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MEASURING MARKET RISK THROUGH VALUE AT RISK. THE ROLE OF FAT-TAIL AND SKEWNESS DISTRIBUTIONS IN VAR ESTIMATE AND LOSS FUNCTIONS IN MODELS COMPARISON

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Y a todos los que no he nombrado y han puesto su granito de arena ….

Contents

List of Abbreviations

AGARCH	Asymmetric GARCH Model
APARCH	Asymmetric Power ARCH Model
APGARCH	Asymmetric Power GARCH Model
ARCD	Autoregressive Conditional Density
ARCH	Autoregressive Conditional Heteroskedasticity Model
AS	Asymmetric Slope Model
BCBS	Basel Committee on Bank Supervision
BMM	Block Maxima Models
BTC	Back-Testing Criterion
CAViaR	Conditional Autoregressive Specification for VaR
CGPD	Conditional Extreme Value Theory
CN	Conditional Normal
CS	Conditional *t*-Student
CVaR	Conditional Value at Risk
DQ	Dynamic Quantile Test
DTARCH	Double Threshold ARCH
EGARCH	Exponential GARCH
EGB2	Exponential Generalized Beta of the Second Kind
ES	Expected Shortfall
EVT	Extreme Value Theory
EWMA	Exponential Weight Moving Average Model
F(r)	Cumulative Distribution Function
FABL	Abad-Benito_López Firm Function
FC	Caporin Firm Function
FHS	Filtered Historical Simulation
FIAPARCH	Fractionally Integrated APARCH Model
FIEGARCH	Fractionally Integrated EGARCH Model
FIGARCH	Fractional Integrated GARCH Model
FS	Sarma Firm Function
G(z)	Distribution Function of Innovations
GARCH	Generalized Autoregressive Conditional Heterocedasticity
GCE	Gram-Charlier Expansion
GED	Generalised Error Distribution
GEV	Generalised Extreme Values Distribution

GJR-GARCH	Glosten-Jagannathan-Runkle GARCH Model
GPD	Generalised Pareto Distribution
GPD	Unconditional Extreme Value Theory
HS	Historical Simulation
IG	Indirect GARCH Model
IGARCH	Integrated GARCH Model
IHS	Inverse Hyperbolic Sign Distribution
KS	Kolmogorov-Smirnov Test
LR	Likelihood Ratio
LR_{cc}	Christoffersen's Likelihood ratio statistic for conditional coverage test
LR_{ind}	Christoffersen's Likelihood ratio statistic for independence of violations
LR_{uc}	Kupiec's likelihood ratio statistic for independence of violations
MS-GARCH	Markov-Switching GARCH Model
NAGARCH	Nonlinear Asymmetric GARCH Model
PGARCH	Power GARCH Model
POT	Peaks Over Threshold Model
Q-GARCH	Quadratic GARCH Model
RC	Caporin Regulator Function
RL	Lineal Regulator Function
RQ	Quadratic Regulator Function
RQL	Quadratic Lopez's Function
RS-APARCH	Regime-Switching Asymmetric Power GARCH Model
RS-GARCH	Regime Switching GARCH Model
RV	Realised Volatility Model
SAV	Symmetric Absolute Value Model
SGED	Skewness Error Generalised Distribution
SGT	Skewness Generalised-*t* Distribution
SQR-GARCH	Square-Root GARCH Model
SSD	Skewness Student-*t* Distribution
ST	*t*-Student Distribution
SV	Stochastic Volatility Model
TGARCH	Threshold GARCH Model
TS-GARCH	Taylor-Schwert GARCH Model
VaR	Value at Risk
VGARCH	Vector GARCH Model
VRate	Ratio of Violation

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Chapter 1

1. Introduction

One of the most important tasks that financial institutions face is to measure any asset exposure to market risk. This risk arises as a result of the changes that may suffer the price of the assets that encompass a portfolio. One of the possible measures to quantify this risk, it is the evaluation of losses likely to be incurred when the price of the portfolio assets falls. This is what Value at Risk (VaR) does. The VaR of a portfolio indicates the maximum amount that an investor may lose over a given time horizon and with a given probability.

The VaR was popularized by J.P.Morgan in the eighties when its risk management methodology, known RiskMetrics, was published. Since then and above all since the Basel Committee on Bank Supervision at the Bank for International Settlements requires the financial institution to meet capital requirements on the basis of VaR estimates, allowing them to use internal models for VaR calculations, this measurement has become a basic market risk management tool for financial institutions

The success of VaR is based on that it is essentially a simple concept, since the VaR reduces the risk associated with a portfolio to a single number. But despite this simplicity, its statistical measurement remains today a challenge. Therefore, over the years different methodologies have been developed for obtaining more accurate VaR estimates.

