.2$  and with  $\pm 1\sigma_{post}$  the recurrent and direct methods both with mean posterior show the first out-of-limits at the first incidence in interval 4.

See Figure 95 showing the evolution for the probability value *p* from the point of view of beta distribution, in intervals 4, 7 and 8 and using a recurrent method with mean posterior.

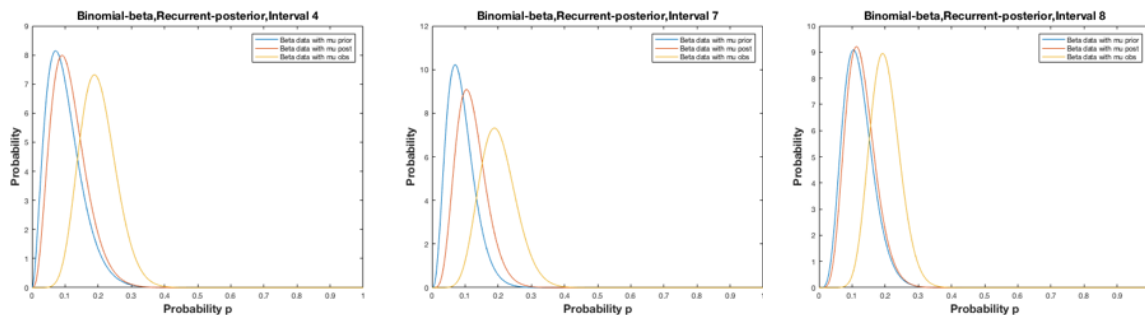


Figure 95.- Evolution of posterior distribution in the binomial-beta inference model, from the point of view of beta distribution, in the intervals 4,7 and 8 using a recurrent method with mean posterior. Being blue:prior; orange: observed; red: posterior.

As observed in the Figure, the posterior data are softly affected for the observed data and maintains values closer to those of the prior distribution.

#### b.- Occupational analysis

The inference models that can be applied are the same as the previously presented. As an example, an application of the Poisson-gamma inference model has been used with a recurrent and mean prior mode with a target for have zero accidents and working with  $\pm 1\sigma_{\text{post}}$ . Table 91.

Inference models	1st interval of detection RM Prior	1st interval of detection RM Posterior	1st interval of detection DM Prior	1st interval of detection DM Posterior	Incidences and intervals	Sigma applied	Min. $\lambda$ variation [posterior: observed]	Max. $\lambda$ variation [posterior: observed]
Poisson-gamma	4	not applied	not applied	not applied	[1:3;1:4;1:6;1:7;1:8]	$\pm 1\sigma_{\text{post}}$	[0.3:0.3] [0.5:0.5]	[0.5:1]

Table 91.- Results for the Poisson-gamma model applied in the occupational analysis measuring the ratio of incidences  $\lambda$  for initiating causes and safety barriers.

In this case the required accident rate value is very low, close for not having. And the control limits  $\pm 1\sigma$  are restrictive in order to highlight the out-of-limits situation well in advance. The model shows the first out-of-limits detection at the second interval incidence, and showed just at the upper limit the first observation, in the industrial application the inference model showed at fourth interval with a 0.25 value and in this occupational analysis showed at third interval with a 0.3 value. As the previous also offers the minimal difference in posterior  $\lambda$  value from the observation at this interval.

See Figure 96 showing the evolution for the value  $\lambda$  from the point of view of gamma distribution, in intervals 4, 6 and 7 using a recurrent method with mean prior.

As previously seen in the process of data collection when initiating causes occur, simultaneous information is being obtained from the status of the operators their equipment and safety barriers, forcing a continuous review; for example, in the interval 3 due to a job self control failure (JSC) and a safety self control fail (SSC) in interval 7 the working and safety procedures must be revised together with sub-functions affecting the safety of the work, the environment of the operator, the characteristics of the operator, the interface of the system, procedures and information and design that affect this kind of workplaces. The fail of the personal equipment protection in the impregnation area in interval 4 must be corrected and revised together to with the origin of the automation failure in the interval 6 in hot pressing. And also the fail in the general



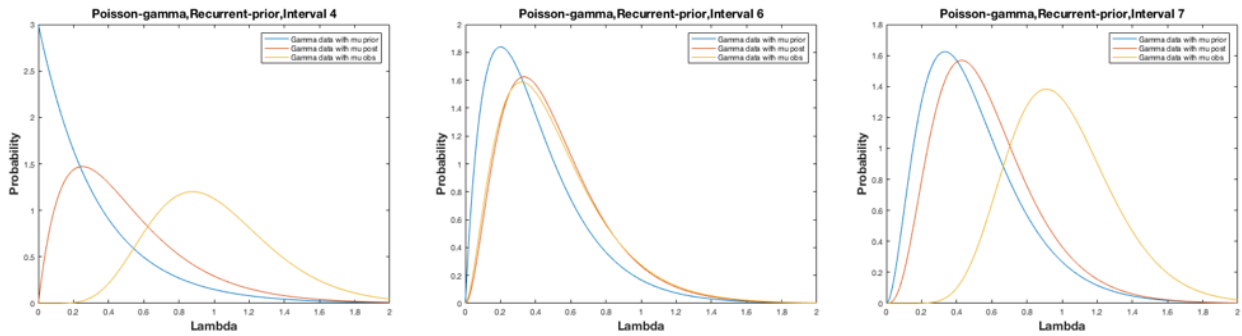


Figure 96.- Evolution of posterior distribution in the Poisson-gamma inference model, from the point of view of gamma distribution, in the intervals 4,6 and 7 using a recurrent method with mean prior. Being blue:prior; orange: observed; red: posterior.

emergency shutdown test requires reviewing the causes, the status of the automatisms and also the interlocks and procedures.

c.- Deviations in cost-time

In the following Table 92 are compared the important parameters of every inference model as a result of their application on the analysis of the deviations in cost-time.

Inference models	1st interval of detection RM Prior	1st interval of detection RM Posterior	1st interval of detection DM Prior	1st interval of detection DM Posterior	Incidences and intervals	Sigma applied	Min. $\mu$ variation [posterior: observed]	Max. $\mu$ variation [posterior: observed]
<b>Cost</b>								
Normal-Normal	4	3	4	4	[1;3;4;5]	$\pm 3\sigma_{post}$	[71:73] all methods	[72:125] all recurrent
Lognormal-Lognormal	4	4	not detected	not detected	[1;3;4;5]	$\pm 3\sigma_{post}$	[77:73] all recurrent	[79:125] all recurrent
Lognormal-gamma	4	4	not detected	not detected	[1;3;4;5]	$\pm 3\sigma_{post}$	[76.5:73] all recurrent	[78:125] all recurrent
Lognormal-uniform	4	4	-	-	[1;3;4;5]	$\pm 1\sigma_{post}$	[72.3:73] all	[97:125] all
<b>Time</b>								
Normal-normal	not applied	not applied	not applied	6	[2;3;6;7]	$\pm 3\sigma_{post}$	[1:1] [2.3:3] [2.7:2]	[2.6:4] [2.4:1]
Exponential-normal	2	not applied	not applied	not applied	[1:2;1:3;1:6; 1:7]	$\pm 3\sigma_{post}$	Min. $\lambda$ [0.33:0.5] [0.83:1]	Max. $\lambda$ [0.67:1] [0.66:0.33]
Normal-gamma I	-	-	4	5	[2;3;6;7]	$\pm 3\sigma_{post}$	Min. $\sigma$ [0.99:1]	Max. $\sigma$ [1.2:1.56]

Table 92.- Comparison of results for inference models applied in the analysis of the cost-time deviation.

For cost inference models, with a target that is coincident with the average value for the planned evolution of costs and with  $\pm 3\sigma_{\text{post}}$ , for the normal-normal, lognormal-lognormal, lognormal-gamma and lognormal-uniform, this last with  $\pm 1\sigma_{\text{post}}$ , show the first out-of-limits at the fourth interval-activity using a recurrent method with mean prior and recurrent method with mean posterior and using this method only the normal-normal detects an out-of-limits at the third interval-activity. Also only the normal-normal show this out-of-limits at the fourth interval using a direct method prior and posterior. From the comparison of the minimal difference in mean  $\mu$  value, all of them present this minimal difference in the second interval-activity, with differences between the different models of  $\pm 4\%$ , and being the most nearest posterior value to the observed the one offered by the normal-normal and lognormal-uniform models. Likewise from the comparison of the maximal difference in mean  $\mu$  value, all of them present this at the fifth interval, in this case the model that approximates better is the lognormal-uniform.

For time inference models, also with a mean target coincident with the mean planned and with  $\pm 3\sigma_{\text{post}}$ , only the exponential-normal shows an out-of-limits at the second activity, followed by the Normal-gamma I inference model detecting an out-of-limits from the sigma planned in the fourth activity, but expressing the gain of 1 day. The minimal difference is presented in different activities so that the normal-normal in the first, third and eighth activities, the exponential-normal in the second and seventh activities and the normal-gamma I also in the second activity. Likewise for the maximum difference presenting it in the activities sixth and tenth for the normal-normal model, in the third and sixth for the exponential-normal and in the fifth for the normal-gamma I.

From both cases, cost and time, the lognormal-uniform in cost application, has a unique representation that is basically using the recurrent method, and the normal-gamma I in time application, due to work with sigma variations, only has direct method.

See Figure 97 showing the evolution for the value  $\mu$  for cost evolution from the point of view of gamma distribution, in the lognormal-gamma inference model in intervals 3, 4 and 5 using a recurrent method with mean prior.

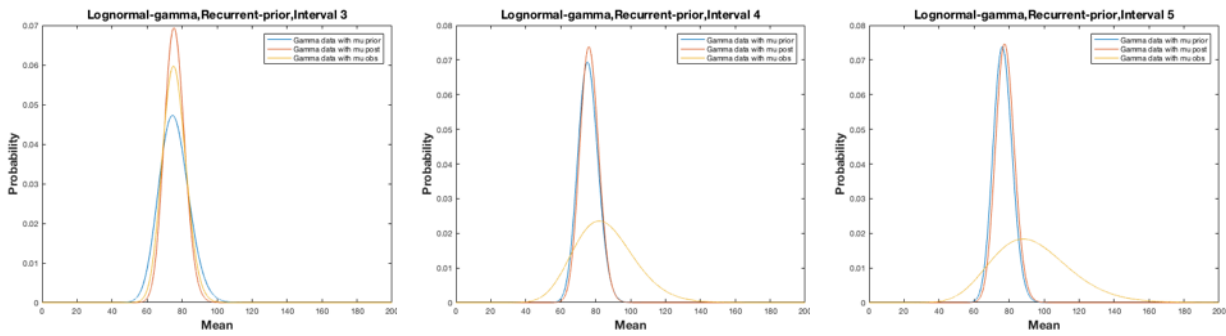


Figure 97.- Evolution of posterior distribution in cost for the lognormal-gamma inference model, from the point of view of gamma distribution, in the intervals 3,4 and 5 using a recurrent method with mean prior. Being blue:prior; orange: observed; red: posterior.

See Figure 98 showing the evolution for the value  $\mu$  for time evolution for the normal-normal inference model in intervals 5, 6 and 7 using a direct method with mean prior.

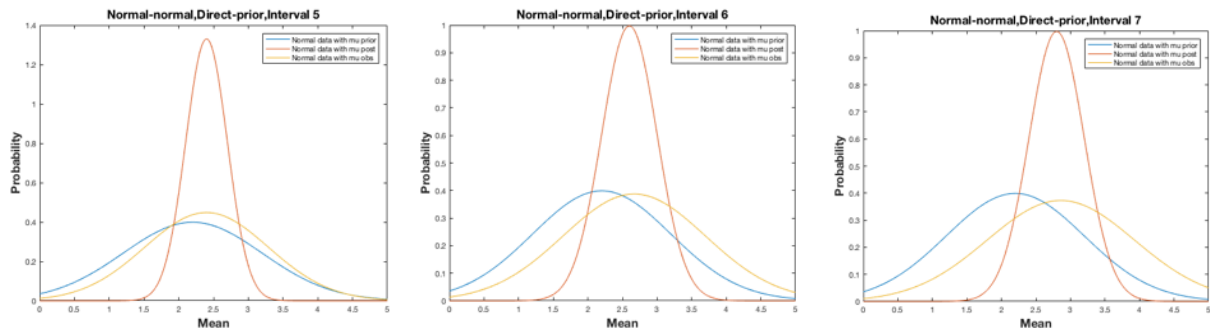


Figure 98.- Evolution of posterior distribution for the normal-normal inference model, in the intervals 5, 6 and 7 using a direct method with mean prior. Being blue:prior; orange: observed; red: posterior.

d.- LOC event with domino effect risk

In the following Table 93 are compared the important parameters of the inference models applied in the analysis of the domino effect risk.

Inference models	1st interval of detection RM	1st interval of detection RM	1st interval of detection DM	1st interval of detection DM	Incidences and intervals	Sigma applied	Min. $\mu$ variation	Max. $\mu$ variation
	Prior	Posterior	Prior	Posterior			[posterior: observed]	[posterior: observed]
Normal-gamma II	2	2	2	2	[1;2;3]	$\pm 2\sigma_{post}$	[9.3;9.3] practically all	[10.1;16.3] practically all
Lognormal-gamma	2	2	2	2	[1;2;3]	$\pm 2\sigma_{post}$	[9.3;9.3] practically all	[9.7;16.3] practically all

Table 93.- Comparison of results for inference models applied in the analysis of the domino effect risk.

All inference models are showing the out-of-limits at interval 2 due to changes in the TOF conditions, with operation and flammability code to 4 and installation behavior (IB) returning to nominal values  $Q3 = 4$  (kg/s);  $P_{in} = 25$  (bar) and  $P_{out} = 7.5$  (bar).

See Figure 99 showing the evolution for the value  $\mu$  for probability evolution from the point of view of lognormal distribution, in the lognormal-gamma inference model in intervals 1, 2 and 3 using a recurrent method with mean prior.

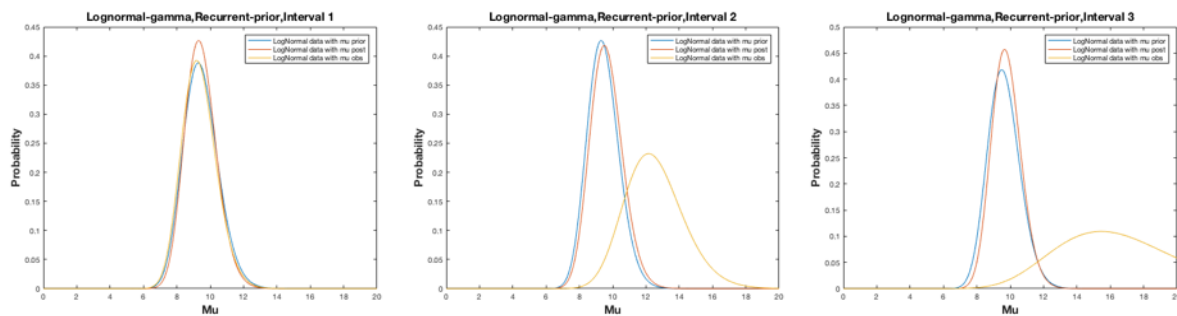


Figure 99.- Evolution of posterior distribution in  $P_{\text{damage,distance}}$  probability mean for the lognormal-gamma inference model, in the intervals 1, 2 and 3 using a recurrent method with mean prior. Being blue: prior; orange: observed; red: posterior.

From the previous Figures showing the posterior evolution of the parameter, can be seen that the posterior, being also a characteristic of the Bayesian inference, has a smaller dispersion than the one that the data collected has.

Of the different results that have been obtained as a result of the application of the different inference models, there is in particular none that stands out especially but it can be seen that in the treatment of incidents through the parameter based on the production ratio,  $\lambda$ , and working with high sensitivity  $\pm 1\sigma_{\text{post}}$ , the exponential-gamma, exponential-normal and Poisson-normal models are presented as the ones that best fit the immediacy characteristic (I) in any of the used recurrent and direct methods. The Poisson-gamma model shows the detection of the out-of-limits at the second incidence interval working in recurrent-prior or direct prior methodologies.

The binomial-beta model, also using it with restrictive limits of  $\pm 1\sigma_{\text{post}}$ , seems to have a better response working with recurrent-posterior or direct-posterior methods.

The models applied in treatment of cost-time deviations show in general better sensibility working with recurrent-posterior or direct-posterior methods, and adjusting the limits from  $\pm 3\sigma_{\text{post}}$ , to  $\pm 1\sigma_{\text{post}}$ , for example the normal-normal model would have already shown in the interval-activity 1 the out-of-limits situation in the two recurrent and direct posterior methods in the cost deviation analysis and in the first incidence in

time deviation analysis, and analogously the lognormal-lognormal and lognormal-normal models would have shown the out-of-limits situation in the interval-activity 3 working with a recurrent-posterior method, and in the interval-activity 4 with the direct-posterior method. The exponential-normal model approaching the deviation in days as incidences arriving at a rate  $\lambda$  shows the out of limits at the first incidence even working with a control limit coefficient of  $\pm 3\sigma_{\text{post}}$ . The normal-gamma I model is working with the sigma deviation as parameter  $p$  in the data collected, and it is highly sensitive but needs to work with low control limits coefficients.

In the treatment of LOC and risk domino effect the lognormal-gamma and normal-gamma II working with parameter  $p$  as a mean value  $\pm 3\sigma_{\text{post}}$ , show the out-of-limits at the second incidence and the normal-gamma II model show that can be applied indistinctly in treatments of cost-time deviation or probability changes.

## 5.2 Metropolis-Hastings sampler.

The Metropolis-Hastings sampling has been done with a random simulation of 4500 values and removing (burning) the first 500 to guarantee the stability of the process, with a repetition cycle of ten times, determining the average value and confidence intervals to obtain the representative posterior values. The resultant traces of the sampling processes also show a correct stochastic variability.

This can be shown using the sample autocorrelation (ACF), and partial autocorrelation functions (PACF) from Annex B. Through the graphical representation of a ACF and PACF seeing their evolution from a time series and including an approximate 95% confidence intervals where the degree of stochasticity is observed when the evolution of the autocorrelation and partial correlation parameters goes mostly within the two 95% confidence bands along the number of lags.

Then analyzing the Markov chain obtained from the Metropolis-Hastings sampling of the exponential-gamma inference model applied in the industrial analysis, from Figure 31, the following graphical evolution for the autocorrelation function ACF is obtained, Figure 100 and Figure 101 for the partial autocorrelation function (PACF). The same analysis for the Weibull-gamma sampling in Figure 36, shows the results in Figure 102 for the autocorrelation function (ACF) and Figure 103 for the partial autocorrelation function (PACF).

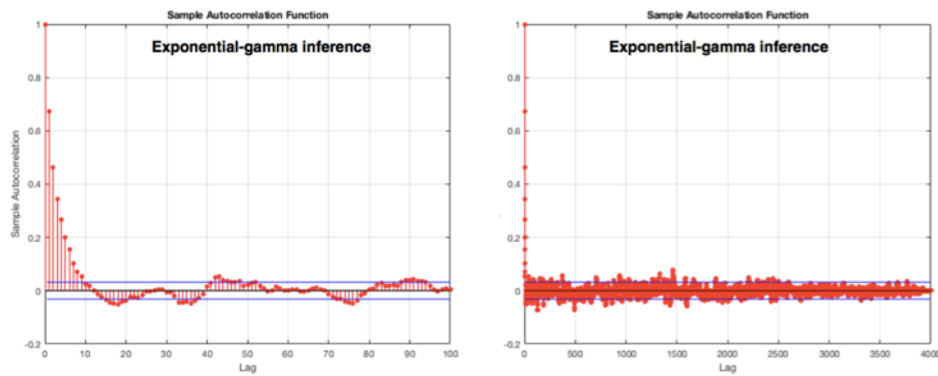


Figure 100.- Evolution of ACF parameter for the chain series from the exponential-gamma inference model sampling. Left in the 100 first lags, there is no seasonality in the data and in the 4000 lags the fluctuation of the parameter goes mainly into de confidence bands.

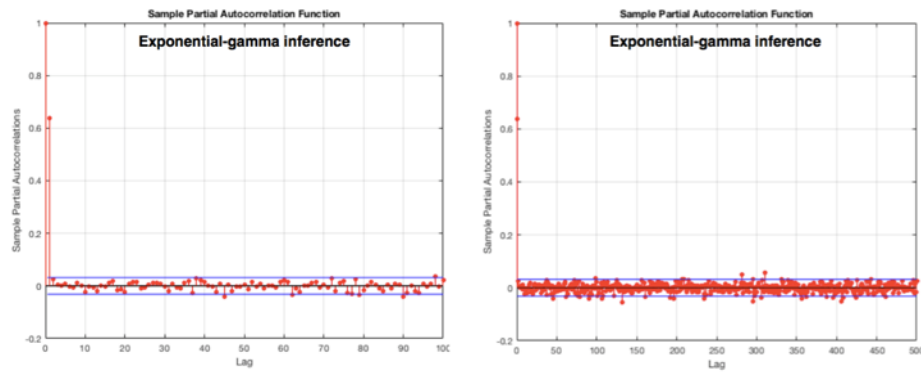


Figure 101.- Evolution of PACF parameter for the chain series from the exponential-gamma inference model sampling. Left in the 100 first lags, there is no seasonality in the data and in the 500 lags the fluctuation of the parameter goes uniformly into de confidence bands.

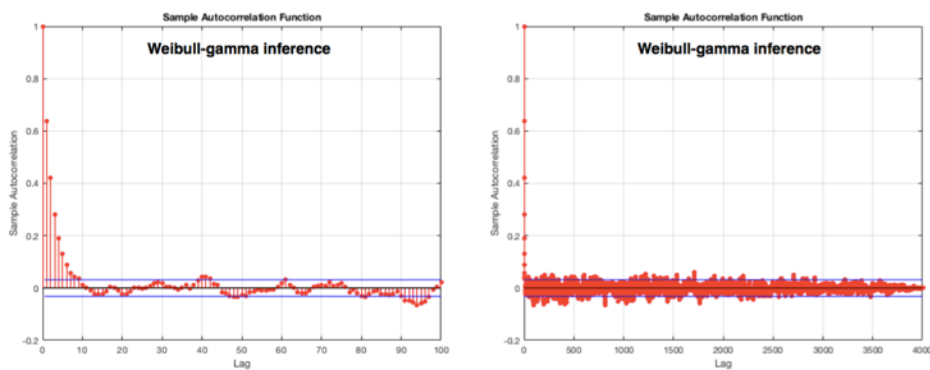


Figure 102.- Evolution of ACF parameter for the chain series from the Weibull-gamma inference model sampling.

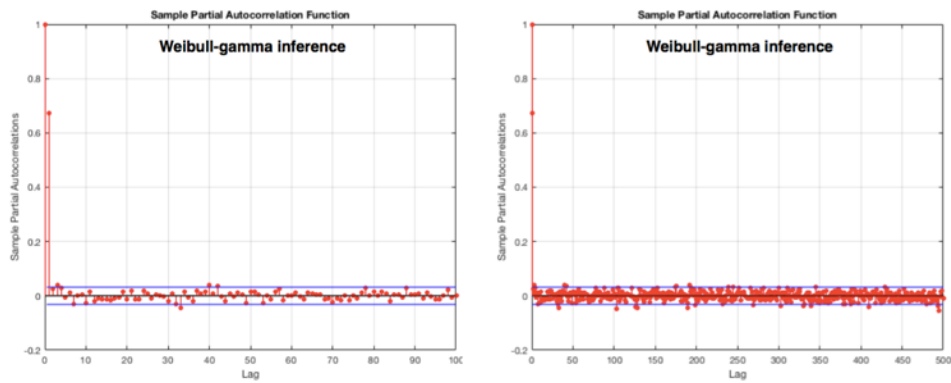


Figure 103.- Evolution of PACF parameter for the chain series from the Weibull-gamma inference model sampling.

Finally the analysis is performed for the lognormal-lognormal sampling in Figure 72, showing the results in Figure 104 for the autocorrelation function (ACF) and Figure 105 for the partial autocorrelation function (PACF).

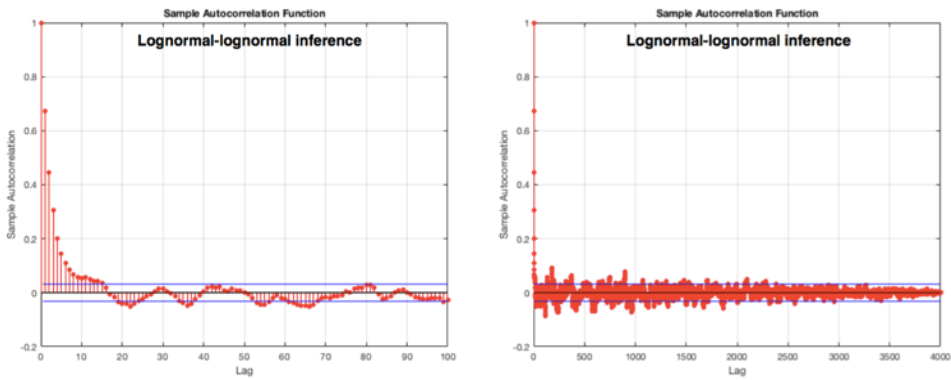


Figure 104.- Evolution of ACF parameter for the chain series from the lognormal-lognormal inference model sampling.

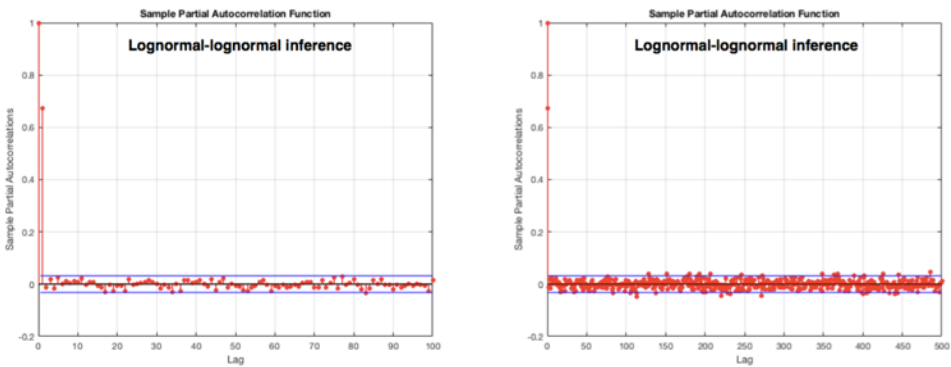


Figure 105.- Evolution of PACF parameter for the chain series from the lognormal-lognormal inference model sampling.

The sampling show acceptance rates (AR) oscillating between 25.7% to 80.8%, see Table 94 being necessary conditions to obtain a good sampling.

Inference models	AR variation [min.-max.]
Exponential-gamma	48.9% - 65.3%
Weibull-gamma	47.6% - 68.1%
Exponential-normal	46.4% - 80.5%
Poisson-normal	53.8% - 80.8%
Lognormal-lognormal	25.7% - 53.4%
Lognormal-gamma	52.5% - 77.6%
Lognormal-uniform	65.9% - 72.3%
Normal-gamma II	47% - 61.9%

Table 94.- Acceptance rates (AR) for sampled inference models.

The higher values of AR is characteristic of sampling at high intervals when the vector of collected data has higher variation in values.

### 5.3 Hidden Markov chain analysis.

The hidden Markov chain procedure allows to follow the situation of the mitigating barriers and obtain a map of their activity. The action of mitigating barriers is critical because when an accident occurs, they have to act with the highest probability that no failures will occur. However, it should be borne in mind that the method is an estimate of reality from the observations made allowing a quick view of which barrier is most used and likely to fail because it is showing high occupations.

According to the experience gathered in the application of the Baum-Welch algorithm the determination of the new transition and emission matrices from observations must have 10 values to obtain representative results.

The algorithm is sensitive to the prior definitions for the transition and emission matrices [187,188], and two procedures have been tested, the direct method, using always the initially defined transition and emission matrices in the actualization of the posterior values from the 10 observations, and the recurrent method, using the transition and emission matrices obtained after each observation, as initial matrices in the next.



The results for observations sequence and number of intervals of passage for the last safety barrier and the last end state, are summarized in the following Table 95;

	Analysis of the mitigative safety barriers observing the end states			Analysis of the end states observing the mitigative safety barriers.		
Case of analysis	Sequences observed of end sates	Recurrent passage intervals for last barrier	Direct passage intervals for last barrier	Sequences observed of active safety barriers	Recurrent passage intervals for last end state	Direct passage intervals for last end state
Industrial processes	[1 1 1 1 1 1 2 3 1 1]	8,9	8;9	[1 1 1 2 1 1 2 2 1 1]	2;1,5;2,5	2;1,5;2,5
	[1 1 1 1 1 2 3 1 1 2]	4;5	4;5	[1 1 2 1 1 2 2 1 1 2]	1;1,5;2,5	1;2;3
Occupational	-	-	-	[1 1 1 1 1 1 2 3 1 1]	18.25;19.30; 20.30;21.30	18.25;19.30; 20.30;21.30
				[1 1 1 1 1 2 3 1 1 2]	10.80;11.80; 12.80;13.80	5;1;2;3
				<b>Sequences observed of active safety barriers</b>	<b>Recurrent passage intervals for 3rd end state</b>	<b>Direct passage intervals for 3rd end state</b>
LOC domino effect	-	-	-	[3 2 3 1 1 1 1 1 3 3]	1;1;6	1;1;6
				[2 3 1 1 1 1 1 3 3 1]	1.5;1;6,5	2;1;18

Table 95.- Comparative of obtained values for passage intervals for the last safety barrier and last or 3rd end states, applying hidden Markov chain analysis in recurrent or direct mode for the studied cases.

In the industrial case application the passage for the last safety barrier is more critical when the observed end states has an increase of state number 2 for a partial operation observed situation. In this case there is no difference in the application of the recurrent or direct mode. The situation, when observing the state of safety barriers, is analogous; an increase in the use of the second barrier because the first one has failed shows a reduction in the passage from the first final state to the last one, and although this value is not modified there are slight variations in the other two using a recurrent or direct method.

In the occupational case, only observations have been made for safety barriers, no observations have been made for the different final states since it would have represented observing personal injuries and accidents, which is precisely what should be avoided. In this case the increase of the second safety barrier activity reduce the time of passage for the last final state, but in direct mode this reduction is more significative.

Finally in the LOC and domino effect risk analysis, also observation have been performed only for the safety barriers. In this case this an increase of the use of the first safety barrier increases slightly the times of passage through the third final state using the recurring method, however, applying the direct method, changes in interval from the first and second end states are practically the same and is pronounced going from the last end state.

Direct method in general seems to have a greater degree of prudence with values closer to the worst case, because in fact the starting hypotheses on the initial transition and emission values are maintained along the Markov analysis and there is no posterior modification that reduces this information.

This fact is important when applying the hidden Markov chain analysis in less variability situations, where the methods can fail obtaining good convergence, as seen in the following Table 96, for the industrial process.

	Analysis of the mitigative safety barriers observing the end states			Analysis of the end states observing the mitigative safety barriers.		
Case of analysis	Sequences observed of end sates	Recurrent passage intervals for last barrier	Direct passage intervals for last barrier	Sequences observed of active safety barriers	Recurrent passage intervals for last end state	Direct passage intervals for last end state
Industrial processes	[1 1 1 1 1 1 1 1 1 1]	80.6;80.3	80.6;80.3	[1 1 1 1 1 1 1 1 1 1]	211;209;202	211;209;2002
	[1 1 1 1 1 1 1 1 1 2]	not	7.3;3.7	[1 1 1 1 1 1 1 1 1 2]	not	not
	[1 1 1 1 2 1 1 1 1 1]	not	3.4;4.4	[1 1 1 1 2 1 1 1 1 1]	not	2;3;5

Table 96.- Comparative of converging results for the posterior determination and last safety barrier and last end states passage intervals, applying hidden Markov chain analysis in recurrent or direct mode for the industrial case.

The algorithm of analysis works well when the sampling shows wit great variation in values for the observed vector of 10 samples, in which the difficulty of appreciation of the different transition states is greater. In Table 96 the intervals of passage through the last safety barrier or final state are evident and intuitive without practically analysis in concordance with the vector of observations, however convergence is more difficult due by one hand to the lack of variability and by the other due to the increase of number of variables to analyze, in these situations the direct method being more effective that the recurrent, even when in previous sampling no convergence has been obtained.

Collecting observations for the Bayesian inference or for the analysis of the safety barriers using the binomial-beta model or the hidden Markov chain, can be carried out by the own company personnel or from the automated control systems, however human failures, in general, must be collected on site by personal observation.

## 5.4 Considerations about the bow-tie and mitigative safety barriers use.

a.- Safety barriers and subsystems analysis.

When the analysis of the safety barriers needs to be carried on, the inference model that allows to obtain a probability of failure is the binomial-beta model or a rate of failure through the Poisson-gamma model. The analysis can not only cover the safety barrier as a complete system but can also be carried out in each of the subsystems that compose it, Figure 106.

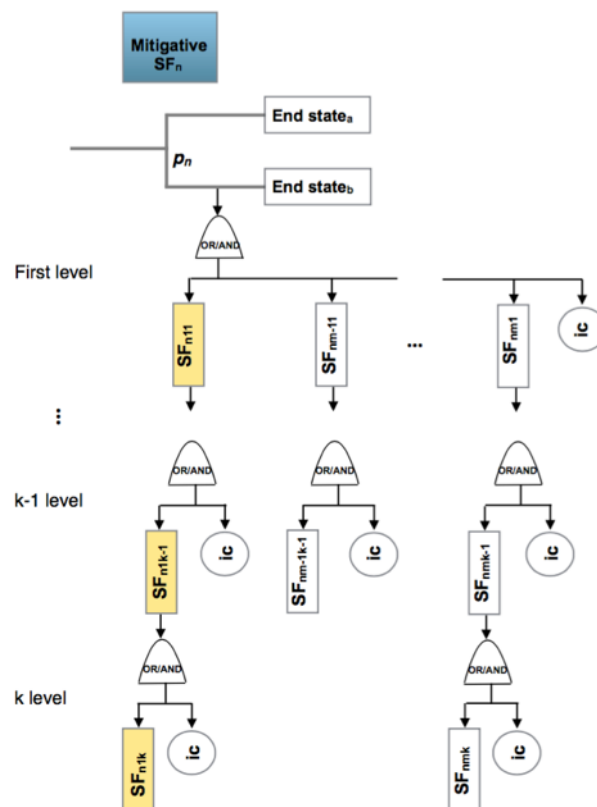


Figure 106.- Generic subsystems conforming a safety barrier. In this case is reflected a mitigative barrier and a critical multilevel chain of subsystems highlighted in yellow.

This represents an important amount of work and can be interesting, once a first data collection has been made and if there is no modification in the composition of the

subsystems, to be able to evaluate the changes that the subsystems represent, at least for the most critical ones, located at a certain level, for different levels or for a mixed composition; and based on the modification of the probability or the incidence ratio in the main barrier taken as a whole.

The tool applied is then a joint distribution, [194], Annex G.

If a binomial-beta distribution has been applied obtaining values of failure probabilities  $p$  in a critical chain of subsystems for a given safety barrier with posterior beta functions defined for the probability of failure of a certain critical multilevel chain, given by the vector;  $(SF_n; SF_{n11}; SF_{n12}; SF_{n1k-1}; SF_{n1k})$ , equal to  $(9.99; 3.91; 3.03; 2.50; 0.63)$  in units of  $10^{-5}$ , see Figure 106, and their beta distribution can be joined to the evolution of the  $SF_n$  probability values, the following Figure 107 is obtained.

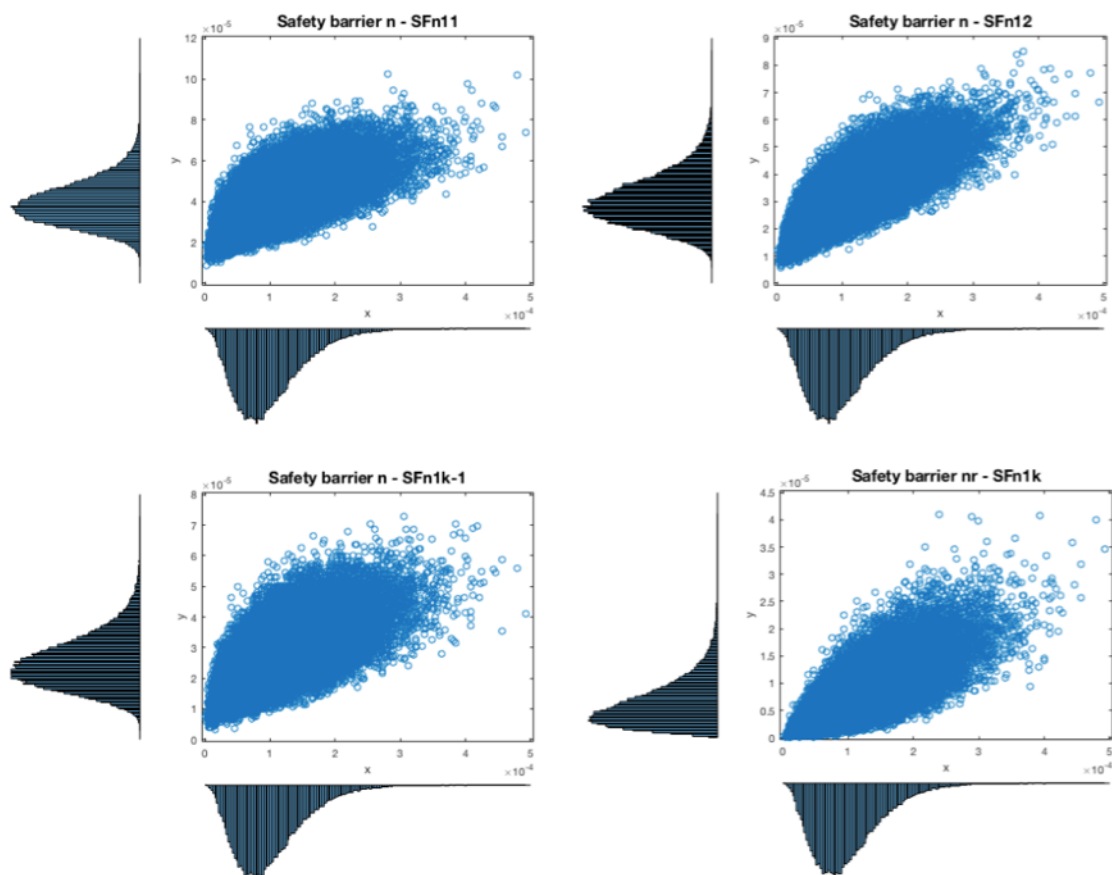


Figure 107.- Joined distributions of beta distribution for the critical subsystems chain from the generic safety barrier n.

With  $\rho = \begin{pmatrix} 1 & 0.7 & 0.8 & 0.7 & 0.8 \\ 0.7 & 1 & 0.7 & 0.7 & 0.8 \\ 0.8 & 0.7 & 1 & 0.7 & 0.6 \\ 0.7 & 0.7 & 0.7 & 1 & 0.6 \\ 0.8 & 0.8 & 0.6 & 0.6 & 1 \end{pmatrix}$  and defined with  $n=100000$  random samples.

If a new value for  $SF_n$  is obtained equal to  $2 \cdot 10^{-4}$  is obtained, the new vector of probabilities is;  $(20;4.99;4.13;3.37;1.14)$  in units of  $10^{-5}$ .

If a Poisson-gamma distribution has been applied obtaining values of incidences rate  $\lambda$  in a critical chain of subsystems for a given safety barrier with posterior functions defined by gamma functions and given by the vector;  $(SF_n; SF_{n11}; SF_{n12}; SF_{n1k-1}; SF_{n1k})$ , equal to  $(2;1.25;0.77;0.21;0.13)$ , see Figure 108, with the same  $\rho$  and  $n=10000$  random samples.

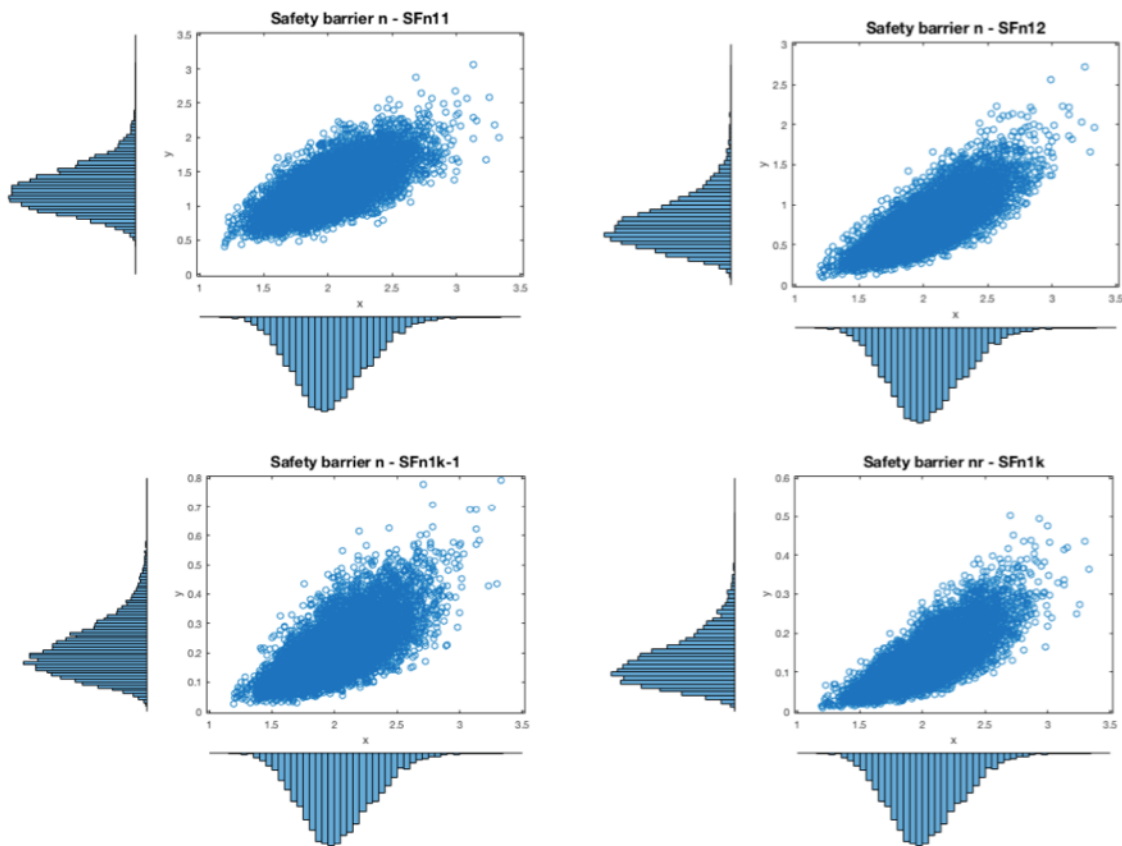


Figure 108.- Jointed distributions of gamma distribution for the critical subsystems chain from the generic safety barrier n.

If a new value for  $SF_n$  is obtained equal to 3 is obtained, the new vector of probabilities is;  $(3;1.84;1.60;0.46;0.31)$ .

b.- Bow-tie and mitigative safety barriers definition

It can cause confusion is the use of the event graph in the bow tie with the objective to analyze possible events and situations when also it is used to reflect the design of safety barriers. And to differentiate these applications, it is needed to state a previous ratio measuring the degree of definition of a safety barrier design scheme on the event tree.

In a general designed system, as from Figure 11, and selecting only the part of the event tree, see Figure 109. Is defined the degree of safety barriers definition (DSBd) as;

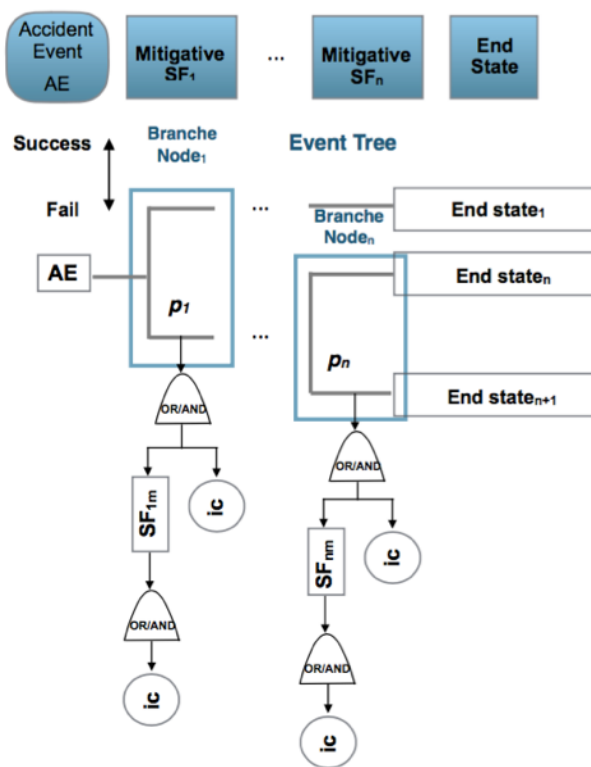


Figure 109.- General event tree part of the bow-tie for the mitigative safety barriers definition.

$$DSBd = \text{number of mitigative safety barriers} / \text{number of branching nodes} \quad (36)$$

This value has to be 1 or close to 1 for a correct definition of the safety barriers scheme.

Two examples are exposed to visualize this difference.

a.- CSTR reactor.

Meel and Seider, [114,195] proposed the analysis of a CSTR reactor and the adapted consequence modeling scheme is presented on the next Figure 110. Where how to react for the high temperature accident event in a continuous stirred tank reactor (CSTR) is analyzed.

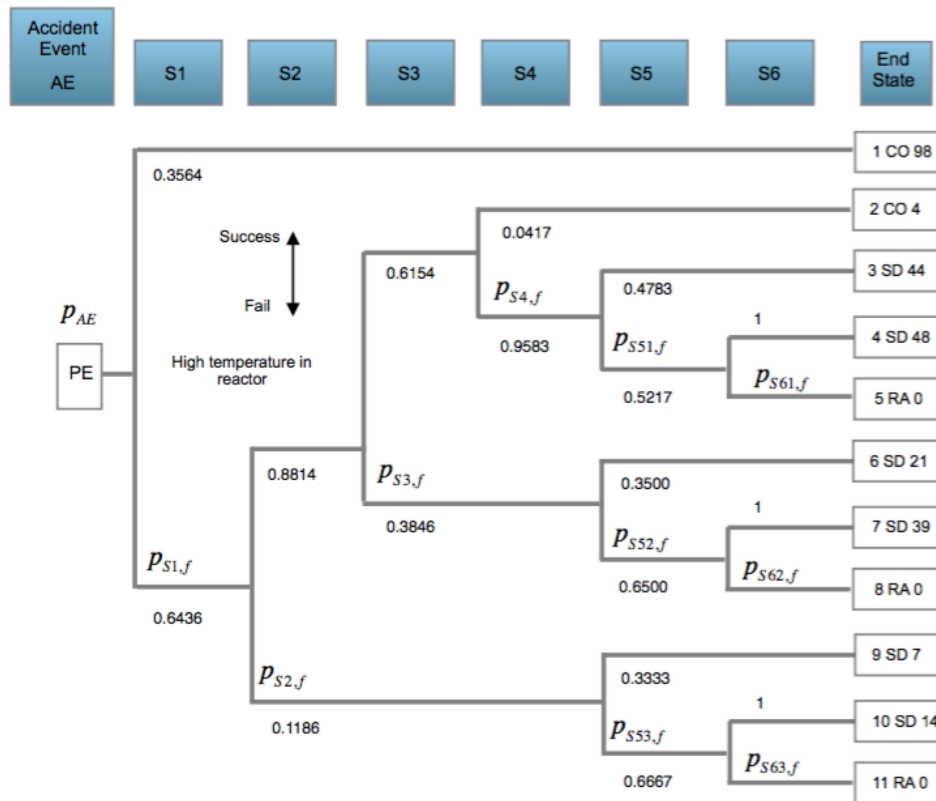


Figure 110.- Consequence scheme for the safety analysis of a CSTR reactor. Adapted from [114].

There are defined 6 reactive safety systems, see Table 97, and 11 end states with three possibilities defined as; CO for continuous operation, SD for manual or automatic shutdown and RA for a catastrophic run away of the reaction. At every end state is indicated the number of times that it is produced from a total of 275, the probabilities of success and failure are defined for every branch in concordance to the end states production.

Safety system ID	Description	Type
S1	First alarm-temperature	Automatic
S2	Operator-confirmation	Human
S3	Operator-evaluation	Human
S4	Operator-action	Human
S5	Second alarm-temperature	Automatic
S6	Second alarm-temperature	Human-manual

Table 97.- Safety systems definition for the CSTR example. Adapted from [114].

The aim is to analyze the behavior of the operator. From the point of view of analysis of the sequences of basic actions is correct and applicable, but from the point of view of application of the designed safety barriers the DSBd is equal to 0.6.

A rational proposal for safety barriers action can be the depicted on Figure 111 where the DSBd is equal to 1, with the safety barriers definition in Table 98.

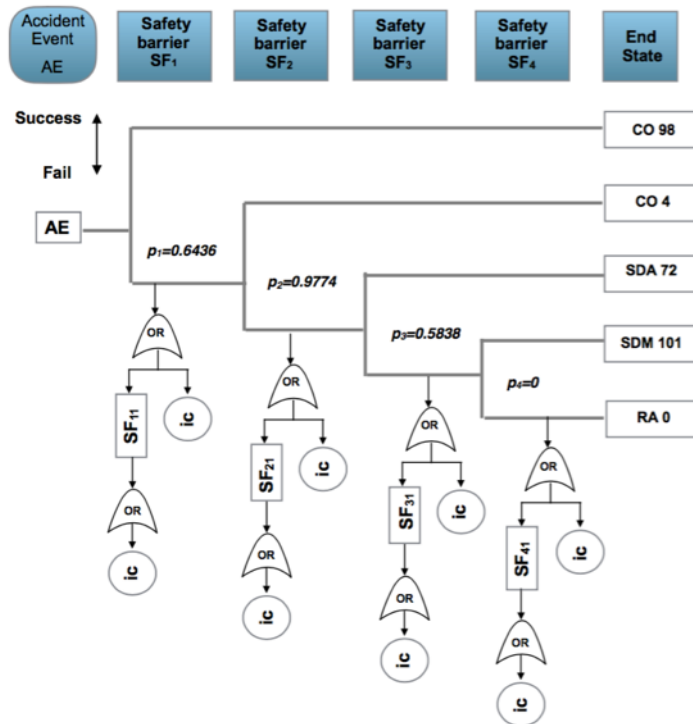


Figure 111.- Safety barrier rational scheme for the CSTR reactor.

Safety system ID	Description	Type
SF <sub>1</sub>	First alarm-temperature	Automatic
SF <sub>2</sub>	Operator-confirmation-evaluation and action	Human
SF <sub>3</sub>	Second alarm-temperature-automatic shutdown	Automatic
SF <sub>4</sub>	Second alarm-temperature-manual shutdown	Human-manual

Table 98.- Safety barriers definition for the CSTR example.

Additionally, but this is not the objective, from this last rational rearrangement it is possible to see that the safety barrier SF<sub>1</sub> is failing with a 64.4% percentage and the



operator safety barrier is failing with a 97.7 %, producing the reported and important amount of shut down's, automatic and manual.

b.- Offshore drilling blowout system.

From the analysis performed by Xue et al. [196] and from Ramzali, Miri Lavasani and Ghodusi [197], an scheme of the end states due to a blowout accident are depicted in the following Figure 112.

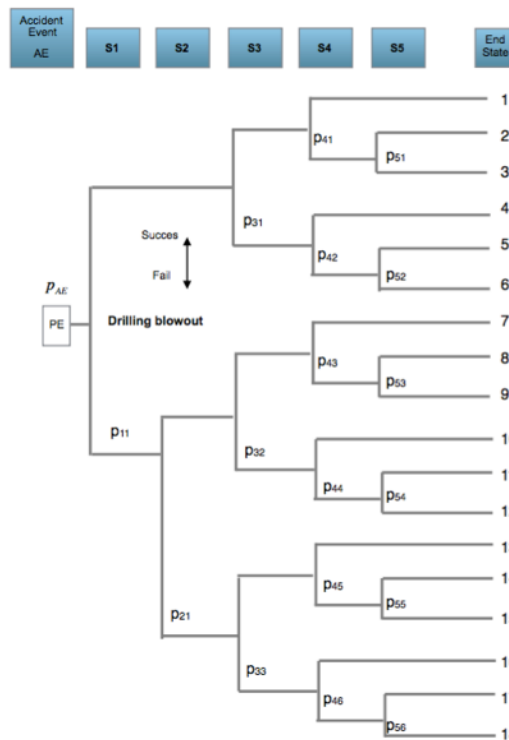


Figure 112.- Scheme for the safety analysis of a drilling blowout accident. Adapted from [196].

With the following definition of end states on Table 99 and safety systems on Table 100.

End state	Description
1	Near miss. Normal operation
2, 8, 13, 14	Incident
3, 9, 15	Accident
4, 5, 6, 10, 11, 12, 16, 17, 18	Catastrophic accident

Table 99.- End states for the drilling blowout consequence analysis [196].

Safety system ID	Description	Type
SF <sub>1</sub>	Ignition prevention	Automatic
SF <sub>2</sub>	Escalation prevention	Automatic, pasive
SF <sub>3</sub>	Emergency response	Automatic
SF <sub>4</sub>	Blowout control	Automatic-manual
SF <sub>5</sub>	Oil spill control	Automatic-manual

Table 100.- Safety systems for the drilling blowout consequence analysis [196].

The aim is to analyze the behavior of the safety systems, but from the point of view of safety barrier design the DSBd is equal to 0.29.

A rational proposal for safety barriers action can be the depicted on Figure 113 where the DSBd is equal to 1, and safety barriers definition in Table 101.

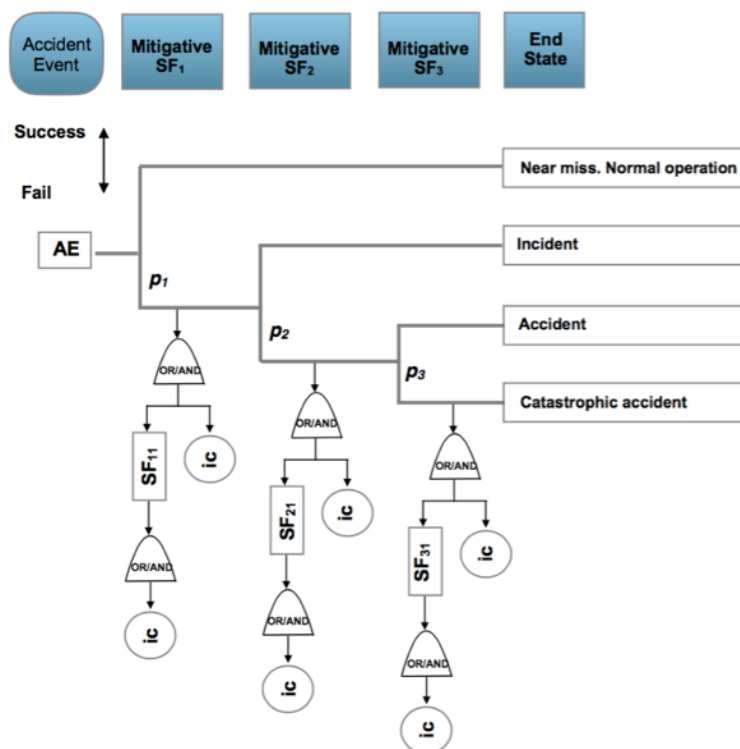


Figure 113.- Safety barrier rational scheme for the drilling blowout accident.

Safety system ID	Description	Type
SF <sub>1</sub>	Ignition prevention and emergency response shutdown	Automatic
SF <sub>2</sub>	Escalation prevention-local oil spill prevention	Automatic, passive
SF <sub>3</sub>	Blow out control - general shutdown and oil spill prevention	Automatic, passive, manual

Table 101.- Safety barriers definition for the drilling blowout accident example.

c.- Pipeline leakage

The analysis performed by Li et al. [198] for a leakage accident in a pipeline are depicted in the following Figure 114.

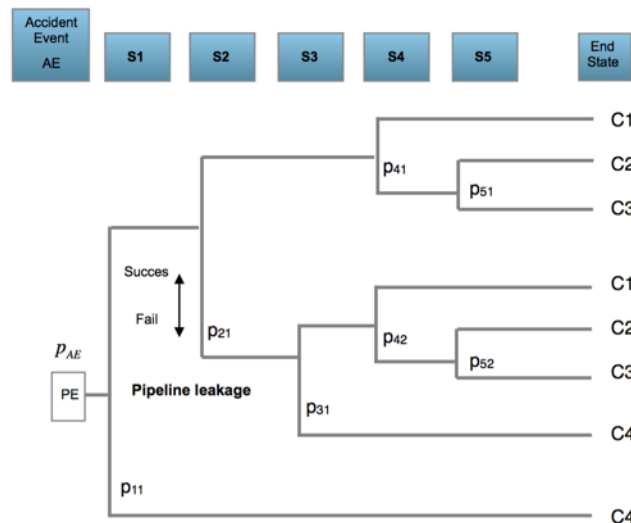


Figure 114.- Scheme for the safety analysis of a pipeline leakage accident. Adapted from [198].

With four different end states and the following definition of safety systems, Table 102.

Safety system ID	Description	Type
SF <sub>1</sub>	Leak detection	Automatic, visual
SF <sub>2</sub>	Emergency shutdown	Automatic, pasive
SF <sub>3</sub>	Manual shutdown	Manual
SF <sub>4</sub>	Repair and scaling	Automatic-manual
SF <sub>5</sub>	Ignition prevention	Automatic-manual

Table 102.- Safety systems for the pipeline leakage consequence analysis [198].

The DSBd is equal to 0.71. The rational proposal is depicted on Figure 115.

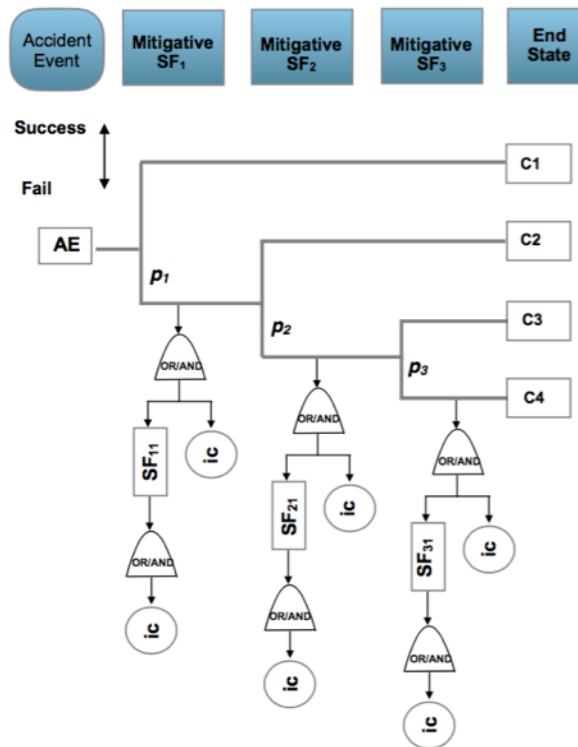


Figure 115.- Safety barrier rational scheme for the pipeline leakage accident.

And safety barriers definition on the following Table 103.

Safety system ID	Description	Type
SF <sub>1</sub>	Detection and shutdown	Automatic
SF <sub>2</sub>	Escalation prevention-local contention	Automatic, passive
SF <sub>3</sub>	Ignition prevention - general shutdown and contention	Automatic, passive, manual

Table 103.- Safety barriers definition for the pipeline leakage accident example.

From the previous examples is clear that safety barriers are systems designed for a fast, effective and comprehensive response to an unexpected situation in order to prevent the spread of the consequences of an accident from the beginning. When, from the point of view of SRC, barriers are examined and their subsystems are being analyzed together in accordance with the actions they must possess, here the objective of the analysis is to guarantee their operation and response according to their design and it is complementary to the analysis works carried out, by their authors in the different

previous examples, whose objective is to detail the behavior by breaking down the different actions, their consequences and the associated risks.

### 5.5 Control chart.

A control chart process could be established without the need to associate it with a Bayesian inference, however, the objective of the new method, in addition to the graphic control of the evolution of risk, allowing simultaneous and immediate corrective actions, is the of having, on the one hand, the objective evaluation of a statistical parameter, which has been obtained from the observations that have been made through a formal procedure, and on the other hand, to be able to have a continuous map of the functioning of the barriers of safety and the causes that affect the safety and health of workers.

It is needed to adjust the distance  $C$  of the control limits from the center line to work with the desired sensitivity.

### 5.6 Fuzzy logic application.

The general procedure follows the general steps as presented in the previous case, with the general expression;

$$P_{\text{damage,distance}} = P_{\text{LOC,distance,TOF}} \cdot f_{\text{Weather,SPW55}} \cdot f_{\text{Behavior,IB}} \quad (13)$$

The process is valid defining the bow-tie for LOC event in liquid and gas, for the determination of  $P_{\text{LOC,distance,TOF}}$  based on toxicity, operation and flammability because this is general, the determination of the  $f_{\text{Weather,SPW55}}$  factor needs to actualize the local weather information and correct the fuzzy logic membership function for it; the  $f_{\text{Behavior,IB}}$  factor needs to analyze every installation where the procedure is applied.

### 5.7 General.

The cases analyzed show that the SRC methodology has the necessary requirements to be able to carry out a treatment of risk situations in different environments.

A sensitivity analysis can also be carried out determining which initiating causes (ic's) or failures in safety barriers are more common and which areas within an industrial process, occupational situations, deviations or LOC and posterior domino effect have more incidence in risk.

About the horizon of data collection of observations it is desirable to work in continuous processes in a day basis, since the different observations can be correlated

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with the day and time of collection. In batch processes, data collection associated with each batch must be carried out. And in general it is desirable to be able to automate as much as possible the collection and analysis of the data using the data acquisition plant tools [199, 200].

Future work is extensive, in a specific scenario the analysis can be carried out considering the entire industrial manufacturing process or occupational area as a whole or analyzing it in deeply and detailed areas.

# Chapter 6

## Conclusions and further research.

### 6.1 Conclusions about research objective.

The objective for to develop a new tool to manage and assess risk situations in industrial processes and occupational accidents, responding to the three characteristics of prevention (P), simultaneity (S) and immediacy (I) has been covered by the new statistical risk control (SRC) methodology offering a new solution being a formal procedure for warning of the existence of a risk and allowing to act in advance correcting their causes.

### 6.2 Conclusions about the existing methodologies.

From the revision of the existing methodologies, it has been verified that all of them have highlights related to the intention on the one hand of establishing guidelines and standards that allow the application of risk policies in organizations, at a general level within the industry , as labor, such as those listed in Table 1. On the other by the use of traditional models based on the search and analysis of risk factors establishing their cause-consequence and the quantification of the probability of occurrence such as those collected in Tables 2 and 3. Or models based on a systemic vision of organizations or cognitive models that allow to explain the errors that occur in these organizations or in jobs, such as those listed in Tables 4 and 5. All of them they have established the characteristic of prevention but to a lesser extent that of simultaneity and do not possess that of immediacy since they are guides or those that allow an earlier analysis of the organizations and jobs and many times later, when an accident has already occurred and they want to analyze the causes that have generated it, but without the objective of still being able to have information on the current status of the scenario analyzed. It is from the development of the dynamic models which adapt to this need to assess the current situation and the immediacy in the information. The new methodology (SRC) follows

the characteristics of this last group, shown in Table 6, with the addition of a formal procedure for to perform the analysis and evaluation of risks in different scenarios considering industrial, occupational, cost-time and LOC-domino effect.

Already at this point it is established that the method, together with most of the existing ones, should not be considered unique in its application but that it is complementary to the usefulness of all of them according to the scenario to be reviewed.

### **6.3 Conclusions about the analyzed cases.**

The cases analyzed have the objective of giving a vision of the operation of the SRC model and how it can be applied in different situations. All of them are based on the author's experience in different industrial projects and applications.

In the cases analyzed at industrial and labor level, the method is applied according to the general approach encompassing the causes that cause the accident and the consequences along with its safety barriers in order to position itself in the worst case situation.

The use in cases of cost and time deviation, apart from its possibility of SRC application as a general method, the case presented of deviations in costs in which the analysis of the deviations themselves was carried out can be interpreted as not necessary, since normally in all the projects systematic controls of the costs and execution times are established compared to the previous planning however, correction of deviations are based on the analysis of their causes at detail and many times, there is no clear vision to indicate whether action is necessary. In this sense it is considered that again the SRC methodology is not a substitute for established methodologies but complementary to its information and useful in cases of cost and time deviation.

A LOC domino effect treatment based on a vision that is easy to implement has been desired. Most of the situations in which this type of risk occurs are due to everyday conditions, such as those produced in production in which they may require an additional temporary space and in which one or several deposits or tanks, even if they are of low volume, they must move; in cases of maintenance work in which containers or elements with flammable products must be incorporated to perform welding tasks, the list may in fact be long; the presented procedure has to take into account the distance to the primary focus susceptible to a loss of content (LOC), the toxicity, operating status and flammability of the components, the meteorological effects by days



of thunderstorms, rainfall and windy days greater than 55km/h, together with the characteristics or status of the critical variables, (temperature, pressure, flow, production ratio, speed) of the installation, in this case the source.

#### **6.4 Conclusions about the applied statistical tools.**

The bow-tie, which is an important element of the new methodology, was first generated by as a framework for the general treatment of risks and followed by adaptation to occupational accidents, cost-time and LOC events.

The content of the initiating causes (ic's) which may be: basic events (ba) such as failures in control systems, equipment or processes; the human risk factors (ha) which are human errors and the potential causes (pot) and the safety barriers have been presented with their general definition.

With these previous schemes it has been desired to collect the widest possible range of Bayesian inference models in order to have a better comparison. Models based on the analysis of an incidence rate (Poisson, exponential, Weibull) using the gamma or normal distributions to express the parameter  $p$ , as well as probability (binomial) using the beta distribution for the parameter, are more suitable for the general treatment of the SRC method. The models based on the analysis of the mean as a parameter (normal, lognormal) and their normal, lognormal, gamma and uniform distributions are adequate in this treatment, both in the case of cost deviations and in the LOC-domino effect analysis. The special case of approximating the values of the differences, in the analysis of the deviations in time, as an incidence rate works correctly. The special application, in time deviation, for the standard deviation as parameter using the Normal-gamma I model requires high sensitivity in the control limit with  $\pm 1/2\sigma_{\text{post}}$ . In general the presented inference models show that they are suitable for this type of analysis.

Using control charts or tables for representing the limits evolution it requires a previous adjustment of the distance of the control limits from the center line in multiples of the standard deviation, in order to raise the required sensitivity.

One could argue that a treatment of charts or control tables can be applied directly without using treatments based on Bayesian inference; however, it should be noted that the objective is not only to inform about risk situations well in advance, but also to gather the information that allows establishing a knowledge base about the evolution of the risk probabilities, whether material, personal, or economic, in the different equipment, facilities, people and projects according to the information collected, using,

for this, a formal treatment of the data collected in the form of causes or failures in security barriers.

The application of the hidden Markov chain for the analysis of reactive safety barriers provides control for these types of barriers, since they are critical in their operation or failure in the event of an accident. The tool, in its application, has the constraint of the initial values dependence, and works better using a direct scheme; but both methods, direct and recurrent it is needed to run and compare in order to take and consider, from both, the worst case.

## **6.5 General conclusions**

Additional applications are emerging as the use of a Bayesian network for assess the situations of leakage accidents [200,201]; the inclusion of the economic valuation into the FMECA procedure [202]; the use of the PETRI-NET tools [203,204]; the identification and monitoring of emerging risks over time [69,70]; or the treatment through neural networks, [205,206,207]. These are new proposals that highlight the importance of risk assessment and the supporting role that the new SRC methodology can provide.

The new method has strengths and weaknesses. The strong point is that the objective of the new methodology is that it is easy to implement and use to quickly get an idea of the overall risk status of the incidences of an industrial process, the workers situation, the cost-time deviation and the risk for a domino effect; at the same time this is a weakness since it alone cannot be enough and must be complemented with other techniques. In addition, an important feature of the new method is that it is based on dynamic risk methodologies, which, with respect to the quantitative risk analysis, allow the risk situation to be updated [104,105], and the weak points are based on the fact that by itself it is not a substitute for other methods.

In conclusion, the SRC method is a formal method that allows us to meet the three characteristics of prevention, simultaneity and immediacy. It is applicable to the assessment of industrial, occupational, cost-time, and LOC-domino effect hazards in different industrial and manufacturing scenarios, and offers an overview of the risk status in the simplest way possible and at the same time it is reliable, in accordance with the established control limits and providing warning in advance to be able to prevent risk.

## **6.6 Further research.**

Considering future works the SRC methodology is linked to the evolution of dynamic analysis of risks techniques and in this regard it is important to monitor the future evolution of this group of techniques considering also its application for different types of industries and situations of occupational risk.

Also, it is necessary to monitor the future evolution of the existing methodologies in order to complement their applicability.



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# Annex A

## Bayesian inference models.

### A.1. Bayes rule.

The starting point for Bayesian inference is Baye's rule. The simplest form of this is;

$$P(A / B) = \frac{P(A)P(B / A)}{P(A)P(B / A) + P(\bar{A})P(B / \bar{A})} \quad (\text{A.1})$$

where  $A, B$  are events such that  $P(B) > 0$ .

This is easily proven by considering that:

$$P(A / B) = \frac{P(AB)}{P(B)} \quad (\text{A.2})$$

Expression obtained from the definition of conditional probability, stated in the next equation

$$P(G / M) = \frac{P(G \cap M)}{P(M)} \quad (\text{A.3})$$

Where:

$P(AB) = P(A)P(B / A)$  by the multiplicative law of probability.

$P(B) = P(AB) + P(\bar{A}B) = P(A)P(B / A) + P(\bar{A})P(B / \bar{A})$  by the law of total probability.

The following terminology is applied:

$P(A)$ ; is the prior probability of  $A$  (meaning the probability of  $A$  before  $B$  is known to have occurred). Also represents the a priori beliefs regarding  $A$ .

$P(A / B)$ ; is the posterior probability of  $A$  given  $B$  (meaning the probability of  $A$  after  $B$  is known to have occurred). Also represents the a posteriori beliefs regarding  $A$ .

More generally, can be considered any event  $B$  such that  $P(B) > 0$  and  $k > 1$  events,  $A_1, \dots, A_k$  which form a partition of any superset of  $B$  (such as the entire space  $S$ ); then for any  $i = 1, \dots, k$  it is true that

$$P(A_i / B) = \frac{P(A_i B)}{P(B)} \quad \text{where } P(B) = \sum_{j=1}^n P(A_j B) \quad \text{and } P(A_j B) = P(A_j)P(B / A_j) \text{ being;}$$

$$P(A_i / B) = \frac{P(A_i B)}{P(B)} = \frac{P(A_i)P(B / A_i)}{\sum_{j=1}^n P(A_j B)} = \frac{P(A_i)P(B / A_i)}{\sum_{j=1}^n P(A_j)P(B / A_j)} \quad (\text{A.4})$$

## A.2. General Bayesian models and the posterior distribution.

Baye's formula can be generalized to statistical models. The basic components of a Bayesian model are:

- the data, denoted by  $y$
- the parameter, denoted by  $\theta$
- the model distribution, given by a specification or distribution for  $f(y / \theta)$  or
- the prior distribution, given by a specification or distribution for  $f(\theta)$  or  $F(\theta)$

Where  $F$ , denotes cumulative distribution function (*cdf*) and  $f$ , denotes probability density function (*pdf*) when applied to a continuous or discrete random variable.

Bayesian inference requires determination of the posterior probability distribution of  $\theta$ , as;

$$f(\theta / y) = \frac{f(\theta)f(y / \theta)}{f(y)} \quad (\text{A.5})$$

Where  $f(y)$  is the unconditional probability distribution function (*pdf*) of  $y$ , given by:

$$f(y) = \int f(y / \theta) dF(\theta) = \begin{cases} \int f(\theta)f(y / \theta) d\theta & \text{for } \theta \text{ continuous} \\ \sum_{\theta} f(\theta)f(y / \theta) & \text{for } \theta \text{ discrete} \end{cases}$$



### A.3. The proportionality expression.

Takes in consideration that  $f(y)$  is a constant with respect to  $\theta$  in the equation (A.5), which means that it is possible to write;

$$f(\theta / y) = c \cdot f(\theta) f(y / \theta) \quad (\text{A.6})$$

Where  $c = \frac{1}{f(y)}$

And from expression (A.6) is usual to write;

$$f(\theta / y) \propto f(\theta) f(y / \theta) \quad (\text{A.7})$$

Then for determining the required posterior distribution, only it is needed to multiply the prior density (or the kernel of that density) by the function of the data evolution  $y$  or model distribution.

### A.4. The binomial - beta model.

In the binomial - beta model; data  $y$  follow a binomial model and probability of risk is the parameter  $p = \theta$ ;  $(y / p) = \text{Binomial}(n, p)$ , and the binomial parameter follows, as a prior, a beta distribution  $p \sim \text{Beta}(\alpha, \beta)$ .

The posterior distribution of  $p$  is defined according the expression (A.7);

$$f(p / y) \propto f(p) f(y / p) = \frac{p^{\alpha-1} (1-p)^{\beta-1}}{B(\alpha, \beta)} \binom{n}{y} p^y (1-p)^{n-y} =$$

ignoring constants which do not depend on  $p$  is

$$= p^{\alpha-1} (1-p)^{\beta-1} p^y (1-p)^{n-y} = p^{(\alpha+y)-1} (1-p)^{(\beta+n-y)-1}, \quad \text{for } 0 < p < 1$$

This is the kernel of the beta density with parameters  $\alpha + y$  and  $\beta + n - y$ ; it follows that the posterior distribution of  $p$  is given by;

$$(p / y) \sim \text{Beta}(\alpha + y, \beta + n - y) \quad (\text{A.8})$$

Being the posterior density of  $p$  a beta distribution, defined by;

$$f(p/y) = \frac{p^{(\alpha+y)-1} (1-p)^{(\beta+n-y)-1}}{B(\alpha+y, \beta+n-y)}, \text{ for } 0 < p < 1 \quad (\text{A.9})$$

### A.5. The normal - normal model.

In the normal - normal model; data  $y$  follow a normal model;  $(y_1, \dots, y_n / \mu) \sim N(\mu, \sigma^2)$ , and the normal parameter  $p = \mu$  follows, as a prior, also a normal distribution  $\mu \sim N(\mu_0, \sigma_0^2)$ . Where  $\sigma^2, \mu_0$  and  $\sigma_0^2$  are known or specified constants.

The posterior density of  $\mu$  given data in the form of the vector  $y$ , is defined accordingly to expression (A.7);

$$\begin{aligned} f(\mu/y) &\propto f(\mu)f(y/\mu) \propto \exp\left(-\frac{1}{2}\left(\frac{\mu-\mu_0}{\sigma_0}\right)^2\right) \cdot \prod_{i=1}^n \exp\left\{-\frac{1}{2}\left(\frac{y_i-\mu}{\sigma}\right)^2\right\} \text{ being equal to} \\ &= \exp\left(-\frac{1}{2}\left[\frac{1}{\sigma_0^2}(\mu^2 - 2\mu\mu_0 + \mu_0^2) + \frac{1}{\sigma^2}\left(\sum_{i=1}^n y_i^2 - 2\mu n\bar{y} + n\mu^2\right)\right]\right) \end{aligned}$$

Where  $\bar{y} = \frac{(y_1 + \dots + y_n)}{n}$  is the sample mean.

The posterior density of  $\mu$  is proportional to the exponent of a quadratic in  $\mu$ , that is;

$$f(\mu/y) \propto \exp\left(-\frac{1}{2\sigma_*^2}(\mu - \mu_*)^2\right) \text{ which implies that; } (\mu/y) \sim N(\mu_*, \sigma_*^2).$$

It remains to find the normal mean and variance parameters  $\mu_*$  and  $\sigma_*^2$ ; that must be functions of the known quantities of  $n, \bar{y}, \sigma, \mu_0$  and  $\sigma_0$ .

Ignoring constants and maintaining the  $\mu$  function, it is possible to write;

$$\begin{aligned} &= \exp\left(-\frac{1}{2}\left[\frac{1}{\sigma_0^2}(\mu^2 - 2\mu\mu_0) + \frac{1}{\sigma^2}(-2\mu n\bar{y} + n\mu^2)\right]\right) \\ &= \exp\left(-\frac{1}{2}\left[\mu^2\left(\frac{1}{\sigma_0^2} + \frac{n}{\sigma^2}\right) - 2\mu\left(\frac{\mu_0}{\sigma_0^2} + \frac{n\bar{y}}{\sigma^2}\right) + c\right]\right) \end{aligned}$$

Where  $c$  is a constant with respect to  $\mu$ ;

Defining ;  $a = \frac{1}{\sigma_0^2} + \frac{n}{\sigma^2}$  and  $b = \frac{\mu_0}{\sigma_0^2} + \frac{n\bar{y}}{\sigma^2}$  is then;

$$\begin{aligned} &= \exp\left(-\frac{1}{2}[a\mu^2 - 2\mu b + c]\right) = \exp\left(-\frac{1}{2}\left[a\left(\mu^2 - 2\frac{b}{a}\mu\right) + c\right]\right) = \exp\left(-\frac{1}{2}\left[a\left(\mu^2 - 2\frac{b}{a}\mu + \left(\frac{b}{a}\right)^2\right) + c'\right]\right) \\ &= \exp\left(-\frac{1}{2}\left[\frac{1}{a}\left(\mu - \frac{b}{a}\right)^2 + c'\right]\right) \end{aligned}$$

Where  $c'$  is a constant with respect to  $\mu$ ; and then is;

$$f(\mu/y) \propto \exp\left(-\frac{1}{2\left(\frac{1}{a}\right)}\left(\mu - \frac{b}{a}\right)^2\right)$$

And equating with

$$f(\mu/y) \propto \exp\left(-\frac{1}{2\sigma_*^2}(\mu - \mu_*)^2\right)$$

The posterior is a normal distribution  $(\mu/y) \sim N(\mu_*, \sigma_*^2)$  with;

$$\sigma_*^2 = \frac{1}{a} = \frac{1}{\frac{1}{\sigma_0^2} + \frac{n}{\sigma^2}} = \frac{\sigma^2 \sigma_0^2}{\sigma^2 + n\sigma_0^2} \quad (\text{A.10})$$

$$\mu_* = \frac{b}{a} = \frac{\frac{\mu_0}{\sigma_0^2} + \frac{n\bar{y}}{\sigma^2}}{\frac{1}{\sigma_0^2} + \frac{n}{\sigma^2}} = \frac{\sigma^2 \mu_0 + n\sigma_0^2 \bar{y}}{\sigma^2 + n\sigma_0^2} \quad (\text{A.11})$$

A little algebra shows that the posterior mean can also be written as;

$$\mu_* = (1-k)\mu_0 + k\bar{y} \quad (\text{A.12})$$

And the posterior variance as;

$$\sigma_*^2 = k \frac{\sigma^2}{n} \quad (\text{A.13})$$

$$\text{where } k = \frac{n}{n + \frac{\sigma^2}{\sigma_0^2}} \quad (\text{A.14})$$

Another way to derive  $\mu_*$  and  $\sigma_*^2$  is to write;

$$f(\mu / y) \propto \exp\left(-\frac{1}{2\sigma_*^2}(\mu - \mu_*)^2\right) = \exp\left(-\frac{1}{2\sigma_*^2}(\mu^2 - 2\mu\mu_* + \mu_*^2)\right)$$

and equating coefficients of powers of  $\mu$  with;

$$f(\mu / y) \propto \exp\left(-\frac{1}{2}\left[\frac{1}{\sigma_0^2}(\mu^2 - 2\mu\mu_0 + \mu_0^2) + \frac{1}{\sigma^2}\left(\sum_{i=1}^n y_i^2 - 2\mu n\bar{y} + n\mu^2\right)\right]\right)$$

Is;

$$\frac{\mu^2}{\sigma_0^2} + \frac{\mu_0^2}{\sigma_0^2} + \frac{n\mu^2}{\sigma^2} = \frac{\mu^2}{\sigma_*^2} + \frac{\mu_*^2}{\sigma_*^2} \quad \text{only } \mu \text{ function} \quad \frac{\mu^2}{\sigma_0^2} + \frac{n\mu^2}{\sigma^2} = \frac{\mu^2}{\sigma_*^2} \rightarrow \frac{1}{\sigma_0^2} + \frac{n}{\sigma^2} = \frac{1}{\sigma_*^2}$$

and

$$-\frac{2\mu\mu_0}{\sigma_0^2} - \frac{2\mu n\bar{y}}{\sigma^2} = -\frac{2\mu\mu_*}{\sigma_*^2} \Rightarrow \frac{\mu_0}{\sigma_0^2} + \frac{n\bar{y}}{\sigma^2} = \frac{\mu_*}{\sigma_*^2}$$

Resolving can get the  $\mu_*$  and  $\sigma_*^2$  values.

## A.6. The normal - gamma I model.

In the normal - gamma model; data  $y$  follow a normal model;  $(y_1, \dots, y_n / \mu, \lambda) \sim N(\mu, 1/\lambda)$ , being  $\mu$  known and the parameter  $\sigma^2 = 1/\lambda$  follows, as a prior, a gamma distribution  $\lambda \sim G(\alpha, \beta)$ .

The posterior density of  $\lambda$  given data in the form of the vector  $y$ , is defined accordingly to expression (A.7);

$$f(\lambda / y) \propto f(y)f(y / \lambda) \propto \lambda^{\alpha-1} e^{-\beta\lambda} \times \prod_{i=1}^n \frac{1}{\sqrt{\lambda}} \exp\left\{-\frac{1}{2}\left(\frac{y_i - \mu}{\sqrt{\lambda}}\right)^2\right\} \text{ being equal to}$$

$$= \lambda^{\alpha-1} e^{-\beta\lambda} \times \lambda^{n/2} \exp\left\{-\frac{\lambda}{2} \sum_{i=1}^n (y_i - \mu)^2\right\} = \lambda^{a-1} e^{-b\lambda} \quad \text{for some } a \text{ and } b.$$

From the previous expression the posterior is  $f(\lambda / y) \sim \text{Gamma}(a, b)$

Where;

$$a = \alpha + \frac{n}{2} \tag{A.15}$$

$$b = \beta + \frac{n}{2} s_{\mu}^2 \tag{A.16}$$

With;

$$s_{\mu}^2 = \frac{1}{n} \sum_{i=1}^n (y_i - \mu)^2 \tag{A.17}$$

### A.7. The Poisson - gamma model.

In the Poisson - gamma model; data  $y$  follow a Poisson model;  $(y_1, \dots, y_n / \lambda) \sim \text{Poisson}(\lambda)$ , and  $\lambda$  follows, as a prior, a gamma distribution  $\lambda \sim G(\alpha, \beta)$ . The posterior density of  $\lambda$

$$f(\lambda / y) \propto f(\lambda) f(y / \lambda) = \frac{\beta^{\alpha} \lambda^{\alpha-1} e^{-\beta\lambda}}{\Gamma(\alpha)} \cdot \prod_{i=1}^n \frac{e^{-\lambda} \lambda^{y_i}}{y_i!} \propto \lambda^{(\alpha+n\bar{y})-1} e^{-\lambda(\beta+n)}$$

That is, the posterior is a gamma distribution;

$$f(\lambda / y) \sim \text{Gam}(\alpha + n\bar{y}, \beta + n) \tag{A.18}$$

### A.8. The exponential - gamma model.

In the exponential-gamma model; data  $y$  follow an exponential model;  $(y_1, \dots, y_n / \lambda) \sim \text{Poisson}(\lambda)$ , and  $\lambda$  follows, as a prior, a gamma distribution  $\lambda \sim G(\alpha, \beta)$ . The posterior density of  $\lambda$  is;

$$f(\lambda / y) \propto f(\lambda) f(y / \lambda) \propto \lambda^{\alpha-1} e^{-y_i(\beta+\lambda)} \tag{A.19}$$

Being a gamma distribution.

### A.9. The Weibull - gamma model.

In the Weibull-gamma model; data  $y$  follow a Weibull model;  $(y_1, \dots, y_n / \lambda, c) \sim Weibull(y_i / \lambda, c)$ , and  $\lambda$  follows, as a prior, a gamma distribution  $\lambda \sim G(\alpha, \beta)$ . The posterior density of  $\lambda$  is;

$$f(\lambda / y) \propto f(\lambda) f(y / \lambda) \propto y_i^{\alpha-1} \cdot e^{-\beta y_i} \cdot \left(\frac{y_i}{\lambda}\right)^{c-1} e^{-\left(\frac{y_i}{\lambda}\right)^c} \quad (A.20)$$

Being a gamma distribution.

When  $c=1$  the failure rate function is constant being equivalent to an Exponential-Uniform model. If  $c>1$  the failure rate function is increasing. If  $0<c<1$  the failure rate function is decreasing.

### A.10. The exponential - normal model.

In the exponential-normal model; data  $y$  follow an exponential model;  $(y_1, \dots, y_n / \lambda) \sim \exp(y_i / \lambda)$ , and  $\lambda$  follows, as a prior, a normal distribution  $\lambda \sim N(\mu, \sigma)$ . The posterior density of  $\lambda$  is;

$$f(\lambda / Data) \propto g(Data / \lambda) f(\lambda) \propto \lambda e^{-\lambda y_i} \cdot e^{-\frac{1}{2} \left(\frac{\lambda - \mu}{\sigma}\right)^2} \quad (A.21)$$

Being a normal distribution.

### A.11. The Poisson - normal model.

In the Poisson-normal model; data  $y$  follow a Poisson model  $(y_1, \dots, y_n / \lambda) \sim Poisson(\lambda)$ , and  $\lambda$  follows, as a prior, a normal distribution  $\lambda \sim N(\mu, \sigma)$ . The posterior density of  $\lambda$  is;

$$f(\lambda / Data) \propto g(Data / \lambda) f(\lambda) \propto \prod_{i=1}^n \frac{e^{-\lambda} \lambda^{y_i}}{y_i!} \cdot e^{-\frac{1}{2} \left(\frac{\lambda - \mu}{\sigma}\right)^2} \quad (A.22)$$

Being a normal distribution.

### A.12. The lognormal - lognormal model.

Using a LogNormal-LogNormal model; data  $(y_1, \dots, y_n)$  follow a lognormal probability distribution function with mean  $\mu$  and standard deviation  $\sigma$  expressed as

$(y_1, \dots, y_n / \mu) \sim \log N(\mu, \sigma^2)$  and a prior for the distribution of  $\mu$  also following a LogNormal distribution expressed as  $\mu \sim \log N(\mu_0, \sigma_0^2)$ . The posterior density of  $\mu$  is expressed as

$$f(\mu / y) \propto f(\mu) f(y / \mu) \propto \frac{1}{\mu} \exp \left\{ -\frac{1}{2} \left( \frac{\mu - \mu_0}{\sigma_0} \right)^2 \right\} \cdot \prod_{i=1}^n \frac{1}{y_i} \exp \left\{ -\frac{1}{2} \left( \frac{\ln y_i - \mu}{\sigma} \right)^2 \right\} \quad (\text{A.23})$$

Being a LogNormal distribution.

### A.13. The lognormal - gamma model.

Using a LogNormal-gamma model; data  $(y_1, \dots, y_n)$  follow a lognormal probability distribution function with mean  $\mu$  expressed as  $(y_1, \dots, y_n / \mu) \sim \log N(\mu, \sigma^2)$  and the prior for the distribution of  $\mu$  follows a gamma distribution expressed as  $\mu \sim G(\alpha, \beta)$ . The posterior density of  $\mu$  is expressed as

$$f(\mu / y) \propto f(\mu) f(y / \mu) \propto \mu^{\alpha-1} e^{-\frac{\mu}{\beta}} \cdot \prod_{i=1}^n \frac{1}{y_i} \exp \left\{ -\frac{1}{2} \left( \frac{\ln y_i - \mu}{\sigma} \right)^2 \right\} \quad (\text{A.24})$$

Being a gamma distribution.

### A.14. The lognormal - uniform model.

Using a LogNormal-uniform model; data  $(y_1, \dots, y_n)$  with mean  $\mu$  follow a lognormal probability distribution function with mean  $\mu$  expressed as  $(y_1, \dots, y_n / \mu) \sim \log N(\mu, \sigma^2)$  and the prior for the distribution of  $\mu$  follows an uniform distribution expressed as  $\mu \sim \text{uniform}(a, b)$ . The posterior density of  $\mu$  is expressed as:

$$f(\mu / y) \propto f(\mu) f(y / \mu) \propto \frac{1}{\mu} \cdot \prod_{i=1}^n \frac{1}{y_i} \exp \left\{ -\frac{1}{2} \left( \frac{\ln y_i - \mu}{\sigma} \right)^2 \right\} \quad (\text{A.25})$$

Being a LogNormal distribution.

### A.15. The normal - gamma II model.

In this normal - gamma model variation; data  $y$  follow a normal model;  $(y_1, \dots, y_n / \mu, \sigma) \sim N(\mu, \sigma^2)$ , being  $\sigma$  known and  $\mu$  follows, as a prior, a gamma distribution  $\mu \sim G(\alpha, \beta)$ .

The posterior density of  $\lambda$  given data in the form of the vector  $y$ , is defined accordingly to expression (A.7);

$$f(\mu / y) \propto f(y) f(y / \mu) \propto \mu^{\alpha-1} e^{-\beta/\mu} \times \prod_{i=1}^n \frac{1}{\sigma} \exp \left\{ -\frac{1}{2} \left( \frac{y_i - \mu}{\sigma} \right)^2 \right\}$$

Being a gamma distribution.



# Annex B

## Metropolis-Hastings sampler.

### B.1. The Metropolis - Hastings sampler.

With the aim to estimate the posterior mean and variance it is needed to sample from a probability distribution function (*pdf*) expressed as  $f(X_n) = f(\theta / y) \propto f(\theta)f(y / \theta)$  and defined on the domains  $S$ ,  $\Theta$  and  $Y$ . With the application of the concept of discrete Markov chain.

A stochastic process  $\{X_n, n \geq 0\}$  on state space  $S$  is defined as a discrete-time Markov Chain (DTMC) if for all  $i$  and  $j$  in  $S$ , if;

$$P(X_{n+1} = j / X_n = i, X_{n-1}, \dots, X_0) = P(X_{n+1} = j / X_n = i) \quad (\text{B.1})$$

For time  $t = n = 0, 1, 2, \dots, N$

Additionally a DTMC  $\{X_n, n \geq 0\}$  is a time homogeneous if for all  $n=0, 1, \dots$ , when;

$$P(X_{n+1} = j / X_n = i) = P(X_1 = j / X_0 = i) \quad (\text{B.2})$$

With the system having the Markov property that is if, given the complete history  $X_0, X_1, \dots, X_{n-1}$ , the next state  $X_n$  depends only upon  $X_{n-1}$ . That is, as far as predicting  $X_{n+1}$  is concerned, the knowledge of  $X_0, X_1, \dots, X_{n-2}$  is redundant if  $X_{n-1}$  is known for all values of  $n$ .

The Metropolis-Hastings procedure begins by specifying an initial value of  $X$ , call it  $X_0$ .

Then a suitable driver distribution which is easy to sample from, is defined being a pdf  $g(t / X_n)$  being the proposal density.

This driver has to be symmetric, in the sense that  $g(t / X_n) = g(X_n / t)$  or more precisely,  $g(t = a / X_n = b) = g(t = b / X_n = a) \quad \forall a, b \in \mathfrak{R}$  and for any  $n$ .

Then the following iteration is performed for each  $j = 1, 2, 3, \dots, J$  where  $J$  is large;

1.- Generate a candidate value of  $X_n$  by sampling  $X'_j \sim g(t / X_{j-1})$  with a defined range of possible values for  $X_n$ . Being  $X'_j$  the proposed value and  $g(t / X_{j-1})$ , as previously indicated, the proposal density.

2.- Calculate the acceptance probability as  $p = \frac{f(X'_j)}{f(X_{j-1})}$ .

If  $p > 1$  then the value  $p = 1$  is taken. Also if  $X'_j$  is outside the range of possible values for the random variable  $X_n$  in the proposal density, then  $f(X'_j) = 0$  and so  $p = 0$ .

3.- Accept the proposed value  $X'_j$  with probability  $p$ , with the following rule;

To determine if  $X'_j$  is accepted, generate a random  $u \sim U(0,1)$  (independently).

If  $u < p$  then accept  $X'_j$ , and otherwise reject  $X'_j$ .

4.- If  $X'_j$  has been accepted then is  $X_j = X'_j$ ; otherwise, if has been rejected, then is  $X_j = X_{j-1}$  (the last value  $X_{j-1}$  is maintained in case of rejection).

This procedure results in the realization of a Markov chain,  $X_0, X_1, X_2, \dots, X_K$ . The last value of this chain  $X_K$ , may be taken as an observation from  $f(X_n)$  at least approximately. The approximation will be extremely good if  $K$  is sufficiently large.

A value of acceptance rate (AR) is obtained as a ratio of the accepted number in front of the total sample  $J$ .

If  $J$  is sufficiently large, stochastic convergence will be achieved within  $B$  iterations, regardless of the point(s)  $X_0$  from which the algorithm is started.

Relabelling the last value,  $x_K$ , in the  $j$ 'th chain as  $x_j$  ( $j = 1, \dots, J$ ) leads the required sample, namely  $x_1, \dots, x_J \sim f(x)$  and then a random sample of size  $J$  from  $f(X_n)$ , is obtained.

The obtention of random samples of size  $J$ , can be repeated  $m$  times to obtain a representative value from the Markov chain at steady state.