Thus, the first goal in this Thesis (Chapter 2) is to conduct a thorough theoretical review of existing methodologies, showing the strengths and weaknesses presented on each of them. Additionally, since there is no consensus on the best approach, a summary of the empirical results obtained by works devoted to the comparison of VaR methodologies is displayed.

As it is shown in the theoretical review, one of the most widely approaches used by financial institutions is parametric approximation approach. This approach assumes that financial returns follow a known distribution, usually the normal distribution.. This assumption simplifies the computation of VaR considerably. However, it is inconsistent with the empirical evidence, which finds that the distribution of asset returns is skewed, fat-tailed, and peaked around the mean (see Bollerslev, 1987). This implies that extreme events are much more likely to occur in practice than would be predicted by the symmetric thinner-tailed normal distribution. Consequently, the normality assumption can produce VaR estimates that are inappropriate measures of the true risk faced by financial institutions.

Since the Student-*t* distribution (ST) has fatter tails than the normal one, this distribution has been commonly used in finance and risk management, particularly to model conditional asset returns (Bollerslev, 1987). The empirical evidence of this distribution performance in estimating VaR is ambiguous. Some papers show that the ST distribution performs better than the normal distribution (see Abad and Benito, 2013; Orhan and Köksal, 2012 and Polanski and Stoja, 2010), while other papers report that the ST distribution overestimates the proportion of exceptions (see Angelidis et al., 2007 and Guermat and Harris, 2002).

The ST distribution can often account well for the excess kurtosis found in common asset returns, but this distribution does not capture the skewness of the returns. Taking this in to account, one direction for research in risk management involves searching for other distribution functions that capture this characteristic.

Thus, the second objective raised in this Thesis (Chapter 3), is the evaluation of the accuracy of some skewed and fat-tail distributions for the purpose of the VaR estimation. In Chapter 3, a comparison of a wide range of symmetric and asymmetric distributions is conducted. For such purpose, an empirical analysis using data of the main European, Americans and Asians stock indices will be performed. The comparative is performed following two directions. First, the distributions are compared in statistical terms to determine which it is the best for fitting financial returns. Second, the distributions are compared in terms of VaR, in order to select which is best for forecasting VaR.

As important as measuring market risk is to analyze the results of estimations generated, i.e. what is known by the term "backtesting". This concept refers to the procedures used to analyze the results obtained from VaR measure. Risk managers need a tool or formal procedure that allows them to analyze the VaR measure results as they are interested in choosing the best model among different alternative VaR measures. As the Basel Committee points out, backtesting is one of the key elements of risk management. In fact, in Basel III (2010), the Committee noted the need to verify the adequacy of the model using frequent backtesting, although not any particular technique was mentioned.

By making a review of the literature, backtesting procedures can be broadly classified into two groups: backtesting based on any statistical test and backtesting based on a loss function. The unconditional coverage test (Kupiec (1995)), the conditional coverage test and the independence test of Christoffersen (1998) and the Backtesting Criterion Statistic are the most usual backtesting procedures based on any statistical test.

The loss function was proposed by Lopez (1998, 1999), who pointed out that it is also important to know the magnitude of losses non-covered.

For the calculation of the uncovered losses, Lopez proposes the use of a loss function. The loss function is based on examining the distance between the observed yields and the VaR measure when losses are non-covered. Banking regulators are concerned about the number of times that losses exceed the VaR and the size of the unfunded losses. Thus, the loss function proposed by Lopez is consistent with the concern shown by regulators.

However, risk managers have a conflict between the objective of security and the goal of profit maximization. Excessive VaR requires them to keep too much capital, imposing large opportunity cost of capital to the company. Considering this fact, Sarma et al. (2003) proposed a firm's loss function, which in addition to measuring the loss non-covered, also takes into account the opportunity cost of the company when the losses are covered, i.e., the estimated VaR is above the current yield.

There are numerous loss functions raised by the literature which evaluate the uncovered losses, aligning themselves with the regulators concerns though there is little evidence of any loss function complying with risk managers' needs. Taking this into account, a new firm's loss function is proposed in this Thesis.

To last, the third goal of the Thesis (Chapter 4) is to examine whether the comparison of VaR models depends on the loss function used for such purpose.

To do so, a comparison of different VaR models using the loss functions proposed by the literature are carried out, taking into account both regulators and company risk managers concerns, and eventually checking if the results of these comparisons are robust to the loss function used.

Additionally, a new firm's loss function has been proposed, which has the advantage of of being more precise estimating the opportunity cost of the firm when the losses are covered.

Finally, the Thesis ends with some concluding remarks shown in the Chapter 5.

Chapter 2*

A Comprehensive Review of Value at Risk Methodologies

2.1. Introduction

Basel I, also called the Basel Accord, is the agreement reached in 1988 in Basel (Switzerland) by the Basel Committee on Bank Supervision (BCBS), involving the chairmen of the central banks of Germany, Belgium, Canada, France, Italy, Japan, Luxembourg, Netherlands, Spain, Sweden, Switzerland, the United Kingdom and the United States of America. This accord provides recommendations on banking regulations with regards to credit, market and operational risks. Its purpose is to ensure that financial institutions hold enough capital on account to meet obligations and absorb unexpected losses.

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For a financial institution measuring the risk it faces is an essential task. In the specific case of market risk, a possible method of measurement is the evaluation of losses likely to be incurred when the price of the portfolio assets falls. This is what Value at Risk (VaR) does. The portfolio VaR represents the maximum amount an investor may lose over a given time period with a given probability. Since the BCBS at the Bank for International Settlements requires a financial institution to meet capital requirements on the basis of VaR estimates, allowing them to use internal models for VaR calculations, this measurement has become a basic market risk management tool for financial institutions[1]. Consequently, it is not surprising that the last decade has witnessed the growth of academic literature comparing alternative modelling approaches and proposing new models for VaR estimations in an attempt to improve upon those already in existence.

Although the VaR concept is very simple, its calculation is not easy. The methodologies initially developed to calculate a portfolio VaR are (i) the variance-covariance approach, also called the Parametric method, (ii) the Historical Simulation (Non-parametric method) and (iii) the Monte Carlo simulation, which is a Semi-parametric method. As is well known, all these methodologies, usually called standard models, have numerous shortcomings, which have led to the development of new proposals (see Jorion (2001)).

Among Parametric approaches, the first model for VaR estimation is Riskmetrics, from J.P. Morgan (1996). The major drawback of this model is the normal distribution assumption for financial returns. Empirical evidence shows that financial returns do not follow a normal distribution. The second relates to the model used to estimate financial

[1] When the Basel I Accord was concluded in 1988, no capital requirement was defined for the market risk. However, regulators soon recognised the risk to a banking system if insufficient capital was held to absorb the large sudden losses from huge exposures in capital markets. During the mid-90s, proposals were tabled for an amendment to the 1988 accord, requiring additional capital over and above the minimum required for credit risk. Finally, a market risk capital adequacy framework was adopted in 1995 for implementation in 1998. The 1995 Basel I Accord amendment provided a menu of approaches for determining the market risk capital requirements.

return conditional volatility. The third involves the assumption that return is independent and identically distributed (iid). There is substantial empirical evidence to demonstrate that standardised financial returns distribution is not iid.

Given these drawbacks research on the Parametric method has moved in several directions. The first involves finding a more sophisticated volatility model capturing the characteristics observed in financial returns volatility. The second line of research involves searching for other density functions that capture skewness and kurtosis of financial returns. Finally, the third line of research considers higher-order conditional moments are time-varying.

In the context of the Non-parametric method, several Non-parametric density estimation methods have been implemented, with improvement on the results obtained by Historical Simulation. In the framework of the Semi-parametric method, new approaches have been proposed: (i) the Filtered Historical Simulation, proposed by Barone-Adesi et al. (1999); (ii) the CAViaR method, proposed by Engle and Manganelli (2004) and (iii) the conditional and unconditional approaches based on the Extreme Value Theory. In this article, we will review the full range of methodologies developed to estimate VaR, from standard models to those recently proposed. We will expose the relative strengths and weaknesses of these methodologies, from both theoretical and practical perspectives. The article's objective is to provide the financial risk researcher with all the models and proposed developments for VaR estimation, bringing him to the limits of knowledge in this field.

The paper is structured as follows. In the next section, we review a full range of methodologies developed to estimate VaR. In subsection 2.1, a non-parametric approach is presented. Parametric approaches are offered in subsection 2.2, and semi-parametric approaches in subsection 2.3. In section 2.3, the procedures for measuring VaR adequacy are described and in section 2.4, the empirical results obtained by papers dedicated to comparing VaR methodologies are shown. In section 2.5, some important topics of VaR are discussed. The last section presents the main conclusions.