Generating a chain of length  $J$  with a sample  $K$  a larger number of times  $m$  may be considered wasteful of computer resources. Typically only one long chain is generated, of length  $J = B + K$ , where  $B$  is sufficiently large for stochastic convergence to be achieved from the starting value  $x_0$  and  $J$  is again the required sample size. But in this thesis this sample obtention is repeated  $m$  times. Discarding the results of the first  $B$  iterations (called the burn-in, including also  $X_0$ ) and relabelling the last  $J$  values of the chain appropriately, the result will be the sample  $X_1, \dots, X_J \sim f(X_n)$ .

A possible problem with this method of generating the sample values is that they will be autocorrelated in a some extent not truly random and then is  $X_1, \dots, X_J \sim f(X_n)$  approximately a random sample. Moreover if  $J$  is sufficiently large, then these values will be effectively independent meaning that a probability histogram of these values will converge to  $f(X_n)$  as  $J$  tends to infinity.

## B.2. Time series. Autocorrelation.

A stochastic process corresponds to a family of time indexed random variables  $X(\omega, t)$  where  $\omega$  belongs to a sample space and  $t$  to an index set. For a fixed  $t$ ,  $X(\omega, t)$  is a random variable and for a given  $\omega$ ,  $X(\omega, t)$  as a function of  $t$  is called a sample function. Then a time series is a sample function from a certain stochastic process. The sample function is then expressed as  $X_t$  or  $X_n$ . And the following definitions are taken;

Mean function of the process:

$$\mu_n = E(X_n) \quad (\text{B.3})$$

Variance function of the process:

$$\text{Var}(X_n) = \sigma_n^2 = E(X_n - \mu_n)^2 \quad (\text{B.4})$$

Covariance function between  $X_{n_1}$  and  $X_{n_2}$  as:

$$\gamma(n_1, n_2) = E(X_{n_1} - \mu_{n_1})(X_{n_2} - \mu_{n_2}) \quad (\text{B.5})$$

Correlation function between  $X_{n_1}$  and  $X_{n_2}$  as:

$$\rho(n_1, n_2) = \frac{\gamma(n_1, n_2)}{\sqrt{\sigma_{n_1}^2} \sqrt{\sigma_{n_2}^2}} \quad (\text{B.6})$$

The sample autocorrelation function (ACF) is then defined as:

$$\rho_k = \frac{\text{Cov}(X_n, X_{n+k})}{\sqrt{\text{Var}(X_n)} \sqrt{\text{Var}(X_{n+k})}} = \hat{\rho}_k = \frac{\sum_{n=1}^{m-k} (X_n - \bar{X})(X_{n+k} - \bar{X})}{\sum_{n=1}^m (X_n - \bar{X})^2} \quad (\text{B.7})$$

Whereis the sample mean of the series is defined as:

$$\bar{X} = \sum_{n=1}^m \frac{Z_n}{m} \quad (\text{B.8})$$

For a stationary process the covariance function  $\gamma(n_1, n_2)$  and the autocorrelation function  $\rho_k$  have the following properties;

1. If  $n_1 = n_2$  that is  $k = 0$ , then is  $\gamma(n, n) = \text{Var}(X_n)$  and  $\rho_k = \rho_0 = 1$
2.  $|\gamma(n_1, n_2)| \leq \gamma(n, n)$  and  $|\rho_k| \leq 1$
3. If  $n_c = n_b + k$ , and  $n_a = n_b - k$  is  $\gamma(n_b, n_c) = \gamma(n_b, n_a)$  and is  $\rho_k = \rho_{-k}$  and the functions are symmetric around the lag  $k$ ; this property follows from the characteristic of the Markov chain, having the Markov property.

An ACF plot can be performed showing the evolution of the (ACF)  $\rho_k$  with  $k$ .

An approximate 95% confidence intervals are drawn at  $\pm 2S\hat{\rho}_k$  when plotting the (ACF) being this parameter the standard error for the auto correlation function, defined as:

$$S\hat{\rho}_k = \sqrt{\frac{1}{m}(1 + 2\hat{\rho}_1^2 + \dots + 2\hat{\rho}_{k-1}^2)} \quad (\text{B.9})$$

For a stochastic process  $\rho_k = \begin{cases} 1, & \text{for } k = 0 \\ 0, & \text{for } k \neq 0 \end{cases}$  and then the sample autocorrelation

function in every lag is,  $2\hat{\rho}_1^2 = \dots = 2\hat{\rho}_{k-1}^2 = 0$  and;

$$S\hat{\rho}_k = \sqrt{\frac{1}{m}} \quad (\text{B.10})$$

Being  $m$  the total Markov chain number of lags.

### B.3. Time series. Partial autocorrelation.

In addition to the autocorrelation between  $X_n$  and  $X_{n+k}$  it is needed to analyze the correlation between  $X_n$  and  $X_{n+k}$  and the intervening variables between them as  $X_{n+1}$ ,  $X_{n+2}$ , ..., and  $X_{n+k-1}$ . The conditional correlation;

$$\text{Corr}(X_n, X_{n+k} \mid X_{n+1}, \dots, X_{n+k-1}) \quad (\text{B.11})$$

is referred as the partial autocorrelation function (PACF).

To analyze this expression it is needed to define what is the linear dependence of  $X_n$  and  $X_{n+k}$  everyone from  $X_{n+1}$ ,  $X_{n+2}$ , ..., and  $X_{n+k-1}$ . Then considering a Markov chain  $X_n$  and assuming that  $E(X_n) = 0$ , is;

a.- Let the linear dependence of  $X_{n+k}$  from  $X_{n+1}$ ,  $X_{n+2}$ , ..., and  $X_{n+k-1}$  be defined as the best linear estimate  $\hat{X}_{n+k}$  and expressed as;

$$\hat{X}_{n+k} = \alpha_1 X_{n+k-1} + \alpha_2 X_{n+k-2} + \dots + \alpha_{k-1} X_{n+1} \quad (\text{B.12})$$

where  $\alpha_i$  ( $1 \leq i \leq k-1$ ) are the mean squared linear regression coefficients obtained from minimizing;

$$E(X_{n+k} - \hat{X}_{n+k})^2 = E(X_{n+k} - \alpha_1 X_{n+k-1} - \alpha_2 X_{n+k-2} - \dots - \alpha_{k-1} X_{n+1})^2 \quad (\text{B.13})$$

Similarly;

b.- The best linear estimate  $\hat{X}_n$  can be expressed as;

$$\hat{X}_n = \beta_1 X_{n+1} + \beta_2 X_{n+2} + \dots + \beta_{k-1} X_{n+k-1} \quad (\text{B.14})$$

where  $\beta_i$  ( $1 \leq i \leq k-1$ ) are the mean squared linear regression coefficients obtained from minimizing;

$$E(X_n - \hat{X}_n)^2 = E(X_n - \beta_1 X_{n+1} - \beta_2 X_{n+2} - \dots - \beta_{k-1} X_{n+k-1})^2 \quad (\text{B.15})$$

The partial autocorrelation function (PACF) between  $X_n$  and  $X_{n+k}$  will equal the ordinary autocorrelation between  $(X_n - \hat{X}_n)$  and  $(X_{n+k} - \hat{X}_{n+k})$  being defined as;

$$p_k = \frac{\text{Cov}[(X_n - \hat{X}_n)(X_{n+k} - \hat{X}_{n+k})]}{\sqrt{\text{Var}(X_n - \hat{X}_n)}\sqrt{\text{Var}(X_{n+k} - \hat{X}_{n+k})}} \quad (\text{B.16})$$

For a stochastic process  $p_k = \begin{cases} 1, & \text{for } k = 0 \\ 0, & \text{for } k \neq 0 \end{cases}$

An ACF plot can be performed showing the evolution of the (PACF)  $p_k$  with  $k$ .

Also an approximate 95% confidence intervals are drawn at  $\pm 2S\hat{p}_k$  when plotting the (PACF) being this parameter the standard error for the partial auto correlation function, defined as:

$$S\hat{p}_k = \sqrt{\frac{1}{n}} \quad (\text{B.17})$$

Being  $n$  the total Markov chain number of lags.

# Annex C

## Hidden time Markov chains.

### C.1. Discrete time Markov chains.

Let  $X_n$  be the state of the system at time  $n$  for a system that is observed at times  $0, 1, 2, \dots, n$ . Suppose the system is currently at time  $n$ , and observations have been made in times  $0, 1, \dots, n-1 \leq n$ . The question is if it is possible to predict, in a probabilistic way, the state of the system at time  $n+1$ . In general,  $X_{n+1}$  depends (in a possibly random fashion) on  $X_0, X_1, \dots, X_{n-1}$ . Considerable simplification occurs if, given the complete history  $X_0, X_1, \dots, X_{n-1}$ , the next state  $X_n$  depends only upon  $X_{n-1}$ . That is, as far as predicting  $X_{n+1}$  is concerned, the knowledge of  $X_0, X_1, \dots, X_{n-2}$  is redundant if  $X_{n-1}$  is known. If the system has this property at all values of  $n$  (and not just for one  $n$  value), it is said that the system has a Markov property.

A stochastic process  $\{X_n, n \geq 0\}$  on state space  $S$  is a discrete-time Markov Chain (DTMC) if for all  $i$  and  $j$  in  $S$ , if;

$$P(X_{n+1} = j / X_n = i, X_{n-1}, \dots, X_0) = P(X_{n+1} = j / X_n = i) \quad (C.1)$$

A DTMC  $\{X_n, n \geq 0\}$  is a time homogeneous if for all  $n=0, 1, \dots$ , when;

$$P(X_{n+1} = j / X_n = i) = P(X_1 = j / X_0 = i) \quad (C.2)$$

It should be kept in mind that according to (C.1) implies that the conditional probability on the left hand side is the same no matter what values  $X_0, X_1, \dots, X_{n-1}$  take. Sometimes this property is described as <given the present state of the system (namely  $X_n$ ), the future state of the DTMC (namely  $X_{n+1}$ ) is independent of its past (namely  $X_0, X_1, \dots, X_{n-1}$ )>. The quantity;

$$P(X_{n+1} = j / X_n = i) \text{ for } i, j = 1, 2, \dots, N \quad (C.3)$$

Is called a *one-step transition probability* of the DTMC at time  $n$ . Equation (C.2) implies that, for time homogeneous DTMCs, the one-step transition probability depends on  $i$  and  $j$  but is the same at all times  $n$ ; hence the terminology *time homogeneous*.

In this thesis only time-homogeneous DTMCs with finite state space  $S=\{1,2,..,N\}$  are considered.

## C.2. Transient distributions.

Let  $\{X_n, n \geq 0\}$  be a time-homogeneous DTMC on state space  $S = \{1, 2, \dots, N\}$  with transition probability matrix  $P$  and initial distribution  $a = [a_1, \dots, a_N]$  of states. Where;

$$a_i = P(X_0 = i) \text{ for } 1 \leq i \leq N.$$

The transient distribution corresponds to the distribution of  $X_n$  for a fixed  $n \geq 0$ . And in general the problem is to know the state  $P(X_n = j)$  for all  $j \in S$  and  $n \geq 0$ . Is;

$$P(X_n = j) = \sum_{i=1}^N P(X_n = j / X_0 = i) \cdot P(X_0 = i) = \sum_{i=1}^N a_i \cdot P(X_n = j / X_0 = i) \quad (C.4)$$

Introducing the notation;

$$a_j^{(n)} = P(X_n = j) \quad (C.5)$$

$$p_{i,j}^{(n)} = P(X_n = j / X_0 = i) \quad (C.6)$$

This quantity is called the  $n$ -step transition probability of the DTMC. Analogous to the one-step transition probability matrix  $P = [p_{i,j}]$ , an  $n$ -step transition probability matrix is defined as follows.

$$P^{(n)} = \begin{bmatrix} P_{1,1}^{(n)} & P_{1,2}^{(n)} & P_{1,3}^{(n)} & \cdots & P_{1,N}^{(n)} \\ P_{2,1}^{(n)} & P_{2,2}^{(n)} & P_{2,3}^{(n)} & \cdots & P_{2,N}^{(n)} \\ P_{3,1}^{(n)} & P_{3,2}^{(n)} & P_{3,3}^{(n)} & \cdots & P_{3,N}^{(n)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ P_{N,1}^{(n)} & P_{N,2}^{(n)} & P_{N,3}^{(n)} & \cdots & P_{N,N}^{(n)} \end{bmatrix}$$

The case for  $P^{(0)}$ , is;  $p_{i,j}^{(0)} = P(X_0 = j / X_0 = i) = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$



This implies that  $P^{(0)} = I$  is the identity matrix, (from (C.3) and (C.6)),

The case for  $P^{(1)}$ , is;  $p_{i,j}^{(1)} = P(X_1 = j / X_0 = i) = p_{i,j}$

And hence

$$P^{(1)} = P$$

Now construct the transient vector;

$a^{(n)} = [a_1^{(n)}, a_2^{(n)}, \dots, a_N^{(n)}]$  so that (C.4) can be written in matrix form, as;

$$a^{(n)} = a \cdot P^{(n)}$$

Note that;  $a^{(0)} = a \cdot P^{(0)} = a \cdot I = a$  the initial distribution of the DTMC

To develop computing methods of the n-step transition probability matrix  $P^{(n)}$  the following theorem is needed;

**Theorem 1.- n-step transition probability matrix.**

$$P^{(n)} = P^n \tag{C.7}$$

Where  $P^n$  is the  $n$ th power of matrix  $P$ .

Proof: Since  $P^0 = I$  and  $P^1 = P$  the theorem is true for  $n = 0, 1$ . Hence let  $n \geq 2$ . Is;

$$\begin{aligned} p_{i,j}^{(n)} &= P(X_n = j / X_0 = i) \\ &= \sum_{k=1}^N P(X_n = j / X_{n-1} = k, X_0 = i) \cdot P(X_{n-1} = k / X_0 = i) \\ &= \sum_{k=1}^N p_{i,k}^{(n-1)} \cdot P(X_n = j / X_{n-1} = k, X_0 = i) \quad \text{from C.6} \\ &= \sum_{k=1}^N p_{i,k}^{(n-1)} \cdot P(X_n = j / X_{n-1} = k) \quad \text{due to the Markov property} \\ &= \sum_{k=1}^N p_{i,k}^{(n-1)} \cdot P(X_1 = j / X_0 = k) \quad \text{due to time homogeneity} \\ &= \sum_{k=1}^N p_{i,k}^{(n-1)} \cdot p_{k,j} \end{aligned}$$

The last sum can be recognized as a matrix multiplication operation, and the equation above, which is valid for all  $1 \leq i, j \leq N$  can be written in a more succinct fashion in matrix terminology as;

$$P^{(n)} = P^{(n-1)} \cdot P \quad (\text{C.8})$$

Using the equation above for  $n = 2$  is  $P^{(2)} = P^{(1)} \cdot P = P \cdot P$

Similarly for  $n = 3$  is  $P^{(3)} = P^{(2)} \cdot P = P \cdot P \cdot P$

In general, using  $P^{(n)}$  as the  $n$ th power of the matrix  $P$ , is (C.7).

### Corollary 1.1

$$a^{(n)} = a \cdot P^{(n)}$$

### Corollary 1.2

$$P(X_{n+m} = j / X_n = i, X_{n-1}, \dots, X_0) = P(X_{n+m} = j / X_n = i) = p_{i,j}^m$$

### Theorem 2.- Chapman-Kolmogorov equation

The  $n$ -step transition probabilities satisfy the following equation, called the Chapman-Kolmogorov equation:

$$p_{i,j}^{(n+m)} = \sum_{k=1}^N p_{i,k}^{(n)} p_{k,j}^{(m)} \quad (\text{C.9})$$

Proof:

$$\begin{aligned} P(X_{n+m} = j / X_0 = i) &= \sum_{k=1}^N P(X_{n+m} = j / X_n = k, X_0 = i) \cdot P(X_n = k / X_0 = i) \\ &= \sum_{k=1}^N P(X_{n+m} = j / X_n = k) \cdot P(X_n = k / X_0 = i) \quad \text{due to Corollary 1.2} \\ &= \sum_{k=1}^N P(X_m = j / X_0 = k) \cdot P(X_n = k / X_0 = i) \quad \text{due to time homogeneity} \\ &= \sum_{k=1}^N P(X_n = k / X_0 = i) \cdot P(X_m = j / X_0 = k) \\ &= \sum_{k=1}^N p_{i,k}^{(n)} \cdot p_{k,j}^{(m)} \end{aligned}$$

In matrix form can be expressed as;

$$P^{(n+m)} = P^{(n)} \cdot P^{(m)} = p^{(m)} \cdot p^{(n)}$$

The equations above imply that the matrices  $P^{(n)}$  and  $P^{(m)}$  commute for all  $n$  and  $m$ ; being an unusual property for matrices. Theorem 2 makes it especially easy to compute the transient distributions in DTMC's since it reduces the computations to matrix power and multiplication.

### C.3. Occupancy times.

Let  $\{X_n, n \geq 0\}$  be a time - homogeneous DTMC on state space  $S = \{1, 2, \dots, N\}$  with transition probability matrix  $P$  and initial distribution  $a = [a_1, \dots, a_N]$ .

The occupancy times are the expected amount of time that the DTMC spends in a given state during a given interval of time. Since the DTMC undergoes one transition per unit time, the occupancy time is the same as the expected number of times it visits a given state in a finite number of transitions. Formally is;

Let  $N_j(n)$  be the number of times the DTMC visits state  $j$  over the time span  $\{0, 1, 2, \dots, n\}$  and let;

$$m_{i,j}(n) = E(N_j(n) / X_0 = i)$$

The quantity  $m_{i,j}(n)$  is called the occupancy time up to  $n$  of state  $j$  starting from state  $i$ .

Let;

$$M(n) = \begin{bmatrix} m_{1,1}(n) & m_{1,2}(n) & m_{1,3}(n) & \cdots & m_{1,N}(n) \\ m_{2,1}(n) & m_{2,2}(n) & m_{2,3}(n) & \cdots & m_{2,N}(n) \\ m_{3,1}(n) & m_{3,2}(n) & m_{3,3}(n) & \cdots & m_{3,N}(n) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ m_{N,1}(n) & m_{N,2}(n) & m_{N,3}(n) & \cdots & m_{N,N}(n) \end{bmatrix}$$

be the occupancy times matrix. The next theorem gives a simple method of computing the occupancy times.

#### Theorem 3.- Occupancy times

The occupancy times matrix is given by;

$$M(n) = \sum_{r=0}^n P^r \tag{C.10}$$

Proof: Let  $Z_n = 1$  if  $X_n = j$  and  $Z_n = 0$  if  $X_n \neq j$ . Then

$$N_j(n) = Z_0 + Z_1 + \dots + Z_n$$

Hence;

$$\begin{aligned} m_{i,j}(n) &= E(N_j(n) / X_0 = i) = E(Z_0 + Z_1 + \dots + Z_n / X_0 = i) = \sum_{r=0}^n E(Z_r / X_0 = i) \\ &= \sum_{r=0}^n P(Z_r = 1 / X_0 = i) = \sum_{r=0}^n P(X_r = j / X_0 = i) = \sum_{r=0}^n p_{i,j}^{(r)} \end{aligned}$$

And writing the previous equivalence in matrix form, the Equation A.10 is obtained.

#### C.4. Limiting behavior.

Let  $\{X_n, n \geq 0\}$  be a time - homogeneous DTMC on state space  $S = \{1, 2, \dots, N\}$  with transition probability matrix  $P$ . The problem to find the limiting behavior of  $X_n$  as  $n$  tends to infinity.

If it does, the limiting or steady-state distribution is denoted by;

$$\pi = [\pi_1, \pi_2, \dots, \pi_N] \quad (C.11)$$

where;

$$\pi_j = \lim_{n \rightarrow \infty} P(X_n = j) \text{ with } j \in S \quad (C.12)$$

And if the limiting distribution exists it has to satisfy the following theorem;

#### Theorem 4.- Limiting distributions

If a limiting distribution  $\pi$  exists it satisfies,

$$\pi_j = \sum_{i=1}^N \pi_i p_{i,j} \text{ with } j \in S \quad \text{balance equations} \quad (C.13)$$

and

$$\sum_{j=1}^N \pi_j = 1 \quad \text{normalizing equation} \quad (C.14)$$

Proof: Using the law of Total Probability

The law of Total Probability, states that if  $\{B_n, n=1,2,3,\dots\}$  is finite or countable infinity partition of a sample space, and each event  $B_n$  is measurable, then for any event  $A$  of the same probability space, is;

$$P(A) = \sum_n P(A \cap B_n) = \sum_n P(A / B_n) \cdot P(B_n) \text{ And is the marginal probability of } A.$$

Then is;

$$P(X_{n+1} = j) = \sum_{i=1}^N P(X_n = i) \cdot p_{i,j} \quad \text{for } j \in S \quad (\text{C.15})$$

Now, let  $n$  tend to infinity on both the right- and left-hand sides. Then, assuming that the limiting distribution exists, is;

$$\lim_{n \rightarrow \infty} P(X_n = j) = \lim_{n \rightarrow \infty} P(X_{n+1} = j) = \pi_j$$

Substituting in (C.15) the equation (C.13) is obtained, and can be written in matrix form as;

$$\pi = \pi \cdot P \quad (\text{C.16})$$

And are called *the balance equations* or *the steady-state equations*. Equation (C.14) is called the *normalizing equation*.

### C.5. First passage times.

The question to answer is what is the random time at which a stochastic process “first passes into” a given subset of the state space.

Let  $\{X_n, n \geq 0\}$  be a DTMC on state space  $S = \{1, 2, \dots, N\}$  with transition probability matrix  $P$ . The first-passage time into state  $N$ , is defined as;

$$T = \min\{n \geq 0 : X_n = N\} \quad (\text{C.17})$$

Note that  $T$  is not the minimum number of steps in which the DTMC can reach state  $N$ . It is the (random) number of steps that it takes to actually visit state  $N$ . Typically  $T$  can take values in  $\{0, 1, 2, 3, \dots\}$ .

Let;

$$m_i = E(T / X_0 = i)$$

Clearly  $m_N = 0$ . The next theorem gives a method of computing  $m_i$  with  $1 \leq i \leq N-1$ .

**Theorem 5.- Expected first-passage times**

Proof:  $\{m_i, 1 \leq i \leq N-1\}$  satisfy;

$$m_i = 1 + \sum_{j=1}^{N-1} p_{i,j} m_j \quad \text{for } 1 \leq i \leq N-1 \quad (\text{C.18})$$

That is, the mean passage time from state  $i$  to state  $j$  (also denoted by  $m_{ij}$ ) is the expected number of transitions before state  $j$  is reached. There are two possibilities to perform these transitions:

Case 1: With probability  $p_{ij}$  it will take one transition to go from state  $i$  to state  $j$ .

Case 2: With probability  $p_{ik}$  it will take one transition to go from  $i$  to state  $k$  ( $k \neq j$ ). From state  $k$  it will take  $m_{kj}$  transitions to go from  $k$  to  $j$ . Therefore, it will take an average of  $1 + m_{kj}$  transitions to go from  $i$  to  $j$ .

Suppose  $X_0 = i$  and  $X_1 = j$  (the transition is from initial state  $i$  to state  $j$ ) because if  $j=N$  then is  $T=1$ , and if  $j \neq N$  then the DTMC has spent time unit to go to state  $j$  and the expected time from then on to reach state  $N$  is given by  $m_j$ , hence we get;

$$E(T / X_0 = i, X_1 = j) = \begin{cases} 1 & \text{if } j = N \\ 1 + m_j & \text{if } j \neq N \end{cases}$$

Then  $m_i$  or  $m_{ij}$  is;

$$m_i = \text{case1} + \text{case2} = p_{ij} \cdot 1 + \sum_{k \neq j} p_{ik} (1 + m_{kj}) = 1 + \sum_{k \neq j} p_{ik} m_{kj}$$

From the definition of  $m_i$ ;

$$\begin{aligned} m_i &= E(T / X_0 = i) = \sum_{j=1}^N E(T / X_0 = i, X_1 = j) \cdot P(X_1 = j / X_0 = i) = \sum_{j=1}^{N-1} (1 + m_j) p_{i,j} + (1) p_{i,N} = \\ &= \sum_{j=1}^N (1) p_{i,j} + \sum_{j=1}^{N-1} p_{i,j} m_j = 1 + \sum_{j=1}^{N-1} p_{i,j} m_j \end{aligned}$$

as desired.

## C.6. Discrete and hidden Markov chain.

As defined before, consider a system which may be described at any time as being in one of a set of  $N$  distinct states.

At discrete times, the system undergoes a change of state (possibly back to the same state) according to a set of probabilities associated with the state. We denote the time instants associated with state changes as  $n=0,1,2,\dots$ , and we denote the actual state at time  $t$  as  $X_n$ . If it has a Markov property.

$$P(X_{n+1} = j / X_n = i, X_{n-1}, \dots, X_0) = P(X_{n+1} = j / X_n = i)$$

considering those processes with time independency and leading a set of state transition probabilities  $p_{i,j}^{(n)}$  of the form;

$$p_{i,j}^{(n)} = P(X_n = j / X_0 = i) \quad 1 \leq i, \quad j \leq N$$

With state transition coefficients having the properties;

$$p_{i,j}^{(n)} \geq 0$$

$$\sum_{j=1}^N p_{i,j}^{(n)} = 1$$

This situation considers Markov chains in which each state correspond to an observable event. There is a case where the observation is also a probabilistic function of the state- $i$  but the resulting chain (which is called a hidden Markov chain) is a doubly embedded stochastic process with an underlying stochastic chain that is not observable (it is hidden). And the hidden underlying sequence can only be deduced through the observations of a visible sequence of variables or symbols related to with the underlying chain.

A hidden Markov chain (HMC) is characterized by the following;

- 1.- The number of the states  $N$  denoted as  $S = \{S_1, S_2, \dots, S_N\}$  and the state at time  $t=n$  as  $X_n$ , of the hidden chain.
- 2.- The number  $M$  of distinct observation symbols. Denoted as  $V = \{V_1, V_2, \dots, V_M\}$ . And their associated observation sequence  $O = O_1 O_2, \dots, O_T$ .

Where each observation  $O_t$  is one of the symbols  $v_k$  from  $V$  and  $T$  is the number of observations in the sequence.

3.- The state transition probability distribution  $P = \{p_{i,j}^{(n)}\}$  where;

$$p_{i,j}^{(n)} = P(X_n = j / X_0 = i) \quad 1 \leq i, \quad j \leq N$$

4.- The observation symbol probability distribution in state  $X_n = j$ ,  $Q = \{q_j(k)\}$  where;

$$q_j(k) = P(v_k \text{ at } t = n / X_n = j) \text{ for } 1 \leq j \leq N \text{ and } 1 \leq k \leq M$$

5.- The initial state distribution  $a = [a_1, \dots, a_N]$  where;

$$a_i = P(X_0 = i) \text{ for } 1 \leq i \leq N$$

A complete specification of an HMC requires the definition of the probabilities;

$$p_{i,j}^{(n)} = P(X_n = j / X_0 = i) \quad 1 \leq i, \quad j \leq N$$

$$q_j(k) = P(v_k \text{ at } t / X_n = j) \text{ for } 1 \leq j \leq N \text{ and } 1 \leq k \leq M$$

and the initial state distribution;

$$a_i = P(X_0 = i) \text{ for } 1 \leq i \leq N$$

To indicate the complete parameter set of a HMC, it is possible to use the compact notation;

$$\psi = (P, Q, a) \tag{C.19}$$

The Basic Problem for HMC applied in this thesis is called the learning: How to adjust parameters of the HMC  $\psi = (P, Q, a)$  to maximize  $P(O / \psi)$ .

There is no any known analytical way to solve this problem. But it is possible choose  $\psi = (P, Q, a)$  such that  $P(O / \psi)$  is locally maximized by using an iterative procedure such as the Baum-Welch method also called the backward-forward method.

First a parameter  $\xi_n(i, j)$  is defined, the probability of being in state  $X_n = i$  at time  $n$  and state  $X_{n+1} = j$  at time  $n+1$ , given a  $\psi = (P, Q, a)$  for a HMC with an observation sequence.



$$\xi_n(i, j) = P(X_n = i, X_{n+1} = j / O, \psi) \quad (C.20)$$

The sequence of events leading to the conditions required is illustrated in Figure C.6.1

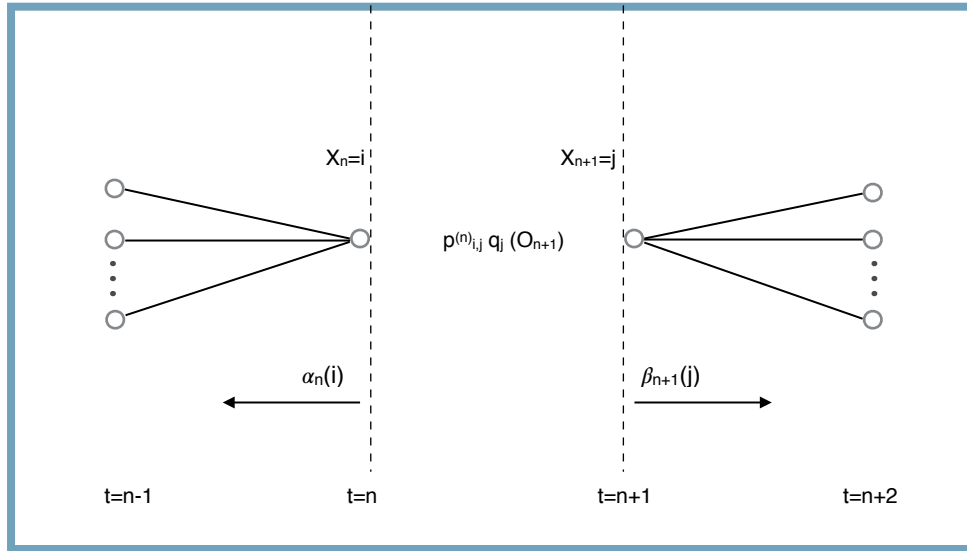


Figure C.6.1.- Backward and Forward variables and sequence of events (adapted from [140]).

From the definitions of forward and backward variables (Figure C.6.1):

forward variable:

$$\alpha_n(i) = P(O_1 O_2 \dots O_t, X_n = i / \psi) \quad (C.21)$$

backward variable:

$$\beta_n(i) = P(O_{t+1}, O_{t+2}, \dots, O_T / X_n = i, \psi) \quad (C.22)$$

It is possible to can write  $\xi_t(i, j)$  in the form,

$$\xi_n(i, j) = \frac{\alpha_n(i) p_{i,j}^{(n)} q_j(O_{n+1}) \beta_{n+1}(j)}{P(O / \psi)} = \frac{\alpha_n(i) p_{i,j}^{(n)} q_j(O_{n+1}) \beta_{n+1}(j)}{\sum_{i=1}^N \sum_{j=1}^N \alpha_n(i) p_{i,j}^{(n)} q_j(O_{n+1}) \beta_{n+1}(j)} \quad (C.23)$$

Where the numerator term is just  $P(X_n = i, X_{n+1} = j, O / \psi)$  and the division by  $P(O / \psi)$  gives the desired probability measure.

Defining  $\gamma_n(i)$  as the probability of being in state  $X_n = i$  at time  $n$  given the observation sequence and the model  $\psi = (P, Q, a)$  it is possible to relate to  $\xi_n(i, j)$  by summing over  $j$ , giving;

$$\gamma_n(i) = \sum_{j=1}^N \xi_n(i, j) \quad (C.24)$$

If we sum  $\gamma_n(i)$  over the time index  $n$ , we get a quantity which can be interpreted as the expected number of times that state  $X_n = i$  is visited, or equivalently, the expected number of transitions made from state  $X_n = i$ . Similarly, summation of  $\xi_n(i, j)$  from  $t=1$  to  $t=n-1$  can be interpreted as the expected number of transitions from state  $X_n = i$  to state  $X_n = j$ , that is:

$$\sum_{n=1}^{n-1} \gamma_n(i) = \text{expected number of transitions from } X_n = i$$

$$\sum_{n=1}^{n-1} \xi_n(i, j) = \text{expected number of transitions from } X_n = i \text{ to } X_n = j$$

Then a set of reasonable re-estimation formulas for  $a$ ,  $P$  and  $Q$  are:

$\bar{a}_i$  = expected frequency in state  $X_0 = i$  at time  $\gamma_0(i)$  ( $n=0$ )

$$\bar{p}_{i,j}^{(n)} = \frac{\text{expected number of transitions from state } X_n = i \text{ to state } X_n = j}{\text{expected number of transitions from state } X_n = i} = \frac{\sum_{n=1}^{n-1} \xi_n(i, j)}{\sum_{n=1}^{n-1} \gamma_n(i)}$$

$$\bar{q}_j(k) = \frac{\text{expected number of times in state } X_n = j \text{ and observing symbol } v_k}{\text{expected number of times in } X_n = j} = \frac{\sum_{n=1}^n \gamma_n(j)}{\sum_{n=1}^n \gamma_n(j)}$$

If the current model is defined as  $\psi = (P, Q, a)$  and it is used to compute the right hand sides of the previous expressions the re-estimate model is defined as  $\bar{\psi} = (\bar{P}, \bar{Q}, \bar{a})$ , and has been proven [140], that:

- 1.- The initial model  $\psi$  defines a critical point in the likelihood function in which case  $\bar{\psi} = \psi$
- 2.- The model  $\bar{\psi}$  is more likely than model  $\psi$  in the sense that  $P(O/\bar{\psi}) > P(O/\psi)$ .

It should be pointed that the forward-backward algorithm leads to local maxima only, and taking in care that in most problems of interest the optimization surface can be complex with many local maxima.



# Annex D

## Control charts.

The purpose of establishing a control chart or Shewhart control chart, is to systematically monitor the activity and determine if it is necessary to carry out corrective actions on it.

They are based on two premises:

- 1.- The measurements or the taking of samples must belong to a population that has a distribution function.
- 2.- The measurements, for each group of samples, must be statistically independent. A measurement should not affect other measurements.

The characteristic or parameter that is measured is a variable that belong to a set of possible values (cost, precursor event, failures, etc.) and their control is based on the analysis of the tendency and variability. The general model on which the Shewhart control chart is based in the measurement of a statistic  $\theta \in \Theta$  for which their mean is  $\mu_{\theta}$  and the variance is  $\sigma_{\theta}^2$  being;

The upper control limit, and low control limits that are defined as:

$$UCL = \mu_{\theta} + C\sigma_{\theta} \quad (D.1)$$

$$LCL = \mu_{\theta} - C\sigma_{\theta} \quad (D.2)$$

Respectively, where  $C$  is the distance of the control limits from the center line in multiples of the standard deviation.



# Annex E

## Tables of LOC probabilities.

### E.1. Stationary and atmospheric vessels.

G1: Instantaneous release of the content.

G2: Continuous release of the content in 10 min at constant rate.

G3: Continuous release from a hole with an effective diameter of 10 mm.

Installation	LOC frequency ( $y^{-1}$ )		
	G1	G2	G3
stationary vessel			
pressure	$5 \cdot 10^{-7}$	$5 \cdot 10^{-7}$	$1 \cdot 10^{-5}$
process	$5 \cdot 10^{-6}$	$5 \cdot 10^{-6}$	$1 \cdot 10^{-4}$
reactor	$5 \cdot 10^{-5}$	$5 \cdot 10^{-5}$	$1 \cdot 10^{-3}$

Table E.1.1.- Frequency  $\lambda$  of LOC episode for stationary vessels, (adapted from [45]).

Installation	LOC frequency ( $y^{-1}$ )		
	G1	G2	G3
atmospheric vessel			
single containment	$5 \cdot 10^{-6}$	$5 \cdot 10^{-6}$	$1 \cdot 10^{-4}$
protective outer shell	$5 \cdot 10^{-7}$	$5 \cdot 10^{-7}$	$1 \cdot 10^{-4}$
double containment	$5 \cdot 10^{-8}$	$5 \cdot 10^{-8}$	$1 \cdot 10^{-4}$
in-ground	$1 \cdot 10^{-8}$	-	-

Table E.1.2.- Frequency  $\lambda$  of LOC episode for atmospheric vessels, (adapted from [45]).

## E.2. Pipes.

G1: Full bore rupture.

G2: Leak.

Installation	LOC frequency ( $y^{-1}$ )		
	pipeline	G1	G2
nominal $\varnothing < 75$ mm	$1 \cdot 10^{-6}$	$5 \cdot 10^{-6}$	-
75 mm $< \varnothing < 150$ mm	$3 \cdot 10^{-7}$	$5 \cdot 10^{-6}$	-
nominal $\varnothing > 150$ mm	$1 \cdot 10^{-7}$	$5 \cdot 10^{-7}$	-

Table E.2.1.- Frequency  $\lambda$  of LOC episode for pipelines, (adapted from [45]).

## E.3. Pumps.

G1: Catastrophic failure with full bore rupture of the largest connecting pipeline.

G2: Leak.

Installation	LOC frequency ( $y^{-1}$ )		
	pump	G1	G2
without additional provisions	$1 \cdot 10^{-4}$	$5 \cdot 10^{-4}$	-
with a steel containment	$5 \cdot 10^{-5}$	$2.5 \cdot 10^{-4}$	-
canned pumps	$1 \cdot 10^{-5}$	$5 \cdot 10^{-5}$	-

Table E.3.1.- Frequency  $\lambda$  of LOC episode for pumps, (adapted from [45]).

## E.4. Heat exchangers.

G1: Instantaneous release of the content.

G2: Continuous release of the content in 10 min at constant rate.

G3: Continuous release from a hole with an effective diameter of 10 mm.

G4: Simultaneous and total rupture of the hole at the entrance and exit of a pipe.

G.5: Full bore rupture of one pipe.

G.6: Leak



Installation	LOC frequency (y <sup>-1</sup> )					
heat exchanger	G1	G2	G3	G4	G5	G6
dangerous substance outside pipes	1·10 <sup>-4</sup>	5·10 <sup>-4</sup>	1·10 <sup>-3</sup>	-	-	-
dangerous substance inside pipes pressure outer shell < pressure of dangerous substance				1.2·10 <sup>-5</sup>	1.2·10 <sup>-3</sup>	1.2·10 <sup>-3</sup>
dangerous substance inside pipes pressure outer shell > pressure of dangerous substance				1.2·10 <sup>-6</sup>	-	-

Table E.4.1.- Frequency  $\lambda$  of LOC episode for heat exchangers, (adapted from [45]).

## E.5. Pressure relief devices.

G1: Discharge of pressure relief device at maximum rate.

Installation	LOC frequency (y <sup>-1</sup> )		
pressure relief device	G1	G2	G3
without additional provisions	2·10 <sup>-5</sup>	-	-

Table E.5.1.- Frequency  $\lambda$  of LOC episode for pressure relief devices, (adapted from [45]).

## E.6. Storage in warehouses.

G1: Handling solids, with dispersion of a fraction of the unit packaging as respirable powder.

G2: Handling liquids, with spill of the complete unit packaging.

S3: Emission of unburned toxics and toxics produced in the fire or explosion.

Installation	LOC frequency (packaging unit <sup>-1</sup> )		EMISSION (y <sup>-1</sup> )
storage in warehouse	G1	G2	G3
storage of substances	1·10 <sup>-5</sup>	1·10 <sup>-5</sup>	8.8·10 <sup>-4</sup> (worst case)

Table E.6.1.- Frequency  $\lambda$  of LOC episode for storage in warehouse, (adapted from [45]).

## E.7. Road tankers.

G1: Instantaneous release of the content.

G2: Continuous release from a hole with the size of the largest connection.

L1a: Full bore rupture of the of the loading / unloading connection.

L2a: Leak of the loading / unloading connection.

L1b: Full bore rupture of the loading / unloading arm.

L2b: Leak of the loading / unloading arm.

E.1: External impact.

S.1: Fire under tank modeled as total and instantaneous release of content.

Installation	LOC frequency (y <sup>-1</sup> )		(h <sup>-1</sup> )				(y <sup>-1</sup> )	
	G1	G2	L1a	L2a	L1b	L2b	E1	S1
with / without pressure	5·10 <sup>-7</sup> (worst case)	5·10 <sup>-7</sup>	4·10 <sup>-5</sup>	4·10 <sup>-5</sup>	3·10 <sup>-8</sup>	3·10 <sup>-7</sup>	1·10 <sup>-6</sup>	1·10 <sup>-5</sup> (worst case)

Table E.7.1.- Frequency  $\lambda$  of LOC episode for road tankers, (adapted from [45]).

## E.8. Ship tankers.

L.1: Full bore rupture of the loading / unloading arm.

L.2: Leak of the loading / unloading arm.

E.1: External impact, large spill.

E.2: External impact, small spill.

Installation	LOC frequency (shipment <sup>-1</sup> )		(y <sup>-1</sup> )	
	L1	L2	E1	E2
all class	6·10 <sup>-5</sup>	6·10 <sup>-4</sup>	0.1f <sub>0</sub> (worst case)	0.2f <sub>0</sub> (worst case)

Table E.8.1.- Frequency  $\lambda$  of LOC episode for ship tankers, where  $f_0 = 6.7 \cdot 10^{-11} \cdot T \cdot t \cdot N$ ; with T total number of ships by year, t average hours of loading / unloading, N number of shipments by year; (adapted from [45]).

# Annex F

## Fuzzy logic.

There are two fuzzy inference systems (FIS) that are possible to apply, the Mamdani-type and the Sugeno-type. Mamdani-type uses the technique of de-fuzzification of the fuzzy output, while Sugeno-type uses the weighted average to compute the output. Sugeno method is computationally efficient and works well with optimization and adaptive techniques, because there is no de-fuzzification step which makes it very attractive in control problems, particularly for dynamic non linear systems.

But Mamdani's fuzzy inference method is the most commonly applied because their rules definition are very close to the human language expression. It was proposed in 1975 by Ebrahim Mamdani as an attempt to control a steam engine and boiler combination by synthesizing a set of linguistic control rules obtained from experienced human operators. In this thesis the Mamdani-type FIS is applied.

The fuzzy inference is a method that interprets the values in the input vector and, based on rules, assigns values to the output vector. (Figure F.1).

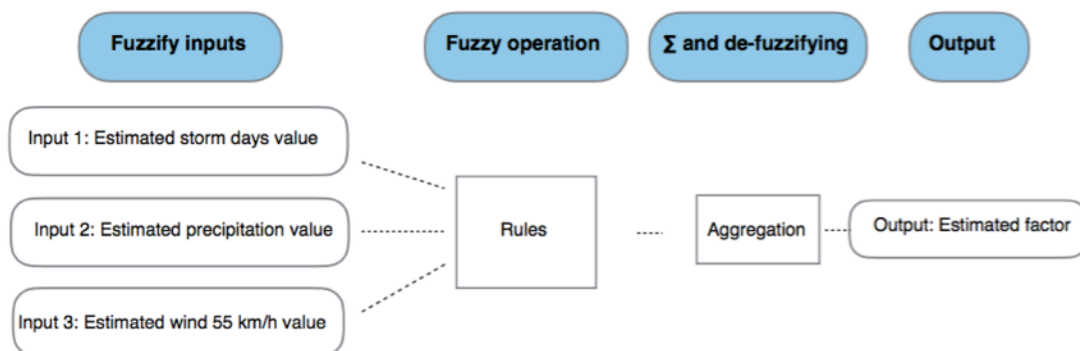


Figure F.1.- Fuzzy inference scheme procedure. Inputs applied for the (SPW55) fuzzy estimation.

Procedure:

1. Define inputs and their intervals of variability. This is the input space.
2. Define the fuzzy sets.

A diffuse set has the characteristic of not having clear and clearly defined limits. Although each of the inputs have their own intervals of action; by interacting with each other, affecting risk can show limits that overlap and are not clearly defined.

3. Define the membership functions (MF).

To reflect the degree of interaction and diffuse limits between the Inputs, the membership functions (MF) are used, which are curves for which the variations of each Input are defined and associated with an interval valued between 0 and 1. These curves can be: piece-wise linear functions, gaussian functions, sigmoid curves or quadratic, cubic-polynomial.

4. Rules are defined.

These rules are logical collecting the Inputs and giving an Output value defined also as a membership function (MF) with a considered interval.

5. Aggregation and de-fuzzy

The logical rules are summarized and de-fuzzified giving the Output value.

6. Obtention of the Output value.

For the case of the fuzzy inference system (FIS) for Storm, Precipitation and Wind55 (SPW55), with the inputs and their intervals of variability are: Storm [0 4] in days; Precipitation [0 120] in mm; and Wind55 [0 14] in days. And the Output [1 2] being this a factor.

The fuzzy sets and the membership functions as triangular distributions are defined. Every input and the output has three sets for low, medium and high. See Figure F.2.

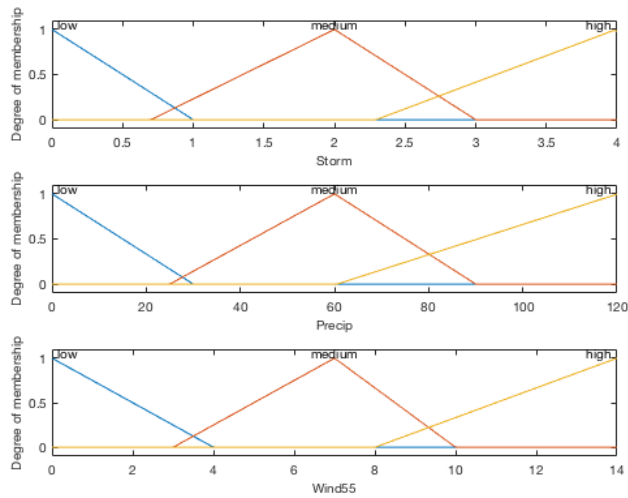


Figure F.2.- Fuzzy sets and memberships definition for the three inputs on the (SPW55) inference.

The definition of rules are;

- 1 If Storm is low or Precipitation is low or Wind55 is low then factor is low
- 2 If Storm is medium or Precipitation is medium or Wind55 is medium then factor is medium.
- 3 If Storm is high or Precipitation is high or Wind 55 is high then factor is high.
- 4 If Storm is high then factor is high.

Every rule is independent from each other.

The aggregation and de-fuzzy process are presented on Figure F.3. In this case when Storm = 2 days, Precipitation = 60 mm and Wind55 = 7 days, the Factor is 1.5.

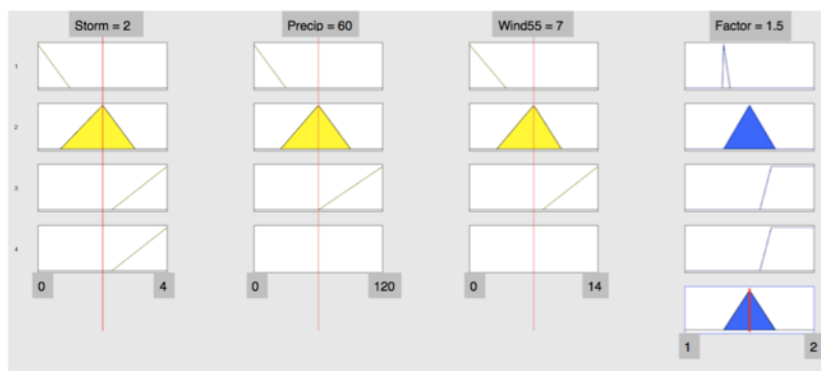


Figure F.3.- Aggregation and output for the (SPW55) fuzzy inference.