2.2. Value at Risk Methods

According to Jorion (2001), "VaR measure is defined as the worst expected loss over a given horizon under normal market conditions at a given level of confidence. For instance, a bank might say that the daily VaR of its trading portfolio is $1 million at the 99 percent confidence level. In other words, under normal market conditions, only one percent of the time, the daily loss will exceed $1 million." In fact the VaR just indicates the most we can expect to lose if no negative event occurs.

The VaR is thus a conditional quantile of the asset return loss distribution. Among the main advantages of VaR are simplicity, wide applicability and universality (see Jorion (1990,1997))[2].

Let $r_1, r_2, r_3,..., r_n$ be identically distributed independent random variables representing the financial returns. Use $F(r)$ to denote the cumulative distribution function, $F(r) = \Pr(r_t < r | \Omega_{t-1})$, conditionally on the information set Ω_{t-1} that is available at time t-1. Assume that $\{r_t\}$ follows the stochastic process:

$$\begin{gathered} r_t = \mu + \varepsilon_t \\ \varepsilon_t = z_t \sigma_t \quad z_t \sim iid\left(0,1\right) \end{gathered} \tag{2.1}$$

[2] There is another market risk measurement, called Expected Shortfall (ES). ES measures the expected value of our losses if we get a loss in excess of VaR. So that, this measure tells us what to expect in a bad estate, while the VaR tells us nothing more than to expect a loss higher than the VaR itself. In section 5, we will formally define this measure besides presenting some criticisms of VaR measurement.

where $\sigma_t^2 = E\left(z_t^2 | \Omega_{t-1}\right)$ and z_t has the conditional distribution function $G(z)$, $G(z) = Pr(z_t < z | \Omega_{t-1})$ The VaR with a given probability $\alpha \in (0,1)$ denoted by VaR(α), is defined as the α quantile of the probability distribution of financial returns:

$$F(VaR(\alpha)) = Pr(r_t < VaR(\alpha)) = \alpha \text{ or } VaR(\alpha) = \inf\left\{v \middle| P(r_t \le v) = \alpha\right\}$$

This quantile can be estimated in two different ways: (1) inverting the distribution function of financial returns, $F(r)$, and (2) inverting the distribution function of innovations, $G(z)$. With regard to the latter, it is also necessary to estimate σ_t^2.

$$VaR(\alpha) = F^{-1}(\alpha) = \mu + \sigma_t G^{-1}(\alpha) \tag{2.2}$$

Hence, a VaR model involves the specifications of $F(r)$ or $G(z)$. The estimation of these functions can be carried out using the following methods: (1) Non-parametric methods (2) Parametric methods and (3) Semi-parametric methods. Below we will describe the methodologies, which have been developed in each of these three cases to estimate VaR[3].

2.2.1 Non-parametric methods

The Non-parametric approaches seek to measure a portfolio VaR without making strong assumptions about returns distribution. The essence of these approaches is to let data speak for themselves as much as possible and to use recent returns empirical distribution - not some assumed theoretical distribution - to estimate VaR.

All Non-parametric approaches are based on the underlying assumption that the near future will be sufficiently similar to the recent past for us to be able to use the data from the recent past to forecast the risk in the near future.

[3] For a more pedagogic review of some of these methodologies see Feria-Domínguez (2005).

The Non-parametric approaches include (a) Historical Simulation and (b) Non-parametric density estimation methods.

2.2.1.1. Historical Simulation

Historical Simulation is the most widely implemented Non-parametric approach. This method uses the empirical distribution of financial returns as an approximation for $F(r)$, thus $\text{VaR}(\alpha)$ is the α quantile of empirical distribution. To calculate the empirical distribution of financial returns, different sizes of samples can be considered.

The advantages and disadvantages of the Historical Simulation have been well documented by Down (2002). The two main advantages are as follows: (1) the method is very easy to implement, and (2) as this approach does not depend on parametric assumptions on the distribution of the return portfolio, it can accommodate wide tails, skewness and any other non-normal features in financial observations. The biggest potential weakness of this approach is that its results are completely dependent on the data set. If our data period is unusually quiet, Historical Simulation will often underestimate risk and if our data period is unusually volatile, Historical Simulation will often overestimate it. In addition, Historical Simulation approaches are sometimes slow to reflect major events, such as the increases in risk associated with sudden market turbulence.

The first papers involving the comparison of VaR methodologies, such as those by Beder (1995), Hendricks (1996), Beder (1996), and Pritsker (1997), reported that the Historical Simulation performed at least as well as the methodologies developed in the early years, the Parametric approach and the Monte Carlo simulation. The main conclusion of these papers is that among the methodologies developed initially, no approach appeared to perform better than the others.

However more recent papers such as those by Abad and Benito (2012), Ashley and Randall (2009), Trenca (2009), Angelidis et al. (2007), Alonso and Arcos (2005), Gento (2001), Danielsson and de Vries (2000) have reported that the Historical Simulation approach produces inaccurate VaR estimates. In comparison with other recently developed methodologies such as the Historical Simulation Filtered, Conditional Extreme Value Theory and Parametric approaches (as we become further separated from normality and consider volatility models more sophisticated than Riskmetrics), Historical Simulation provides a very poor VaR estimate.

2.2.1.2. Non-parametric density estimation methods

Unfortunately, the Historical Simulation approach does not best utilise the information available. It also has the practical drawback that it only gives VaR estimates at discrete confidence intervals determined by the size of our data set[4]. The solution to this problem is to use the theory of Non-parametric density estimation. The idea behind Non-parametric density is to treat our data set as if it were drawn from some unspecific or unknown empirical distribution function. One simple way to approach this problem is to draw straight lines connecting the mid-points at the top of each histogram bar. With these lines drawn the histogram bars can be ignored and the area under the lines treated as though it was a probability density function (pdf) for VaR estimation at any confidence level. However, we could draw overlapping smooth curves and so on. This approach conforms exactly to the theory of non-parametric density estimation, which leads to important decisions about the width of bins and where bins should be centred. These decisions can therefore make a difference to our results (for a discussion, see Butler and Schachter (1998) or Rudemo (1982)).

[4] Thus, if we have, e.g., 100 observations, it allows us to estimate VaR at the 95% confidence level but not the VaR at the 95.1% confidence level. The VaR at the 95% confidence level is given by the sixth largest loss, but the VaR at the 95.1% confidence level is a problem because there is no loss observation to accompany it.

A kernel density estimator (Silverman (1986), Sheather and Marron (1990)) is a method for generalising a histogram constructed with the sample data. A histogram results in a density that is piecewise constant where a kernel estimator results in smooth density. Smoothing the data can be performed with any continuous shape spread around each data point. As the sample size grows, the net sum of all the smoothed points approaches the true pdf whatever that may be irrespective of the method used to smooth the data.

The smoothing is accomplished by spreading each data point with a kernel, usually a pdf centred on the data point, and a parameter called the bandwidth. A common choice of bandwidth is that proposed by Silverman (1986). There are many kernels or curves to spread the influence of each point, such as the Gaussian kernel density estimator, the Epanechnikov kernel, the biweight kernel, an isosceles triangular kernel and an asymmetric triangular kernel. From the kernel, we can calculate the percentile or estimate of the VaR.

2.2.2. Parametric method

Parametric approaches measure risk by fitting probability curves to the data and then inferring the VaR from the fitted curve. Among Parametric approaches, the first model to estimate VaR was Riskmetrics from J.P. Morgan (1996). This model assumes that the return portfolio and/or the innovations of return follow a normal distribution. Under this assumption, the VaR of a portfolio at an 1-α% confidence level is calculated as $VaR(\alpha)=\mu+\sigma_t G^{-1}(\alpha)$, where $G^{-1}(\alpha)$ is the α quantile of the standard normal distribution and σ_t is the conditional standard deviation of the return portfolio. To estimate σ_t, J.P. Morgan uses an Exponential Weight Moving Average Model (EWMA). The expression of this model is as follows:

$$\sigma_t^2=(1-\lambda)\sum_{j=0}^{N-1}\lambda^j\left(\varepsilon_{t-j}\right)^2 \qquad (2.3)$$

where $\lambda = 0.94$ and the window size (N) is 74 days for daily data.

The major drawbacks of Riskmetrics are related to the normal distribution assumption for financial returns and/or innovations. Empirical evidence shows that financial returns do not follow normal distribution. The skewness coefficient is in most cases negative and statistically significant, implying that the financial return distribution is skewed to the left. This result is not in accord with the properties of a normal distribution, which is symmetric. Also, empirical distribution of financial return has been documented to exhibit significantly excessive kurtosis (fat tails and peakness) (see Bollerslev (1987)). Consequently, the size of the actual losses is much higher than that predicted by a normal distribution.

The second drawback of Riskmetrics involves the model used to estimate the conditional volatility of the financial return. The EWMA model captures some non-linear characteristics of volatility, such as varying volatility and cluster volatility, but does not take into account asymmetry and the leverage effect (see Black (1976) and Pagan and Schwert (1990)). In addition, this model is technically inferior to the GARCH family models in modelling the persistence of volatility.