# Annex G

## Joint distributions.

Most statistics applications require analyzing more than one random variable in a simultaneous way. Let  $X$  and  $Y$  denote two random variables.

Their joint cumulative distribution function is defined as;

$$F(x,y) = P(X \leq x, Y \leq y) \quad (\text{G.1})$$

And then their joint density function is defined as;

$$f(x,y) = \frac{\partial^2 F(x,y)}{\partial x \partial y} \quad (\text{G.2})$$

Their covariance is defined as;

$$\text{Cov}(X,Y) = E[(X - E(X)) \cdot (Y - E(Y))] = E(XY) - E(X)E(Y) \quad (\text{G.3})$$

Measuring the degree of coordination in the way  $X$  and  $Y$  might change.

Their correlation coefficient is defined as;

$$\rho(X,Y) = \frac{\text{Cov}(X,Y)}{\sqrt{\text{Var}(X)\text{Var}(Y)}} \quad (\text{G.4})$$

Measuring, in this case, the dependence between two random variables.

For the case of more than two random variables. Let  $X = (X_1, \dots, X_n)^T$  a random column vector of dimension  $n$ .

The joint cumulative distribution function can be defined as;

$$F(x_1, \dots, x_n) = P(X_1 \leq x_1, \dots, X_n \leq x_n) \quad (\text{G.5})$$

And the joint density function as;

$$f(x_1, \dots, x_n) = \frac{\partial^n F(x_1, \dots, x_n)}{\partial x_1 \dots \partial x_n} \quad (\text{G.6})$$

With covariance matrix;

$$\sigma_{ij} = \text{Cov}(X_i, X_j) = E[(X_i - E(X_i)) \cdot (X_j - E(X_j))] \quad (\text{G.7})$$

With its elements defined as pairwise covariances, and being, for  $i = j$  the diagonal elements.

$$\sigma_{ii} = \text{Cov}(X_i, X_i) = \text{Var}(X_i) \quad (\text{G.8})$$

And correlation matrix;

$$\rho_{ij} = \frac{\sigma_{ij}}{\sqrt{\sigma_{ii} \sigma_{jj}}} \quad (\text{G.9})$$

To enhance the needs for good a data fit appear the concept of Joint Distribution-Copula. But before is needed to define the concept of marginal distribution.

If from the two variables X, Y only one, say X, is important for the analysis and the values of the other Y are irrelevant or not enough information is available on them, then it is possible to consider a marginal density function of X and define it as the integral of the joint density over  $y$ , as;

$$f_X(x) = \int_{-\infty}^{\infty} f(x, y) dy \quad (\text{G.10})$$

And, accordingly, the marginal cumulative distribution function is;

$$F_X(x) = \lim_{y \rightarrow \infty} F(x, y) \quad (\text{G.11})$$

If on the contrary, the specific value  $y$  is relevant and known, from a Bayesian point of view, it is possible to consider the conditional cumulative distribution of X given Y, as;

$$F_{X|Y}(x|y) = P(X \leq x | Y = y) \quad (\text{G.12})$$

With density function;

$$f_{X|Y}(x|y) = \frac{f(x, y)}{f_Y(y)} \quad (\text{G.13})$$



Then,

- a. modeling marginal cumulative distributions  $F(x)$  and  $G(y)$ .
- b. modeling the joint distribution as

$$H(x,y) = C(F(x),G(y)) \quad (\text{G.14})$$

That is “mixing” the marginal distributions with the help of a special bivariate function  $C$ , reflecting the relation pattern. This class of functions  $C$  is known as Joint Distributions - Copulas.

### G.1. Quasi-monotonicity.

Introducing the notation:  $I = [0,1]$  is the unit segment;  $I^2 = [0,1] \times [0,1]$  is the unit square. Then for any  $u_1 \leq u_2, v_1 \leq v_2$  and  $B = [u_1, u_2] \times [v_1, v_2]$  is a rectangular region in the plane as shown in Figure G.1.1.

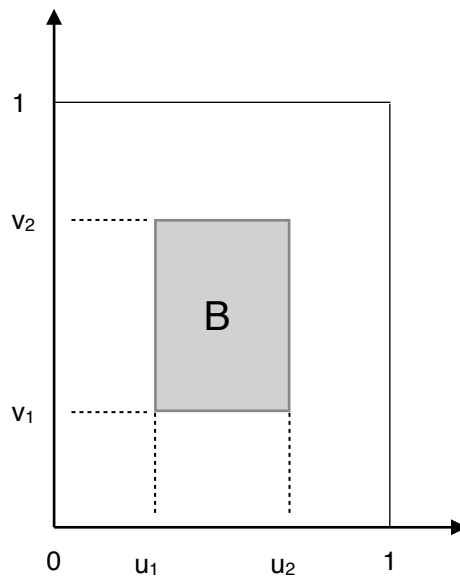


Figure G.1.1.- Rectangular area (adapted from [194]).

Let  $A(u,v)$  be a function from  $I^2$  to  $I$  and  $B$  be a rectangular region in the unit square. The  $A$ -volume of the region  $B$  is defined as;

$$V_A(B) = A(u_2, v_2) - A(u_1, v_2) - A(u_2, v_1) + A(u_1, v_1) \quad (\text{G.15})$$

Let  $\Delta_{u_1}^{u_2}$  and  $\Delta_{v_1}^{v_2}$  represent the first consecutive difference, as;

$$\Delta_{u_1}^{u_2} A(u, v) = A(u_2, v) - A(u_1, v) \quad \text{and} \quad \Delta_{v_1}^{v_2} A(u, v) = A(u, v_2) - A(u, v_1) \quad (\text{G.16})$$

The  $A$ -volume is then;

$$V_A(B) = \Delta_{v_1}^{v_2} \Delta_{u_1}^{u_2} A(u, v) \quad (\text{G.17})$$

Then the function  $A(u, v)$  is called quasi-monotone if for any rectangular area  $B$  in the unit square its  $A$ -volume is nonnegative.

Additionally, the function  $A(u, v)$  is grounded on  $I^2$  if  $A(0, v) = A(u, 0) = 0$

## G.2. Definition of Copula.

The function  $C$  is called a copula if it satisfies;

1.  $C : I^2 \rightarrow I$ .
2. For any  $u, v \in [0, 1]$   $C(0, v) = C(u, 0) = 0$ .
3. For any  $u, v \in [0, 1]$   $C(1, v) = v$ ;  $C(u, 1) = u$ .
4. For any  $0 \leq u_1 \leq u_2 \leq 1$  and  $0 \leq v_1 \leq v_2 \leq 1$  is

$$V_C(B) = C(u_2, v_2) - C(u_1, v_2) - C(u_2, v_1) + C(u_1, v_1) \geq 0$$

For any copula  $C(u, v)$  partial derivatives  $\frac{\partial C}{\partial u}$  and  $\frac{\partial C}{\partial v}$  exist for almost all  $u, v \in [0, 1]$ .

Let  $\frac{\partial^2 C}{\partial u \partial v}$  and  $\frac{\partial^2 C}{\partial v \partial u}$  exist and be continuous on  $I^2$ , then the copula density is;

$$c(u, v) = \frac{\partial^2 C}{\partial u \partial v} = \frac{\partial^2 C}{\partial v \partial u} \quad (\text{G.18})$$

From the definition, it is evident that if  $u = F(x)$  and  $v = G(y)$  are two distribution functions, then any copula  $C(u, v) = C(F(x), G(y))$  is a valid bivariate distribution function, and the joint probability density function of  $X$  and  $Y$  can be represented as;

$$f(x, y) = \frac{\partial^2 C}{\partial u \partial v} \cdot \frac{dF}{dx} \cdot \frac{dG}{dy} \quad (\text{G.19})$$

Where  $\frac{dF}{dx}$  and  $\frac{dG}{dy}$  are marginal densities of X and Y.

### G.3. Elliptical copulas. Gaussian. Construction.

In this thesis, the kind of copulas applied corresponds to the so-called elliptical being its exponent the Gaussian or normal.

One method of building a Gaussian copula is the method of inverses. This uses the fact that for  $u, v \in [0,1]$  is uniformly distributed on  $[0,1]$ , their inverse transforms  $\Phi^{-1}(u)$  and  $\Phi^{-1}(v)$  are two independent random variables that have the same cumulative distribution function in  $[0,1]$  and allowing for an effective separation of the marginal distributions  $u = F(x)$  and  $v = F(y)$ , being;

$$H(x,y) = \Phi_{\rho}[\Phi^{-1}(F(x)), \Phi^{-1}(G(y))] \quad (\text{G.20})$$

Where  $\rho$  is the correlation matrix. For example a Gaussian  $N(0,1)$  joint bivariate distribution with  $\rho = \begin{pmatrix} 1 & 0.7 \\ 0.7 & 1 \end{pmatrix}$  is defined with  $n=100000$  random samples. Their histogram is, Figure G.3.1;

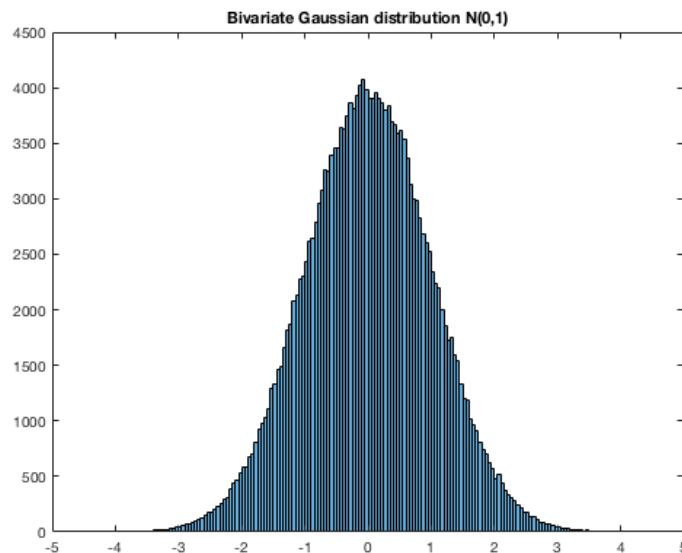


Figure G.3.1.- Bivariate Gaussian distribution.

And their cumulated distribution is  $u, v \in [0,1]$ , Figure G.3.2.

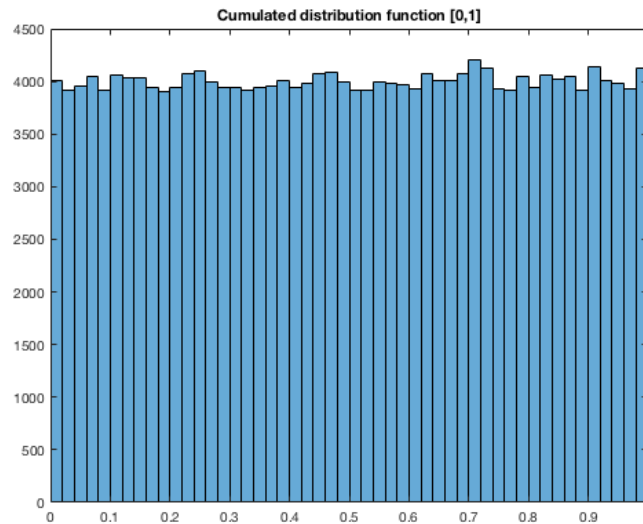


Figure G.3.2.- Cumulated distribution function.

Two marginal distributions are defined, being;  $F(x) = \text{gamma}(2,1)$  and  $G(x) = t - \text{Student}(5)$  with joint distribution-copula defined, Figure G.3.3, according to (F.20), as;

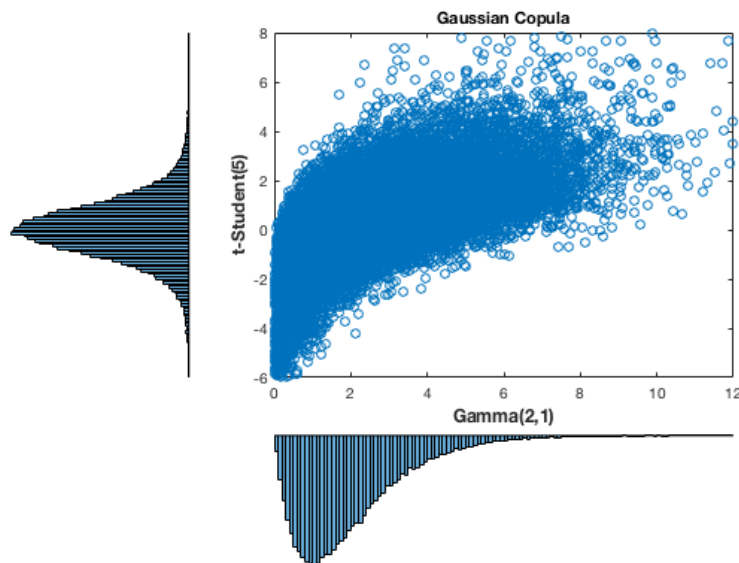


Figure G.3.3.- Gaussian joint distribution-copula for Gamma(2.1) and t-Student(5) marginals.

# Annex H

## Applied programs.

The Matlab programs applied in the different cases are collected in this annex.

### H.1. Poisson-gamma.

a.- Recurrent method with mean prior.

```
clear
%% Recurrent with mean prior
%Prior time 0
alfa0=0.001; beta0=0.001; mean0=0.001;

%Interval 1

yi=[0]; yiobs1=[0]; st=sum(yi); Nt=length(yi); alfa1=alfa0+st; beta1=beta0+Nt; lambda1=alfa1/beta1;
sigma1=sqrt(alfa1/beta1^2);
CILM1 = mean0-sigma1;
if CILM1<0
    CILM1=0;
end
MEAN1 = mean0; CIUM1 = mean0+sigma1;
xx1=[0 yiobs1]; lim1 = [CILM1 MEAN1 CIUM1];
figure
controlchart(xx1,'chart','i','limits',lim1,'specs',[0 0.5],'width',1); title('interval t=1')

% Interval 2

yi=[0]; yiobs2=[0]; st=sum(yi); Nt=length(yi); alfa2=alfa1+st; beta2=beta1+Nt; lambda2=alfa2/beta2;
sigma2=sqrt(alfa2/beta2^2);
CILM2 = mean0-sigma2;
if CILM2<0
    CILM2=0;
end
MEAN2 = mean0; CIUM2 = mean0+sigma2;
xx2=[yiobs1 yiobs2]; lim2 = [CILM2 MEAN2 CIUM2];
figure
controlchart(xx2,'chart','i','limits',lim2,'specs',[0 0.5],'width',1); title('interval t=2')

% Interval 3

yi=[0]; yiobs3=[0]; st=sum(yi); Nt=length(yi); alfa3=alfa2+st; beta3=beta2+Nt; lambda3=alfa3/beta3;
sigma3=sqrt(alfa3/beta3^2);
CILM3 = mean0-sigma3;
if CILM3<0
    CILM3=0;
end
MEAN3 = mean0; CIUM3 = mean0+sigma3;
xx3=[yiobs1 yiobs2 yiobs3]; lim3 = [CILM3 MEAN3 CIUM3];
figure
controlchart(xx3,'chart','i','limits',lim3,'specs',[0 0.5],'width',1); title('interval t=3')
```

```
% Interval 4
```

```
yi=[1]; yiobs4=[1/4]; st=sum(yi); Nt=length(yi); alfa4=alfa3+st; beta4=beta3+Nt; lambda4=alfa4/beta4;
sigma4=sqrt(alfa4/beta4^2);
CILM4 = mean0-sigma4;
if CILM4<0
    CILM4=0;
end
MEAN4 = mean0; CIUM4 = mean0+sigma4;
xx4=[yiobs1 yiobs2 yiobs3 yiobs4]; lim4 = [CILM4 MEAN4 CIUM4];
figure
controlchart(xx4,'chart',{'i'},'limits',lim4,'specs',[0 0.5],'width',1); title('interval t=4')
```

```
% Interval 5
```

```
yi=[0]; yiobs5=[0]; st=sum(yi); Nt=length(yi); alfa5=alfa4+st; beta5=beta4+Nt; lambda5=alfa5/beta5;
sigma5=sqrt(alfa5/beta5^2);
CILM5 = mean0-sigma5;
if CILM5<0
    CILM5=0;
end
MEAN5 = mean0; CIUM5 = mean0+sigma5;
xx5=[yiobs1 yiobs2 yiobs3 yiobs4 yiobs5]; lim5 = [CILM5 MEAN5 CIUM5];
figure
controlchart(xx5,'chart',{'i'},'limits',lim5,'specs',[0 0.5],'width',1); title('interval t=5')
```

```
% Interval 6
```

```
yi=[0]; yiobs6=[0]; st=sum(yi); Nt=length(yi); alfa6=alfa5+st; beta6=beta5+Nt; lambda6=alfa6/beta6;
sigma6=sqrt(alfa6/beta6^2);
CILM6 = mean0-sigma6;
if CILM6<0
    CILM6=0;
end
MEAN6 = mean0; CIUM6 = mean0+sigma6;
xx6=[0 yiobs1 yiobs2 yiobs3 yiobs4 yiobs5 yiobs6]; lim6 = [CILM6 MEAN6 CIUM6];
figure
controlchart(xx6,'chart',{'i'},'limits',lim6,'specs',[0 0.5],'width',1); title('interval t=6')
```

```
% Interval 7
```

```
yi=[1]; yiobs7=[1/3]; st=sum(yi); Nt=length(yi); alfa7=alfa6+st; beta7=beta6+Nt; lambda7=alfa7/beta7;
sigma7=sqrt(alfa7/beta7^2);
CILM7 = mean0-sigma7;
if CILM7<0
    CILM7=0;
end
MEAN7 = mean0; CIUM7 = mean0+sigma7;
xx7=[yiobs1 yiobs2 yiobs3 yiobs4 yiobs5 yiobs6 yiobs7]; lim7 = [CILM7 MEAN7 CIUM7];
figure
controlchart(xx7,'chart',{'i'},'limits',lim7,'specs',[0 0.5],'width',1); title('interval t=7')
```

```
% Interval 8
```

```
yi=[1]; yiobs8=[1/1]; st=sum(yi); Nt=length(yi); alfa8=alfa7+st; beta8=beta7+Nt; lambda8=alfa8/beta8;
sigma8=sqrt(alfa8/beta8^2);
CILM8 = mean0-sigma8;
if CILM8<0
    CILM8=0;
end
MEAN8 = mean0; CIUM8 = mean0+sigma8;
xx8=[yiobs1 yiobs2 yiobs3 yiobs4 yiobs5 yiobs6 yiobs7 yiobs8]; lim8 = [CILM8 MEAN8 CIUM8];
figure
controlchart(xx8,'chart',{'i'},'limits',lim8,'specs',[0 0.5],'width',1); title('interval t=8')
```

```
% Interval 9
```

```

yi=[0]; yiobs9=[0]; st=sum(yi); Nt=length(yi); alfa9=alfa8+st; beta9=beta8+Nt; lambda9=alfa9/beta9;
sigma9=sqrt(alfa9/beta9^2);
CILM9 = mean0-sigma9;
if CILM9<0
    CILM9=0;
end
MEAN9 = mean0; CIUM9 = mean0+sigma9;
xx9=[yiobs1 yiobs2 yiobs3 yiobs4 yiobs5 yiobs6 yiobs7 yiobs8 yiobs9]; lim9 = [CILM9 MEAN9 CIUM9];
figure
controlchart(xx9,'chart',{'i'},'limits',lim9,'specs',[0 0.5],'width',1); title('interval t=9')

```

% Interval 10

```

yi=[0]; yiobs10=[0]; st=sum(yi); Nt=length(yi); alfa10=alfa9+st; beta10=beta9+Nt; lambda10=alfa10/beta10;
sigma10=sqrt(alfa10/beta10^2);
CILM10 = mean0-sigma10;
if CILM10<0
    CILM10=0;
end
MEAN10 = mean0; CIUM10 = mean0+sigma10;
xx10=[yiobs1 yiobs2 yiobs3 yiobs4 yiobs5 yiobs6 yiobs7 yiobs8 yiobs9 yiobs10];
lim10 = [CILM10 MEAN10 CIUM10];
figure
controlchart(xx10,'chart',{'i'},'limits',lim10,'specs',[0 0.5],'width',1); title('interval t=10')

```

## b.- Recurrent method with mean posterior.

```

clear
%% Recurrent with Mean posterior
%%Prior time 0
alfa0=0.001; beta0=0.001; mean0=0.001;

%Interval 1

yi=[0]; yiobs1=[0]; st=sum(yi); Nt=length(yi); alfa1=alfa0+st; beta1=beta0+Nt; lambda1=alfa1/beta1;
sigma1=sqrt(alfa1/beta1^2);
CILM1 = lambda1-sigma1;
if CILM1<0
    CILM1=0;
end
MEAN1 = lambda1; CIUM1 = lambda1+sigma1;
xx1=[0 yiobs1]; lim1 = [CILM1 MEAN1 CIUM1];
figure
controlchart(xx1,'chart',{'i'},'limits',lim1,'specs',[0 0.5],'width',1); title('interval t=1')

% Interval 2

yi=[0]; yiobs2=[0]; st=sum(yi); Nt=length(yi); alfa2=alfa1+st; beta2=beta1+Nt; lambda2=alfa2/beta2;
sigma2=sqrt(alfa2/beta2^2);
CILM2 = lambda2-sigma2;
if CILM2<0
    CILM2=0;
end
MEAN2 = lambda2; CIUM2 = lambda2+sigma2;
xx2=[yiobs1 yiobs2]; lim2 = [CILM2 MEAN2 CIUM2];
figure
controlchart(xx2,'chart',{'i'},'limits',lim2,'specs',[0 0.5],'width',1); title('interval t=2')

% Interval 3

yi=[0]; yiobs3=[0]; st=sum(yi); Nt=length(yi); alfa3=alfa2+st; beta3=beta2+Nt; lambda3=alfa3/beta3;
sigma3=sqrt(alfa3/beta3^2);
CILM3 = lambda3-sigma3;
if CILM3<0
    CILM3=0;
end

```

```

MEAN3 = lambda3; CIUM3 = lambda3+sigma3;
xx3=[yiobs1 yiobs2 yiobs3]; lim3 = [CILM3 MEAN3 CIUM3];
figure
controlchart(xx3,'chart',{'i'},'limits',lim3,'specs',[0 0.5],'width',1); title('interval t=3')

% Interval 4

yi=[1]; yiobs4=[1/4]; st=sum(yi);Nt=length(yi); alfa4=alfa3+st; beta4=beta3+Nt; lambda4=alfa4/beta4;
sigma4=sqrt(alfa4/beta4^2);
CILM4 = lambda4-sigma4;
if CILM4<0
    CILM4=0;
end
MEAN4 = lambda4; CIUM4 = lambda4+sigma4;
xx4=[yiobs1 yiobs2 yiobs3 yiobs4]; lim4 = [CILM4 MEAN4 CIUM4];
figure
controlchart(xx4,'chart',{'i'},'limits',lim4,'specs',[0 0.5],'width',1); title('interval t=4')

% Interval 5

yi=[0]; yiobs5=[0]; st=sum(yi); Nt=length(yi); alfa5=alfa4+st; beta5=beta4+Nt; lambda5=alfa5/beta5;
sigma5=sqrt(alfa5/beta5^2);
CILM5 = lambda5-sigma5;
if CILM5<0
    CILM5=0;
end
MEAN5 = lambda5; CIUM5 = lambda5+sigma5;
xx5=[yiobs1 yiobs2 yiobs3 yiobs4 yiobs5]; lim5 = [CILM5 MEAN5 CIUM5];
figure
controlchart(xx5,'chart',{'i'},'limits',lim5,'specs',[0 0.5],'width',1); title('interval t=5')

% Interval 6

yi=[0]; yiobs6=[0]; st=sum(yi); Nt=length(yi); alfa6=alfa5+st; beta6=beta5+Nt; lambda6=alfa6/beta6;
sigma6=sqrt(alfa6/beta6^2);
CILM6 = lambda6-sigma6;
if CILM6<0
    CILM6=0;
end
MEAN6 = lambda6; CIUM6 = lambda6+sigma6;
xx6=[yiobs1 yiobs2 yiobs3 yiobs4 yiobs5 yiobs6]; lim6 = [CILM6 MEAN6 CIUM6];
figure
controlchart(xx6,'chart',{'i'},'limits',lim6,'specs',[0 0.5],'width',1); title('interval t=6')

% Interval 7

yi=[1]; yiobs7=[1/3]; st=sum(yi); Nt=length(yi); alfa7=alfa6+st; beta7=beta6+Nt; lambda7=alfa7/beta7;
sigma7=sqrt(alfa7/beta7^2);
CILM7 = lambda7-sigma7;
if CILM7<0
    CILM7=0;
end
MEAN7 = lambda7; CIUM7 = lambda7+sigma7;
xx7=[yiobs1 yiobs2 yiobs3 yiobs4 yiobs5 yiobs6 yiobs7]; lim7 = [CILM7 MEAN7 CIUM7];
figure
controlchart(xx7,'chart',{'i'},'limits',lim7,'specs',[0 0.5],'width',1); title('interval t=7')

% Interval 8

yi=[1]; yiobs8=[1/1]; st=sum(yi); Nt=length(yi); alfa8=alfa7+st; beta8=beta7+Nt; lambda8=alfa8/beta8;
sigma8=sqrt(alfa8/beta8^2);
CILM8 = lambda8-sigma8;
if CILM8<0
    CILM8=0;
end
MEAN8 = lambda8; CIUM8 = lambda8+sigma8;
xx8=[yiobs1 yiobs2 yiobs3 yiobs4 yiobs5 yiobs6 yiobs7 yiobs8]; lim8 = [CILM8 MEAN8 CIUM8];

```



```

figure
controlchart(xx8,'chart',{'i'},'limits',lim8,'specs',[0 0.5],'width',1); title('interval t=8')

% Interval 9

yi=[0]; yiobs9=[0]; st=sum(yi); Nt=length(yi); alfa9=alfa8+st; beta9=beta8+Nt; lambda9=alfa9/beta9;
sigma9=sqrt(alfa9/beta9^2);
CILM9 = lambda9-sigma9;
if CILM9<0
    CILM9=0;
end
MEAN9 = lambda9; CIUM9 = lambda9+sigma9
xx9=[yiobs1 yiobs2 yiobs3 yiobs4 yiobs5 yiobs6 yiobs7 yiobs8 yiobs9]; lim9 = [CILM9 MEAN9 CIUM9];
figure
controlchart(xx9,'chart',{'i'},'limits',lim9,'specs',[0 0.5],'width',1); title('interval t=9')

% Interval 10

yi=[0]; yiobs10=[0]; st=sum(yi); Nt=length(yi); alfa10=alfa9+st; beta10=beta9+Nt; lambda10=alfa10/beta10;
sigma10=sqrt(alfa10/beta10^2);
CILM10 = lambda10-sigma10;
if CILM10<0
    CILM10=0;
end
MEAN10 = lambda10; CIUM10 = lambda10+sigma10;
xx10=[yiobs1 yiobs2 yiobs3 yiobs4 yiobs5 yiobs6 yiobs7 yiobs8 yiobs9 yiobs10];
lim10 = [CILM10 MEAN10 CIUM10];
figure
controlchart(xx10,'chart',{'i'},'limits',lim10,'specs',[0 0.5],'width',1); title('interval t=10')

```

### c.- Direct method with mean prior.

```

clear
%% Direct with Mean prior constant
%Prior time 0
alfa0=0.001; beta0=0.001; mean0=0.001;

%Interval 1

yi=[0]; yiobs1=[0]; st=sum(yi); Nt=length(yi); alfa1=alfa0+st; beta1=beta0+Nt; lambda1=alfa1/beta1;
sigma1=sqrt(alfa1/beta1^2);
CILM1 = mean0-sigma1;
if CILM1<0
    CILM1=0;
end
MEAN1 = mean0; CIUM1 = mean0+sigma1; yiobs1= yiobs1(Nt);
xx1=[0 yiobs1]; lim1 = [CILM1 MEAN1 CIUM1];
figure
controlchart(xx1,'chart',{'i'},'limits',lim1,'specs',[0 0.5],'width',1); title('interval t=1')

% Interval 2

yi=[0 0]; yiobs2=[0 0]; st=sum(yi); Nt=length(yi); alfa2=alfa0+st; beta2=beta0+Nt; lambda2=alfa2/beta2;
sigma2=sqrt(alfa2/beta2^2);
CILM2 = mean0-sigma2;
if CILM2<0
    CILM2=0;
end
MEAN2 = mean0; CIUM2 = mean0+sigma2; yiobs2= yiobs2(Nt);
xx2=[yiobs1 yiobs2]; lim2 = [CILM2 MEAN2 CIUM2];
figure
controlchart(xx2,'chart',{'i'},'limits',lim2,'specs',[0 0.5],'width',1); title('interval t=2')

% Interval 3

yi=[0 0 0]; yiobs3=[0 0 0]; st=sum(yi); Nt=length(yi); alfa3=alfa0+st; beta3=beta0+Nt; lambda3=alfa3/beta3;

```

```

sigma3=sqrt(alfa3/beta3^2);
CILM3 = mean0-sigma3;
if CILM3<0
    CILM3=0;
end
MEAN3 = mean0; CIUM3 = mean0+sigma3; yiobs3= yiobs3(Nt);
xx3=[yiobs1 yiobs2 yiobs3]; lim3 = [CILM3 MEAN3 CIUM3];
figure
controlchart(xx3,'chart',{'i'},'limits',lim3,'specs',[0 0.5],'width',1); title('interval t=3')

% Interval 4

yi=[0 0 0 1]; yiobs4=[0 0 0 1/4]; st=sum(yi); Nt=length(yi); alfa4=alfa0+st; beta4=beta0+Nt; lambda4=alfa4/beta4;
sigma4=sqrt(alfa4/beta4^2);
CILM4 = mean0-sigma4;
if CILM4<0
    CILM4=0;
end
MEAN4 = mean0; CIUM4 = mean0+sigma4; yiobs4= yiobs4(Nt);
xx4=[yiobs1 yiobs2 yiobs3 yiobs4]; lim4 = [CILM4 MEAN4 CIUM4];
figure
controlchart(xx4,'chart',{'i'},'limits',lim4,'specs',[0 0.5],'width',1); title('interval t=4')

% Interval 5

yi=[0 0 0 1 0]; yiobs5=[0 0 0 1/4 0]; st=sum(yi); Nt=length(yi); alfa5=alfa0+st; beta5=beta0+Nt;
lambda5=alfa5/beta5; sigma5=sqrt(alfa5/beta5^2);
CILM5 = mean0-sigma5;
if CILM5<0
    CILM5=0;
end
MEAN5 = mean0; CIUM5 = mean0+sigma5; yiobs5= yiobs5(Nt);
xx5=[yiobs1 yiobs2 yiobs3 yiobs4 yiobs5]; lim5 = [CILM5 MEAN5 CIUM5];
figure
controlchart(xx5,'chart',{'i'},'limits',lim5,'specs',[0 0.5],'width',1); title('interval t=5')

% Interval 6

yi=[0 0 0 1 0 0]; yiobs6=[0 0 0 1/4 0 0]; st=sum(yi); Nt=length(yi); alfa6=alfa0+st; beta6=beta0+Nt;
lambda6=alfa6/beta6; sigma6=sqrt(alfa6/beta6^2);
CILM6 = mean0-sigma6;
if CILM6<0
    CILM6=0;
end
MEAN6 = mean0; CIUM6 = mean0+sigma6; yiobs6= yiobs6(Nt);
xx6=[yiobs1 yiobs2 yiobs3 yiobs4 yiobs5 yiobs6]; lim6 = [CILM6 MEAN6 CIUM6];
figure
controlchart(xx6,'chart',{'i'},'limits',lim6,'specs',[0 0.5],'width',1); title('interval t=6')

% Interval 7

yi=[0 0 0 1 0 0 1]; yiobs7=[0 0 0 1/4 0 0 1/3]; st=sum(yi); Nt=length(yi); alfa7=alfa0+st; beta7=beta0+Nt;
lambda7=alfa7/beta7; sigma7=sqrt(alfa7/beta7^2);
CILM7 = mean0-sigma7;
if CILM7<0
    CILM7=0;
end
MEAN7 = mean0; CIUM7 = mean0+sigma7; yiobs7= yiobs7(Nt);
xx7=[yiobs1 yiobs2 yiobs3 yiobs4 yiobs5 yiobs6 yiobs7]; lim7 = [CILM7 MEAN7 CIUM7];
figure
controlchart(xx7,'chart',{'i'},'limits',lim7,'specs',[0 0.5],'width',1); title('interval t=7')

% Interval 8

yi=[0 0 0 1 0 0 1 1]; yiobs8=[0 0 0 1/4 0 0 1/3 1/1]; st=sum(yi); Nt=length(yi); alfa8=alfa0+st; beta8=beta0+Nt;
lambda8=alfa8/beta8; sigma8=sqrt(alfa8/beta8^2);
CILM8 = mean0-sigma8;

```

```

if CILM8<0
    CILM8=0;
end
MEAN8 = mean0; CIUM8 = mean0+sigma8; yiobs8= yiobs8(Nt);
xx8=[yiobs1 yiobs2 yiobs3 yiobs4 yiobs5 yiobs6 yiobs7 yiobs8]; lim8 = [CILM8 MEAN8 CIUM8];
figure
controlchart(xx8,'chart',{'i'},'limits',lim8,'specs',[0 0.5],'width',1); title('interval t=8')

% Interval 9

yi=[0 0 0 1 0 0 1 1 0]; yiobs9=[0 0 0 1/4 0 0 1/3 1/1 0]; st=sum(yi); Nt=length(yi); alfa9=alfa0+st; beta9=beta0+Nt;
lambda9=alfa9/beta9; sigma9=sqrt(alfa9/beta9^2);
CILM9 = mean0-sigma9;
if CILM9<0
    CILM9=0;
end
MEAN9 = mean0; CIUM9 = mean0+sigma9; yiobs9= yiobs9(Nt);
xx9=[yiobs1 yiobs2 yiobs3 yiobs4 yiobs5 yiobs6 yiobs7 yiobs8 yiobs9]; lim9 = [CILM9 MEAN9 CIUM9];
figure
controlchart(xx9,'chart',{'i'},'limits',lim9,'specs',[0 0.5],'width',1); title('interval t=9')

% Interval 10

yi=[0 0 0 1 0 0 1 1 0 0]; yiobs10=[0 0 0 1/4 0 0 1/3 1/1 0 0]; st=sum(yi); Nt=length(yi); alfa10=alfa0+st;
beta10=beta0+Nt; lambda10=alfa10/beta10; sigma10=sqrt(alfa10/beta10^2);
CILM10 = mean0-sigma10;
if CILM10<0
    CILM10=0;
end
MEAN10 = mean0; CIUM10 = mean0+sigma10; yiobs10= yiobs10(Nt);
xx10=[yiobs1 yiobs2 yiobs3 yiobs4 yiobs5 yiobs6 yiobs7 yiobs8 yiobs9 yiobs10];
lim10 = [CILM10 MEAN10 CIUM10];
figure
controlchart(xx10,'chart',{'i'},'limits',lim10,'specs',[0 0.5],'width',1); title('interval t=10')

```

#### d.- Direct method with mean posterior.

```

clear
%% Direct with Mean prior posterior
%Prior time 0
alfa0=0.001; beta0=0.001; mean0=0.001;

%Interval 1

yi=[0]; yiobs1=[0]; st=sum(yi); Nt=length(yi); alfa1=alfa0+st; beta1=beta0+Nt; lambda1=alfa1/beta1;
sigma1=sqrt(alfa1/beta1^2);
CILM1 = lambda1-sigma1;
if CILM1<0
    CILM1=0;
end
MEAN1 = lambda1; CIUM1 = lambda1+sigma1; yiobs1= yiobs1(Nt);
xx1=[0 yiobs1]; lim1 = [CILM1 MEAN1 CIUM1];
figure
controlchart(xx1,'chart',{'i'},'limits',lim1,'specs',[0 0.5],'width',1); title('interval t=1')

% Interval 2

yi=[0 0]; yiobs2=[0 0]; st=sum(yi); Nt=length(yi); alfa2=alfa0+st; beta2=beta0+Nt; lambda2=alfa2/beta2;
sigma2=sqrt(alfa2/beta2^2);
CILM2 = lambda2-sigma2;
if CILM2<0
    CILM2=0;
end
MEAN2 = lambda2; CIUM2 = lambda2+sigma2; yiobs2= yiobs2(Nt);
xx2=[yiobs1 yiobs2]; lim2 = [CILM2 MEAN2 CIUM2];
figure

```

```

controlchart(xx2,'chart',{'i'},'limits',lim2,'specs',[0 0.5],'width',1); title('interval t=2')

% Interval 3
yi=[0 0 0]; yiobs3=[0 0 0]; st=sum(yi); Nt=length(yi); alfa3=alfa0+st; beta3=beta0+Nt; lambda3=alfa3/beta3;
sigma3=sqrt(alfa3/beta3^2);
CILM3 = lambda3-sigma3;
if CILM3<0
    CILM3=0;
end
MEAN3 = lambda3; CIUM3 = lambda3+sigma3; yiobs3= yiobs3(Nt);
xx3=[yiobs1 yiobs2 yiobs3]; lim3 = [CILM3 MEAN3 CIUM3];
figure
controlchart(xx3,'chart',{'i'},'limits',lim3,'specs',[0 0.5],'width',1); title('interval t=3')

% Interval 4
yi=[0 0 0 1]; yiobs4=[0 0 0 1/4]; st=sum(yi); Nt=length(yi); alfa4=alfa0+st; beta4=beta0+Nt; lambda4=alfa4/beta4;
sigma4=sqrt(alfa4/beta4^2);
CILM4 = lambda4-sigma4;
if CILM4<0
    CILM4=0;
end
MEAN4 = lambda4; CIUM4 = lambda4+sigma4; yiobs4= yiobs4(Nt);
xx4=[yiobs1 yiobs2 yiobs3 yiobs4]; lim4 = [CILM4 MEAN4 CIUM4];
figure
controlchart(xx4,'chart',{'i'},'limits',lim4,'specs',[0 0.5],'width',1); title('interval t=4')

% Interval 5
yi=[0 0 0 1 0]; yiobs5=[0 0 0 1/4 0]; st=sum(yi); Nt=length(yi); alfa5=alfa0+st; beta5=beta0+Nt;
lambda5=alfa5/beta5; sigma5=sqrt(alfa5/beta5^2);
CILM5 = lambda5-sigma5;
if CILM5<0
    CILM5=0;
end
MEAN5 = lambda5; CIUM5 = lambda5+sigma5; yiobs5= yiobs5(Nt);
xx5=[yiobs1 yiobs2 yiobs3 yiobs4 yiobs5]; lim5 = [CILM5 MEAN5 CIUM5];
figure
controlchart(xx5,'chart',{'i'},'limits',lim5,'specs',[0 0.5],'width',1); title('interval t=5')

% Interval 6
yi=[0 0 0 1 0 0]; yiobs6=[0 0 0 1/4 0 0]; st=sum(yi); Nt=length(yi); alfa6=alfa0+st; beta6=beta0+Nt;
lambda6=alfa6/beta6; sigma6=sqrt(alfa6/beta6^2);
CILM6 = lambda6-sigma6;
if CILM6<0
    CILM6=0;
end
MEAN6 = lambda6; CIUM6 = lambda6+sigma6; yiobs6= yiobs6(Nt);
xx6=[yiobs1 yiobs2 yiobs3 yiobs4 yiobs5 yiobs6]; lim6 = [CILM6 MEAN6 CIUM6];
figure
controlchart(xx6,'chart',{'i'},'limits',lim6,'specs',[0 0.5],'width',1); title('interval t=6')

% Interval 7
yi=[0 0 0 1 0 0 1]; yiobs7=[0 0 0 1/4 0 0 1/3]; st=sum(yi); Nt=length(yi); alfa7=alfa0+st; beta7=beta0+Nt;
lambda7=alfa7/beta7; sigma7=sqrt(alfa7/beta7^2);
CILM7 = lambda7-sigma7;
if CILM7<0
    CILM7=0;
end
MEAN7 = lambda7; CIUM7 = lambda7+sigma7; yiobs7= yiobs7(Nt);
xx7=[yiobs1 yiobs2 yiobs3 yiobs4 yiobs5 yiobs6 yiobs7]; lim7 = [CILM7 MEAN7 CIUM7];
figure
controlchart(xx7,'chart',{'i'},'limits',lim7,'specs',[0 0.5],'width',1); title('interval t=7')

```

```

% Interval 8
yi=[0 0 0 1 0 0 1 1]; yiobs8=[0 0 0 1/4 0 0 1/3 1/1]; st=sum(yi); Nt=length(yi); alfa8=alfa0+st; beta8=beta0+Nt;
lambda8=alfa8/beta8; sigma8=sqrt(alfa8/beta8^2);
CILM8 = lambda8-sigma8;
if CILM8<0
    CILM8=0;
end
MEAN8 = lambda8; CIUM8 = lambda8+sigma8; yiobs8= yiobs8(Nt);
xx8=[yiobs1 yiobs2 yiobs3 yiobs4 yiobs5 yiobs6 yiobs7 yiobs8]; lim8 = [CILM8 MEAN8 CIUM8];
figure
controlchart(xx8,'chart',{'i'},'limits',lim8,'specs',[0 0.5],'width',1); title('interval t=8')

% Interval 9
yi=[0 0 0 1 0 0 1 1 0]; yiobs9=[0 0 0 1/4 0 0 1/3 1/1 0]; st=sum(yi); Nt=length(yi); alfa9=alfa0+st; beta9=beta0+Nt;
lambda9=alfa9/beta9; sigma9=sqrt(alfa9/beta9^2);
CILM9 = lambda9-sigma9;
if CILM9<0
    CILM9=0;
end
MEAN9 = lambda9; CIUM9 = lambda9+sigma9; yiobs9= yiobs9(Nt);
xx9=[yiobs1 yiobs2 yiobs3 yiobs4 yiobs5 yiobs6 yiobs7 yiobs8 yiobs9]; lim9 = [CILM9 MEAN9 CIUM9];
figure
controlchart(xx9,'chart',{'i'},'limits',lim9,'specs',[0 0.5],'width',1); title('interval t=9')

% Interval 10
yi=[0 0 0 1 0 0 1 1 0 0]; yiobs10=[0 0 0 1/4 0 0 1/3 1/1 0 0]; st=sum(yi); Nt=length(yi); alfa10=alfa0+st;
beta10=beta0+Nt; lambda10=alfa10/beta10; sigma10=sqrt(alfa10/beta10^2);
CILM10 = lambda10-sigma10;
if CILM10<0
    CILM10=0;
end
MEAN10 = lambda10; CIUM10 = lambda10+sigma10; yiobs10= yiobs10(Nt);
xx10=[yiobs1 yiobs2 yiobs3 yiobs4 yiobs5 yiobs6 yiobs7 yiobs8 yiobs9 yiobs10];
lim10 = [CILM10 MEAN10 CIUM10];
figure
controlchart(xx10,'chart',{'i'},'limits',lim10,'specs',[0 0.5],'width',1); title('interval t=10')

```

## H.2. Exponential-gamma.

### a.- Recurrent method.

```

clear
%%% Init
%Parameters specification
x0=0.2; c=0.1; B=500; K=4000; J=B+K; a=0; r=0; mu=0; n=10;
% Initial t=0 specification 1/20
t=20;
%Values for gamma
alfa0=0.5; beta0=10; meangamma0=alfa0/beta0; sigmagamma0=sqrt(alfa0/beta0^2); object=meangamma00;

for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
    x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
    if (x(j,1)<0) || (x(j,1)>3) %>2
        p(j,1)=0;
    else p(j,1)=(gampdf(x(j,1),alfa0,1/beta0)*exppdf(object,x(j,1)))/...