The third drawback of the traditional Parametric approach involves the iid return assumption. There is substantial empirical evidence that the standardised distribution of financial returns is not iid (see Hansen (1994), Harvey and Siddique (1999), Jondeau and Rockinger (2003), Bali and Weinbaum (2007) and Brooks et al. (2005)).

Given these drawbacks research on the Parametric method has been made in several directions. The first attempts searched for a more sophisticated volatility model capturing the characteristics observed in financial returns volatility. Here, three families of volatility

models have been considered: (i) the GARCH, (ii) Stochastic Volatility and (iii) Realised volatility. The second line of research investigated other density functions that capture the skew and kurtosis of financial returns. Finally, the third line of research considered that the higher-order conditional moments are time-varying.

Using the Parametric method but with a different approach, McAleer et al. (2010a) proposed a risk management strategy consisting of choosing from among different combinations of alternative risk models to measure VaR. As the authors remark, given that a combination of forecast models is also a forecast model, this model is a novel method for estimating the VaR. With such an approach McAleer et al. (2010b) suggest using a combination of VaR forecasts to obtain a crisis robust risk management strategy. McAleer et al. (2011) present cross-country evidence to support the claim that the median point forecast of VaR is generally robust to a Global Financial Crisis.

2.2.2.1. Volatility models

The volatility models proposed in literature to capture the characteristics of financial returns can be divided into three groups: the GARCH family, the stochastic volatility models and realised volatility-based models. As to the GARCH family, Engle (1982) proposed the Autoregressive Conditional Heterocedasticity (ARCH), which featured a variance that does not remain fixed but rather varies throughout a period. Bollerslev (1986) further extended the model by inserting the ARCH generalised model (GARCH). This model specifies and estimates two equations: the first depicts the evolution of returns in accordance with past returns, whereas the second patterns the evolving volatility of returns. The most generalised formulation for the GARCH models is the GARCH (p, q) model represented by the following expression:

$$r_t = \mu_t + \varepsilon_t$$
$$\sigma_t^2 = \alpha_o + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-1}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-1}^2 \tag{2.4}$$

In the GARCH (1,1) model, the empirical applications conducted on financial series detect hat $\alpha_1 + \beta_1$ is observed to be very close to the unit. The integrated GARCH model (IGARCH) of Engle and Bollerslev (1986)[5] is then obtained forcing the condition that the addition is equal to the unit in expression (4). The conditional variance properties of the IGARCH model are not very attractive from the empirical point of view due to the very slow phasing out of the shock impact upon the conditional variance (volatility persistence). Nevertheless, the impacts that fade away show exponential behaviour, which is how the fractional integrated GARCH model (FIGARCH) proposed by Baillie, Bollerslev and Mikkelsen (1996) behaves, with the simplest specification, FIGARCH (1,d,0), being:

$$\sigma_t^2 = \frac{\alpha_o}{1-\beta_1} + \left(1 - \frac{(1-L)^d}{(1-\beta_1 L)}\right) r_t^2 . \tag{2.5}$$

If the parameters comply with the setting conditions $\alpha_0 > 0,\ 0 \le \beta_1 < d \le 1$, the conditional variance of the model is most likely positive for all t cases. With this model, there is a likelihood that the r_t^2 effect upon σ_{t+k}^2 will trigger a decline over the hyperbolic rate while k surges.

The models previously mentioned do not completely reflect the nature posed by the volatility of the financial times series because, although they accurately characterise the volatility clustering properties, they do not take into account the asymmetric performance of yields before positive or negative shocks (leverage effect). Because previous models depend on the square errors, the effect caused by positive innovations is the same as the effect

[5] The EWMA model is equivalent to the IGARCH model with the intercept α_0 being restricted to be zero, the autoregressive parameter β being set at a pre-specific value λ, and the coefficient of ε_{t-1}^2 being equal to 1-λ.

produced by negative innovations of equal absolute value. Nonetheless, reality shows that in financial time series, the existence of the leverage effect is observed, which means that volatility increases at a higher rate when yields are negative compared with when they are positive. In order to capture the leverage effect several non linear GARCH formulations have been proposed. In Table 2.1 we present some of the most popular. For a detailed review of the asymmetric GARCH models see Bollerslev (2009).