```

```

    (gampdf(x(j-1,1),alfa0,1/beta0)*exp-pdf(object,x(j-1,1)));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
    x(j,1)=x(j,1);
    a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
    r=r+1;
end
end
%Burn - in
for j=B:J
    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu0(i)=mean(xx); vaa0(i)=var(xx); stdd0(i)=sqrt(vaa0(i));
end

%Acceptance Rate AR
[muhat0,sigmahat0,muci0,sigmaci0] = normfit(muu0); AR0=mean(a)/J; muu00=mean(muu0); vaa00=mean(vaa0);
std00=sqrt(vaa00);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 0','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 0','FontSize',15);
%Histogram
figure
h = histogram(xx);
counts = h.Values; phat=gamfit(xx); alfa00 = phat(1); beta00 = 1/phat(2); meangamma0=phat(1)/(1/phat(2));
sigmagamma0=sqrt(phat(1)/(1/phat(2)^2));
hold on
xxx=0:0.01:2;
yyy=gampdf(xxx,phat(1),phat(2));
plot(xxx,yyy*max(counts)/max(yyy),'DisplayName','y=f(x) gamma')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 0','FontSize',15);
hold off
legend('Location','northeast')

%Lambda post, sigma post
Lambda00=muu00; Sigma00=sigmatgamma0;

% First incident Interval t=4
%Parameters specification
x0=0.2; c=0.2; B=500; K=4000; J=B+K; a=0; r=0; mu=0; n=10;
%Data
yi =[1]; t=4; yiP1 = [1/4];
%Values for gamma
alfa1=alfa0; beta1=beta0; meangamma1=alfa1/beta1; sigmagamma1=sqrt(alfa1/beta1^2);

for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
    x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
    if (x(j,1)<0) || (x(j,1)>3)
        p(j,1)=0;
    end
end
end

```

```

else p(j,1)=(gampdf(x(j,1),alfa1,1/beta1)*exppdf(yiP1,x(j,1)))/...
(gampdf(x(j-1,1),alfa1,1/beta1)*exppdf(yiP1,x(j-1,1)));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
x(j,1)=x(j,1);
a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
r=r+1;
end
end
%Burn - in
for j=B:J
xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu1(i)=mean(xx); vaa1(i)=var(xx); stdd1(i)=sqrt(vaa1(i));
end

%Acceptance Rate AR
[muhat1,sigmat1,muci1,sigmaci1] = normfit(muu1); AR1=mean(a)/J; muu11=mean(muu1); vaa11=mean(vaa1);
std11=sqrt(vaa11);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 4','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 4','FontSize',15);
%Histogram
figure
h = histogram(xx);
counts = h.Values; phat1=gamfit(xx); alfa11 = phat1(1); beta11 = 1/phat1(2); meangamma1=phat1(1)/(1/phat1(2));
sigmagamma1=sqrt(phat1(1)/(1/phat1(2)^2));
hold on
xxx=0:0.01:2;
yyy=gampdf(xxx,phat1(1),phat1(2));
plot(xxx,yyy*max(counts)/max(yyy),'DisplayName','y=f(x) gamma')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 4','FontSize',15);
hold off
legend('Location','northeast')

%Lambda post, sigma post
Lambda11=muu11; Sigma11=sigmat1;

% Second incident
%Parameters specification
x0=0.2; c=0.2; B=500; K=4000; J=B+K; a=0; r=0; mu=0; n =10;
%Data
% Incidence in t=7, 7-4=3, one incident. Interval t=3
yi =[1]; t=3; yiP2=[1/3];
%Values for gamma
alfa2=alfa11; beta2=beta11; meangamma2=alfa2/beta2; sigmagamma2=sqrt(alfa2/beta2^2);

for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);

```

```

if (x(j,1)<0) || (x(j,1)>3)
    p(j,1)=0;
else p(j,1)=(gampdf(x(j,1),alfa2,1/beta2)*exppdf(yiP2,x(j,1)))/...
    (gampdf(x(j-1,1),alfa2,1/beta2)*exppdf(yiP2,x(j-1,1)));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
    x(j,1)=x(j,1);
    a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
    r=r+1;
end
end
%Burn - in
for j=B:J
    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu2(i)=mean(xx); vaa2(i)=var(xx); stdd2(i)=sqrt(vaa2(i));
end

%Acceptance Rate AR
[muhat2,sigmahat2,muci2,sigmaci2] = normfit(muu2); AR2=mean(a)/J; muu22=mean(muu2); vaa22=mean(vaa2);
stdd22=sqrt(vaa22);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 7','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(1,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 7','FontSize',15);
%Histogram
figure
h = histogram(xx);
counts = h.Values; phat2=gamfit(xx); alfa22 = phat2(1); beta22 = 1/phat2(2); meangamma2=phat2(1)/(1/phat2(2));
sigmagamma2=sqrt(phat2(1)/(1/phat2(2)^2));
hold on
xxx=0:0.01:2;
yyy=gampdf(xxx,phat2(1),phat2(2));
plot(xxx,yyy*max(counts)/max(yyy),'DisplayName','y=f(x) gamma')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 7','FontSize',15);
hold off
legend('Location','northeast')

%Lambda post, sigma post
Lambda22=muu22; Sigma22=sigmatgamma2;

% Third incident
% Incident in t=8, 8-7=1. Parameters specification
x0=0.2; c=0.2; B=500; K=4000; J=B+K; a=0; r=0; mu=0;
%Data
yi =[1]; t=1; yiP3=[1/1];
%Values for gamma
alfa3=alfa22; beta3=beta22; meangamma3=alfa3/beta3; sigmagamma3=sqrt(alfa3/beta3^2);

for i=1:n
    a(i)=0;
    %Candidate value generation
    x(1,1)=x0;
    for j=2:J

```



```

x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
if (x(j,1)<0 || (x(j,1)>3)
    p(j,1)=0;
else p(j,1)=(gampdf(x(j,1),alfa3,1/beta3)*expPDF(yiP3,x(j,1)))/...
    (gampdf(x(j-1,1),alfa3,1/beta3)*expPDF(yiP3,x(j-1,1)));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
    x(j,1)=x(j,1);
    a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
    r=r+1;
end
end
%Burn - in
for j=B:J
    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu3(i)=mean(xx); vaa3(i)=var(xx); stdd3(i)=sqrt(vaa3(i));
end

%Acceptance Rate AR
[muhat3,sigmahat3,muci3,sigmaci3] = normfit(muu3); AR3=mean(a)/J; muu33=mean(muu3); vaa33=mean(vaa3);
stdd33=sqrt(vaa33);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 8','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 8','FontSize',15);
%Histogram
figure
h = histogram(xx);
counts = h.Values; phat3=gamfit(xx); alfa33 = phat3(1); beta33 = 1/phat3(2); meangamma3=phat3(1)/(1/phat3(2));
sigmagamma3=sqrt(phat3(1)/(1/phat3(2)^2));
hold on
xxx=0:0.01:2;
yyy=gampdf(xxx,phat3(1),phat3(2));
plot(xxx,yyy*max(counts)/max(yyy),'DisplayName','y=f(x) gamma')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 8','FontSize',15);
hold off
legend('Location','northeast')

%Lambda post, sigma post
Lambda33=muu33; Sigma33=sigmatgamma3;

```

## b.- Direct method.

```

clear
%% Init
%Parameters specification
x0=0.2; c=0.2; B=500; K=4000; J=B+K; a=0; r=0; mu=0; n=10;
% Initial t=0, specification 1/20
t=20;
%Values for gamma
alfa0=0.5; beta0=10; meangamma00=alfa0/beta0; sigmagamma00=sqrt(alfa0/beta0^2); object=meangamma00;

```

```

for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
if (x(j,1)<0) || (x(j,1)>3) %>2
p(j,1)=0;
else p(j,1)=(gampdf(x(j,1),alfa0,1/beta0)*exppdf(object,x(j,1)))/...
(gampdf(x(j-1,1),alfa0,1/beta0)*exppdf(object,x(j-1,1)));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
x(j,1)=x(j,1);
a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
r=r+1;
end
end
%Burn - in
for j=B:J
xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu0(i)=mean(xx); vaa0(i)=var(xx); stdd0(i)=sqrt(vaa0(i));
end

%Acceptance Rate AR
[muhat0,sigmahat0,muci0,sigmaci0] = normfit(muu0); AR0=mean(a)/J; muu00=mean(muu0); vaa00=mean(vaa0);
std00=sqrt(vaa00);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 0','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 0','FontSize',15);
%Histogram
figure
h = histogram(xx);
counts = h.Values; phat=gamfit(xx); alfa00 = phat(1); beta00 = 1/phat(2); meangamma0=phat(1)/(1/phat(2));
sigmagamma0=sqrt(phat(1)/(1/phat(2)^2));
hold on
xxx=0:0.01:2;
yyy=gampdf(xxx,phat(1),phat(2));
plot(xxx,yyy*max(counts)/max(yyy),'DisplayName','y=f(x) gamma')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 0','FontSize',15);
hold off
legend('Location','northeast')

%Lambda post, sigma post
Lambda00=muu00; Sigma00=sigmatgamma0;

% First incident. Interval t=4
%Parameters specification
x0=0.2; c=0.2; B=500; K=4000; J=B+K; a=0; r=0; mu=0; n=10;
%Data
yi =[1]; t=4; yiP1 = [1/4];
%Values for gamma
alfa1=alfa0; beta1=beta0; meangamma1=alfa1/beta1; sigmagamma1=sqrt(alfa1/beta1^2);

```

```

for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
if (x(j,1)<0) || (x(j,1)>3)
p(j,1)=0;
else p(j,1)=(gampdf(x(j,1),alfa1,1/beta1)*expPDF(yiP1,x(j,1)))/...
(gampdf(x(j-1,1),alfa1,1/beta1)*expPDF(yiP1,x(j-1,1)));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
x(j,1)=x(j,1);
a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
r=r+1;
end
end
%Burn - in
for j=B:J
xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu1(i)=mean(xx); vaa1(i)=var(xx); stdd1(i)=sqrt(vaa1(i));
end

%Acceptance Rate AR
[muhat1,sigmahat1,muc1,sigmac1] = normfit(muu1); AR1=mean(a)/J; muu11=mean(muu1); vaa11=mean(vaa1);
std11=sqrt(vaa11);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 4','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 4','FontSize',15);
%Histogram
figure
h = histogram(xx);
counts = h.Values; phat1=gamfit(xx); alfa11 = phat1(1); beta11 = 1/phat1(2); meangamma1=phat1(1)/(1/phat1(2));
sigmagamma1=sqrt(phat1(1)/(1/phat1(2)^2));
hold on
xxx=0:0.01:2;
yyy=gampdf(xxx,phat1(1),phat1(2));
plot(xxx,yyy*max(counts)/max(yyy),'DisplayName','y=f(x) gamma')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 4','FontSize',15);
hold off

%Lambda post, sigma post
Lambda11=muu11; Sigma11=sigmatgamma1;

% Second incident
%Parameters specification
x0=0.2; c=0.2; B=500; K=4000; J=B+K; a=0; r=0; mu=0; n =10;
%Data. Incident in t=7, 7-4=3 un incident
yi =[1]; t=3; yiP2=[1/3];
%Values for gamma
alfa2=alfa0; beta2=beta0; meangamma2=alfa2/beta2; sigmagamma2=sqrt(alfa2/beta2^2);

```

```

for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
if (x(j,1)<0) || (x(j,1)>3)
p(j,1)=0;
else p(j,1)=(gampdf(x(j,1),alfa2,1/beta2)*expPDF(yiP2,x(j,1)))/...
(gampdf(x(j-1,1),alfa2,1/beta2)*expPDF(yiP2,x(j-1,1)));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
x(j,1)=x(j,1);
a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
r=r+1;
end
end
%Burn - in
for j=B:J
xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu2(i)=mean(xx); vaa2(i)=var(xx); stdd2(i)=sqrt(vaa2(i));
end

%Acceptance Rate AR
[muhat2,sigmat2,muci2,sigmaci2] = normfit(muu2); AR2=mean(a)/J; muu22=mean(muu2); vaa22=mean(vaa2);
stdd22=sqrt(vaa22);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold');title('Trace interval 7','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 7','FontSize',15);
%Histogram
figure
h = histogram(xx);
counts = h.Values; phat2=gamfit(xx); alfa22 = phat2(1); beta22 = 1/phat2(2); meangamma2=phat2(1)/(1/phat2(2));
sigmagamma2=sqrt(phat2(1)/(1/phat2(2)^2));
hold on
xxx=0:0.01:2;
yyy=gampdf(xxx,phat2(1),phat2(2));
plot(xxx,yyy*max(counts)/max(yyy),'DisplayName','y=f(x) gamma')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 7','FontSize',15);
hold off
legend('Location','northeast')

%Lambda post, sigma post
Lambda22=muu22; Sigma22=sigmat2;

% Third incident. Incident in t=8, 8-7=1
%Parameters specification
x0=0.2; c=0.2; B=500; K=4000; J=B+K; a=0; r=0; mu=0;
%Data
yi =[1]; t=1; yiP3=[1/1];
%Values for gamma

```

```

alfa3=alfa0; beta3=beta0; meangamma3=alfa3/beta3; sigmagamma3=sqrt(alfa3/beta3^2);

for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
    x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
    if (x(j,1)<0) || (x(j,1)>3)
        p(j,1)=0;
    else p(j,1)=(gampdf(x(j,1),alfa3,1/beta3)*exppdf(yiP3,x(j,1)))/...
        (gampdf(x(j-1,1),alfa3,1/beta3)*exppdf(yiP3,x(j-1,1)));
    end
    u=random('Uniform',0,1,1,1);
    if u<p(j,1)
        x(j,1)=x(j,1);
        a(i)=a(i)+1;
    else x(j,1)=x(j-1,1);
        r=r+1;
    end
end
end
%Burn - in
for j=B:J
    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu3(i)=mean(xx); vaa3(i)=var(xx); stdd3(i)=sqrt(vaa3(i));
end

%Acceptance Rate AR
[muhat3,sigmat3,muci3,sigmaci3] = normfit(muu3); AR3=mean(a)/J; muu33=mean(muu3); vaa33=mean(vaa3);
stdd33=sqrt(vaa33);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 8','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 8','FontSize',15);
%Histogram
figure
h = histogram(xx);
counts = h.Values; phat3=gamfit(xx); alfa33 = phat3(1); beta33 = 1/phat3(2); meangamma3=phat3(1)/(1/phat3(2));
sigmagamma3=sqrt(phat3(1)/(1/phat3(2)^2));
hold on
xxx=0:0.01:2;
yyy=gampdf(xxx,phat3(1),phat3(2));
plot(xxx,yyy*max(counts)/max(yyy),'DisplayName','y=f(x) gamma')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 8','FontSize',15);
hold off
legend('Location','northeast')

%Lambda post, sigma post
Lambda33=muu33; Sigma33=sigmat3;

```

### H.3. Weibull-gamma.

#### a.- Recurrent method.

```

clear
%Parameters specification
x0=0.2; c=0.2; B=500; K=4000; J=B+K; a=0; r=0; mu=0; n=10;
% Initial t=0. Specification 1/20
t=20;
%Values for gamma
alfa0=0.5; beta0=10; meangamma0=alfa0/beta0; sigmagamma0=sqrt(alfa0/beta0^2); object=meangamma0;
%Weibull
aw=1.5;

for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
if (x(j,1)<0) || (x(j,1)>3)
p(j,1)=0;
else p(j,1)=(gampdf(x(j,1),alfa0,1/beta0)*wblpdf(object,x(j,1),aw))/...
(gampdf(x(j-1,1),alfa0,1/beta0)*wblpdf(object,x(j-1,1),aw));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
x(j,1)=x(j,1);
a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
r=r+1;
end
end
%Burn - in
for j=B:J
xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu0(i)=mean(xx); vaa0(i)=var(xx); stdd0(i)=sqrt(vaa0(i));
end

%Acceptance Rate AR
[muhat0,sigmahat0,muci0,sigmaci0] = normfit(muu0); AR0=mean(a)/J; muu00=mean(muu0); vaa00=mean(vaa0);
std00=sqrt(vaa00);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 0','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 0','FontSize',15);
%Histogram
figure
h = histogram(xx);
counts = h.Values; phat=gamfit(xx); alfa00 = phat(1); beta00 = 1/phat(2); meangamma0=phat(1)/(1/phat(2));
sigmagamma0=sqrt(phat(1)/(1/phat(2)^2));
hold on
xxx=0:0.01:2;
yyy=gampdf(xxx,phat(1),phat(2));

```

```

plot(xxx,yyy*max(counts)/max(yyy),'DisplayName','y=f(x) gamma')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 0','FontSize',15);
hold off
legend('Location','northeast')

%Lambda post, sigma post
Lambda00=muu00; Sigma00=sigmagamma0;

% First incident
%Parameters specification
x0=0.2; c=0.2; B=500; K=4000; J=B+K; a=0; r=0; mu=0;
%Data. Interval t=4
yi =[1]; t=4; yiP1=[1/4];
% Values for gamma
alfa1=alfa0; beta1=beta0; meangamma1=alfa1/beta1; sigmagamma1=sqrt(alfa1/beta1^2);
%Weibull
aw=1.5;

for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
if (x(j,1)<0) || (x(j,1)>3)
p(j,1)=0;
else p(j,1)=(gampdf(x(j,1),alfa1,1/beta1)*wblpdf(yiP1,x(j,1),aw))/...
(gampdf(x(j-1,1),alfa1,1/beta1)*wblpdf(yiP1,x(j-1,1),aw));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
x(j,1)=x(j,1);
a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
r=r+1;
end
end
%Burn - in
for j=B:J
xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu1(i)=mean(xx); vaa1(i)=var(xx); stdd1(i)=sqrt(vaa1(i));
end

%Acceptance Rate AR
[muhat1,sigmahat1,muci1,sigmaci1] = normfit(muu1); AR1=mean(a)/J; muu11=mean(muu1); vaa11=mean(vaa1);
std11=sqrt(vaa11);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 4','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 4','FontSize',15);
%Histogram
figure
h = histogram(xx);
counts = h.Values; phat1=gamfit(xx); alfa11 = phat1(1); beta11 = 1/phat1(2); meangamma1=phat1(1)/(1/phat1(2));
sigmagamma1=sqrt(phat1(1)/(1/phat1(2)^2));

```

```

hold on
xxx=0:0.01:2;
yyy=gampdf(xxx,phat1(1),phat1(2));
plot(xxx,yyy*max(counts)/max(yyy),'DisplayName','y=f(x) gamma')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 4','FontSize',15);
hold off
legend('Location','northeast')

%Lambda post, sigma post
Lambda11=muu11; Sigma11=sigmagamma1;

% Second incident
%Parameters specification
x0=0.2; c=0.2; B=500; K=4000; J=B+K; a=0; r=0; mu=0;
%Data. Incident in t=7, 7-4=3. Interval t=3
yi =[1]; t=3; yiP2=[1/3];
%Values for gamma
alfa2=alfa11; beta2=beta11; meangamma2=alfa2/beta2; sigmagamma2=sqrt(alfa2/beta2^2);
%Weibull
aw=1.5;

for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
if (x(j,1)<0) || (x(j,1)>3)
p(j,1)=0;
else p(j,1)=(gampdf(x(j,1),alfa2,1/beta2)*wblpdf(yiP2,x(j,1),aw))/...
(gampdf(x(j-1,1),alfa2,1/beta2)*wblpdf(yiP2,x(j-1,1),aw));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
x(j,1)=x(j,1);
a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
r=r+1;
end
end
%Burn - in
for j=B:J
xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu2(i)=mean(xx); vaa2(i)=var(xx); stdd2(i)=sqrt(vaa2(i));
end

%Acceptance Rate AR
[muhat2,sigmahat2,muci2,sigmaci2] = normfit(muu2); AR2=mean(a)/J; muu22=mean(muu2); vaa22=mean(vaa2);
stdd22=sqrt(vaa22);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 7','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 7','FontSize',15);
%Histogram
figure

```



```

h = histogram(xx);
counts = h.Values; phat2=gamfit(xx); alfa22 = phat2(1); beta22 = 1/phat2(2); meangamma2=phat2(1)/(1/phat2(2));
sigmagamma2=sqrt(phat2(1)/(1/phat2(2)^2));
hold on
xxx=0:0.01:2;
yyy=gampdf(xxx,phat2(1),phat2(2));
plot(xxx,yyy*max(counts)/max(yyy),'DisplayName','y=f(x) gamma')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 7','FontSize',15);
hold off
legend('Location','northeast')

%Lambda post, sigma post
Lambda22=muu22; Sigma22=sigmatgamma2;

% Third incident
%Parameters specification
x0=0.2; c=0.2; B=500; K=4000; J=B+K; a=0; r=0; mu=0;
%Data. Incident in t=8, 8-7=1. Interval t=1
yi =[1]; t=1; yiP3=[1/1];
%Values for gamma
alfa3=alfa22; beta3=beta22; meangamma3=alfa3/beta3; sigmagamma3=sqrt(alfa3/beta3^2);
%Weibull
aw=1.5;

for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
if (x(j,1)<0) || (x(j,1)>3)
p(j,1)=0;
else p(j,1)=(gampdf(x(j,1),alfa3,1/beta3)*wblpdf(yiP3,x(j,1),aw))/...
(gampdf(x(j-1,1),alfa3,1/beta3)*wblpdf(yiP3,x(j-1,1),aw));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
x(j,1)=x(j,1);
a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
r=r+1;
end
end
%Burn - in
for j=B:J
xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu3(i)=mean(xx); vaa3(i)=var(xx); stdd3(i)=sqrt(vaa3(i));
end

%Acceptance Rate AR
[muhat3,sigmahat3,muci3,sigmaci3] = normfit(muu3); AR3=mean(a)/J; muu33=mean(muu3); vaa33=mean(vaa3);
stdd33=sqrt(vaa33);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 8','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');

```

```

ylabel('Mean value', 'FontSize', 15, 'FontWeight', 'bold'); title('Burn-in interval 8', 'FontSize', 15);
%Histogram
figure
h = histogram(xx);
counts = h.Values; phat3=gamfit(xx); alfa33 = phat3(1); beta33 = 1/phat3(2); meangamma3=phat3(1)/(1/phat3(2));
sigmagamma3=sqrt(phat3(1)/(1/phat3(2)^2));
hold on
xxx=0:0.01:2;
yyy=gampdf(xxx,phat3(1),phat3(2));
plot(xxx,yyy*max(counts)/max(yyy),'DisplayName','y=f(x) gamma')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 8','FontSize',15);
hold off
legend('Location','northeast')

%Lambda post, sigma post
Lambda33=muu33; Sigma33=sigmatgamma3;

```

### b.- Direct method.

```

clear
%Parameters specification
x0=0.2; c=0.2; B=500; K=4000; J=B+K; a=0; r=0; mu=0; n=10;
% Initial t=0. Specification 1/20
t=20;
%Values for gamma
alfa0=0.5; beta0=10; meangamma0=alfa0/beta0; sigmagamma0=sqrt(alfa0/beta0^2); object=meangamma0;
%Weibull
aw=1.5;

for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
if (x(j,1)<0) || (x(j,1)>3)
p(j,1)=0;
else p(j,1)=(gampdf(x(j,1),alfa0,1/beta0)*wblpdf(object,x(j,1),aw))/...
(gampdf(x(j-1,1),alfa0,1/beta0)*wblpdf(object,x(j-1,1),aw));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
x(j,1)=x(j,1);
a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
r=r+1;
end
end
%Burn - in
for j=B:J
xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu0(i)=mean(xx); vaa0(i)=var(xx); stdd0(i)=sqrt(vaa0(i));
end

%Acceptance Rate AR
[muhat0,sigmahat0,muci0,sigmaci0] = normfit(muu0); AR0=mean(a)/J; muu00=mean(muu0); vaa00=mean(vaa0);
std00=sqrt(vaa00);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)

```

```

xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 0','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(1,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 0','FontSize',15);
%Histogram
figure
h = histogram(xx);
counts = h.Values; phat=gamfit(xx); alfa00 = phat(1); beta00 = 1/phat(2); meangamma0=phat(1)/(1/phat(2));
sigmagamma0=sqrt(phat(1)/(1/phat(2)^2));
hold on
xxx=0:0.01:2;
yyy=gampdf(xxx,phat(1),phat(2));
plot(xxx,yyy*max(counts)/max(yyy),'DisplayName','y=f(x) gamma')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 0','FontSize',15);
hold off
legend('Location','northeast')

%Lambda post, sigma post
Lambda00=muu00; Sigma00=sigmatgamma0;

% First incident
%Parameters specification
x0=0.2; c=0.2; B=500; K=4000; J=B+K; a=0; r=0; mu=0;
%Data. Interval t=4
yi =[1]; t=4; yiP1=[1/4];
%Values for gamma
alfa1=alfa0; beta1=beta0; meangamma1=alfa1/beta1; sigmagamma1=sqrt(alfa1/beta1^2);
%Weibull
aw=1.5;

for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
if (x(j,1)<0) || (x(j,1)>3)
p(j,1)=0;
else p(j,1)=(gampdf(x(j,1),alfa1,1/beta1)*wblpdf(yiP1,x(j,1),aw))/...
(gampdf(x(j-1,1),alfa1,1/beta1)*wblpdf(yiP1,x(j-1,1),aw));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
x(j,1)=x(j,1);
a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
r=r+1;
end
end
%Burn - in
for j=B:J
xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu1(i)=mean(xx); vaa1(i)=var(xx); stdd1(i)=sqrt(vaa1(i));
end

%Acceptance Rate AR
[muhat1,sigmahat1,muci1,sigmaci1] = normfit(muu1); AR1=mean(a)/J; muu11=mean(muu1); vaa11=mean(vaa1);
stdl1=sqrt(vaa11);

%Graphics
figure

```

```

k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 4','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 4','FontSize',15);
%Histogram
figure
h = histogram(xx);
counts = h.Values; phat1=gamfit(xx); alfa11 = phat1(1); beta11 = 1/phat1(2); meangamma1=phat1(1)/(1/phat1(2));
sigmagamma1=sqrt(phat1(1)/(1/phat1(2)^2));
hold on
xxx=0:0.01:2;
yyy=gampdf(xxx,phat1(1),phat1(2));
plot(xxx,yyy*max(counts)/max(yyy),'DisplayName','y=f(x) gamma')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 4','FontSize',15);
hold off
legend('Location','northeast')

%Lambda post, sigma post
Lambda11=muu11; Sigma11=sigmatgamma1;

% Second incident
%Parameters specification
x0=0.2; c=0.2; B=500; K=4000; J=B+K; a=0; r=0; mu=0;
%Data. Incident in t=7, 7-4=3. Interval t=3
yi =[1]; t=3; yiP2=[1/3];
%Values for gamma
alfa2=alfa0; beta2=beta0; meangamma2=alfa2/beta2; sigmagamma2=sqrt(alfa2/beta2^2);
%Weibull
aw=1.5;

for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
if (x(j,1)<0) || (x(j,1)>3)
p(j,1)=0;
else p(j,1)=(gampdf(x(j,1),alfa2,1/beta2)*wblpdf(yiP2,x(j,1),aw))/...
(gampdf(x(j-1,1),alfa2,1/beta2)*wblpdf(yiP2,x(j-1,1),aw));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
x(j,1)=x(j,1);
a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
r=r+1;
end
end
%Burn - in
for j=B:J
xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu2(i)=mean(xx); vaa2(i)=var(xx); stdd2(i)=sqrt(vaa2(i));
end

%Acceptance Rate AR
[muhat2,sigmahat2,muci2,sigmaci2] = normfit(muu2); AR2=mean(a)/J; muu22=mean(muu2); vaa22=mean(vaa2);
stdd22=sqrt(vaa22);

```

```

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 7','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 7','FontSize',15);
%Histogram
figure
h = histogram(xx);
counts = h.Values; phat2=gamfit(xx); alfa22 = phat2(1); beta22 = 1/phat2(2); meangamma2=phat2(1)/(1/phat2(2));
sigmagamma2=sqrt(phat2(1)/(1/phat2(2)^2));
hold on
xxx=0:0.01:2;
yyy=gampdf(xxx,phat2(1),phat2(2));
plot(xxx,yyy*max(counts)/max(yyy),'DisplayName','y=f(x) gamma')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 7','FontSize',15);
hold off
legend('Location','northeast')

%Lambda post, sigma post
Lambda22=muu22; Sigma22=sigmatgamma2;

% Third incident
%Parameters specification
x0=0.2; c=0.2; B=500; K=4000; J=B+K; a=0; r=0; mu=0;
%Data. Incident in t=8, 8-7=1. Interval t=1
yi =[1]; t=1; yiP3=[1/1];
%Values for gamma
alfa3=alfa0; beta3=beta0; meangamma3=alfa3/beta3; sigmagamma3=sqrt(alfa3/beta3^2);
%Weibull
aw=1.5;

for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
if (x(j,1)<0) || (x(j,1)>3)
p(j,1)=0;
else p(j,1)=(gampdf(x(j,1),alfa3,1/beta3)*wblpdf(yiP3,x(j,1),aw))/...
(gampdf(x(j-1,1),alfa3,1/beta3)*wblpdf(yiP3,x(j-1,1),aw));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
x(j,1)=x(j,1);
a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
r=r+1;
end
end
%Burn - in
for j=B:J
xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu3(i)=mean(xx); vaa3(i)=var(xx); stdd3(i)=sqrt(vaa3(i));
end

```

```

%Acceptance Rate AR
[muhat3,sigmahat3,muci3,sigmaci3] = normfit(muu3); AR3=mean(a)/J; muu33=mean(muu3); vaa33=mean(vaa3);
stdd33=sqrt(vaa33);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 8','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 8','FontSize',15);
%Histogram
figure
h = histogram(xx);
counts = h.Values; phat3=gamfit(xx); alfa33 = phat3(1); beta33 = 1/phat3(2); meangamma3=phat3(1)/(1/phat3(2));
sigmagamma3=sqrt(phat3(1)/(1/phat3(2)^2));
hold on
xxx=0:0.01:2;
yyy=gampdf(xxx,phat3(1),phat3(2));
plot(xxx,yyy*max(counts)/max(yyy),'DisplayName','y=f(x) gamma')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 8','FontSize',15);
hold off
legend('Location','northeast')

%Lambda post, sigma post
Lambda33=muu33; Sigma33=sigmatgamma3;

```

## H.4. Exponential-normal.

### a.- Recurrent method.

```

clear
%First incident t=4
%Parameters specification
x0=1; c=0.2; B=500; K=4000; J=B+K; a=0; r=0; mu=0; n=10; object=0.05;
%Vector of incidences
yi =[0 0 0 1]; sumY=sum(yi); nn=length(yi);
%Vector of parameters
yiP=[0 0 0 1/4]; SiP = std(yiP,1); yii = sumY/nn; S = std(yi,1); MU = mean(yi,2);

for i=1:n
%Candidate value generation
a(i)=0;
x(1,1)=x0;
for j=2:J
x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
if (x(j,1)<0) || (x(j,1)>20)
p(j,1)=0;
else p(j,1)=(exp-pdf(yiP(nn),x(j,1))*norm-pdf(object,x(j,1),SiP))/...
(exp-pdf(yiP(nn),x(j-1,1))*norm-pdf(object,x(j-1,1),SiP));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
x(j,1)=x(j,1);
a(i)=a(i)+1;
else x(j,1)=x(j-1,1);

```

```

    r=r+1;
    end
end
%Burn - in
for j=B:J
    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu1(i)=mean(xx); vaa1(i)=var(xx); stdd1(i)=sqrt(vaa1(i));
end

%Acceptance Rate AR
[muhat1,sigmahat1,muci1,sigmaci1] = normfit(muu1); AR1=mean(a)/J; muu11=mean(muu1); vaa11=mean(vaa1);
std11=sqrt(vaa11);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 4','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 4','FontSize',15);

%Histogram
figure
h = histogram(xx);
%Normal fitting
counts = h.Values; [muhat,sigmahat]=normfit(xx);
hold on
xxx=0:0.01:2;
yyyy=normpdf(xxx,muhat,sigmahat);
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) normal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 4','FontSize',15);
hold off
legend('Location','northeast')

%Lambda post, sigma post
Lambda11=muu11; Sigma11=sigmahat;

% Second incident in t=7; t=7-4=3
%Parameters specification
x0=1; c=0.2; B=500; K=4000; J=B+K; a=0; r=0; mu=0; n=10; object=0.05;
%Vector of incidences
yi =[0 0 0 1 0 0 1]; sumY=sum(yi); nn=length(yi);
% Vector of parameters
yiP=[0 0 0 1/4 0 0 1/3]; SiP = std(yiP,1); yii = sumY/nn; S = std(yi,1); MU = mean(yi,2);

for i=1:n
%Candidate value generation
a(i)=0;
x(1,1)=x0;
for j=2:J
    x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
    if (x(j,1)<0) || (x(j,1)>20)
        p(j,1)=0;
    else p(j,1)=(exppdf(yiP(nn),x(j,1))*normpdf(muu11,x(j,1),SiP))/...
        (exppdf(yiP(nn),x(j-1,1))*normpdf(muu11,x(j-1,1),SiP));
    end
    u=random('Uniform',0,1,1,1);
    if u<p(j,1)
        x(j,1)=x(j,1);
    end
end
end

```

```

    a(i)=a(i)+1;
    else x(j,1)=x(j-1,1);
        r=r+1;
    end
end
%Burn - in
for j=B:J
    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu2(i)=mean(xx); vaa2(i)=var(xx); stdd2(i)=sqrt(vaa2(i));
end

%Acceptance Rate AR
[muhat2,sigmahat2,muci2,sigmaci2] = normfit(muu2); AR2=mean(a)/J; muu22=mean(muu2); vaa22=mean(vaa2);
std22=sqrt(vaa22);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 7','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 7','FontSize',15);
%Histogram
figure
h = histogram(xx);
%Normal fitting
counts = h.Values; [muhat,sigmahat]=normfit(xx);
hold on
xxx=0:0.01:3;
yyyy=normpdf(xxx,muhat,sigmahat);
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) normal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 7','FontSize',15);
hold off
legend('Location','northeast')

%Lambda post, sigma post
Lambda22=muu22; Sigma22=sigmahat;

%Third incident in t=8; t=8-7=1
%Parameters specification
x0=1; c=0.2; B=500; K=4000; J=B+K; a=0; r=0; mu=0; n=10; object=0.05;
%Vector of incidences
yi =[0 0 0 1 0 0 1 1]; sumY=sum(yi); nn=length(yi);
%Vector of parameters
yiP=[0 0 0 1/4 0 0 1/3 1/1]; SiP = std(yiP,1); yii = sumY/nn; S = std(yi,1); MU = mean(yi,2);

for i=1:n
%Candidate value generation
a(i)=0;
x(1,1)=x0;
for j=2:J
    x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
    if (x(j,1)<0) || (x(j,1)>20)
        p(j,1)=0;
    else p(j,1)=(expPDF(yiP(nn),x(j,1))*normPDF(muu22,x(j,1),SiP))/...
        (expPDF(yiP(nn),x(j-1,1))*normPDF(muu22,x(j-1,1),SiP));
    end
    u=random('Uniform',0,1,1,1);
    if u<p(j,1)

```



```

    x(j,1)=x(j,1);
    a(i)=a(i)+1;
    else x(j,1)=x(j-1,1);
        r=r+1;
    end
end
%Burn - in
for j=B:J
    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu3(i)=mean(xx); vaa3(i)=var(xx); stdd3(i)=sqrt(vaa3(i));
end

%Acceptance Rate AR
[muhat3,sigmahat3,muci3,sigmaci3] = normfit(muu3); AR3=mean(a)/J; muu33=mean(muu3); vaa33=mean(vaa3);
std33=sqrt(vaa33);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 8','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 8','FontSize',15);
%Histogram
figure
h = histogram(xx);
%Normal fitting
counts = h.Values; [muhat,sigmahat]=normfit(xx);
hold on
xxx=0:0.01:3;
yyyy=normpdf(xxx,muhat,sigmahat);
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) normal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 8','FontSize',15);
hold off
legend('Location','northeast')

%Lambda post, sigma post
Lambda33=muu33; Sigma33=sigmahat;

```

## b.- Direct method.

```

clear
%First incident t=4
%Parameters specification
x0=1; c=0.2; B=500; K=4000; J=B+K; a=0; r=0; mu=0; n=10; object=0.05;
%Vector de incidences
yi =[0 0 0 1]; sumY=sum(yi); nn=length(yi);
%Vector of parameters
yiP=[0 0 0 1/4]; SiP = std(yiP,1); yii = sumY/nn; S = std(yi,1); MU = mean(yi,2);

for i=1:n
%Candidate value generation
a(i)=0;
x(1,1)=x0;
for j=2:J
    x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
    if (x(j,1)<0) || (x(j,1)>20)
        p(j,1)=0;
    end
end
end

```

```

else p(j,1)=(expmpdf(yiP(nn),x(j,1))*normpdf(object,x(j,1),SiP))/...
      (expmpdf(yiP(nn),x(j-1,1))*normpdf(object,x(j-1,1),SiP));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
    x(j,1)=x(j,1);
    a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
    r=r+1;
end
end
%Burn - in
for j=B:J
    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu1(i)=mean(xx); vaa1(i)=var(xx); stdd1(i)=sqrt(vaa1(i));
end

%Acceptance Rate AR
[muhat1,sigmahat1,muci1,sigmaci1] = normfit(muu1); AR1=mean(a)/J; muu11=mean(muu1); vaa11=mean(vaa1);
std11=sqrt(vaa11);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 4','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 4','FontSize',15);
%Histogram
figure
h = histogram(xx);
%Normal fitting
counts = h.Values; [muhat,sigmahat]=normfit(xx);
hold on
xxx=0:0.01:2;
yyyy=normpdf(xxx,muhat,sigmahat);
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) normal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 4','FontSize',15);
hold off
legend('Location','northeast')

%Lambda post, sigma post
Lambda11=muu11; Sigma11=sigmahat;

% Second incident in t=7; t=7-4=3
%Parameters specification
x0=1; c=0.2; B=500; K=4000; J=B+K; a=0; r=0; mu=0; n=10; object=0.05;
%Vector of incidences
yi =[0 0 0 1 0 0 1]; sumY=sum(yi); nn=length(yi);
%Vector of parameters
yiP=[0 0 0 1/4 0 0 1/3]; SiP = std(yiP,1); yii = sumY/nn; S = std(yi,1); MU = mean(yi,2);

for i=1:n
%Candidate value generation
a(i)=0;
x(1,1)=x0;
for j=2:J
    x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
    if (x(j,1)<0) || (x(j,1)>20)

```

```

    p(j,1)=0;
    else p(j,1)=(expddf(yiP(nn),x(j,1))*normpdf(object,x(j,1),SiP))/...
        (expddf(yiP(nn),x(j-1,1))*normpdf(object,x(j-1,1),SiP));
    end
    u=random('Uniform',0,1,1,1);
    if u<p(j,1)
        x(j,1)=x(j,1);
        a(i)=a(i)+1;
    else x(j,1)=x(j-1,1);
        r=r+1;
    end
end
%Burn - in
for j=B:J
    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu2(i)=mean(xx); vaa2(i)=var(xx); stdd2(i)=sqrt(vaa2(i));
end

%Acceptance Rate AR
[muhat2,sigmahat2,muci2,sigmaci2] = normfit(muu2); AR2=mean(a)/J; muu22=mean(muu2); vaa22=mean(vaa2);
std22=sqrt(vaa22);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 7','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 7','FontSize',15);
%Histogram
figure
h = histogram(xx);
%Normal fitting
counts = h.Values; [muhat,sigmahat]=normfit(xx);
hold on
xxx=0:0.01:3;
yyyy=normpdf(xxx,muhat,sigmahat);
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) normal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 7','FontSize',15);
hold off
legend('Location','northeast')

%Lambda post, sigma post
Lambda22=muu22; Sigma22=sigmahat;

%Third incident in t= 8, t=8-7=1
%Parameters specification
x0=1; c=0.2; B=500; K=4000; J=B+K; a=0; r=0; mu=0; n=10; object=0.05;
%Vector of incidences
yi =[0 0 0 1 0 0 1 1]; sumY=sum(yi); nn=length(yi);
%Vector of parameters
yiP=[0 0 0 1/4 0 0 1/3 1/1]; SiP = std(yiP,1); yii = sumY/nn; S = std(yi,1); MU = mean(yi,2);

for i=1:n
    %Candidate value generation
    a(i)=0;
    x(1,1)=x0;
    for j=2:J
        x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
    end
end

```

```

if (x(j,1)<0) || (x(j,1)>20)
    p(j,1)=0;
else p(j,1)=(exppdf(yiP(nn),x(j,1))*normpdf(object,x(j,1),SiP))/...
    (exppdf(yiP(nn),x(j-1,1))*normpdf(object,x(j-1,1),SiP));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
    x(j,1)=x(j,1);
    a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
    r=r+1;
end
end
%Burn - in
for j=B:J
    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu3(i)=mean(xx); vaa3(i)=var(xx); stdd3(i)=sqrt(vaa3(i));
end

%Acceptance Rate AR
[muhat3,sigmahat3,muci3,sigmaci3] = normfit(muu3); AR3=mean(a)/J; muu33=mean(muu3); vaa33=mean(vaa3);
std33=sqrt(vaa33);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 8','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(1,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 8','FontSize',15);
%Histogram
figure
h = histogram(xx);
%Normal fitting
counts = h.Values; [muhat,sigmahat]=normfit(xx);
hold on
xxx=0:0.01:3;
yyyy=normpdf(xxx,muhat,sigmahat);
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) normal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 8','FontSize',15);
hold off
legend('Location','northeast')

%Lambda post, sigma post
Lambda33=muu33; Sigma33=sigmahat;

```

## H.5. Poisson-normal.

### a. - Recurrent method.

```

clear
%First incident
%Parameters specification
x0=1; c=0.2; B=500; K=4000; J=B+K; a=0; r=0; mu=0; n=10; object=0.05;

```

```

%Vector of incidences
yi =[0 0 0 1]; sumY=sum(yi); nn=length(yi); S = std(yi,1); MU = mean(yi,2);
%Vector of parameters
yiP=[0 0 0 1/4]; SiP = std(yiP,1); yii = sumY/nn;

for i=1:n;
%Candidate value generation
a(i)=0;
x(1,1)=x0;
for j=2:J
    x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
    if (x(j,1)<0) || (x(j,1)>20)
        p(j,1)=0;
    else p(j,1)=(poisspdf(yi(nn),x(j,1))*normpdf(object,x(j,1),SiP))/...
        (poisspdf(yi(nn),x(j-1,1))*normpdf(object,x(j-1,1),SiP));
    end
    u=random('Uniform',0,1,1,1);
    if u<p(j,1)
        x(j,1)=x(j,1);
        a(i)=a(i)+1;
    else x(j,1)=x(j-1,1);
        r=r+1;
    end
end
%Burn - in
for j=B:J
    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu1(i)=mean(xx); vaa1(i)=var(xx); stdd1(i)=sqrt(vaa1(i));
end

%Acceptance Rate AR
[muhat1,sigmahat1,muci1,sigmaci1] = normfit(muu1); AR1=mean(a)/J; muu11=mean(muu1); vaa11=mean(vaa1);
std11=sqrt(vaa11);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 4','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 4','FontSize',15);
%Histogram
figure
h = histogram(xx);
%Normal fitting
counts = h.Values; [muhat,sigmahat]=normfit(xx);
hold on
xxx=0:0.01:3;
yyyy=normpdf(xxx,muhat,sigmahat);
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) normal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 4','FontSize',15);
hold off
legend('Location','northeast')

%Lambda post, sigma post
Lambda11=muu11; Sigma11=sigmahat;

%Second incident in t=7; t=7-4=3
%Parameters specification

```

```

x0=1; c=0.2; B=500; K=4000; J=B+K; a=0; r=0; mu=0; n=10; object=0.05;
%Vector of incidences
yi =[0 0 0 1 0 0 1]; sumY=sum(yi); nn=length(yi); S = std(yi,1); MU = mean(yi,2);
%Vector of parameters
yiP=[0 0 0 1/4 0 0 1/3]; SiP = std(yiP,1); yii = sumY/nn;

for i=1:n;
%Candidate value generation
a(i)=0;
x(1,1)=x0;
for j=2:J
    x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
    if (x(j,1)<0) || (x(j,1)>20)
        p(j,1)=0;
    else p(j,1)=(poisspdf(yi(nn),x(j,1))*normpdf(mu11,x(j,1),SiP))/...
        (poisspdf(yi(nn),x(j-1,1))*normpdf(mu11,x(j-1,1),SiP));
    end
    u=random('Uniform',0,1,1,1);
    if u<p(j,1)
        x(j,1)=x(j,1);
        a(i)=a(i)+1;
    else x(j,1)=x(j-1,1);
        r=r+1;
    end
end
%Burn - in
for j=B:J
    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu2(i)=mean(xx); vaa2(i)=var(xx); stdd2(i)=sqrt(vaa2(i));
end

%Acceptance Rate AR
[muhat2,sigmahat2,muci2,sigmaci2] = normfit(muu2); AR2=mean(a)/J; muu22=mean(muu2); vaa22=mean(vaa2);
std22=sqrt(vaa22);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 7','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 7','FontSize',15);
%Histogram
figure
h = histogram(xx);
%Normal fitting
counts = h.Values; [muhat,sigmahat]=normfit(xx);
hold on
xxx=0:0.01:2;
yyyy=normpdf(xxx,muhat,sigmahat);
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) normal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 7','FontSize',15);
hold off
legend('Location','northeast')

%Lambda post, sigma post
Lambda22=muu22; Sigma22=sigmahat;

%Third incident in t=8; t=8-7=1

```

```

%Parameters specification
x0=1; c=0.2; B=500; K=4000; J=B+K; a=0; r=0; mu=0; n=10; object=0.05;
%Vector of incidences
yi=[0 0 0 1 0 0 1 1]; sumY=sum(yi); nn=length(yi); S = std(yi,1); MU = mean(yi,2);
%Vector of parameters
yiP=[0 0 0 1/4 0 0 1/3 1/1]; SiP = std(yiP,1); yii = sumY/nn;

for i=1:n;
%Candidate value generation
a(i)=0;
x(1,1)=x0;
for j=2:J
    x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
    if (x(j,1)<0) || (x(j,1)>20)
        p(j,1)=0;
    else p(j,1)=(poisspdf(yi(nn),x(j,1))*normpdf(mu22,x(j,1),SiP))/...
        (poisspdf(yi(nn),x(j-1,1))*normpdf(mu22,x(j-1,1),SiP));
    end
    u=random('Uniform',0,1,1,1);
    if u<p(j,1)
        x(j,1)=x(j,1);
        a(i)=a(i)+1;
    else x(j,1)=x(j-1,1);
        r=r+1;
    end
end
end
%Burn - in
for j=B:J
    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu3(i)=mean(xx); vaa3(i)=var(xx); stdd3(i)=sqrt(vaa3(i));
end

%Acceptance Rate AR
[muhat3,sigmahat3,muci3,sigmaci3] = normfit(muu3); AR3=mean(a)/J; muu33=mean(muu3); vaa33=mean(vaa3);
std33=sqrt(vaa33);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 8','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 8','FontSize',15);
%Histogram
figure
h = histogram(xx);
%Normal fitting
counts = h.Values; [muhat,sigmahat]=normfit(xx);
hold on
xxx=0:0.01:3;
yyyy=normpdf(xxx,muhat,sigmahat);
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) normal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 8','FontSize',15);
hold off
legend('Location','northeast')

%Lambda post, sigma post
Lambda33=muu33; Sigma33=sigmahat;

```

## b.- Direct method.

```

clear
%First incident
%Parameters specification
x0=1; c=0.2; B=500; K=4000; J=B+K; a=0; r=0; mu=0; n=10; object=0.05;
%Vector of incidences
yi =[0 0 0 1]; sumY=sum(yi); nn=length(yi); S = std(yi,1); MU = mean(yi,2);
%Vector of parameters
yiP=[0 0 0 1/4]; SiP = std(yiP,1); yii = sumY/nn;

for i=1:n;
%Candidate value generation
a(i)=0;
x(1,1)=x0;
for j=2:J
    x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
    if (x(j,1)<0) || (x(j,1)>20)
        p(j,1)=0;
    else p(j,1)=(poisspdf(yi(nn),x(j,1))*normpdf(object,x(j,1),SiP))/...
        (poisspdf(yi(nn),x(j-1,1))*normpdf(object,x(j-1,1),SiP));
    end
    u=random('Uniform',0,1,1,1);
    if u<p(j,1)
        x(j,1)=x(j,1);
        a(i)=a(i)+1;
    else x(j,1)=x(j-1,1);
        r=r+1;
    end
end
end
%Burn - in
for j=B:J
    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu1(i)=mean(xx); vaa1(i)=var(xx); stdd1(i)=sqrt(vaa1(i));
end

%Acceptance Rate AR
[muhat1,sigmahat1,muci1,sigmaci1] = normfit(muu1); AR1=mean(a)/J; muu11=mean(muu1); vaa11=mean(vaa1);
std11=sqrt(vaa11);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 4','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 4','FontSize',15);
%Histogram
figure
h = histogram(xx);
%Normal fitting
counts = h.Values; [muhat,sigmahat]=normfit(xx);
hold on
xxx=0:0.01:3;
yyyy=normpdf(xxx,muhat,sigmahat);
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) normal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 4','FontSize',15);
hold off

```



```

legend('Location','northeast')

%Lambda post, sigma post
Lambda11=muu11; Sigma11=sigmahat;

%Second incident in t=7; t=7-4=3
%Parameters specification
x0=1; c=0.2; B=500; K=4000; J=B+K; a=0; r=0; mu=0; n=10; object=0.05;
%Vector of incidencias
yi=[0 0 0 1 0 0 1]; sumY=sum(yi); nn=length(yi); S = std(yi,1); MU = mean(yi,2);
%Vector of parameters
yiP=[0 0 0 1/4 0 0 1/3]; SiP = std(yiP,1); yii = sumY/nn;

for i=1:n;
%Candidate value generation
a(i)=0;
x(1,1)=x0;
for j=2:J
    x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
    if (x(j,1)<0) || (x(j,1)>20)
        p(j,1)=0;
    else p(j,1)=(poisspdf(yi(nn),x(j,1))*normpdf(object,x(j,1),SiP))/...
        (poisspdf(yi(nn),x(j-1,1))*normpdf(object,x(j-1,1),SiP));
    end
    u=random('Uniform',0,1,1,1);
    if u<p(j,1)
        x(j,1)=x(j,1);
        a(i)=a(i)+1;
    else x(j,1)=x(j-1,1);
        r=r+1;
    end
end
end
%Burn - in
for j=B:J
    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu2(i)=mean(xx); vaa2(i)=var(xx); stdd2(i)=sqrt(vaa2(i));
end

%Acceptance Rate AR
[muhat2,sigmahat2,muci2,sigmaci2] = normfit(muu2); AR2=mean(a)/J; muu22=mean(muu2); vaa22=mean(vaa2);
std22=sqrt(vaa22);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 7','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 7','FontSize',15);
%Histogram
figure
h = histogram(xx);
%Normal fitting
counts = h.Values; [muhat,sigmahat]=normfit(xx);
hold on
xxx=0:0.01:2;
yyyy=normpdf(xxx,muhat,sigmahat);
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) normal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 7','FontSize',15);

```

```

hold off
legend('Location','northeast')

%Lambda post, sigma post
Lambda22=muu22; Sigma22=sigmahat;

%Third incident in t=8; t=8-7=1
%Parameters specification
x0=1; c=0.2; B=500; K=4000; J=B+K; a=0; r=0; mu=0; n=10; object=0.05;
%Vector of incidences
yi=[0 0 0 1 0 0 1 1]; sumY=sum(yi); nn=length(yi); S = std(yi,1); MU = mean(yi,2);
%Vector of parameters
yiP=[0 0 0 1/4 0 0 1/3 1/1]; SiP = std(yiP,1); yii = sumY/nn;

for i=1:n;
%Candidate value generation
a(i)=0;
x(1,1)=x0;
for j=2:J
    x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
    if (x(j,1)<0) || (x(j,1)>20)
        p(j,1)=0;
    else p(j,1)=(poisspdf(yi(nn),x(j,1))*normpdf(object,x(j,1),SiP))/...
        (poisspdf(yi(nn),x(j-1,1))*normpdf(object,x(j-1,1),SiP));
    end
    u=random('Uniform',0,1,1,1);
    if u<p(j,1)
        x(j,1)=x(j,1);
        a(i)=a(i)+1;
    else x(j,1)=x(j-1,1);
        r=r+1;
    end
end
%Burn - in
for j=B:J
    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu3(i)=mean(xx); vaa3(i)=var(xx); stdd3(i)=sqrt(vaa3(i));
end

%Acceptance Rate AR
[muhat3,sigmahat3,muci3,sigmaci3] = normfit(muu3); AR3=mean(a)/J; muu33=mean(muu3); vaa33=mean(vaa3);
std33=sqrt(vaa33);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 8','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 8','FontSize',15);
%Histogram
figure
h = histogram(xx);
%Normal fitting
counts = h.Values; [muhat,sigmahat]=normfit(xx);
hold on
xxx=0:0.01:3;
yyyy=normpdf(xxx,muhat,sigmahat);
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) normal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');

```

```
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 8','FontSize',15);
hold off
legend('Location','northeast')

%Lambda post, sigma post
Lambda33=muu33; Sigma33=sigmahat;
```

## H.6. Binomial-beta.

### a.- Recurrent method with mean prior.

```
clear
%Prior time 0
alfa0=3; beta0=12; mean0=alfa0/(alfa0+beta0); sigma0=sqrt(alfa0*beta0/(((alfa0+beta0)^2)*(alfa0+beta0+1)));
CILM0 = mean0-sigma0;
if CILM0<=0
    CILM0=0;
end
MEAN0 = mean0; CIUM0 = mean0+sigma0;
xx0=[mean0 mean0]; lim0 = [CILM0 MEAN0 CIUM0];
figure
controlchart(xx0,'chart',{'i'},'limits',lim0,'specs',[0.1 0.15],'width',1); title('interval t=0')

%Interval 1

n1=5; K1=0; p1=K1/n1; L1=n1-K1; alfa1=alfa0+K1; beta1=beta0+n1-K1; mean1=alfa1/(alfa1+beta1);
var1=alfa1*beta1/(((alfa1+beta1)^2)*(alfa1+beta1+1));
sigma1=sqrt(alfa1*beta1/(((alfa1+beta1)^2)*(alfa1+beta1+1)));
CILM1 = mean0-sigma1;
if CILM1<=0
    CILM1=0;
end
MEAN1 = mean0; CIUM1 = mean0+sigma1;
xx1=[mean0 p1]; lim1 = [CILM1 MEAN1 CIUM1];
figure
controlchart(xx1,'chart',{'i'},'limits',lim1,'specs',[0.1 0.15],'width',1); title('interval t=1')

% Interval 2

n2=5; K2=0; p2=K2/n2; L2=n2-K2; alfa2=alfa1+K2; beta2=beta1+n2-K2; mean2=alfa2/(alfa2+beta2);
sigma2=sqrt(alfa2*beta2/(((alfa2+beta2)^2)*(alfa2+beta2+1)));
CILM2 = mean0-sigma2;
if CILM2<=0
    CILM2=0;
end
MEAN2 = mean0; CIUM2 = mean0+sigma2;
xx2=[p1 p2]; lim2 = [CILM2 MEAN2 CIUM2];
figure
controlchart(xx2,'chart',{'i'},'limits',lim2,'specs',[0.1 0.15],'width',1); title('interval t=2')

% Interval 3

n3=5; K3=0; p3=K3/n3; L3=n3-K3; alfa3=alfa2+K3; beta3=beta2+n3-K3; mean3=alfa3/(alfa3+beta3);
sigma3=sqrt(alfa3*beta3/(((alfa3+beta3)^2)*(alfa3+beta3+1)));
CILM3 = mean0-sigma3;
if CILM3<=0
    CILM3=0;
end
MEAN3 = mean0; CIUM3 = mean0+sigma3;
xx3=[p1 p2 p3]; lim3 = [CILM3 MEAN3 CIUM3];
figure
controlchart(xx3,'chart',{'i'},'limits',lim3,'specs',[0.1 0.15],'width',1); title('interval t=3')

% Interval 4
```

```

n4=5; K4=1; p4=K4/n4; L4=n4-K4; alfa4=alfa3+K4; beta4=beta3+n4-K4; mean4=alfa4/(alfa4+beta4);
sigma4=sqrt(alfa4*beta4/(((alfa4+beta4)^2)*(alfa4+beta4+1)));
CILM4 = mean0-sigma4;
if CILM4<=0
    CILM4=0;
end
MEAN4 = mean0; CIUM4 = mean0+sigma4;
xx4=[p1 p2 p3 p4]; lim4 = [CILM4 MEAN4 CIUM4];
figure
controlchart(xx4,'chart','i','limits',lim4,'specs',[0.1 0.15],'width',1); title('interval t=4')

```

```
% Interval 5
```

```

n5=5; K5=0; p5=K5/n5; L5=n5-K5; alfa5=alfa4+K5; beta5=beta4+n5-K5; mean5=alfa5/(alfa5+beta5);
sigma5=sqrt(alfa5*beta5/(((alfa5+beta5)^2)*(alfa5+beta5+1)));
CILM5 = mean0-sigma5;
if CILM5<=0
    CILM5=0;
end
MEAN5 = mean0; CIUM5 = mean0+sigma5;
xx5=[p1 p2 p3 p4 p5]; lim5 = [CILM5 MEAN5 CIUM5];
figure
controlchart(xx5,'chart','i','limits',lim5,'specs',[0.1 0.15],'width',1); title('interval t=5')

```

```
% Interval 6
```

```

n6=5; K6=0; p6=K6/n6; L6=n6-K6; alfa6=alfa5+K6; beta6=beta5+n6-K6; mean6=alfa6/(alfa6+beta6);
sigma6=sqrt(alfa6*beta6/(((alfa6+beta6)^2)*(alfa6+beta6+1)));
CILM6 = mean0-sigma6;
if CILM6<=0
    CILM6=0;
end
MEAN6 = mean0; CIUM6 = mean0+sigma6;
xx6=[p1 p2 p3 p4 p5 p6]; lim6 = [CILM6 MEAN6 CIUM6];
figure
controlchart(xx6,'chart','i','limits',lim6,'specs',[0.1 0.15],'width',1); title('interval t=6')

```

```
% Interval 7
```

```

n7=5; K7=2; p7=K7/n7; L7=n7-K7; alfa7=alfa6+K7; beta7=beta6+n7-K7; mean7=alfa7/(alfa7+beta7);
sigma7=sqrt(alfa7*beta7/(((alfa7+beta7)^2)*(alfa7+beta7+1)));
CILM7 = mean0-sigma7;
if CILM7<=0
    CILM7=0;
end
MEAN7 = mean0; CIUM7 = mean0+sigma7;
xx7=[p1 p2 p3 p4 p5 p6 p7]; lim7 = [CILM7 MEAN7 CIUM7];
figure
controlchart(xx7,'chart','i','limits',lim7,'specs',[0.1 0.15],'width',1); title('interval t=7')

```

```
% Interval 8
```

```

n8=5; K8=1; p8=K8/n8; L8=n8-K8; alfa8=alfa7+K8; beta8=beta7+n8-K8; mean8=alfa8/(alfa8+beta8);
sigma8=sqrt(alfa8*beta8/(((alfa8+beta8)^2)*(alfa8+beta8+1)));
CILM8 = mean0-sigma8;
if CILM8<=0
    CILM8=0;
end
MEAN8 = mean0; CIUM8 = mean0+sigma8;
xx8=[p1 p2 p3 p4 p5 p6 p7 p8]; lim8 = [CILM8 MEAN8 CIUM8];
figure
controlchart(xx8,'chart','i','limits',lim8,'specs',[0.1 0.15],'width',1); title('interval t=8')

```

```
% Interval 9
```

```
n9=5; K9=0; p9=K9/n9; L9=n9-K9; alfa9=alfa8+K9; beta9=beta8+n9-K9; mean9=alfa9/(alfa9+beta9);
```

```

sigma9=sqrt(alfa9*beta9/(((alfa9+beta9)^2)*(alfa9+beta9+1)));
CILM9 = mean0-sigma9;
if CILM9<=0
    CILM9=0;
end
MEAN9 = mean0; CIUM9 = mean0+sigma9;
xx9=[p1 p2 p3 p4 p5 p6 p7 p8 p9]; lim9 = [CILM9 MEAN9 CIUM9];
figure
controlchart(xx9,'chart',{'i'},'limits',lim9,'specs',[0.1 0.15],'width',1); title('interval t=9')

% Interval 10

n10=5; K10=0; p10=K10/n10; L10=n10-K10; alfa10=alfa9+K10; beta10=beta9+n10-K10;
mean10=alfa10/(alfa10+beta10);
sigma10=sqrt(alfa10*beta10/(((alfa10+beta10)^2)*(alfa10+beta10+1)));

CILM10 = mean0-sigma10;
if CILM10<=0
    CILM10=0;
end
MEAN10 = mean0; CIUM10 = mean0+sigma10;
xx10=[p1 p2 p3 p4 p5 p6 p7 p8 p9 p10]; lim10 = [CILM10 MEAN10 CIUM10];
figure
controlchart(xx10,'chart',{'i'},'limits',lim10,'specs',[0.1 0.15],'width',1); title('interval t=10')

```

### b.- Recurrent method with mean posterior.

```

clear
%Prior time 0

alfa0=3; beta0=12; mean0=alfa0/(alfa0+beta0); sigma0=sqrt(alfa0*beta0/(((alfa0+beta0)^2)*(alfa0+beta0+1)));
CILM0 = mean0-sigma0;
if CILM0<=0
    CILM0=0;
end
MEAN0 = mean0; CIUM0 = mean0+sigma0;
xx0=[mean0 mean0]; lim0 = [CILM0 MEAN0 CIUM0];
figure
controlchart(xx0,'chart',{'i'},'limits',lim0,'specs',[0.1 0.15],'width',1); title('interval t=0')

%Interval 1

n1=5; K1=0; p1=K1/n1; L1=n1-K1; alfa1=alfa0+K1; beta1=beta0+n1-K1; mean1=alfa1/(alfa1+beta1);
var1=alfa1*beta1/(((alfa1+beta1)^2)*(alfa1+beta1+1));
sigma1=sqrt(alfa1*beta1/(((alfa1+beta1)^2)*(alfa1+beta1+1)));
CILM1 = mean1-sigma1;
if CILM1<=0
    CILM1=0;
end
MEAN1 = mean1; CIUM1 = mean1+sigma1;
xx1=[mean0 p1]; lim1 = [CILM1 MEAN1 CIUM1];
figure
controlchart(xx1,'chart',{'i'},'limits',lim1,'specs',[0.1 0.15],'width',1); title('interval t=1')

% Interval 2

n2=5; K2=0; p2=K2/n2; L2=n2-K2; alfa2=alfa1+K2; beta2=beta1+n2-K2; mean2=alfa2/(alfa2+beta2);
sigma2=sqrt(alfa2*beta2/(((alfa2+beta2)^2)*(alfa2+beta2+1)));
CILM2 = mean2-sigma2;
if CILM2<=0
    CILM2=0;
end
MEAN2 = mean2; CIUM2 = mean2+sigma2;
xx2=[p1 p2]; lim2 = [CILM2 MEAN2 CIUM2];
figure
controlchart(xx2,'chart',{'i'},'limits',lim2,'specs',[0.1 0.15],'width',1); title('interval t=2')

```

```
% Interval 3
```

```
n3=5; K3=0; p3=K3/n3; L3=n3-K3; alfa3=alfa2+K3; beta3=beta2+n3-K3; mean3=alfa3/(alfa3+beta3);
sigma3=sqrt(alfa3*beta3/(((alfa3+beta3)^2)*(alfa3+beta3+1)));
CILM3 = mean3-sigma3;
if CILM3<=0
    CILM3=0;
end
MEAN3 = mean3; CIUM3 = mean3+sigma3;
xx3=[p1 p2 p3]; lim3 = [CILM3 MEAN3 CIUM3];
figure
controlchart(xx3,'chart',{'i'},'limits',lim3,'specs',[0.1 0.15],'width',1); title('interval t=3')
```

```
% Interval 4
```

```
n4=5; K4=1; p4=K4/n4; L4=n4-K4; alfa4=alfa3+K4; beta4=beta3+n4-K4; mean4=alfa4/(alfa4+beta4);
sigma4=sqrt(alfa4*beta4/(((alfa4+beta4)^2)*(alfa4+beta4+1)));
CILM4 = mean4-sigma4;
if CILM4<=0
    CILM4=0;
end
MEAN4 = mean4; CIUM4 = mean4+sigma4;
xx4=[p1 p2 p3 p4]; lim4 = [CILM4 MEAN4 CIUM4];
figure
controlchart(xx4,'chart',{'i'},'limits',lim4,'specs',[0.1 0.15],'width',1); title('interval t=4')
```

```
% Interval 5
```

```
n5=5; K5=0; p5=K5/n5; L5=n5-K5; alfa5=alfa4+K5; beta5=beta4+n5-K5; mean5=alfa5/(alfa5+beta5);
sigma5=sqrt(alfa5*beta5/(((alfa5+beta5)^2)*(alfa5+beta5+1)));
CILM5 = mean5-sigma5;
if CILM5<=0
    CILM5=0;
end
MEAN5 = mean5; CIUM5 = mean5+sigma5;
xx5=[p1 p2 p3 p4 p5]; lim5 = [CILM5 MEAN5 CIUM5];
figure
controlchart(xx5,'chart',{'i'},'limits',lim5,'specs',[0.1 0.15],'width',1); title('interval t=5')
```

```
% Interval 6
```

```
n6=5; K6=0; p6=K6/n6; L6=n6-K6; alfa6=alfa5+K6; beta6=beta5+n6-K6; mean6=alfa6/(alfa6+beta6);
sigma6=sqrt(alfa6*beta6/(((alfa6+beta6)^2)*(alfa6+beta6+1)));
CILM6 = mean6-sigma6;
if CILM6<=0
    CILM6=0;
end
MEAN6 = mean6; CIUM6 = mean6+sigma6;
xx6=[p1 p2 p3 p4 p5 p6]; lim6 = [CILM6 MEAN6 CIUM6];
figure
controlchart(xx6,'chart',{'i'},'limits',lim6,'specs',[0.1 0.15],'width',1); title('interval t=6')
```

```
% Interval 7
```

```
n7=5; K7=2; p7=K7/n7; L7=n7-K7; alfa7=alfa6+K7; beta7=beta6+n7-K7; mean7=alfa7/(alfa7+beta7);
sigma7=sqrt(alfa7*beta7/(((alfa7+beta7)^2)*(alfa7+beta7+1)));
CILM7 = mean7-sigma7;
if CILM7<=0
    CILM7=0;
end
MEAN7 = mean7; CIUM7 = mean7+sigma7;
xx7=[p1 p2 p3 p4 p5 p6 p7]; lim7 = [CILM7 MEAN7 CIUM7];
figure
controlchart(xx7,'chart',{'i'},'limits',lim7,'specs',[0.1 0.15],'width',1); title('interval t=7')
```

```
% Interval 8
```

```

n8=5; K8=1; p8=K8/n8; L8=n8-K8; alfa8=alfa7+K8; beta8=beta7+n8-K8; mean8=alfa8/(alfa8+beta8);
sigma8=sqrt(alfa8*beta8/(((alfa8+beta8)^2)*(alfa8+beta8+1)));
CILM8 = mean8-sigma8;
if CILM8<=0
    CILM8=0;
end
MEAN8 = mean8; CIUM8 = mean8+sigma8;
xx8=[p1 p2 p3 p4 p5 p6 p7 p8]; lim8 = [CILM8 MEAN8 CIUM8];
figure
controlchart(xx8,'chart',{i},'limits',lim8,'specs',[0.1 0.15],'width',1); title('interval t=8')

```

% Interval 9

```

n9=5; K9=0; p9=K9/n9; L9=n9-K9; alfa9=alfa8+K9; beta9=beta8+n9-K9; mean9=alfa9/(alfa9+beta9);
sigma9=sqrt(alfa9*beta9/(((alfa9+beta9)^2)*(alfa9+beta9+1)));
CILM9 = mean9-sigma9;
if CILM9<=0
    CILM9=0;
end
MEAN9 = mean9; CIUM9 = mean9+sigma9;
xx9=[p1 p2 p3 p4 p5 p6 p7 p8 p9]; lim9 = [CILM9 MEAN9 CIUM9];
figure
controlchart(xx9,'chart',{i},'limits',lim9,'specs',[0.1 0.15],'width',1); title('interval t=9')

```

% Interval 10

```

n10=5; K10=0; p10=K10/n10; L10=n10-K10; alfa10=alfa9+K10; beta10=beta9+n10-K10;
mean10=alfa10/(alfa10+beta10);
sigma10=sqrt(alfa10*beta10/(((alfa10+beta10)^2)*(alfa10+beta10+1)));
CILM10 = mean10-sigma10;
if CILM10<=0
    CILM10=0;
end
MEAN10 = mean10; CIUM10 = mean10+sigma10;
xx10=[p1 p2 p3 p4 p5 p6 p7 p8 p9 p10]; lim10 = [CILM10 MEAN10 CIUM10];
figure
controlchart(xx10,'chart',{i},'limits',lim10,'specs',[0.1 0.15],'width',1); title('interval t=10')

```

### c.- Direct method with mean prior.

```

clear
%Prior time 0
alfa0=3; beta0=12; mean0=alfa0/(alfa0+beta0); sigma0=sqrt(alfa0*beta0/(((alfa0+beta0)^2)*(alfa0+beta0+1)));
CILM0 = mean0-sigma0;
if CILM0<=0
    CILM0=0;
end
MEAN0 = mean0; CIUM0 = mean0+sigma0;
xx0=[mean0 mean0]; lim0 = [CILM0 MEAN0 CIUM0];
figure
controlchart(xx0,'chart',{i},'limits',lim0,'specs',[0.1 0.15],'width',1); title('interval t=0')

%Interval 1
%Observed
n=5; K=0; p11=K/n; n1=n; K1=K; L1=n1-K1; alfa1=alfa0+K1; beta1=beta0+n1-K1; mean1=alfa1/(alfa1+beta1);
var1=alfa1*beta1/(((alfa1+beta1)^2)*(alfa1+beta1+1));
sigma1=sqrt(alfa1*beta1/(((alfa1+beta1)^2)*(alfa1+beta1+1)));
CILM1 = mean0-sigma1;
if CILM1<=0
    CILM1=0;
end
MEAN1 = mean0; CIUM1 = mean0+sigma1;
xx1=[mean0 p11]; lim1 = [CILM1 MEAN1 CIUM1];
figure
controlchart(xx1,'chart',{i},'limits',lim1,'specs',[0.1 0.15],'width',1); title('interval t=1')

```

```

% Interval 2
%Observed
n=5; K=0; p22=K/n; n2=n+n1; K2=K+K1; L2=n2-K2; alfa2=alfa0+K2; beta2=beta0+n2-K2;
mean2=alfa2/(alfa2+beta2); sigma2=sqrt(alfa2*beta2/(((alfa2+beta2)^2)*(alfa2+beta2+1)));
CILM2 = mean0-sigma2;
if CILM2<=0
    CILM2=0;
end
MEAN2 = mean0; CIUM2 = mean0+sigma2;
xx2=[p11 p22]; lim2 = [CILM2 MEAN2 CIUM2];
figure
controlchart(xx2,'chart',{'i'},'limits',lim2,'specs',[0.1 0.15],'width',1); title('interval t=2')

% Interval 3
%Observed
n=5; K=0; p33=K/n; n3=n+n2; K3=K+K2; L3=n3-K3; alfa3=alfa0+K3; beta3=beta0+n3-K3;
mean3=alfa3/(alfa3+beta3); sigma3=sqrt(alfa3*beta3/(((alfa3+beta3)^2)*(alfa3+beta3+1)));
CILM3 = mean0-sigma3;
if CILM3<=0
    CILM3=0;
end
MEAN3 = mean0; CIUM3 = mean0+sigma3;
xx3=[p11 p22 p33]; lim3 = [CILM3 MEAN3 CIUM3];
figure
controlchart(xx3,'chart',{'i'},'limits',lim3,'specs',[0.1 0.15],'width',1); title('interval t=3')

% Interval 4
%Observed
n=5; K=1; p44=K/n; n4=n+n3; K4=K+K3; L4=n4-K4; alfa4=alfa0+K4; beta4=beta0+n4-K4;
mean4=alfa4/(alfa4+beta4); sigma4=sqrt(alfa4*beta4/(((alfa4+beta4)^2)*(alfa4+beta4+1)));
CILM4 = mean0-sigma4;
if CILM4<=0
    CILM4=0;
end
MEAN4 = mean0; CIUM4 = mean0+sigma4;
xx4=[p11 p22 p33 p44]; lim4 = [CILM4 MEAN4 CIUM4];
figure
controlchart(xx4,'chart',{'i'},'limits',lim4,'specs',[0.1 0.15],'width',1); title('interval t=4')

% Interval 5
%Observed
n=5; K=0; p55=K/n; n5=n+n4; K5=K+K4; L5=n5-K5; alfa5=alfa0+K5; beta5=beta0+n5-K5;
mean5=alfa5/(alfa5+beta5); sigma5=sqrt(alfa5*beta5/(((alfa5+beta5)^2)*(alfa5+beta5+1)));
CILM5 = mean0-sigma5;
if CILM5<=0
    CILM5=0;
end
MEAN5 = mean0; CIUM5 = mean0+sigma5;
xx5=[p11 p22 p33 p44 p55]; lim5 = [CILM5 MEAN5 CIUM5];
figure
controlchart(xx5,'chart',{'i'},'limits',lim5,'specs',[0.1 0.15],'width',1); title('interval t=5')

% Interval 6
%Observed
n=5; K=0; p66=K/n; n6=n+n5; K6=K+K5; L6=n6-K6; alfa6=alfa0+K6; beta6=beta0+n6-K6;
mean6=alfa6/(alfa6+beta6); sigma6=sqrt(alfa6*beta6/(((alfa6+beta6)^2)*(alfa6+beta6+1)));
CILM6 = mean0-sigma6;
if CILM6<=0
    CILM6=0;
end
MEAN6 = mean0; CIUM6 = mean0+sigma6;
xx6=[p11 p22 p33 p44 p55 p66]; lim6 = [CILM6 MEAN6 CIUM6];
figure
controlchart(xx6,'chart',{'i'},'limits',lim6,'specs',[0.1 0.15],'width',1); title('interval t=6')

% Interval 7

```



```
%Observed
n=5; K=2; p77=K/n; n7=n+n6; K7=K+K6; L7=n7-K7; alfa7=alfa0+K7; beta7=beta0+n7-K7;
mean7=alfa7/(alfa7+beta7); sigma7=sqrt(alfa7*beta7/(((alfa7+beta7)^2)*(alfa7+beta7+1)));
CILM7 = mean0-sigma7;
if CILM7<=0
    CILM7=0;
end
MEAN7 = mean0; CIUM7 = mean0+sigma7;
xx7=[p11 p22 p33 p44 p55 p66 p77]; lim7 = [CILM7 MEAN7 CIUM7];
figure
controlchart(xx7,'chart',{'i'},'limits',lim7,'specs',[0.1 0.15],'width',1); title('interval t=7')
```

```
% Interval 8
%Observed
n=5; K=1; p88=K/n; n8=n+n7; K8=K+K7; L8=n8-K8; alfa8=alfa0+K8; beta8=beta0+n8-K8;
mean8=alfa8/(alfa8+beta8); sigma8=sqrt(alfa8*beta8/(((alfa8+beta8)^2)*(alfa8+beta8+1)));
CILM8 = mean0-sigma8;
if CILM8<=0
    CILM8=0;
end
MEAN8 = mean0; CIUM8 = mean0+sigma8;
xx8=[p11 p22 p33 p44 p55 p66 p77 p88]; lim8 = [CILM8 MEAN8 CIUM8];
figure
controlchart(xx8,'chart',{'i'},'limits',lim8,'specs',[0.1 0.15],'width',1); title('interval t=8')
```

```
% Interval 9
%Observed
n=5; K=0; p99=K/n; n9=n+n8; K9=K+K8; L9=n9-K9; alfa9=alfa0+K9; beta9=beta0+n9-K9;
mean9=alfa9/(alfa9+beta9); sigma9=sqrt(alfa9*beta9/(((alfa9+beta9)^2)*(alfa9+beta9+1)));
CILM9 = mean0-sigma9;
if CILM9<=0
    CILM9=0;
end
MEAN9 = mean0; CIUM9 = mean0+sigma9;
xx9=[p11 p22 p33 p44 p55 p66 p77 p88 p99]; lim9 = [CILM9 MEAN9 CIUM9];
figure
controlchart(xx9,'chart',{'i'},'limits',lim9,'specs',[0.1 0.15],'width',1); title('interval t=9')
```

```
% Interval 10
%Observed
n=5; K=0; p1010=K/n; n10=n+n9; K10=K+K9; L10=n10-K10; alfa10=alfa0+K10; beta10=beta0+n10-K10;
mean10=alfa10/(alfa10+beta10); sigma10=sqrt(alfa10*beta10/(((alfa10+beta10)^2)*(alfa10+beta10+1)));
CILM10 = mean0-sigma10;
if CILM10<=0
    CILM10=0;
end
MEAN10 = mean0; CIUM10 = mean0+sigma10;
xx10=[p11 p22 p33 p44 p55 p66 p77 p88 p99 p1010]; lim10 = [CILM10 MEAN10 CIUM10];
figure
controlchart(xx10,'chart',{'i'},'limits',lim10,'specs',[0.1 0.15],'width',1); title('interval t=10')
```

### c.- Direct method with mean posterior.

```
clear
%Prior time 0
alfa0=3; beta0=12; mean0=alfa0/(alfa0+beta0); sigma0=sqrt(alfa0*beta0/(((alfa0+beta0)^2)*(alfa0+beta0+1)));
CILM0 = mean0-sigma0;
if CILM0<=0
    CILM0=0;
end
MEAN0 = mean0; CIUM0 = mean0+sigma0;
xx0=[mean0 mean0]; lim0 = [CILM0 MEAN0 CIUM0];
figure
controlchart(xx0,'chart',{'i'},'limits',lim0,'specs',[0.1 0.15],'width',1); title('interval t=0')
```

```
%Interval 1
```

```

%Observed
n=5; K=0; p11=K/n; n1=n; K1=K; L1=n1-K1; alfa1=alfa0+K1; beta1=beta0+n1-K1; mean1=alfa1/(alfa1+beta1);
var1=alfa1*beta1/(((alfa1+beta1)^2)*(alfa1+beta1+1));
sigma1=sqrt(alfa1*beta1/(((alfa1+beta1)^2)*(alfa1+beta1+1)));
CILM1 = mean1-sigma1;
if CILM1<=0
    CILM1=0;
end
MEAN1 = mean1; CIUM1 = mean1+sigma1;
xx1=[mean0 p11]; lim1 = [CILM1 MEAN1 CIUM1];
figure
controlchart(xx1,'chart',{'i'},'limits',lim1,'specs',[0.1 0.15],'width',1); title('interval t=1')

```

```

% Interval 2
%Observed
n=5; K=0; p22=K/n; n2=n+n1; K2=K+K1; L2=n2-K2; alfa2=alfa0+K2; beta2=beta0+n2-K2;
mean2=alfa2/(alfa2+beta2); sigma2=sqrt(alfa2*beta2/(((alfa2+beta2)^2)*(alfa2+beta2+1)));
CILM2 = mean2-sigma2;
if CILM2<=0
    CILM2=0;
end
MEAN2 = mean2; CIUM2 = mean2+sigma2;
xx2=[p11 p22]; lim2 = [CILM2 MEAN2 CIUM2];
figure
controlchart(xx2,'chart',{'i'},'limits',lim2,'specs',[0.1 0.15],'width',1); title('interval t=2')

```

```

% Interval 3
%Observed
n=5; K=0; p33=K/n; n3=n+n2; K3=K+K2; L3=n3-K3; alfa3=alfa0+K3; beta3=beta0+n3-K3;
mean3=alfa3/(alfa3+beta3); sigma3=sqrt(alfa3*beta3/(((alfa3+beta3)^2)*(alfa3+beta3+1)));
CILM3 = mean3-sigma3;
if CILM3<=0
    CILM3=0;
end
MEAN3 = mean3; CIUM3 = mean3+sigma3;
xx3=[p11 p22 p33]; lim3 = [CILM3 MEAN3 CIUM3];
figure
controlchart(xx3,'chart',{'i'},'limits',lim3,'specs',[0.1 0.15],'width',1); title('interval t=3')

```

```

% Interval 4
%Observed
n=5; K=1; p44=K/n; n4=n+n3; K4=K+K3; L4=n4-K4; alfa4=alfa0+K4; beta4=beta0+n4-K4;
mean4=alfa4/(alfa4+beta4); sigma4=sqrt(alfa4*beta4/(((alfa4+beta4)^2)*(alfa4+beta4+1)));
CILM4 = mean4-sigma4;
if CILM4<=0
    CILM4=0;
end
MEAN4 = mean4; CIUM4 = mean4+sigma4;
xx4=[p11 p22 p33 p44]; lim4 = [CILM4 MEAN4 CIUM4];
figure
controlchart(xx4,'chart',{'i'},'limits',lim4,'specs',[0.1 0.15],'width',1); title('interval t=4')

```

```

% Interval 5
%Observed
n=5; K=0; p55=K/n; n5=n+n4; K5=K+K4; L5=n5-K5; alfa5=alfa0+K5; beta5=beta0+n5-K5;
mean5=alfa5/(alfa5+beta5); sigma5=sqrt(alfa5*beta5/(((alfa5+beta5)^2)*(alfa5+beta5+1)));
CILM5 = mean5-sigma5;
if CILM5<=0
    CILM5=0;
end
MEAN5 = mean5; CIUM5 = mean5+sigma5;
xx5=[p11 p22 p33 p44 p55]; lim5 = [CILM5 MEAN5 CIUM5];
figure
controlchart(xx5,'chart',{'i'},'limits',lim5,'specs',[0.1 0.15],'width',1); title('interval t=5')

```

```

% Interval 6
%Observed

```

```

n=5; K=0; p66=K/n; n6=n+n5; K6=K+K5; L6=n6-K6; alfa6=alfa0+K6; beta6=beta0+n6-K6;
mean6=alfa6/(alfa6+beta6); sigma6=sqrt(alfa6*beta6/(((alfa6+beta6)^2)*(alfa6+beta6+1)));
CILM6 = mean6-sigma6;
if CILM6<=0
    CILM6=0;
end
MEAN6 = mean6; CIUM6 = mean6+sigma6;
xx6=[p11 p22 p33 p44 p55 p66]; lim6 = [CILM6 MEAN6 CIUM6];
figure
controlchart(xx6,'chart',{'i'},'limits',lim6,'specs',[0.1 0.15],'width',1); title('interval t=6')

% Interval 7
%Observed
n=5; K=2; p77=K/n; n7=n+n6; K7=K+K6; L7=n7-K7; alfa7=alfa0+K7; beta7=beta0+n7-K7;
mean7=alfa7/(alfa7+beta7); sigma7=sqrt(alfa7*beta7/(((alfa7+beta7)^2)*(alfa7+beta7+1)));
CILM7 = mean7-sigma7;
if CILM7<=0
    CILM7=0;
end
MEAN7 = mean7; CIUM7 = mean7+sigma7;
xx7=[p11 p22 p33 p44 p55 p66 p77]; lim7 = [CILM7 MEAN7 CIUM7];
figure
controlchart(xx7,'chart',{'i'},'limits',lim7,'specs',[0.1 0.15],'width',1); title('interval t=7')

% Interval 8
%Observed
n=5; K=1; p88=K/n; n8=n+n7; K8=K+K7; L8=n8-K8; alfa8=alfa0+K8; beta8=beta0+n8-K8;
mean8=alfa8/(alfa8+beta8); sigma8=sqrt(alfa8*beta8/(((alfa8+beta8)^2)*(alfa8+beta8+1)));
CILM8 = mean8-sigma8;
if CILM8<=0
    CILM8=0;
end
MEAN8 = mean8; CIUM8 = mean8+sigma8;
xx8=[p11 p22 p33 p44 p55 p66 p77 p88]; lim8 = [CILM8 MEAN8 CIUM8];
figure
controlchart(xx8,'chart',{'i'},'limits',lim8,'specs',[0.1 0.15],'width',1); title('interval t=8')

% Interval 9
%Observed
n=5; K=0; p99=K/n; n9=n+n8; K9=K+K8; L9=n9-K9; alfa9=alfa0+K9; beta9=beta0+n9-K9;
mean9=alfa9/(alfa9+beta9); sigma9=sqrt(alfa9*beta9/(((alfa9+beta9)^2)*(alfa9+beta9+1)));
CILM9 = mean9-sigma9;
if CILM9<=0
    CILM9=0;
end
MEAN9 = mean9; CIUM9 = mean9+sigma9;
xx9=[p11 p22 p33 p44 p55 p66 p77 p88 p99]; lim9 = [CILM9 MEAN9 CIUM9];
figure
controlchart(xx9,'chart',{'i'},'limits',lim9,'specs',[0.1 0.15],'width',1); title('interval t=9')

% Interval 10
%Observed
n=5; K=0; p1010=K/n; n10=n+n9; K10=K+K9; L10=n10-K10; alfa10=alfa0+K10; beta10=beta0+n10-K10;
mean10=alfa10/(alfa10+beta10); sigma10=sqrt(alfa10*beta10/(((alfa10+beta10)^2)*(alfa10+beta10+1)));
CILM10 = mean10-sigma10;
if CILM10<=0
    CILM10=0;
end
MEAN10 = mean10; CIUM10 = mean10+sigma10;
xx10=[p11 p22 p33 p44 p55 p66 p77 p88 p99 p1010]; lim10 = [CILM10 MEAN10 CIUM10];
figure
controlchart(xx10,'chart',{'i'},'limits',lim10,'specs',[0.1 0.15],'width',1); title('interval t=10')

```

## H.7. Hidden Markov chain.

### a.- Recurrent method

```

clear
TRANS=[0.8 0.15 0.03 0.02;0.5 0.4 0.05 0.05;0.2 0.3 0.4 0.1;0.15 0.4 0.4 0.05];
EMIS=[0.7 0.2 0.1;0.4 0.4 0.2;0.3 0.3 0.4;0.2 0.2 0.6];

% A random sequence and a matrix of states
[seqsim0,statessim0] = hmmgenerate(10000,TRANS,EMIS);
figure
%Evolution of the initial defined sequence of end states
histogram(seqsim0);
xlabel('Observed safety barriers','FontSize',15,'FontWeight','bold');
title('Evolution of defined emission matrix','FontSize',15);
figure
%Evolution of initial defined transitions for the three safety barriers
histogram(statessim0);
xlabel('End states','FontSize',15,'FontWeight','bold');
title('Evolution of defined transition matrix','FontSize',15);

% Observed sequence
seqobs = [1 1 1 1 1 1 2 3 1 1];

%Estimated Transition and Emission Matrices using a guess using Baum-Welch algorithm
TRANS_GUESS = TRANS;
EMIS_GUESS = EMIS;
[TRANS_ESTobsBW, EMIS_ESTobsBW] = hmmtrain(seqobs,TRANS_GUESS, EMIS_GUESS);

% A random sequence and a matrix of states
[seqsim1,statessim1] = hmmgenerate(10000,TRANS_ESTobsBW,EMIS_ESTobsBW);
figure
%Evolution of the initial defined sequence of end states
histogram(seqsim1);
xlabel('Observed safety barriers','FontSize',15,'FontWeight','bold');
title('Evolution of defined emission matrix','FontSize',15);
figure
%Evolution of initial defined transitions for the three safety barriers
histogram(statessim1);
xlabel('End states','FontSize',15,'FontWeight','bold');
title('Evolution of defined transition matrix','FontSize',15);

%Relative limiting steady states
r=35;
pi=TRANS^r;
pestobsBW=TRANS_ESTobsBW^r;

%Occupancy times
ss=length(seqobs);
%MestBW=TRANS_ESTobsBW^0;
MestBW = zeros(length(TRANS_ESTobsBW(1,:)));
for i=1:1:ss
    E=TRANS_ESTobsBW^i;
    MestBW=MestBW+E;
end

%First passage for the fourth transition state
e=[1;1;1];
TRANSFPobs1BW=[TRANS_ESTobsBW(1,:);TRANS_ESTobsBW(2,:);TRANS_ESTobsBW(3,:)];
TRANSFPobsBW=[TRANSFPobs1BW(:,1) TRANSFPobs1BW(:,2) TRANSFPobs1BW(:,3)];
fun3 = @(z)e+TRANSFPobsBW*z-z;
options = optimoptions('fsolve','Display','off');
z0 = [1;1;1];
%Call fsolve and obtain information about the solution process.
[z,fval,exitflag,output] = fsolve(fun3,z0,options)

```

```

%Second observation
seqobs2 = [1 1 1 1 1 2 3 1 1 2];

%Estimated Transition and Emission Matrices using a guess using Baum-Welch algorithm
TRANS_GUESS2 = TRANS_ESTobsBW;
EMIS_GUESS2 = EMIS_ESTobsBW;
[TRANS_ESTobsBW2, EMIS_ESTobsBW2] = hmmtrain(seqobs2,TRANS_GUESS2, EMIS_GUESS2);

% A random sequence and a matrix of states
[seqsim2,statessim2] = hmmgenerate(10000,TRANS_ESTobsBW2, EMIS_ESTobsBW2);
figure
%Evolution of the initial defined sequence of end states
histogram(seqsim2);
xlabel('Observed safety barriers','FontSize',15,'FontWeight','bold');
title('Evolution of defined emission matrix','FontSize',15);
figure
%Evolution of initial defined transitions for the three safety barriers
histogram(statessim2);
xlabel('End states','FontSize',15,'FontWeight','bold');
title('Evolution of defined transition matrix','FontSize',15);

%Relative limiting steady states
pestobsBW2=TRANS_ESTobsBW2^r;

%Occupancy times
sss=length(seqobs2);
%MestBW2=TRANS_ESTobsBW2^0;
MestBW2 = zeros(length(TRANS_ESTobsBW2(1,:)));
for i=1:1:sss
    E2=TRANS_ESTobsBW2^i;
    MestBW2=MestBW2+E2;
end

%First passage for the fourth transition state
e=[1;1;1];
TRANSFPobs1BW2=[TRANS_ESTobsBW2(1,:);TRANS_ESTobsBW2(2,:);TRANS_ESTobsBW2(3,:)];
TRANSFPobsBW2=[TRANSFPobs1BW2(:,1) TRANSFPobs1BW2(:,2) TRANSFPobs1BW2(:,3)];
fun4 = @(y)e+TRANSFPobsBW2*y-y;
options = optimoptions('fsolve','Display','off');
y0 = [1;1;1];
%Call fsolve and obtain information about the solution process.
[y,fval,exitflag,output] = fsolve(fun4,y0,options)

```

## b.- Direct method

```

clear
TRANS=[0.8 0.15 0.03 0.02;0.5 0.4 0.05 0.05;0.2 0.3 0.4 0.1;0.15 0.4 0.4 0.05];
EMIS=[0.7 0.2 0.1;0.4 0.4 0.2;0.3 0.3 0.4;0.2 0.2 0.6];

% A random sequence and a matrix of states
[seqsim0,statessim0] = hmmgenerate(10000,TRANS,EMIS);
figure
%Evolution of the initial defined sequence of end states
histogram(seqsim0);
xlabel('Observed safety barriers','FontSize',15,'FontWeight','bold');
title('Evolution of defined emission matrix','FontSize',15);
figure
%Evolution of initial defined transitions for the three safety barriers
histogram(statessim0);
xlabel('End states','FontSize',15,'FontWeight','bold');
title('Evolution of defined transition matrix','FontSize',15);

%Observed sequence
seqobs = [1 1 1 1 1 2 3 1 1];

```

```

%Estimation of Transition and Emission Matrices using a guess using Baum-Welch algorithm
TRANS_GUESS = TRANS;
EMIS_GUESS = EMIS;
[TRANS_ESTobsBW, EMIS_ESTobsBW] = hmmtrain(seqobs,TRANS_GUESS, EMIS_GUESS);

% A random sequence and a matrix of states
[seqsim1,statessim1] = hmmgenerate(10000,TRANS_ESTobsBW,EMIS_ESTobsBW);
figure
%Evolution of the initial defined sequence of end states
histogram(seqsim1);
xlabel('Observed safety barriers','FontSize',15,'FontWeight','bold');
title('Evolution of defined emission matrix','FontSize',15);
figure
%Evolution of initial defined transitions for the three safety barriers
histogram(statessim1);
xlabel('End states','FontSize',15,'FontWeight','bold');
title('Evolution of defined transition matrix','FontSize',15);

%Relative limiting steady states
r=35;
pi=TRANS^r;
pestobsBW=TRANS_ESTobsBW^r;

%Occupancy times
ss=length(seqobs);
%MestBW=TRANS_ESTobsBW^0;
MestBW = zeros(length(TRANS_ESTobsBW(1,:)));
for i=1:1:ss
    E=TRANS_ESTobsBW^i;
    MestBW=MestBW+E;
end

%First passage for the fourth transition state
e=[1;1;1];
TRANSFPobs1BW=[TRANS_ESTobsBW(1,:);TRANS_ESTobsBW(2,:);TRANS_ESTobsBW(3,:)];
TRANSFPobsBW=[TRANSFPobs1BW(:,1) TRANSFPobs1BW(:,2) TRANSFPobs1BW(:,3)];
fun3 = @(z)e+TRANSFPobsBW*z-z;
options = optimoptions('fsolve','Display','off');
z0 = [1;1;1];
%Call fsolve and obtain information about the solution process.
[z,fval,exitflag,output] = fsolve(fun3,z0,options)

%Second observation
seqobs2 = [1 1 1 1 1 2 3 1 1 2];

%Estimation of Transition and Emission Matrices using a guess using Baum-Welch algorithm
TRANS_GUESS2 = TRANS;
EMIS_GUESS2 = EMIS;
[TRANS_ESTobsBW2, EMIS_ESTobsBW2] = hmmtrain(seqobs2,TRANS_GUESS2, EMIS_GUESS2);

% A random sequence and a matrix of states
[seqsim2,statessim2] = hmmgenerate(10000,TRANS_ESTobsBW2, EMIS_ESTobsBW2);
figure
%Evolution of the initial defined sequence of end states
histogram(seqsim2);
xlabel('Observed safety barriers','FontSize',15,'FontWeight','bold');
title('Evolution of defined emission matrix','FontSize',15);
figure
%Evolution of initial defined transitions for the three safety barriers
histogram(statessim2);
xlabel('End states','FontSize',15,'FontWeight','bold');
title('Evolution of defined transition matrix','FontSize',15);

%Relative limiting 2 steady states
pestobsBW2=TRANS_ESTobsBW2^r;

%Occupancy times in the next interval from observed intervals Baum-Welch

```

```

sss=length(seqobs2);
%MestBW2=TRANS_ESTobsBW2^0;
MestBW2 = zeros(length(TRANS_ESTobsBW2(1,:)));
for i=1:1:sss
    E2=TRANS_ESTobsBW2^i;
    MestBW2=MestBW2+E2;
end

%First passage for the fourth transition state
e=[1;1;1];
TRANSFPobs1BW2=[TRANS_ESTobsBW2(1,:);TRANS_ESTobsBW2(2,:);TRANS_ESTobsBW2(3,:)];
TRANSFPobsBW2=[TRANSFPobs1BW2(:,1) TRANSFPobs1BW2(:,2) TRANSFPobs1BW2(:,3)];
fun4 = @(y)e+TRANSFPobsBW2*y-y;
options = optimoptions('fsolve','Display','off');
y0 = [1;1;1];
%Call fsolve and obtain information about the solution process.
[y,fval,exitflag,output] = fsolve(fun4,y0,options)

```

## H.8. Normal-normal.

### a.- Recurrent method with mean prior.

```

clear
%% Interval 1
%Prior
r = [68 74 82 110 115 93 86]; [muhat,sigmahat] = normfit(r); mu0=muhat; sigma0=sigmahat;

%Data vector
yi=[69]; n1=length(yi); ymean1=mean(yi); ysigma1=sqrt(var(yi,1));
% Posterior
k1=n1/(n1+(ysigma1^2/sigma0^2)); mupost1=(1-k1)*mu0+(k1*ymean1); sigmapost1=sqrt(k1*ysigma1^2/n1);

% Límits
CILM1 =(muhat-3*sigmapost1); MEAN1 = muhat; CIUM1 =(muhat+3*sigmapost1);
xx1=[yi]; lim1 = [CILM1 MEAN1 CIUM1];
%figure
%controlchart(xx1,'chart',{i},'limits',lim1,'specs',[90 107],'width',1); title('interval t=1')

%% Interval 2
%Data vector
yi=[69 73]; n2=length(yi); ymean2=mean(yi); ysigma2=sqrt(var(yi,1));
% Prior recurrent
mu0=mupost1; sigma0=sigmahat;
% Posterior
k2=n2/(n2+(ysigma2^2/sigma0^2)); mupost2=(1-k2)*mu0+(k2*ymean2); sigmapost2=sqrt(k2*ysigma2^2/n2);

% Límits
CILM2 =(muhat-3*sigmapost2); MEAN2 = muhat; CIUM2 =(muhat+3*sigmapost2);
xx2=[yi]; lim2 = [CILM2 MEAN2 CIUM2];
figure
controlchart(xx2,'chart',{i},'limits',lim2,'specs',[90 90],'width',1); title('interval t=2')

%% Interval 3
%Data vector
yi=[69 73 85]; n3=length(yi); ymean3=mean(yi); ysigma3=sqrt(var(yi,1));
% Prior recurrent
mu0=mupost2; sigma0=sigmapost2;
% Posterior
k3=n3/(n3+(ysigma3^2/sigma0^2)); mupost3=(1-k3)*mu0+(k3*ymean3); sigmapost3=sqrt(k3*ysigma3^2/n3);

% Límits
CILM3 =(muhat-3*sigmapost3); MEAN3 = muhat; CIUM3 =(muhat+3*sigmapost3);
xx3=[yi]; lim3 = [CILM3 MEAN3 CIUM3];
figure