Table 2.1. Asymmetric GARCH

	Formulations	**Restrictions**
EGARCH (1,1)	$log(\sigma_t^2)=\alpha_0+\gamma\left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right)+\alpha_1\left(\left\|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right\|-\sqrt{\frac{2}{\pi}}\right)+\beta\, log(\sigma_{t-1}^2)$	$\alpha_1+\beta<1$
GJR-GARCH (1,1)	$\sigma_t^2=\alpha_o+\alpha_1\varepsilon_{t-1}^2-\gamma\,\varepsilon_{t-1}^2S_{t-1}^-+\beta\sigma_{t-1}^2$ $S_{t-1}^-=1 \quad for \quad \varepsilon_{t-1}<0 \quad and \quad S_{t-1}^-=0 \quad otherwiese$	$\alpha_o\geq 0,\ \alpha_1,\beta>0$ $\alpha_1+\beta<1$
TS GARCH (1,1)	$\sigma_t=\alpha_o+\alpha_1\|\varepsilon_{t-1}\|+\beta\sigma_{t-1}$	$\alpha_o>0,\ \alpha_1,\beta>0$ $\alpha_1+\beta<1$
TGARCH	$\sigma_t=\alpha_o+\alpha_1\|\varepsilon_{t-1}\|+\gamma\,\varepsilon_{t-1}S_{t-1}^-+\beta\sigma_{t-1}$ $S_{t-1}^-=1 \quad for \quad \varepsilon_{t-1}<0 \quad and \quad S_{t-1}^-=0 \quad otherwiese$	$\alpha_o>0,\ \alpha_1,\beta>0$ $\alpha_1+\beta<1$
PGARCH (1,1)	$\sigma_t^\delta=\omega+\alpha\|\varepsilon_{t-1}\|^\delta+\beta\sigma_{t-1}^\delta$	$\omega>0,\ \alpha\geq 0$ $\beta\geq 0,\ \delta>0$
APGARCH (1,1)	$\sigma_t^\delta=\alpha_0+\alpha_1(\|\varepsilon_{t-1}\|+\gamma\varepsilon_{t-1})^\delta+\beta\sigma_{t-1}^\delta$	$\alpha_o>0,\ \alpha_i\geq 0,\beta>0,$ $\delta>0-1<\gamma<1$
AGARCH (1,1)	$\sigma_t^2=\alpha_o+\alpha_1\varepsilon_{t-1}^2+\gamma\varepsilon_{t-1}^2+\beta\sigma_{t-1}^2$	$\alpha_0>0,\ \alpha_1,\beta>0$ $\alpha_1+\beta<1$
SQR-GARCH	$\sigma_t^2=\alpha_0+\alpha_1(\gamma\sigma_{t-1}+\varepsilon_{t-1}/\sigma_{t-1})^2+\beta\sigma_{t-1}^2$	$\alpha_0>0,\ \alpha_1,\beta>0$ $\alpha_1+\beta<1$
Q-GARCH	$\sigma_t^2=\alpha_0+\alpha_1(\varepsilon_{t-1}+\gamma\sigma_{t-1})^2+\beta\sigma_{t-1}^2$	$\alpha_0>0,\ \alpha_1,\beta>0$ $\alpha_1+\beta<1$
VGARCH	$\sigma_t^2=\alpha_0+\alpha_1(\gamma+\sigma_{t-1}^{-1}\varepsilon_{t-1})^2+\beta\sigma_{t-1}^2$	$\alpha_0>0,\ 0<\beta<1$ $0<\alpha_1<1$
NAGARCH (1,1)	$\sigma_t^2=\alpha_0+\alpha_1(\varepsilon_{t-1}+\gamma\sigma_{t-1})^2+\beta\sigma_{t-1}^2$	$\alpha_0>0,\ \alpha_1,\beta>0$ $\alpha_1+\beta<1$
MS-GARCH (1,1)	$r_t=\mu_{s_t}+\varepsilon_t=\mu_{s_t}+\sigma_t z_t \quad with \quad z_t\ iid\ N(0,1)$ $\sigma_t^2=\omega_{s_t}+\alpha_{s_t}\varepsilon_{t-1}^2+\beta_{s_t}\sigma_{t-1}^2$	s_t: state of the process at the t $\omega_{s_t}>0,\alpha_{s_t}\geq 0,$ and $\beta_{s_t}\geq 0.$
RS-APARCH	$\sigma_{s_t}^{\delta_{St}}=\omega_{s_t}+\alpha_{s_t}\left(\|\varepsilon_{t-1}\|+\gamma_{s_t1}\varepsilon_{t-1}\right)^{\delta_{St}}+\beta_{s_t2}\sigma_{t-1}^{\delta_{St}}$	$\omega_{s_t}>0,\alpha_{s_t}\geq 0,\beta_{s_t}>0$ $\delta>0 \quad -1<\gamma_{s_t1}<1$

In all models presented in this table, γ is the leverage parameter. A negative value of γ means that past negative shocks have a deeper impact on current conditional volatility than past positive shocks. Thus, we expect the parameter to be negative ($\gamma<0$). The persistence of volatility is captured by the β parameter. As for the EGARCH model, the volatility of return also depends on the size of innovations. If α_1 is positive, the innovations superior to the mean have a deeper impact on current volatility than those inferior.