```

```

controlchart(xx3,'chart',{'i'},'limits',lim3,'specs',[90 90],'width',1);
title('interval t=3')

%% Interval 4
%Data vector
yi=[69 73 85 115]; n4=length(yi); ymean4=mean(yi); ysigma4=sqrt(var(yi,1));
% Prior recurrent
mu0=mupost3; sigma0=sigmapost3;
% Posterior
k4=n4/(n4+(ysigma4^2/sigma0^2)); mupost4=(1-k4)*mu0+(k4*ymean4); sigmapost4=sqrt(k4*ysigma4^2/n4);

% Limits
CILM4 =(muhat-3*sigmapost4); MEAN4 = muhat; CIUM4 =(muhat+3*sigmapost4);
xx4=[yi]; lim4 = [CILM4 MEAN4 CIUM4];
figure
controlchart(xx4,'chart',{'i'},'limits',lim4,'specs',[90 90],'width',1); title('interval t=4')

%% Interval 5
%Data vector
yi=[69 73 85 115 125]; n5=length(yi); ymean5=mean(yi); ysigma5=sqrt(var(yi,1));
% Prior recurrent
mu0=mupost4; sigma0=sigmapost4;
% Posterior
k5=n5/(n5+(ysigma5^2/sigma0^2)); mupost5=(1-k5)*mu0+(k5*ymean5); sigmapost5=sqrt(k5*ysigma5^2/n5);

% Limits
CILM5 =(muhat-3*sigmapost5); MEAN5 = muhat; CIUM5 =(muhat+3*sigmapost5);
xx5=[yi]; lim5 = [CILM5 MEAN5 CIUM5];
figure
controlchart(xx5,'chart',{'i'},'limits',lim5,'specs',[90 90],'width',1); title('interval t=5')

%% Interval 6
%Data vector
yi=[69 73 85 115 125 90]; n6=length(yi); ymean6=mean(yi); ysigma6=sqrt(var(yi,1));
% Prior recurrent
mu0=mupost5; sigma0=sigmapost5;
% Posterior
k6=n6/(n6+(ysigma6^2/sigma0^2)); mupost6=(1-k6)*mu0+(k6*ymean6); sigmapost6=sqrt(k6*ysigma6^2/n6);

% Limits
CILM6 =(muhat-3*sigmapost6); MEAN6 = muhat; CIUM6 =(muhat+3*sigmapost6);
xx6=[yi]; lim6 = [CILM6 MEAN6 CIUM6];
figure
controlchart(xx6,'chart',{'i'},'limits',lim6,'specs',[90 90],'width',1); title('interval t=6')

%% Interval 7
%Data vector
yi=[69 73 85 115 125 90 79]; n7=length(yi); ymean7=mean(yi); ysigma7=sqrt(var(yi,1));
% Prior recurrent
mu0=mupost6; sigma0=sigmapost6;
% Posterior
k7=n7/(n7+(ysigma7^2/sigma0^2)); mupost7=(1-k7)*mu0+(k7*ymean7); sigmapost7=sqrt(k7*ysigma7^2/n7);

% Limits
CILM7 =(muhat-3*sigmapost7); MEAN7 = muhat; CIUM7 =(muhat+3*sigmapost7);
xx7=[yi]; lim7 = [CILM7 MEAN7 CIUM7];
figure
controlchart(xx7,'chart',{'i'},'limits',lim7,'specs',[90 90],'width',1); title('interval t=7')

```

### b.- Recurrent method with mean posterior.

```

clear
%% Interval 1
%Prior
r = [68 74 82 110 115 93 86]; [muhat,sigmahat] = normfit(r); mu0=muhat; sigma0=sigmahat;
%Data vector

```



```

yi=[69]; n1=length(yi); ymean1=mean(yi); ysigma1=sqrt(var(ymean1,1));
% Posterior
k1=n1/(n1+(ysigma1^2/sigma0^2)); mupost1=(1-k1)*mu0+(k1*ymean1); sigmapost1=sqrt(k1*ysigma1^2/n1);

% Limits
CILM1 =(mupost1-3*sigmapost1); MEAN1 = mupost1; CIUM1 =(mupost1+3*sigmapost1);
xx1=[yi]; lim1 = [CILM1 MEAN1 CIUM1];
%figure
%controlchart(xx1,'chart',{'i'},'limits',lim1,'specs',[90 107],'width',1); title('interval t=1')

%% Interval 2
%Data vector
yi=[69 73]; n2=length(yi); ymean2=mean(yi); ysigma2=sqrt(var(yi,1));
% Prior recurrent
mu0=mupost1; sigma0=sigmahat;
% Posterior
k2=n2/(n2+(ysigma2^2/sigma0^2)); mupost2=(1-k2)*mu0+(k2*ymean2); sigmapost2=sqrt(k2*ysigma2^2/n2);

% Limits
CILM2 =(mupost2-3*sigmapost2); MEAN2 = mupost2; CIUM2 =(mupost2+3*sigmapost2);
xx2=[yi]; lim2 = [CILM2 MEAN2 CIUM2];
figure
controlchart(xx2,'chart',{'i'},'limits',lim2,'specs',[90 90],'width',1); title('interval t=2')

%% Interval 3
%Data vector
yi=[69 73 85]; n3=length(yi); ymean3=mean(yi); ysigma3=sqrt(var(yi,1));
% Prior recurrent
mu0=mupost2; sigma0=sigmapost2;
% Posterior
k3=n3/(n3+(ysigma3^2/sigma0^2)); mupost3=(1-k3)*mu0+(k3*ymean3); sigmapost3=sqrt(k3*ysigma3^2/n3);

% Limits
CILM3 =(mupost3-3*sigmapost3); MEAN3 = mupost3; CIUM3 =(mupost3+3*sigmapost3);
xx3=[yi]; lim3 = [CILM3 MEAN3 CIUM3];
figure
controlchart(xx3,'chart',{'i'},'limits',lim3,'specs',[90 90],'width',1); title('interval t=3')

%% Interval 4
%Data vector
yi=[69 73 85 115]; n4=length(yi); ymean4=mean(yi); ysigma4=sqrt(var(yi,1));
% Prior recurrent
mu0=mupost3; sigma0=sigmapost3;
% Posterior
k4=n4/(n4+(ysigma4^2/sigma0^2)); mupost4=(1-k4)*mu0+(k4*ymean4); sigmapost4=sqrt(k4*ysigma4^2/n4);

% Limits
CILM4 =(mupost4-3*sigmapost4); MEAN4 = mupost4; CIUM4 =(mupost4+3*sigmapost4);
xx4=[yi]; lim4 = [CILM4 MEAN4 CIUM4];
figure
controlchart(xx4,'chart',{'i'},'limits',lim4,'specs',[90 90],'width',1); title('interval t=4')

%% Interval 5
%Data vector
yi=[69 73 85 115 125]; n5=length(yi); ymean5=mean(yi); ysigma5=sqrt(var(yi,1));
% Prior recurrent
mu0=mupost4; sigma0=sigmapost4;
% Posterior
k5=n5/(n5+(ysigma5^2/sigma0^2)); mupost5=(1-k5)*mu0+(k5*ymean5); sigmapost5=sqrt(k5*ysigma5^2/n5);

% Limits
CILM5 =(mupost5-3*sigmapost5); MEAN5 = mupost5; CIUM5 =(mupost5+3*sigmapost5);
xx5=[yi]; lim5 = [CILM5 MEAN5 CIUM5];
figure
controlchart(xx5,'chart',{'i'},'limits',lim5,'specs',[90 90],'width',1); title('interval t=5')

%% Interval 6

```

```

% Data vector
yi=[69 73 85 115 125 90]; n6=length(yi); ymean6=mean(yi); ysigma6=sqrt(var(yi,1));
% Prior recurrent
mu0=mupost5; sigma0=sigmapost5;
% Posterior
k6=n6/(n6+(ysigma6^2/sigma0^2)); mupost6=(1-k6)*mu0+(k6*ymean6); sigmapost6=sqrt(k6*ysigma6^2/n6);

% Limits
CILM6 =(mupost6-3*sigmapost6); MEAN6 = mupost6; CIUM6 =(mupost6+3*sigmapost6);
xx6=[yi]; lim6 = [CILM6 MEAN6 CIUM6];
figure
controlchart(xx6,'chart','i','limits',lim6,'specs',[90 90],'width',1); title('interval t=6')

%% Interval 7
% Data vector
yi=[69 73 85 115 125 90 79]; n7=length(yi); ymean7=mean(yi); ysigma7=sqrt(var(yi,1));
% Prior recurrent
mu0=mupost6; sigma0=sigmapost6;
% Posterior
k7=n7/(n7+(ysigma7^2/sigma0^2)); mupost7=(1-k7)*mu0+(k7*ymean7); sigmapost7=sqrt(k7*ysigma7^2/n7);

% Limits
CILM7 =(mupost7-3*sigmapost7); MEAN7 = mupost7; CIUM7 =(mupost7+3*sigmapost7);
xx7=[yi]; lim7 = [CILM7 MEAN7 CIUM7];
figure
controlchart(xx7,'chart','i','limits',lim7,'specs',[90 90],'width',1); title('interval t=7')

```

### c.- Direct method with mean prior.

```

clear
%% Interval 1
% Prior
r = [68 74 82 110 115 93 86]; [muhat,sigmahat] = normfit(r); mu0=muhat; sigma0=sigmahat;
% Data vector
yi=[69]; n1=length(yi); ymean1=mean(yi); ysigma1=sqrt(var(yi,1));
% Posterior
k1=n1/(n1+(ysigma1^2/sigma0^2)); mupost1=(1-k1)*mu0+(k1*ymean1); sigmapost1=sqrt(k1*ysigma1^2/n1);

% Limits
CILM1 =(mu0-3*sigmapost1); MEAN1 = mu0; CIUM1 =(mu0+3*sigmapost1);
xx1=[yi]; lim1 = [CILM1 MEAN1 CIUM1];
% figure
% controlchart(xx1,'chart','i','limits',lim1,'specs',[90 107],'width',1); title('interval t=1')

%% Interval 2
% Data vector
yi=[69 73]; n2=length(yi); ymean2=mean(yi); ysigma2=sqrt(var(yi,1));
% Posterior
k2=n2/(n2+(ysigma2^2/sigma0^2)); mupost2=(1-k2)*mu0+(k2*ymean2); sigmapost2=sqrt(k2*ysigma2^2/n2);

% Limits
CILM2 =(mu0-3*sigmapost2); MEAN2 = mu0; CIUM2 =(mu0+3*sigmapost2);
xx2=[yi]; lim2 = [CILM2 MEAN2 CIUM2];
figure
controlchart(xx2,'chart','i','limits',lim2,'specs',[90 90],'width',1); title('interval t=2')

%% Interval 3
% Data vector
yi=[69 73 85]; n3=length(yi); ymean3=mean(yi); ysigma3=sqrt(var(yi,1));
% Posterior
k3=n3/(n3+(ysigma3^2/sigma0^2)); mupost3=(1-k3)*mu0+(k3*ymean3); sigmapost3=sqrt(k3*ysigma3^2/n3);

% Limits
CILM3 =(mu0-3*sigmapost3); MEAN3 = mu0; CIUM3 =(mu0+3*sigmapost3);
xx3=[yi]; lim3 = [CILM3 MEAN3 CIUM3];
figure

```

```

controlchart(xx3,'chart',{'i'},'limits',lim3,'specs',[90 90],'width',1); title('interval t=3')

%% Interval 4
%Data vector
yi=[69 73 85 115]; n4=length(yi); ymean4=mean(yi); ysigma4=sqrt(var(yi,1));
%Posterior
k4=n4/(n4+(ysigma4^2/sigma0^2)); mupost4=(1-k4)*mu0+(k4*ymean4); sigmapost4=sqrt(k4*ysigma4^2/n4);

%Limits
CILM4 =(mu0-3*sigmapost4); MEAN4 = mu0; CIUM4 =(mu0+3*sigmapost4);
xx4=[yi]; lim4 = [CILM4 MEAN4 CIUM4];
figure
controlchart(xx4,'chart',{'i'},'limits',lim4,'specs',[90 90],'width',1); title('interval t=4')

%% Interval 5
%Data vector
yi=[69 73 85 115 125]; n5=length(yi); ymean5=mean(yi); ysigma5=sqrt(var(yi,1));
%Posterior
k5=n5/(n5+(ysigma5^2/sigma0^2)); mupost5=(1-k5)*mu0+(k5*ymean5); sigmapost5=sqrt(k5*ysigma5^2/n5);

%Limits
CILM5 =(mu0-3*sigmapost5); MEAN5 = mu0; CIUM5 =(mu0+3*sigmapost5);
xx5=[yi]; lim5 = [CILM5 MEAN5 CIUM5];
figure
controlchart(xx5,'chart',{'i'},'limits',lim5,'specs',[90 90],'width',1); title('interval t=5')

%% Interval 6
%Data vector
yi=[69 73 85 115 125 90]; n6=length(yi); ymean6=mean(yi); ysigma6=sqrt(var(yi,1));
%Posterior
k6=n6/(n6+(ysigma6^2/sigma0^2)); mupost6=(1-k6)*mu0+(k6*ymean6); sigmapost6=sqrt(k6*ysigma6^2/n6);

%Limits
CILM6 =(mu0-3*sigmapost6); MEAN6 = mu0; CIUM6 =(mu0+3*sigmapost6);
xx6=[yi]; lim6 = [CILM6 MEAN6 CIUM6];
figure
controlchart(xx6,'chart',{'i'},'limits',lim6,'specs',[90 90],'width',1); title('interval t=6')

%% Interval 7
%Data vector
yi=[69 73 85 115 125 90 79]; n7=length(yi); ymean7=mean(yi); ysigma7=sqrt(var(yi,1));
%Posterior
k7=n7/(n7+(ysigma7^2/sigma0^2)); mupost7=(1-k7)*mu0+(k7*ymean7); sigmapost7=sqrt(k7*ysigma7^2/n7);

%Limits
CILM7 =(mu0-3*sigmapost7); MEAN7 = mu0; CIUM7 =(mu0+3*sigmapost7);
xx7=[yi]; lim7 = [CILM7 MEAN7 CIUM7];
figure
controlchart(xx7,'chart',{'i'},'limits',lim7,'specs',[90 90],'width',1); title('interval t=7')

```

#### d.- Direct method with mean posterior.

```

clear
%% Interval 1
%Prior
r = [68 74 82 110 115 93 86]; [muhat,sigmahat] = normfit(r); mu0=muhat; sigma0=sigmahat;
%Data vector
yi=[69]; n1=length(yi); ymean1=mean(yi); ysigma1=std(yi,1);
% Posterior
k1=n1/(n1+(ysigma1^2/sigma0^2)); mupost1=(1-k1)*mu0+(k1*ymean1); sigmapost1=sqrt(k1*ysigma1^2/n1);

%Limits
CILM1 =(mupost1-3*sigmapost1); MEAN1 = mupost1; CIUM1 =(mupost1+3*sigmapost1);
xx1=[yi]; lim1 = [CILM1 MEAN1 CIUM1];
%figure
%controlchart(xx1,'chart',{'i'},'limits',lim1,'specs',[90 107],'width',1); title('interval t=1')

```

```

%% Interval 2
%Data vector
yi=[69 73]; n2=length(yi); ymean2=mean(yi); ysigma2=std(yi,1);
%Posterior
k2=n2/(n2+(ysigma2^2/sigma0^2)); mupost2=(1-k2)*mu0+(k2*ymean2); sigmapost2=sqrt(k2*ysigma2^2/n2);

%Limits
CILM2 =(mupost2-3*sigmapost2); MEAN2 = mupost2; CIUM2 =(mupost2+3*sigmapost2);
xx2=[yi]; lim2 = [CILM2 MEAN2 CIUM2];
figure
controlchart(xx2,'chart','i','limits',lim2,'specs',[90 90],'width',1); title('interval t=2')

%% Interval 3
%Data vector
yi=[69 73 85]; n3=length(yi); ymean3=mean(yi); ysigma3=std(yi,1);
%Posterior
k3=n3/(n3+(ysigma3^2/sigma0^2)); mupost3=(1-k3)*mu0+(k3*ymean3); sigmapost3=sqrt(k3*ysigma3^2/n3);

%Limits
CILM3 =(mupost3-3*sigmapost3); MEAN3 = mupost3; CIUM3 =(mupost3+3*sigmapost3);
xx3=[yi]; lim3 = [CILM3 MEAN3 CIUM3];
figure
controlchart(xx3,'chart','i','limits',lim3,'specs',[90 90],'width',1); title('interval t=3')

%% Interval 4
%Data vector
yi=[69 73 85 115]; n4=length(yi); ymean4=mean(yi); ysigma4=std(yi,1);
%Posterior
k4=n4/(n4+(ysigma4^2/sigma0^2)); mupost4=(1-k4)*mu0+(k4*ymean4); sigmapost4=sqrt(k4*ysigma4^2/n4);

%Limits
CILM4 =(mupost4-3*sigmapost4); MEAN4 = mupost4; CIUM4 =(mupost4+3*sigmapost4);
xx4=[yi]; lim4 = [CILM4 MEAN4 CIUM4];
figure
controlchart(xx4,'chart','i','limits',lim4,'specs',[90 90],'width',1); title('interval t=4')

%% Interval 5
%Data vector
yi=[69 73 85 115 125]; n5=length(yi); ymean5=mean(yi); ysigma5=std(yi,1);
%Posterior
k5=n5/(n5+(ysigma5^2/sigma0^2)); mupost5=(1-k5)*mu0+(k5*ymean5); sigmapost5=sqrt(k5*ysigma5^2/n5);

%Limits
CILM5 =(mupost5-3*sigmapost5); MEAN5 = mupost5; CIUM5 =(mupost5+3*sigmapost5);
xx5=[yi]; lim5 = [CILM5 MEAN5 CIUM5];
figure
controlchart(xx5,'chart','i','limits',lim5,'specs',[90 90],'width',1); title('interval t=5')

%% Interval 6
%Data vector
yi=[69 73 85 115 125 90]; n6=length(yi); ymean6=mean(yi); ysigma6=std(yi,1);
%Posterior
k6=n6/(n6+(ysigma6^2/sigma0^2)); mupost6=(1-k6)*mu0+(k6*ymean6); sigmapost6=sqrt(k6*ysigma6^2/n6);

%Limits
CILM6 =(mupost6-3*sigmapost6); MEAN6 = mupost6; CIUM6 =(mupost6+3*sigmapost6);
xx6=[yi]; lim6 = [CILM6 MEAN6 CIUM6];
figure
controlchart(xx6,'chart','i','limits',lim6,'specs',[90 90],'width',1); title('interval t=6')

%% Interval 7
%Data vector
yi=[69 73 85 115 125 90 79]; n7=length(yi); ymean7=mean(yi); ysigma7=std(yi,1);
%Posterior
k7=n7/(n7+(ysigma7^2/sigma0^2)); mupost7=(1-k7)*mu0+(k7*ymean7); sigmapost7=sqrt(k7*ysigma7^2/n7);

```

```
%Limits
CILM7 =(mupost7-3*sigmapost7); MEAN7 = mupost7; CIUM7 =(mupost7+3*sigmapost7);
xx7=[yi]; lim7 = [CILM7 MEAN7 CIUM7];
figure
controlchart(xx7,'chart',{'i'},'limits',lim7,'specs',[90 90],'width',1); title('interval t=7')
```

## H.9. Lognormal-lognormal.

### a.- Recurrent method.

```
clear

%% First interval
%Parameters specification
x0=0.2; c=65; B=500; K=4000; J=B+K; a=0; r=0; n=10;
%Data
y0=[68 74 82 110 115 93 86]; y=[69]; meanY=mean(y);
%Initial values
mu0=mean(y0); sigma0=std(log(y0)); %Sigma of data y, adopted y0; sigma1=std(log(y0));

for i=1:n
    a(i)=0;
    %Candidate value generation
    x(1,1)=x0;
    for j=2:J
        x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
        if x(j,1)<0 || x(j,1)>400
            p(j,1)=0;
        else p(j,1)=lognpdf(x(j,1),log(mu0),sigma0)*lognpdf(meanY,log(x(j,1)),sigma1)/...
            (lognpdf(x(j-1,1),log(mu0),sigma0)*lognpdf(meanY,log(x(j-1,1)),sigma1));
        end
        u=random('Uniform',0,1,1,1);
        if u<p(j,1)
            x(j,1)=x(j,1);
            a(i)=a(i)+1;
        else x(j,1)=x(j-1,1);
            r=r+1;
        end
    end
end
%Burn - in
for j=B:J
    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev
muu1(i)=mean(xx); vaa1(i)=var(xx); stdd1(i)=sqrt(vaa1(i));
end

%Acceptance Rate AR
[muhat1,sigmahat1,muci1,sigmaci1] = normfit(muu1); AR1=mean(a)/J; muu11=mean(muu1); vaa11=mean(vaa1);
std11=sqrt(vaa11);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 1','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 1','FontSize',15);
%Histogram
```

```

figure
h = histogram(xx);
counts = h.Values;
%parameters from the LogNormal
parmhat=lognfit(xx);
hold on
xxx=0:0.1:250;
yyyy=lognpdf(xxx,parmhat(1),parmhat(2));
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) LogNormal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 1','FontSize',15);
hold off
legend('Location','northeast')

%Mean post, sigma post
[M1,V1]=lognstat(parmhat(1),parmhat(2)); Mean11=muu11; Sigma11=sqrt(V1);

%Parameters post for substituting mu0 and sigma0 in the new prior
mu111=muu11; sigma111=parmhat(2);

%% Second interval
%Parameters specification
x0=0.2; c=65; B=500; K=4000; J=B+K; a=0; r=0;
%Data
y0=[68 74 82 110 115 93 86]; y=[69 73]; meanY=mean(y);
%Initial values
mu0=mean(y0); sigma0=std(log(y0)); %Sigma of data y, still adopted y0; sigma1=std(log(y0));

for i=1:n
    a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
    x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
    if x(j,1)<0 || x(j,1)>400
        p(j,1)=0;
    else p(j,1)=lognpdf(x(j,1),log(muu11),sigma111)*lognpdf(meanY,log(x(j,1)),sigma1)/...
        (lognpdf(x(j-1,1),log(muu11),sigma111)*lognpdf(meanY,log(x(j-1,1)),sigma1));
    end
    u=random('Uniform',0,1,1,1);
    if u<p(j,1)
        x(j,1)=x(j,1);
        a(i)=a(i)+1;
    else x(j,1)=x(j-1,1);
        r=r+1;
    end
end
end
%Burn - in
for j=B:J
    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev
muu2(i)=mean(xx); vaa2(i)=var(xx); stdd2(i)=sqrt(vaa2(i));
end

%Acceptance Rate AR
[muhat2,sigmahat2,muci2,sigmaci2] = normfit(muu2); AR2=mean(a)/J; muu22=mean(muu2); vaa22=mean(vaa2);
std22=sqrt(vaa22);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 2','FontSize',15);
l=B:J;

```

```

subplot(2,1,2);
plot(1,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 2','FontSize',15);
%Histogram
figure
h = histogram(xx);
counts = h.Values;
%parameters from the LogNormal
parmhat=lognfit(xx);
hold on
xxx=0:0.1:250;
yyy=lognpdf(xxx,parmhat(1),parmhat(2));
plot(xxx,yyy*max(counts)/max(yyy),'DisplayName','y=f(x) LogNormal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 2','FontSize',15);
hold off
legend('Location','northeast')

%Mean post, sigma post
[M2,V2]=lognstat(parmhat(1),parmhat(2)); Mean22=muu22; Sigma22=sqrt(V2);

%Parameters post for substituting mu0 and sigma0 in the new prior
mu222=muu22; sigma222=parmhat(2);

%% Third interval
%Parameters specification
x0=0.2; c=65; B=500; K=4000; J=B+K; a=0; r=0;
%Data
y0=[68 74 82 110 115 93 86]; y=[69 73 85]; meanY=mean(y);
%Initial values
mu0=mean(y0); sigma0=std(log(y0)); sigma1=std(log(y));

for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
if x(j,1)<0 || x(j,1)>400
p(j,1)=0;
else p(j,1)=lognpdf(x(j,1),log(muu22),sigma222)*lognpdf(meanY,log(x(j,1)),sigma1)/...
(lognpdf(x(j-1,1),log(muu22),sigma222)*lognpdf(meanY,log(x(j-1,1)),sigma1));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
x(j,1)=x(j,1);
a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
r=r+1;
end
end
%Burn - in
for j=B:J
xx(j-B+1,1)=x(j,1);
end
%Mean and Dev
muu3(i)=mean(xx); vaa3(i)=var(xx); stdd3(i)=sqrt(vaa3(i));
end

%Acceptance Rate AR
[muhat3,sigmahat3,muci3,sigmaci3] = normfit(muu3); AR3=mean(a)/J; muu33=mean(muu3); vaa33=mean(vaa3);
std33=sqrt(vaa33);

%Graphics
figure
k=1:J;

```

```

subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 3','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 3','FontSize',15);

%Histogram
figure
h = histogram(xx);
counts = h.Values;
%parameters from the LogNormal
parmhat=lognfit(xx);
hold on
xxx=0:0.1:250;
yyyy=lognpdf(xxx,parmhat(1),parmhat(2));
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) LogNormal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 3','FontSize',15);
hold off
legend('Location','northeast')

%Mean post, sigma post
[M3,V3]=lognstat(parmhat(1),parmhat(2)); Mean33=muu33; Sigma33=sqrt(V3);

%Parameters post for substituting mu0 and sigma0 in the new prior
mu333=muu33; sigma333=parmhat(2);

%% Fourth interval
%Parameters specification
x0=0.2; c=35; B=500; K=4000; J=B+K; a=0; r=0;
%Data
y0=[68 74 82 110 115 93 86]; y=[69 73 85 115]; meanY=mean(y);
%Initial values
mu0=mean(y0); sigma0=std(log(y0)); sigma1=std(log(y));

for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
if x(j,1)<0 || x(j,1)>400
p(j,1)=0;
else p(j,1)=lognpdf(x(j,1),log(muu33),sigma333)*lognpdf(meanY,log(x(j,1)),sigma1)/...
(lognpdf(x(j-1,1),log(muu33),sigma333)*lognpdf(meanY,log(x(j-1,1)),sigma1));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
x(j,1)=x(j,1);
a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
r=r+1;
end
end
%Burn - in
for j=B:J
xx(j-B+1,1)=x(j,1);
end
%Mean and Dev
muu4(i)=mean(xx); vaa4(i)=var(xx); stdd4(i)=sqrt(vaa4(i));
end

%Acceptance Rate AR

```



```
[muhat4,sigmahat4,muci4,sigmaci4] = normfit(muu4); AR4=mean(a)/J; muu44=mean(muu4); vaa44=mean(vaa4);
std44=sqrt(vaa44);
```

```
%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 4','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 4','FontSize',15);
```

```
%Histogram
figure
h = histogram(xx);
counts = h.Values;
%parameters from the LogNormal
parmhat=lognfit(xx);
hold on
xxx=0:0.1:250;
yyyy=lognpdf(xxx,parmhat(1),parmhat(2));
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) LogNormal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 4','FontSize',15);
hold off
legend('Location','northeast')
```

```
%Mean post, sigma post
[M4,V4]=lognstat(parmhat(1),parmhat(2)); Mean44=muu44; Sigma44=sqrt(V4);
```

```
%Parameters post for substituting mu0 and sigma0 in the new prior
mu444=muu44; sigma444=parmhat(2);
```

```
%% Fifth interval
```

```
%Parameters specification
x0=0.2; c=35; B=500; K=4000; J=B+K; a=0; r=0;
%Data
y0=[68 74 82 110 115 93 86]; y=[69 73 85 115 125]; meanY=mean(y);
%Initial values
mu0=mean(y0); sigma0=std(log(y0)); sigma1=std(log(y));
```

```
for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
if x(j,1)<0 || x(j,1)>400
p(j,1)=0;
else p(j,1)=lognpdf(x(j,1),log(muu44),sigma444)*lognpdf(meanY,log(x(j,1)),sigma1)/...
(lognpdf(x(j-1,1),log(muu44),sigma444)*lognpdf(meanY,log(x(j-1,1)),sigma1));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
x(j,1)=x(j,1);
a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
r=r+1;
end
end
%Burn - in
for j=B:J
```

```

    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev
muu5(i)=mean(xx); vaa5(i)=var(xx); stdd5(i)=sqrt(vaa5(i));
end

%Acceptance Rate AR
[muhat5,sigmahat5,muci5,sigmaci5] = normfit(muu5); AR5=mean(a)/J; muu55=mean(muu5); vaa55=mean(vaa5);
std55=sqrt(vaa55);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 5','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 5','FontSize',15);
%Histogram
figure
h = histogram(xx);
counts = h.Values;
%parameters from the LogNormal
parmhat=lognfit(xx);
hold on
xxx=0:0.1:250;
yyyy=lognpdf(xxx,parmhat(1),parmhat(2));
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) LogNormal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 5','FontSize',15);
hold off
legend('Location','northeast')

%Mean post, sigma post
[M5,V5]=lognstat(parmhat(1),parmhat(2)); Mean55=muu55; Sigma55=sqrt(V5);

%Parameters post for substituting mu0 and sigma0 in the new prior
mu555=muu55; sigma555=parmhat(2);

%% Sixth interval
%Parameters specification
x0=0.2; c=35; B=500; K=4000; J=B+K; a=0; r=0;
%Data
y0=[68 74 82 110 115 93 86]; y=[69 73 85 115 125 90]; meanY=mean(y);
%Initial values
mu0=mean(y0); sigma0=std(log(y0)); sigma1=std(log(y));

for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
    x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
    if x(j,1)<0 || x(j,1)>400
        p(j,1)=0;
    else p(j,1)=lognpdf(x(j,1),log(muu55),sigma555)*lognpdf(meanY,log(x(j,1)),sigma1)/...
        (lognpdf(x(j-1,1),log(muu55),sigma555)*lognpdf(meanY,log(x(j-1,1)),sigma1));
    end
    u=random('Uniform',0,1,1,1);
    if u<p(j,1)
        x(j,1)=x(j,1);
        a(i)=a(i)+1;
    else x(j,1)=x(j-1,1);

```

```

    r=r+1;
    end
end
%Burn - in
for j=B:J
    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev
muu6(i)=mean(xx); vaa6(i)=var(xx); stdd6(i)=sqrt(vaa6(i));
end

%Acceptance Rate AR
[muhat6,sigmahat6,muci6,sigmaci6] = normfit(muu6); AR6=mean(a)/J; muu66=mean(muu6); vaa66=mean(vaa6);
std66=sqrt(vaa66);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 6','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 6','FontSize',15);
%Histogram
figure
h = histogram(xx);
counts = h.Values;
%parameters from the LogNormal
parmhat=lognfit(xx);
hold on
xxx=0:0.1:250;
yyyy=lognpdf(xxx,parmhat(1),parmhat(2));
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) LogNormal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 6','FontSize',15);
hold off
legend('Location','northeast')

%Mean post, sigma post
[M6,V6]=lognstat(parmhat(1),parmhat(2)); Mean66=muu66; Sigma66=sqrt(V6);

%Parameters post for substituting mu0 and sigma0 in the new prior
mu666=muu66; sigma666=parmhat(2);

%% Seventh interval
%Parameters specification
x0=0.2; c=35; B=500; K=4000; J=B+K; a=0; r=0;
%Data
y0=[68 74 82 110 115 93 86]; y=[69 73 85 115 125 90 79]; meanY=mean(y);
%Initial values
mu0=mean(y0); sigma0=std(log(y0)); sigma1=std(log(y));

for i=1:n
    a(i)=0;
    %Candidate value generation
    x(1,1)=x0;
    for j=2:J
        x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
        if x(j,1)<0 || x(j,1)>400
            p(j,1)=0;
        else p(j,1)=lognpdf(x(j,1),log(muu66),sigma666)*lognpdf(meanY,log(x(j,1)),sigma1)/...
            (lognpdf(x(j-1,1),log(muu66),sigma666)*lognpdf(meanY,log(x(j-1,1)),sigma1));
        end
    end
end

```

```

u=random('Uniform',0,1,1,1);
if u<p(j,1)
    x(j,1)=x(j,1);
    a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
    r=r+1;
end
end
%Burn - in
for j=B:J
    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev
muu7(i)=mean(xx); vaa7(i)=var(xx); stdd7(i)=sqrt(vaa7(i));
end

%Acceptance Rate AR
[muhat7,sigmahat7,muci7,sigmaci7] = normfit(muu7); AR7=mean(a)/J; muu77=mean(muu7); vaa77=mean(vaa7);
std77=sqrt(vaa77);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 7','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 7','FontSize',15);
%Histogram
figure
h = histogram(xx);
counts = h.Values;
%parameters from the LogNormal
parmhat=lognfit(xx);
hold on
xxx=0:0.1:250;
yyyy=lognpdf(xxx,parmhat(1),parmhat(2));
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) LogNormal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 7','FontSize',15);
hold off
legend('Location','northeast')

%Mean post, sigma post
[M7,V7]=lognstat(parmhat(1),parmhat(2)); Mean77=muu77; Sigma77=sqrt(V7);

%Parameters post for substituting mu0 and sigma0 in the new prior
mu777=muu77; sigma777=parmhat(2);

```

### b.- Direct method.

```

clear
%% First interval
%Parameters specification
x0=0.2; c=65; B=500; K=4000; J=B+K; a=0; r=0; n=10;
%Data
y0=[68 74 82 110 115 93 86]; y=[69]; meanY=mean(y);
%Initial values
mu0=mean(y0); sigma0=std(log(y0)); %Sigma of data y, adopted y0; sigma1=std(log(y0));

for i=1:n
    a(i)=0;

```

```

%Candidate value generation
x(1,1)=x0;
for j=2:J
    x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
    if x(j,1)<0 || x(j,1)>400
        p(j,1)=0;
    else p(j,1)=lognpdf(x(j,1),log(mu0),sigma0)*lognpdf(meanY,log(x(j,1)),sigma1)/...
        (lognpdf(x(j-1,1),log(mu0),sigma0)*lognpdf(meanY,log(x(j-1,1)),sigma1));
    end
    u=random('Uniform',0,1,1,1);
    if u<p(j,1)
        x(j,1)=x(j,1);
        a(i)=a(i)+1;
    else x(j,1)=x(j-1,1);
        r=r+1;
    end
end
%Burn - in
for j=B:J
    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev
muu1(i)=mean(xx);vaa1(i)=var(xx); stdd1(i)=sqrt(vaa1(i));
end

%Acceptance Rate AR
[muhat1,sigmahat1,muci1,sigmaci1] = normfit(muu1); AR1=mean(a)/J; muu11=mean(muu1); vaa11=mean(vaa1);
std11=sqrt(vaa11);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 1','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 1','FontSize',15);
%Histogram
figure
h = histogram(xx);
counts = h.Values;
%parameters from the LogNormal
parmhat=lognfit(xx);
hold on
xxx=0:0.1:250;
yyyy=lognpdf(xxx,parmhat(1),parmhat(2));
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) LogNormal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 1','FontSize',15);
hold off
legend('Location','northeast')

%Mean post, sigma post
[M1,V1]=lognstat(parmhat(1),parmhat(2)); Mean11=muu11; Sigma11=sqrt(V1);

%Parameters post for substituting mu0 and sigma0 in the new prior
mu111=muu11; sigma111=parmhat(2);

%% Second interval
%Parameters specification
x0=0.2; c=65; B=500; K=4000; J=B+K; a=0; r=0;
%Data
y0=[68 74 82 110 115 93 86]; y=[69 73]; meanY=mean(y);

```

```

%Initial values
mu0=mean(y0); sigma0=std(log(y0)); %Sigma of data y, still adopted y0; sigma1=std(log(y0));

for i=1:n
    a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
    x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
    if x(j,1)<0 || x(j,1)>400
        p(j,1)=0;
    else p(j,1)=lognpdf(x(j,1),log(mu0),sigma0)*lognpdf(meanY,log(x(j,1)),sigma1)/...
        (lognpdf(x(j-1,1),log(mu0),sigma0)*lognpdf(meanY,log(x(j-1,1)),sigma1));
    end
    u=random('Uniform',0,1,1,1);
    if u<p(j,1)
        x(j,1)=x(j,1);
        a(i)=a(i)+1;
    else x(j,1)=x(j-1,1);
        r=r+1;
    end
end
%Burn - in
for j=B:J
    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev
muu2(i)=mean(xx); vaa2(i)=var(xx); stdd2(i)=sqrt(vaa2(i));
end

%Acceptance Rate AR
[muhat2,sigmahat2,muci2,sigmaci2] = normfit(muu2); AR2=mean(a)/J; muu22=mean(muu2); vaa22=mean(vaa2);
std22=sqrt(vaa22);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 2','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 2','FontSize',15);

%Histogram
figure
h = histogram(xx);
counts = h.Values;
%parameters from the LogNormal
parmhat=lognfit(xx);
hold on
xxx=0:0.1:250;
yyyy=lognpdf(xxx,parmhat(1),parmhat(2));
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) LogNormal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 2','FontSize',15);
hold off
legend('Location','northeast')

%Mean post, sigma post
[M2,V2]=lognstat(parmhat(1),parmhat(2)); Mean22=muu22; Sigma22=sqrt(V2);

%Parameters post for substituting mu0 and sigma0 in the new prior
mu222=muu22; sigma222=parmhat(2);

```

```

%% Third interval
%Parameters specification
x0=0.2; c=35; B=500; K=4000; J=B+K; a=0; r=0;
%Data
y0=[68 74 82 110 115 93 86]; y=[69 73 85]; meanY=mean(y);
%Initial values
mu0=mean(y0); sigma0=std(log(y0)); %Sigma of data y; sigma1=std(log(y));

for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
if x(j,1)<0 || x(j,1)>400
p(j,1)=0;
else p(j,1)=lognpdf(x(j,1),log(mu0),sigma0)*lognpdf(meanY,log(x(j,1)),sigma1)/...
(lognpdf(x(j-1,1),log(mu0),sigma0)*lognpdf(meanY,log(x(j-1,1)),sigma1));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
x(j,1)=x(j,1);
a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
r=r+1;
end
end
%Burn - in
for j=B:J
xx(j-B+1,1)=x(j,1);
end
%Mean and Dev
muu3(i)=mean(xx); vaa3(i)=var(xx); stdd3(i)=sqrt(vaa3(i));
end

%Acceptance Rate AR
[muhat3,sigmahat3,muci3,sigmaci3] = normfit(muu3); AR3=mean(a)/J; muu33=mean(muu3); vaa33=mean(vaa3);
std33=sqrt(vaa33);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 3','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 3','FontSize',15);
%Histogram
figure
h = histogram(xx);
counts = h.Values;
%parameters from the LogNormal
parmhat=lognfit(xx);
hold on
xxx=0:0.1:250;
yyyy=lognpdf(xxx,parmhat(1),parmhat(2));
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) LogNormal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 3','FontSize',15);
hold off
legend('Location','northeast')

```

```

%Mean post, sigma post
[M3,V3]=lognstat(parmhat(1),parmhat(2)); Mean33=muu33; Sigma33=sqrt(V3);

%Parameters post for substituting mu0 and sigma0 in the new prior
mu333=muu33; sigma333=parmhat(2);

%% Fourth interval
%Parameters specification
x0=0.2; c=35; B=500; K=4000; J=B+K; a=0; r=0;
%Data
y0=[68 74 82 110 115 93 86]; y=[69 73 85 115]; meanY=mean(y);
%Initial values
mu0=mean(y0); sigma0=std(log(y0)); %Sigma of data y; sigma1=std(log(y));

for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
if x(j,1)<0 || x(j,1)>400
p(j,1)=0;
else p(j,1)=lognpdf(x(j,1),log(mu0),sigma0)*lognpdf(meanY,log(x(j,1)),sigma1)/...
(lognpdf(x(j-1,1),log(mu0),sigma0)*lognpdf(meanY,log(x(j-1,1)),sigma1));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
x(j,1)=x(j,1);
a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
r=r+1;
end
end
%Burn - in
for j=B:J
xx(j-B+1,1)=x(j,1);
end
%Mean and Dev
muu4(i)=mean(xx); vaa4(i)=var(xx); stdd4(i)=sqrt(vaa4(i));
end

%Acceptance Rate AR
[muhat4,sigmahat4,muci4,sigmaci4] = normfit(muu4); AR4=mean(a)/J; muu44=mean(muu4); vaa44=mean(vaa4);
std44=sqrt(vaa44);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 4','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 4','FontSize',15);
%Histogram
figure
h = histogram(xx);
counts = h.Values;
%parameters from the LogNormal
parmhat=lognfit(xx);
hold on
xxx=0:0.1:250;
yyyy=lognpdf(xxx,parmhat(1),parmhat(2));
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) LogNormal')

```



```

xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 4','FontSize',15);
hold off
legend('Location','northeast')

%Mean post, sigma post
[M4,V4]=lognstat(parmhat(1),parmhat(2)); Mean44=muu44; Sigma44=sqrt(V4);

%Parameters post for substituting mu0 and sigma0 in the new prior
mu444=muu44; sigma444=parmhat(2);

%% Fifth interval
%Parameters specification
x0=0.2; c=35; B=500; K=4000; J=B+K; a=0; r=0;
%Data
y0=[68 74 82 110 115 93 86]; y=[69 73 85 115 125]; meanY=mean(y);
%Initial values
mu0=mean(y0); sigma0=std(log(y0)); %Sigma of data y; sigma1=std(log(y));

for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
if x(j,1)<0 || x(j,1)>400
p(j,1)=0;
else p(j,1)=lognpdf(x(j,1),log(mu0),sigma0)*lognpdf(meanY,log(x(j,1)),sigma1)/...
(lognpdf(x(j-1,1),log(mu0),sigma0)*lognpdf(meanY,log(x(j-1,1)),sigma1));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
x(j,1)=x(j,1);
a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
r=r+1;
end
end
%Burn - in
for j=B:J
xx(j-B+1,1)=x(j,1);
end
%Mean and Dev
muu5(i)=mean(xx); vaa5(i)=var(xx); stdd5(i)=sqrt(vaa5(i));
end

%Acceptance Rate AR
[muhat5,sigmahat5,muci5,sigmaci5] = normfit(muu5); AR5=mean(a)/J; muu55=mean(muu5); vaa55=mean(vaa5);
std55=sqrt(vaa55);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 5','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 5','FontSize',15);
%Histogram
figure
h = histogram(xx);
counts = h.Values;
%parameters from the LogNormal

```

```

parmhat=lognfit(xx);
hold on
xxx=0:0.1:250;
yyyy=lognpdf(xxx,parmhat(1),parmhat(2));
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) LogNormal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 5','FontSize',15);
hold off
legend('Location','northeast')

%Mean post, sigma post
[M5,V5]=lognstat(parmhat(1),parmhat(2)); Mean55=muu55; Sigma55=sqrt(V5);

%Parameters post for substituting mu0 and sigma0 in the new prior
mu555=muu55; sigma555=parmhat(2);

%% Sixth interval
%Parameters specification
x0=0.2; c=35; B=500; K=4000; J=B+K; a=0; r=0;
%Data
y0=[68 74 82 110 115 93 86]; y=[69 73 85 115 125 90]; meanY=mean(y);
%Initial values
mu0=mean(y0); sigma0=std(log(y0)); %Sigma of data y; sigma1=std(log(y));

for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
if x(j,1)<0 || x(j,1)>400
p(j,1)=0;
else p(j,1)=lognpdf(x(j,1),log(mu0),sigma0)*lognpdf(meanY,log(x(j,1)),sigma1)/...
(lognpdf(x(j-1,1),log(mu0),sigma0)*lognpdf(meanY,log(x(j-1,1)),sigma1));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
x(j,1)=x(j,1);
a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
r=r+1;
end
end
%Burn - in
for j=B:J
xx(j-B+1,1)=x(j,1);
end
%Mean and Dev
muu6(i)=mean(xx); vaa6(i)=var(xx); stdd6(i)=sqrt(vaa6(i));
end

%Acceptance Rate AR
[muhat6,sigmahat6,muci6,sigmaci6] = normfit(muu6); AR6=mean(a)/J; muu66=mean(muu6); vaa66=mean(vaa6);
std66=sqrt(vaa66);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 6','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 6','FontSize',15);

```

```

%Histogram
figure
h = histogram(xx);
counts = h.Values;
%parameters from the LogNormal
parmhat=lognfit(xx);
hold on
xxx=0:0.1:250;
yyyy=lognpdf(xxx,parmhat(1),parmhat(2));
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) LogNormal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 6','FontSize',15);
hold off
legend('Location','northeast')

%Mean post, sigma post
[M6,V6]=lognstat(parmhat(1),parmhat(2)); Mean66=muu66; Sigma66=sqrt(V6);

%Parameters post for substituting mu0 and sigma0 in the new prior
mu666=muu66; sigma666=parmhat(2);

%% Seventh interval
%Parameters specification
x0=0.2; c=35; B=500; K=4000; J=B+K; a=0; r=0;
%Data
y0=[68 74 82 110 115 93 86]; y=[69 73 85 115 125 90 79]; meanY=mean(y);
%Initial values
mu0=mean(y0); sigma0=std(log(y0)); %Sigma of data y; sigma1=std(log(y));

for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
if x(j,1)<0 || x(j,1)>400
p(j,1)=0;
else p(j,1)=lognpdf(x(j,1),log(mu0),sigma0)*lognpdf(meanY,log(x(j,1)),sigma1)/...
(lognpdf(x(j-1,1),log(mu0),sigma0)*lognpdf(meanY,log(x(j-1,1)),sigma1));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
x(j,1)=x(j,1);
a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
r=r+1;
end
end
end
%Burn - in
for j=B:J
xx(j-B+1,1)=x(j,1);
end
%Mean and Dev
muu7(i)=mean(xx); vaa7(i)=var(xx); stdd7(i)=sqrt(vaa7(i));
end

%Acceptance Rate AR
[muhat7,sigmahat7,muci7,sigmaci7] = normfit(muu7); AR7=mean(a)/J; muu77=mean(muu7); vaa77=mean(vaa7);
std77=sqrt(vaa77);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 7','FontSize',15);

```

```

l=B:J;
subplot(2,1,2);
plot(1,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 7','FontSize',15);
%Histogram
figure
h = histogram(xx);
counts = h.Values;
%parameters from the LogNormal
parmhat=lognfit(xx);
hold on
xxx=0:0.1:250;
yyyy=lognpdf(xxx,parmhat(1),parmhat(2));
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) LogNormal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 7','FontSize',15);
hold off
legend('Location','northeast')

%Mean post, sigma post
[M7,V7]=lognstat(parmhat(1),parmhat(2)); Mean77=muu77; Sigma77=sqrt(V7);

%Parameters post for substituting mu0 and sigma0 in the new prior
mu777=muu77; sigma777=parmhat(2);

```

## H.10. Lognormal-uniform.

```

clear
%% First interval
%Parameters specification
x0=0.2; c=65; B=1000; K=4000; J=B+K; a=0; r=0; n=10;
%Data
y0=[68 74 82 110 115 93 86]; y=[69]; meanY=mean(y);
%Initial values
mu0=mean(y0); sigma0=std(log(y0)); %Sigma of data y, adopted y0; sigma1=std(log(y0));

for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
if x(j,1)<0 || x(j,1)>400 %<0 >400
p(j,1)=0;
else p(j,1)=1/x(j,1)*lognpdf(meanY,log(x(j,1)),sigma1)/...
(1/x(j-1,1)*lognpdf(meanY,log(x(j-1,1)),sigma1));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
x(j,1)=x(j,1);
a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
r=r+1;
end
end
%Burn - in
for j=B:J
xx(j-B+1,1)=x(j,1);
end
%Mean and Dev
muu1(i)=mean(xx); vaa1(i)=var(xx); stdd1(i)=sqrt(vaa1(i));
end

%Acceptance Rate AR
[muhat1,sigmahat1,muci1,sigmaci1] = normfit(muu1); AR1=mean(a)/J; muu11=mean(muu1); vaa11=mean(vaa1);

```

```

std11=sqrt(vaa11);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 1','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 1','FontSize',15);
%Histogram
figure
h = histogram(xx);
counts = h.Values;
%parameters from the LogNormal
parmhat=lognfit(xx);
hold on
xxx=0:0.1:250;
yyyy=lognpdf(xxx,parmhat(1),parmhat(2));
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) LogNormal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 1','FontSize',15);
hold off
legend('Location','northeast')

%Mean post, sigma post
[M1,V1]=lognstat(parmhat(1),parmhat(2)); Mean11=muu11; Sigma11=sqrt(V1);

%% Second interval
%Parameters specification
x0=0.2; c=35; B=1000; K=4000; J=B+K; a=0; r=0;
%Data
y0=[68 74 82 110 115 93 86]; y=[69 73]; meanY=mean(y);
%Initial values
mu0=mean(y0); sigma0=std(log(y0)); %Sigma of data y, adopted y0; sigma1=std(log(y0));

for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
if x(j,1)<0 || x(j,1)>400 %<0 >400
p(j,1)=0;
else p(j,1)=1/x(j,1)*lognpdf(meanY,log(x(j,1)),sigma1)/...
(1/x(j-1,1)*lognpdf(meanY,log(x(j-1,1)),sigma1));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
x(j,1)=x(j,1);
a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
r=r+1;
end
end
%Burn - in
for j=B:J
xx(j-B+1,1)=x(j,1);
end
%Mean and Dev
muu2(i)=mean(xx); vaa2(i)=var(xx); stdd2(i)=sqrt(vaa2(i));
end

```

```

%Acceptance Rate AR
[muhat2,sigmahat2,muci2,sigmaci2] = normfit(muu2); AR2=mean(a)/J; muu22=mean(muu2); vaa22=mean(vaa2);
std22=sqrt(vaa22);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 2','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 2','FontSize',15);
%Histogram
figure
h = histogram(xx);
counts = h.Values;
%parameters from the LogNormal
parmhat=lognfit(xx);
hold on
xxx=0:0.1:250;
yyyy=lognpdf(xxx,parmhat(1),parmhat(2));
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) LogNormal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 2','FontSize',15);
hold off
legend('Location','northeast')

%Mean post, sigma post
[M2,V2]=lognstat(parmhat(1),parmhat(2)); Mean22=muu22; Sigma22=sqrt(V2);

%% Third interval
%Parameters specification
x0=0.2; c=35; B=1000; K=4000; J=B+K; a=0; r=0;
%Data
y0=[68 74 82 110 115 93 86]; y=[69 73 85]; meanY=mean(y);
%Initial values
mu0=mean(y0); sigma0=std(log(y0)); %Sigma of data y; sigma1=std(log(y));

for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
if x(j,1)<0 || x(j,1)>400 %<0 >400
p(j,1)=0;
else p(j,1)=1/x(j,1)*lognpdf(meanY,log(x(j,1)),sigma1)/...
(1/x(j-1,1)*lognpdf(meanY,log(x(j-1,1)),sigma1));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
x(j,1)=x(j,1);
a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
r=r+1;
end
end
%Burn - in
for j=B:J
xx(j-B+1,1)=x(j,1);
end
%Mean and Dev
muu3(i)=mean(xx); vaa3(i)=var(xx); stdd3(i)=sqrt(vaa3(i));

```

```

end

%Acceptance Rate AR
[muhat3,sigmahat3,muci3,sigmaci3] = normfit(muu3); AR3=mean(a)/J; muu33=mean(muu3); vaa33=mean(vaa3);
std33=sqrt(vaa33);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 3','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 3','FontSize',15);
%Histogram
figure
h = histogram(xx);
counts = h.Values;
%parameters from the LogNormal
parmhat=lognfit(xx);
hold on
xxx=0:0.1:250;
yyyy=lognpdf(xxx,parmhat(1),parmhat(2));
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) LogNormal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 3','FontSize',15);
hold off
legend('Location','northeast')

%Mean post, sigma post
[M3,V3]=lognstat(parmhat(1),parmhat(2)); Mean33=muu33; Sigma33=sqrt(V3);

%% Fourth interval
%Parameters specification
x0=0.2; c=35; B=1000; K=4000; J=B+K; a=0; r=0;
%Data
y0=[68 74 82 110 115 93 86]; y=[69 73 85 115]; meanY=mean(y);
%Initial values
mu0=mean(y0); sigma0=std(log(y0)); %Sigma of data y; sigma1=std(log(y));

for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
if x(j,1)<0 || x(j,1)>400 %<0 >400
p(j,1)=0;
else p(j,1)=1/x(j,1)*lognpdf(meanY,log(x(j,1)),sigma1)/...
(1/x(j-1,1)*lognpdf(meanY,log(x(j-1,1)),sigma1));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
x(j,1)=x(j,1);
a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
r=r+1;
end
end
%Burn - in
for j=B:J
xx(j-B+1,1)=x(j,1);
end

```

```

%Mean and Dev
muu4(i)=mean(xx); vaa4(i)=var(xx); stdd4(i)=sqrt(vaa4(i));
end

%Acceptance Rate AR
[muhat4, sigmahat4, mucic4, sigmacic4] = normfit(muu4); AR4=mean(a)/J; muu44=mean(muu4); vaa44=mean(vaa4);
std44=sqrt(vaa44);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 4','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 4','FontSize',15);
%Histogram
figure
h = histogram(xx);
counts = h.Values;
%parameters from the LogNormal
parmhat=lognfit(xx);
hold on
xxx=0:0.1:250;
yyyy=lognpdf(xxx,parmhat(1),parmhat(2));
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) LogNormal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 4','FontSize',15);
hold off
legend('Location','northeast')

%Mean post, sigma post
[M4,V4]=lognstat(parmhat(1),parmhat(2)); Mean44=muu44; Sigma44=sqrt(V4);

%% Fifth interval
%Parameters specification
x0=0.2; c=35; B=1000; K=4000; J=B+K; a=0; r=0;
%Data
y0=[68 74 82 110 115 93 86]; y=[69 73 85 115 125]; meanY=mean(y);
%Initial values
mu0=mean(y0); sigma0=std(log(y0)); %Sigma of data y; sigma1=std(log(y));

for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
if x(j,1)<0 || x(j,1)>400 %<0 >400
p(j,1)=0;
else p(j,1)=1/x(j,1)*lognpdf(meanY,log(x(j,1)),sigma1)/...
(1/x(j-1,1)*lognpdf(meanY,log(x(j-1,1)),sigma1));
end
u=random('Uniform',0,1,1,1);
if u<p(j,1)
x(j,1)=x(j,1);
a(i)=a(i)+1;
else x(j,1)=x(j-1,1);
r=r+1;
end
end
%Burn - in
for j=B:J

```



```

    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev
muu5(i)=mean(xx); vaa5(i)=var(xx); stdd5(i)=sqrt(vaa5(i));
end

%Acceptance Rate AR
[muhat5,sigmahat5,muci5,sigmaci5] = normfit(muu5); AR5=mean(a)/J; muu55=mean(muu5); vaa55=mean(vaa5);
std55=sqrt(vaa55);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 5','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 5','FontSize',15);
%Histogram
figure
h = histogram(xx);
counts = h.Values;
%parameters from the LogNormal
parmhat=lognfit(xx);
hold on
xxx=0:0.1:250;
yyyy=lognpdf(xxx,parmhat(1),parmhat(2));
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) LogNormal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 5','FontSize',15);
hold off
legend('Location','northeast')

%Mean post, sigma post
[M5,V5]=lognstat(parmhat(1),parmhat(2)); Mean55=muu55; Sigma55=sqrt(V5);

%% Sixth interval
%Parameters specification
x0=0.2; c=35; B=1000; K=4000; J=B+K; a=0; r=0;
%Data
y0=[68 74 82 110 115 93 86]; y=[69 73 85 115 125 90]; meanY=mean(y);
%Initial values
mu0=mean(y0); sigma0=std(log(y0)); %Sigma of data y; sigma1=std(log(y));

for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
    x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
    if x(j,1)<0 || x(j,1)>400 %<0 >400
        p(j,1)=0;
    else p(j,1)=1/x(j,1)*lognpdf(meanY,log(x(j,1)),sigma1)/...
        (1/x(j-1,1)*lognpdf(meanY,log(x(j-1,1)),sigma1));
    end
    u=random('Uniform',0,1,1,1);
    if u<p(j,1)
        x(j,1)=x(j,1);
        a(i)=a(i)+1;
    else x(j,1)=x(j-1,1);
        r=r+1;
    end
end
end
end

```

```

%Burn - in
for j=B:J
    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev
muu6(i)=mean(xx); vaa6(i)=var(xx); stdd6(i)=sqrt(vaa6(i));
end

%Acceptance Rate AR
[muhat6,sigmahat6,muci6,sigmaci6] = normfit(muu6); AR6=mean(a)/J; muu66=mean(muu6); vaa66=mean(vaa6);
std66=sqrt(vaa66);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 6','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 6','FontSize',15);
%Histogram
figure
h = histogram(xx);
counts = h.Values;
%parameters from the LogNormal
parmhat=lognfit(xx);
hold on
xxx=0:0.1:250;
yyyy=lognpdf(xxx,parmhat(1),parmhat(2));
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) LogNormal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 6','FontSize',15);
hold off
legend('Location','northeast')

%Mean post, sigma post
[M6,V6]=lognstat(parmhat(1),parmhat(2)); Mean66=muu66; Sigma66=sqrt(V6);

%% Seventh interval
%Parameters specification
x0=0.2; c=35; B=1000; K=4000; J=B+K; a=0; r=0;
%Data
y0=[68 74 82 110 115 93 86]; y=[69 73 85 115 125 90 79]; meanY=mean(y);
%Initial values
mu0=mean(y0); sigma0=std(log(y0)); %Sigma of data y; sigma1=std(log(y));

for i=1:n
a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
    x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
    if x(j,1)<0 || x(j,1)>400 %<0 >400
        p(j,1)=0;
    else p(j,1)=1/x(j,1)*lognpdf(meanY,log(x(j,1)),sigma1)/...
        (1/x(j-1,1)*lognpdf(meanY,log(x(j-1,1)),sigma1));
    end
    u=random('Uniform',0,1,1,1);
    if u<p(j,1)
        x(j,1)=x(j,1);
        a(i)=a(i)+1;
    else x(j,1)=x(j-1,1);
        r=r+1;
    end
end

```

```

end
end
%Burn - in
for j=B:J
    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev
muu7(i)=mean(xx); vaa7(i)=var(xx); stdd7(i)=sqrt(vaa7(i));
end

%Acceptance Rate AR
[muhat7,sigmahat7,muci7,sigmaci7] = normfit(muu7); AR7=mean(a)/J; muu77=mean(muu7); vaa77=mean(vaa7);
std77=sqrt(vaa77);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 7','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 7','FontSize',15);
%Histogram
figure
h = histogram(xx);
counts = h.Values;
%parameters from the LogNormal
parmhat=lognfit(xx);
hold on
xxx=0:0.1:250;
yyyy=lognpdf(xxx,parmhat(1),parmhat(2));
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) LogNormal')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 7','FontSize',15);
hold off
legend('Location','northeast')

%Mean post, sigma post
[M7,V7]=lognstat(parmhat(1),parmhat(2)); Mean77=muu77; Sigma77=sqrt(V7);

```

## H.11. Normal-gamma I.

### a.- Direct method with mean prior.

```

clear
%% Interval 1
% Prior
alfa0=2; beta0=2; muprior0=alfa0/beta0; mu0N=2.2; sigma0N=1; lambda0=1/sigma0N^2;
sigmaprior0=sqrt(alfa0/beta0^2);
%Chart control
C=0.5;
% Data
yi=[1]; n1=length(yi); s1=smu(n1,yi,mu0N); a1=alfa0+n1/2; b1=beta0+n1/2*s1; lambda1=a1/b1;
sigmap1=sqrt(a1/b1^2); sigmapostnormal1=1/sqrt(lambda1); SNormal1=std(yi,1);
lambdanormalobs1=1/SNormal1^2;
yyi=[lambdanormalobs1];
CILM1 = lambda0-C*sigmap1; MEAN1 = lambda0; CIUM1 = lambda0+C*sigmap1;
xx1=[yyi]; lim1 = [CILM1 MEAN1 CIUM1];
%figure

```

```

%controlchart(xx1,'chart',{'i'},'limits',lim1,'specs',[lambda0 lambda0],'width',1); title('interval t=1')

%% Interval 2
% Data
yi=[1 3]; n2=length(yi); s2=smu(n2,yi,mu0N); a2=alfa0+n2/2; b2=beta0+n2/2*s2; lambda2=a2/b2;
sigmap2=sqrt(a2/b2^2); sigmapostnormal2=1/sqrt(lambda2); SNormal2=std(yi,1);
lambdanormalobs2=1/SNormal2^2;
yyi=[lambdanormalobs1 lambdanormalobs2];
CILM2 = lambda0-C*sigmap2; MEAN2 = lambda0; CIUM2 = lambda0+C*sigmap2;
xx2=[yyi]; lim2 = [CILM2 MEAN2 CIUM2];
figure
controlchart(xx2,'chart',{'i'},'limits',lim2,'specs',[lambda0 lambda0],'width',1); title('interval t=2')

%% Interval 3
% Data
yi=[1 3 3]; n3=length(yi); s3=smu(n3,yi,mu0N); a3=alfa0+n3/2; b3=beta0+n3/2*s3; lambda3=a3/b3;
sigmap3=sqrt(a3/b3^2); sigmapostnormal3=1/sqrt(lambda3); SNormal3=std(yi,1);
lambdanormalobs3=1/SNormal3^2;
yyi=[lambdanormalobs1 lambdanormalobs2 lambdanormalobs3];
CILM3 = lambda0-C*sigmap3; MEAN3 = lambda0; CIUM3 = lambda0+C*sigmap3;
xx3=[yyi]; lim3 = [CILM3 MEAN3 CIUM3];
figure
controlchart(xx3,'chart',{'i'},'limits',lim3,'specs',[lambda0 lambda0],'width',1); title('interval t=3')

%% Interval 4
% Data
yi=[1 3 3 3]; n4=length(yi); s4=smu(n4,yi,mu0N); a4=alfa0+n4/2; b4=beta0+n4/2*s4; lambda4=a4/b4;
sigmap4=sqrt(a4/b4^2); sigmapostnormal4=1/sqrt(lambda4); SNormal4=std(yi,1);
lambdanormalobs4=1/SNormal4^2;
yyi=[lambdanormalobs1 lambdanormalobs2 lambdanormalobs3 lambdanormalobs4];
CILM4 = lambda0-C*sigmap4; MEAN4 = lambda0; CIUM4 = lambda0+C*sigmap4;
xx4=[yyi]; lim4 = [CILM4 MEAN4 CIUM4];
figure
controlchart(xx4,'chart',{'i'},'limits',lim4,'specs',[lambda0 lambda0],'width',1); title('interval t=4')

%% Interval 5
% Data
yi=[1 3 3 3 2]; n5=length(yi); s5=smu(n5,yi,mu0N); a5=alfa0+n5/2; b5=beta0+n5/2*s5; lambda5=a5/b5;
sigmap5=sqrt(a5/b5^2); sigmapostnormal5=1/sqrt(lambda5); SNormal5=std(yi,1);
lambdanormalobs5=1/SNormal5^2;
yyi=[lambdanormalobs1 lambdanormalobs2 lambdanormalobs3 lambdanormalobs4 lambdanormalobs5];
CILM5 = lambda0-C*sigmap5; MEAN5 = lambda0; CIUM5 = lambda0+C*sigmap5;
xx5=[yyi]; lim5 = [CILM5 MEAN5 CIUM5];
figure
controlchart(xx5,'chart',{'i'},'limits',lim5,'specs',[lambda0 lambda0],'width',1); title('interval t=5')

%% Interval 6
% Data
yi=[1 3 3 3 2 4]; n6=length(yi); s6=smu(n6,yi,mu0N); a6=alfa0+n6/2; b6=beta0+n6/2*s6; lambda6=a6/b6;
sigmap6=sqrt(a6/b6^2); sigmapostnormal6=1/sqrt(lambda6); SNormal6=std(yi,1);
lambdanormalobs6=1/SNormal6^2;
yyi=[lambdanormalobs1 lambdanormalobs2 lambdanormalobs3 lambdanormalobs4 lambdanormalobs5 ...
lambdanormalobs6];
CILM6 = lambda0-C*sigmap6; MEAN6 = lambda0; CIUM6 = lambda0+C*sigmap6;
xx6=[yyi]; lim6 = [CILM6 MEAN6 CIUM6];
figure
controlchart(xx6,'chart',{'i'},'limits',lim6,'specs',[lambda0 lambda0],'width',1); title('interval t=6')

%% Interval 7
% Data
yi=[1 3 3 3 2 4 4]; n7=length(yi); s7=smu(n7,yi,mu0N); a7=alfa0+n7/2; b7=beta0+n7/2*s7; lambda7=a7/b7;
sigmap7=sqrt(a7/b7^2); sigmapostnormal7=1/sqrt(lambda7); SNormal7=std(yi,1);
lambdanormalobs7=1/SNormal7^2;
yyi=[lambdanormalobs1 lambdanormalobs2 lambdanormalobs3 lambdanormalobs4 lambdanormalobs5 ...
lambdanormalobs6 lambdanormalobs7];
CILM7 = lambda0-C*sigmap7; MEAN7 = lambda0; CIUM7 = lambda0+C*sigmap7;
xx7=[yyi]; lim7 = [CILM7 MEAN7 CIUM7];

```

```

figure
controlchart(xx7,'chart',{'i'},'limits',lim7,'specs',[lambda0 lambda0],'width',1); title('interval t=7')

%% Interval 8
% Data
yi=[1 3 3 3 2 4 4 2]; n8=length(yi); s8=smu(n8,yi,mu0N); a8=alfa0+n8/2; b8=beta0+n8/2*s8; lambda8=a8/b8;
sigmap8=sqrt(a8/b8^2); sigmapostnormal8=1/sqrt(lambda8); SNormal8=std(yi,1);
lambdanormalobs8=1/SNormal8^2;
yyi=[lambdanormalobs1 lambdanormalobs2 lambdanormalobs3 lambdanormalobs4 lambdanormalobs5 ...
lambdanormalobs6 lambdanormalobs7 lambdanormalobs8];
CILM8 = lambda0-C*sigmap8; MEAN8 = lambda0; CIUM8 = lambda0+C*sigmap8;
xx8=[yyi]; lim8 = [CILM8 MEAN8 CIUM8];
figure
controlchart(xx8,'chart',{'i'},'limits',lim8,'specs',[lambda0 lambda0],'width',1); title('interval t=8')

%% Interval 9
% Data
yi=[1 3 3 3 2 4 4 2 1]; n9=length(yi); s9=smu(n9,yi,mu0N); a9=alfa0+n9/2; b9=beta0+n9/2*s9; lambda9=a9/b9;
sigmap9=sqrt(a9/b9^2); sigmapostnormal9=1/sqrt(lambda9); SNormal9=std(yi,1);
lambdanormalobs9=1/SNormal9^2;
yyi=[lambdanormalobs1 lambdanormalobs2 lambdanormalobs3 lambdanormalobs4 lambdanormalobs5 ...
lambdanormalobs6 lambdanormalobs7 lambdanormalobs8 lambdanormalobs9];
CILM9 = lambda0-C*sigmap9; MEAN9 = lambda0; CIUM9 = lambda0+C*sigmap9;
xx9=[yyi]; lim9 = [CILM9 MEAN9 CIUM9];
figure
controlchart(xx9,'chart',{'i'},'limits',lim9,'specs',[lambda0 lambda0],'width',1); title('interval t=9')

%% Interval 10
% Data
yi=[1 3 3 3 2 4 4 2 1 1]; n10=length(yi); s10=smu(n10,yi,mu0N); a10=alfa0+n10/2; b10=beta0+n10/2*s10;
lambda10=a10/b10; sigmap10=sqrt(a10/b10^2); sigmapostnormal10=1/sqrt(lambda10); SNormal10=std(yi,1);
lambdanormalobs10=1/SNormal10^2;
yyi=[lambdanormalobs1 lambdanormalobs2 lambdanormalobs3 lambdanormalobs4 lambdanormalobs5 ...
lambdanormalobs6 lambdanormalobs7 lambdanormalobs8 lambdanormalobs9 lambdanormalobs10];
CILM10 = lambda0-C*sigmap10; MEAN10 = lambda0; CIUM10 = lambda0+C*sigmap10;
xx10=[yyi]; lim10 = [CILM10 MEAN10 CIUM10];
figure
controlchart(xx10,'chart',{'i'},'limits',lim10,'specs',[lambda0 lambda0],'width',1); title('interval t=10')

%% Function s
function y = smu(n,x,mu)
sum = 0;
for i = 1:n
    sum = sum+(x(i)-mu)^2;
    y = sum./n;
end
end

```

## b.- Direct method with mean posterior.

```

clear
%% Interval 1
% Prior
alfa0=2; beta0=2; muprior0=alfa0/beta0; mu0N=2.2; sigma0N=1; lambda0=1/sigma0N^2;
sigmaprior0=sqrt(alfa0/beta0^2);
%Chart control
C=0.5;
% Data
yi=[1]; n1=length(yi); s1=smu(n1,yi,mu0N); a1=alfa0+n1/2; b1=beta0+n1/2*s1; lambda1=a1/b1;
sigmap1=sqrt(a1/b1^2); sigmapostnormal1=1/sqrt(lambda1); SNormal1=std(yi,1);
lambdanormalobs1=1/SNormal1^2;
yyi=[lambdanormalobs1];
CILM1 = lambda1-C*sigmap1; MEAN1 = lambda1; CIUM1 = lambda1+C*sigmap1;
xx1=[yyi]; lim1 = [CILM1 MEAN1 CIUM1];
%figure
%controlchart(xx1,'chart',{'i'},'limits',lim1,'specs',[lambda0 lambda0],'width',1); title('interval t=1')

```

```

%% Interval 2
% Data
yi=[1 3]; n2=length(yi); s2=smu(n2,yi,mu0N); a2=alfa0+n2/2; b2=beta0+n2/2*s2; lambda2=a2/b2;
sigmap2=sqrt(a2/b2^2); sigmapostnormal2=1/sqrt(lambda2); SNormal2=std(yi,1);
lambdanormalobs2=1/SNormal2^2;
yyi=[lambdanormalobs1 lambdanormalobs2];
CILM2 = lambda2-C*s2; MEAN2 = lambda2; CIUM2 = lambda2+C*s2;
xx2=[yyi]; lim2 = [CILM2 MEAN2 CIUM2];
figure
controlchart(xx2,'chart',{'i'},'limits',lim2,'specs',[lambda0 lambda0], 'width',1); title('interval t=2')

%% Interval 3
% Data
yi=[1 3 3]; n3=length(yi); s3=smu(n3,yi,mu0N); a3=alfa0+n3/2; b3=beta0+n3/2*s3; lambda3=a3/b3;
sigmap3=sqrt(a3/b3^2); sigmapostnormal3=1/sqrt(lambda3); SNormal3=std(yi,1);
lambdanormalobs3=1/SNormal3^2;
yyi=[lambdanormalobs1 lambdanormalobs2 lambdanormalobs3];
CILM3 = lambda3-C*s3; MEAN3 = lambda3; CIUM3 = lambda3+C*s3;
xx3=[yyi]; lim3 = [CILM3 MEAN3 CIUM3];
figure
controlchart(xx3,'chart',{'i'},'limits',lim3,'specs',[lambda0 lambda0], 'width',1); title('interval t=3')

%% Interval 4
% Data
yi=[1 3 3 3]; n4=length(yi); s4=smu(n4,yi,mu0N); a4=alfa0+n4/2; b4=beta0+n4/2*s4; lambda4=a4/b4;
sigmap4=sqrt(a4/b4^2); sigmapostnormal4=1/sqrt(lambda4); SNormal4=std(yi,1);
lambdanormalobs4=1/SNormal4^2;
yyi=[lambdanormalobs1 lambdanormalobs2 lambdanormalobs3 lambdanormalobs4];
CILM4 = lambda4-C*s4; MEAN4 = lambda4; CIUM4 = lambda4+C*s4;
xx4=[yyi]; lim4 = [CILM4 MEAN4 CIUM4];
figure
controlchart(xx4,'chart',{'i'},'limits',lim4,'specs',[lambda0 lambda0], 'width',1); title('interval t=4')

%% Interval 5
% Data
yi=[1 3 3 3 2]; n5=length(yi); s5=smu(n5,yi,mu0N); a5=alfa0+n5/2; b5=beta0+n5/2*s5; lambda5=a5/b5;
sigmap5=sqrt(a5/b5^2); sigmapostnormal5=1/sqrt(lambda5); SNormal5=std(yi,1);
lambdanormalobs5=1/SNormal5^2;
yyi=[lambdanormalobs1 lambdanormalobs2 lambdanormalobs3 lambdanormalobs4 lambdanormalobs5];
CILM5 = lambda5-C*s5; MEAN5 = lambda5; CIUM5 = lambda5+C*s5;
xx5=[yyi]; lim5 = [CILM5 MEAN5 CIUM5];
figure
controlchart(xx5,'chart',{'i'},'limits',lim5,'specs',[lambda0 lambda0], 'width',1); title('interval t=5')

%% Interval 6
% Data
yi=[1 3 3 3 2 4]; n6=length(yi); s6=smu(n6,yi,mu0N); a6=alfa0+n6/2; b6=beta0+n6/2*s6; lambda6=a6/b6;
sigmap6=sqrt(a6/b6^2); sigmapostnormal6=1/sqrt(lambda6); SNormal6=std(yi,1);
lambdanormalobs6=1/SNormal6^2;
yyi=[lambdanormalobs1 lambdanormalobs2 lambdanormalobs3 lambdanormalobs4 lambdanormalobs5 ...
lambdanormalobs6];
CILM6 = lambda6-C*s6; MEAN6 = lambda6; CIUM6 = lambda6+C*s6;
xx6=[yyi]; lim6 = [CILM6 MEAN6 CIUM6];
figure
controlchart(xx6,'chart',{'i'},'limits',lim6,'specs',[lambda0 lambda0], 'width',1); title('interval t=6')

%% Interval 7
% Data
yi=[1 3 3 3 2 4 4]; n7=length(yi); s7=smu(n7,yi,mu0N); a7=alfa0+n7/2; b7=beta0+n7/2*s7; lambda7=a7/b7;
sigmap7=sqrt(a7/b7^2); sigmapostnormal7=1/sqrt(lambda7); SNormal7=std(yi,1);
lambdanormalobs7=1/SNormal7^2;
yyi=[lambdanormalobs1 lambdanormalobs2 lambdanormalobs3 lambdanormalobs4 lambdanormalobs5 ...
lambdanormalobs6 lambdanormalobs7];
CILM7 = lambda7-C*s7; MEAN7 = lambda7; CIUM7 = lambda7+C*s7;
xx7=[yyi]; lim7 = [CILM7 MEAN7 CIUM7];
figure

```

```

controlchart(xx7,'chart',{'i'},'limits',lim7,'specs',[lambda0 lambda0],'width',1); title('interval t=7')

%% Interval 8
% Data
yi=[1 3 3 3 2 4 4 2]; n8=length(yi); s8=smu(n8,yi,mu0N); a8=alfa0+n8/2; b8=beta0+n8/2*s8; lambda8=a8/b8;
sigmap8=sqrt(a8/b8^2); sigmapostnormal8=1/sqrt(lambda8); SNormal8=std(yi,1);
lambdanormalobs8=1/SNormal8^2;
yyi=[lambdanormalobs1 lambdanormalobs2 lambdanormalobs3 lambdanormalobs4 lambdanormalobs5 ...
lambdanormalobs6 lambdanormalobs7 lambdanormalobs8];
CILM8 = lambda8-C*sigmap8; MEAN8 = lambda8; CIUM8 = lambda8+C*sigmap8;
xx8=[yyi]; lim8 = [CILM8 MEAN8 CIUM8];
figure
controlchart(xx8,'chart',{'i'},'limits',lim8,'specs',[lambda0 lambda0],'width',1); title('interval t=8')

%% Interval 9
% Data
yi=[1 3 3 3 2 4 4 2 1]; n9=length(yi); s9=smu(n9,yi,mu0N); a9=alfa0+n9/2; b9=beta0+n9/2*s9; lambda9=a9/b9;
sigmap9=sqrt(a9/b9^2); sigmapostnormal9=1/sqrt(lambda9); SNormal9=std(yi,1);
lambdanormalobs9=1/SNormal9^2;
yyi=[lambdanormalobs1 lambdanormalobs2 lambdanormalobs3 lambdanormalobs4 lambdanormalobs5 ...
lambdanormalobs6 lambdanormalobs7 lambdanormalobs8 lambdanormalobs9];
CILM9 = lambda9-C*sigmap9; MEAN9 = lambda9; CIUM9 = lambda9+C*sigmap9;
xx9=[yyi]; lim9 = [CILM9 MEAN9 CIUM9];
figure
controlchart(xx9,'chart',{'i'},'limits',lim9,'specs',[lambda0 lambda0],'width',1); title('interval t=9')

%% Interval 10
% Data
yi=[1 3 3 3 2 4 4 2 1 1]; n10=length(yi); s10=smu(n10,yi,mu0N); a10=alfa0+n10/2; b10=beta0+n10/2*s10;
lambda10=a10/b10; sigmap10=sqrt(a10/b10^2); sigmapostnormal10=1/sqrt(lambda10); SNormal10=std(yi,1);
lambdanormalobs10=1/SNormal10^2;
yyi=[lambdanormalobs1 lambdanormalobs2 lambdanormalobs3 lambdanormalobs4 lambdanormalobs5 ...
lambdanormalobs6 lambdanormalobs7 lambdanormalobs8 lambdanormalobs9 lambdanormalobs10];
CILM10 = lambda10-C*sigmap10; MEAN10 = lambda10; CIUM10 = lambda10+C*sigmap10;
xx10=[yyi]; lim10 = [CILM10 MEAN10 CIUM10];
figure
controlchart(xx10,'chart',{'i'},'limits',lim10,'specs',[lambda0 lambda0],'width',1); title('interval t=10')

%% Function s
function y = smu(n,x,mu)
sum = 0;
for i = 1:n
    sum = sum+(x(i)-mu)^2;
    y = sum./n;
end
end

```

## H.12. Normal-gamma II.

### a.- Recurrent method

```

clear
%Parameters specification
x0=0.2; c=2; B=500; K=4000; J=B+K; a=0; r=0; nn=10;
%% Patern
y0=[8.05 9.4349 8.1254 8.3369 10.1147 10.1950 10.2759 9.8897 10.8090 8.8172]; mean0=mean(y0); std0=std(y0,1);
ppp = gamfit(y0); alfasp = ppp(1); betasp = ppp(2);
% Prior
Meanprior = alfasp*betasp; Varianceprior = alfasp*betasp^2; Stdprior = sqrt(Varianceprior);
% Data
yi =[9.3]; n=length(yi); MU1 = mean(yi); ST1 = std(yi,1); %Adopted ST1 = Stdprior; ST1=Stdprior;

for i=1:nn
    a(i)=0;

```

```

    %Candidate value generation
x(1,1)=x0;
for j=2:J
    x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
    if (x(j,1)<0) || (x(j,1)>15)           % >20
        p(j,1)=0;
    else p(j,1)=(gampdf(x(j,1),alfasp,betasp)*normpdf(yi(n),x(j,1),ST1))/...
        (gampdf(x(j-1,1),alfasp,betasp)*normpdf(yi(n),x(j-1,1),ST1));
    end
    u=random('Uniform',0,1,1,1);
    if u<p(j,1)
        x(j,1)=x(j,1);
        a(i)=a(i)+1;
    else x(j,1)=x(j-1,1);
        r=r+1;
    end
end
%Burn - in
for j=B:J
    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu1(i)=mean(xx); vaa1(i)=var(xx); stdd1(i)=sqrt(vaa1(i));
end

%Acceptance Rate AR
[muhat1,sigmahat1,muci1,sigmaci1] = normfit(muu1); AR1=mean(a)/J; muu11=mean(muu1); vaa11=mean(vaa1);
std11=sqrt(vaa11);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 1','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 1','FontSize',15);
%Histogram
figure
h = histogram(xx);
%Ajuste a gamma
counts = h.Values;
parmhat=gamfit(xx);
hold on
xxx=0:0.01:15;
%yyy=gampdf(xxx,phat(1),phat(2));
yyyy=gampdf(xxx,parmhat(1),parmhat(2));
%plot(xxx,yyy*max(counts)/max(yyy),'DisplayName','y=f(x) gamma')
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) gamma')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 1','FontSize',15);
hold off
legend('Location','northeastoutside')

%Data posterior
MU11=muu11; STD11=parmhat(1)*parmhat(2)^2;

%Data for prior recurrent
aa11=parmhat(1); bb11=parmhat(2);

%% Second
%Parameters specification
x0=0.2; c=2; B=500; K=4000; J=B+K; a=0; r=0; nn=10;

```



```

% Data
yi =[9.3 12.4]; n=length(yi); MU2 = mean(yi); ST2 = std(yi,1);

for i=1:nn
    a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
    x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
    if (x(j,1)<0) || (x(j,1)>15)
        p(j,1)=0;
    else p(j,1)=(gampdf(x(j,1),aa11,bb11)*normpdf(yi(n),x(j,1),ST2))/...
        (gampdf(x(j-1,1),aa11,bb11)*normpdf(yi(n),x(j-1,1),ST2));
    end
    u=random('Uniform',0,1,1,1);
    if u<p(j,1)
        x(j,1)=x(j,1);
        a(i)=a(i)+1;
    else x(j,1)=x(j-1,1);
        r=r+1;
    end
end
%Burn - in
for j=B:J
    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu2(i)=mean(xx); vaa2(i)=var(xx); stdd2(i)=sqrt(vaa2(i));
end

%Acceptance Rate AR
[muhat2,sigmahat2,muci2,sigmaci2] = normfit(muu2); AR2=mean(a)/J; muu22=mean(muu2); vaa22=mean(vaa2);
std22=sqrt(vaa22);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 2','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 2','FontSize',15);
%Histogram
figure
h = histogram(xx);
%Ajuste a gamma
counts = h.Values;
parmhat=gamfit(xx);
hold on
xxx=0:0.01:15;
%yyy=gampdf(xxx,phat(1),phat(2));
yyy=gampdf(xxx,parmhat(1),parmhat(2));
%plot(xxx,yyy*max(counts)/max(yyy),'DisplayName','y=f(x) gamma')
plot(xxx,yyy*max(counts)/max(yyy),'DisplayName','y=f(x) gamma2')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 2','FontSize',15);
hold off
legend('Location','northeastoutside')

%Data posterior
MU22=muu22; STD22=parmhat(1)*parmhat(2)^2;

%Data for prior recurrent

```

```

aa22=parmhat(1); bb22=parmhat(2);

%% Third
%Parameters specification
x0=0.2; c=2; B=500; K=4000; J=B+K; a=0; r=0; nn=10;
%Data
yi = [9.3 12.4 16.3]; n=length(yi); MU3 = mean(yi); ST3 = std(yi,1);

for i=1:nn
    a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
    x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
    if (x(j,1)<0) || (x(j,1)>20)
        p(j,1)=0;
    else p(j,1)=(gampdf(x(j,1),aa22,bb22)*normpdf(yi(n),x(j,1),ST3))/...
        (gampdf(x(j-1,1),aa22,bb22)*normpdf(yi(n),x(j-1,1),ST3));
    end
    u=random('Uniform',0,1,1,1);
    if u<p(j,1)
        x(j,1)=x(j,1);
        a(i)=a(i)+1;
    else x(j,1)=x(j-1,1);
        r=r+1;
    end
end
%Burn - in
for j=B:J
    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu3(i)=mean(xx); vaa3(i)=var(xx); stdd3(i)=sqrt(vaa3(i));
end

%Acceptance Rate AR
[muhat3,sigmahat3,muci3,sigmaci3] = normfit(muu3); AR3=mean(a)/J; muu33=mean(muu3); vaa33=mean(vaa3);
std33=sqrt(vaa33);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 3','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 3','FontSize',15);
%Histogram
figure
h = histogram(xx);
%Ajuste a gamma
counts = h.Values;
parmhat=gamfit(xx);
hold on
xxx=0:0.01:20;
%yyy=gampdf(xxx,phat(1),phat(2));
yyy=gampdf(xxx,parmhat(1),parmhat(2));
%plot(xxx,yyy*max(counts)/max(yyy),'DisplayName','y=f(x) gamma')
plot(xxx,yyy*max(counts)/max(yyy),'DisplayName','y=f(x) gamma3')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 3','FontSize',15);
hold off
legend('Location','northeastoutside')

```

```
%Data posterior
MU33=muu33; STD33=parmhat(1)*parmhat(2)^2;
```

```
%Data for prior recurrent
aa33=parmhat(1); bb33=parmhat(2);
```

## b.- Direct method

```
clear
%Parameters specification
x0=0.2; c=2; B=500; K=4000; J=B+K; a=0; r=0; nn=10;
%% Patern
y0=[8.05 9.4349 8.1254 8.3369 10.1147 10.1950 10.2759 9.8897 10.8090 8.8172]; mean0=mean(y0); std0=std(y0,1);
ppp = gamfit(y0); alfasp = ppp(1); betasp = ppp(2);
% Prior
Meanprior = alfasp*betasp; Varianceprior = alfasp*betasp^2; Stdprior = sqrt(Varianceprior);
% Data
yi =[9.3]; n=length(yi); MU1 = mean(yi); ST1 = std(yi,1); %Adopted ST1 = Stdprior; ST1=Stdprior;

for i=1:nn
    a(i)=0;
    %Candidate value generation
    x(1,1)=x0;
    for j=2:J
        x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
        if (x(j,1)<0) || (x(j,1)>15) % >20
            p(j,1)=0;
        else p(j,1)=(gampdf(x(j,1),alfasp,betasp)*normpdf(yi(n),x(j,1),ST1))/...
            (gampdf(x(j-1,1),alfasp,betasp)*normpdf(yi(n),x(j-1,1),ST1));
        end
        u=random('Uniform',0,1,1,1);
        if u<p(j,1)
            x(j,1)=x(j,1);
            a(i)=a(i)+1;
        else x(j,1)=x(j-1,1);
            r=r+1;
        end
    end
end
%Burn - in
for j=B:J
    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu1(i)=mean(xx); vaa1(i)=var(xx); stdd1(i)=sqrt(vaa1(i));
end

%Acceptance Rate AR
[muhat1,sigmahat1,muci1,sigmaci1] = normfit(muu1); AR1=mean(a)/J; muu11=mean(muu1); vaa11=mean(vaa1);
std11=sqrt(vaa11);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 1','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 1','FontSize',15);
%Histogram
figure
h = histogram(xx);
```

```

%Ajuste a gamma
counts = h.Values;
parmhat=gamfit(xx);
hold on
xxx=0:0.01:15;
%yyy=gampdf(xxx,phat(1),phat(2));
yyyy=gampdf(xxx,parmhat(1),parmhat(2));
%plot(xxx,yyy*max(counts)/max(yyy),'DisplayName','y=f(x) gamma')
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) gamma1')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 1','FontSize',15);
hold off
legend('Location','northeastoutside')

%Data posterior
MU11=muu11; STD11=parmhat(1)*parmhat(2)^2;

%Data for prior recurrent
aa11=parmhat(1); bb11=parmhat(2);

%% Second
%Parameters specification
x0=0.2; c=2; B=500; K=4000; J=B+K; a=0; r=0; nn=10;
%Data
yi =[9.3 12.4]; n=length(yi); MU2 = mean(yi); ST2 = std(yi,1);

for i=1:nn
    a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
    x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
    if (x(j,1)<0) || (x(j,1)>15)
        p(j,1)=0;
    else p(j,1)=(gampdf(x(j,1),alfasp,betasp)*normpdf(yi(n),x(j,1),ST2))/...
        (gampdf(x(j-1,1),alfasp,betasp)*normpdf(yi(n),x(j-1,1),ST2));
    end
    u=random('Uniform',0,1,1,1);
    if u<p(j,1)
        x(j,1)=x(j,1);
        a(i)=a(i)+1;
    else x(j,1)=x(j-1,1);
        r=r+1;
    end
end
%Burn - in
for j=B:J
    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu2(i)=mean(xx); vaa2(i)=var(xx); stdd2(i)=sqrt(vaa2(i));
end

%Acceptance Rate AR
[muhat2,sigmahat2,muci2,sigmaci2] = normfit(muu2); AR2=mean(a)/J; muu22=mean(muu2); vaa22=mean(vaa2);
std22=sqrt(vaa22);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 2','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(l,xx)

```

```

xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 2','FontSize',15);
%Histogram
figure
h = histogram(xx);
%Ajuste a gamma
counts = h.Values;
parmhat=gamfit(xx);
hold on
xxx=0:0.01:15;
%yyy=gampdf(xxx,phat(1),phat(2));
yyyy=gampdf(xxx,parmhat(1),parmhat(2));
%plot(xxx,yyy*max(counts)/max(yyy),'DisplayName','y=f(x) gamma')
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) gamma2')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 2','FontSize',15);
hold off
legend('Location','northeastoutside')

%Data posterior
MU22=muu22; STD22=parmhat(1)*parmhat(2)^2;

%Data for prior recurrent
aa22=parmhat(1); bb22=parmhat(2);

%% Third
%Parameters specification
x0=0.2; c=2; B=500; K=4000; J=B+K; a=0; r=0; nn=10;
%Data
yi = [9.3 12.4 16.3]; n=length(yi); MU3 = mean(yi); ST3 = std(yi,1);

for i=1:nn
    a(i)=0;
%Candidate value generation
x(1,1)=x0;
for j=2:J
    x(j,1)=random('Uniform',x(j-1,1)-c,x(j-1,1)+c,1,1);
    if (x(j,1)<0) || (x(j,1)>20)
        p(j,1)=0;
    else p(j,1)=(gampdf(x(j,1),alfasp,betasp)*normpdf(yi(n),x(j,1),ST3))/...
        (gampdf(x(j-1,1),alfasp,betasp)*normpdf(yi(n),x(j-1,1),ST3));
    end
    u=random('Uniform',0,1,1,1);
    if u<p(j,1)
        x(j,1)=x(j,1);
        a(i)=a(i)+1;
    else x(j,1)=x(j-1,1);
        r=r+1;
    end
end
%Burn - in
for j=B:J
    xx(j-B+1,1)=x(j,1);
end
%Mean and Dev.
muu3(i)=mean(xx); vaa3(i)=var(xx); stdd3(i)=sqrt(vaa3(i));
end

%Acceptance Rate AR
[muhat3,sigmahat3,muci3,sigmaci3] = normfit(muu3); AR3=mean(a)/J; muu33=mean(muu3); vaa33=mean(vaa3);
std33=sqrt(vaa33);

%Graphics
figure
k=1:J;
subplot(2,1,1);
plot(k,x)

```

```

xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Trace interval 3','FontSize',15);
l=B:J;
subplot(2,1,2);
plot(1,xx)
xlabel('Counts','FontSize',15,'FontWeight','bold');
ylabel('Mean value','FontSize',15,'FontWeight','bold'); title('Burn-in interval 3','FontSize',15);
%Histogram
figure
h = histogram(xx);
%Ajuste a gamma
counts = h.Values;
parmhat=gamfit(xx);
hold on
xxx=0:0.01:20;
%yyy=gampdf(xxx,phat(1),phat(2));
yyyy=gampdf(xxx,parmhat(1),parmhat(2));
%plot(xxx,yyy*max(counts)/max(yyy),'DisplayName','y=f(x) gamma')
plot(xxx,yyyy*max(counts)/max(yyyy),'DisplayName','y=f(x) gamma3')
xlabel('Mean posterior','FontSize',15,'FontWeight','bold');
ylabel('Counts','FontSize',15,'FontWeight','bold'); title('Interval 3','FontSize',15);
hold off
legend('Location','northeastoutside')

%Data posterior
MU33=muu33; STD33=parmhat(1)*parmhat(2)^2;

%Data for prior recurrent
aa33=parmhat(1); bb33=parmhat(2);

```

# *Curriculum vitae*

## **Technical Skills**

- Project management and engineering design in power energy, oil & gas, chemical and petrochemical fields. Engineering and maintenance management.
- Cost development and controlling. Process design. SHE framework.
- Production, quality control and logistics.
- Public administration and companies relations and negotiations.
- Plant automation and Information Systems. Consulting and organizational services.
- Organic synthesis and laboratory equipment (IR, UV, GC, HPLC, AAS, MS).
- Administration.
- Application: ASME, NORSOK, GMP and 21 CFR part 11, ISO 9001 and 14000.

## **Education**

- Engineering (Mechanical and Chemical)–Bachelor. 1973-1985.
- MBA – EAE and Nottingham Trent University.
- Engineering MSc. 2011-2015. Modelling and optimization of thermal desalination processes combined with a CCC power plant (UNED).
- Actually working in my PhD in Industrial Technologies Engineering (UNED Madrid).

## **Certifications**

- NEBOSH International General Certificate. Occupational Safety and Health. (Elinor-RRC International). Since 2016.
- Generell Godkjenning av Utenlandsk Utdanning - General recognition of foreign education - Norway- NOKUT . Since 2012.
- Basic Offshore Survival and Safety training applied to Norwegian laws. (Falck-Nutec. STAVANGER). Since 2012.

## **Volunteer experience**

- Spanish Red Cross. Since 1975.
- Ingeniører Uten Grenser - Norge. Since 2018.

## **Linkedin. URL.**

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