Finally, it must be pointed out that there are some models that capture the leverage effect and the non-persistence memory effect. For example, Bollerslev and Mikkelsen (1996) insert the FIEGARCH model, which aims to account for both the leveraging effect (EGARCH) and the long memory (FIGARCH) effect. The simplest expression of this family of models is the FIEGARCH (1, d, 0):

$$(1-\phi L)(1-L)^d log(\sigma_t^2) = \alpha_0 + \gamma\left(\frac{r_{t-1}}{\sigma_{t-1}}\right) + \alpha_1\left(\left|\frac{r_{t-1}}{\sigma_{t-1}}\right| - \sqrt{\frac{2}{\pi}}\right) \tag{2.6}$$

Some applications of the family of GARCH models in VaR literature can be found in the following studies: Abad and Benito (2012), Sener et al. (2012), Chen et al. (2011), Chen et al. (2009), Sajjad et al. (2008), Bali and Theodossiou (2007), Angelidis et al. (2007), Haas et al. (2004), Li and Lin (2004), Carvalho et al. (2006), González-Rivera et al. (2004), Giot and Laurent (2004) and Mittnik and Paolella (2000) among others. Although there is no evidence of an overpowering model, the results obtained in these papers seem to indicate that asymmetric GARCH models produce better outcomes.

An alternative path to the GARCH models to represent the temporal changes over volatility is through the stochastic volatility (SV) models proposed by Taylor (1982, 1986). Here volatility in *t* does not depend on the past observations of the series but rather on a non-observable variable, which is usually an autoregressive stochastic process. To ensure the positiveness of the variance, the volatility equation is defined following the logarithm of the variance as in the EGARCH model.

The stochastic volatility (SV) model proposed by Taylor (1982) can be written as:

$$\begin{aligned} r_t &= \mu_t + \sqrt{h_t} z_t \quad z_t \sim N(0,1) \\ \log h_{t+1} &= \alpha + \phi \log h_t + \eta_t \qquad \eta_t \sim N(0,\sigma_\eta) \end{aligned} \tag{2.7}$$

where μ_t represents the conditional mean of the financial return, h_t represents the conditional variance, and z_t and η_t are stochastic white-noise processes.

The basic properties of the model can be found in Taylor (1986, 1994). As in the GARCH family, alternative and more complex models have been developed for the stochastic volatility models to allow for the pattern of both the large memory (see the model of Harvey (1998) and Breidt et al. (1998)) and the leverage effect (see the models of Harvey and Shephard (1996) and So, Li and Lam (2002)). Some applications of the SV model to measure VaR can be found in Fleming and Kirby (2003), Lehar et al. (2002), Chen et al. (2011) and González-Rivera et al. (2004).

The third group of volatility models is Realised Volatility (RV). The origin of the realised volatility concept is certainly not recent. Merton (1980) had already mentioned this concept, showing the likelihood of the latent volatility approximation by the addition of N intra-daily square yields over a t period, thus implying that the addition of square yields could be used for the variance estimation. Taylor and Xu (1997) showed that the daily realised volatility can be easily crafted by adding the intra-daily square yields. Assuming that a day is divided into equidistant N periods and if $r_{i,t}$ represents the intra-daily yield of the i-interval of day t, the daily volatility for day t can be expressed as:

$$\text{RV}=\left[\sum_{i=1}^{N} r_{i,t}\right]^2=\sum_{i=1}^{N} r_{i,t}^2+2\sum_{i=1}^{N}\sum_{j=i+1}^{N} r_{j,t}r_{j-i,t} \qquad (2.8)$$

In the event of yields with ¨zero¨ mean and no correlation whatsoever, then $E\left[\sum_{i=1}^{N} r_{i,t}^2\right]$ is a consistent estimator of the daily variance σ_t^2. Andersen et al. (2001a, 2001b) upheld that this measure significantly improves the forecast compared with the standard procedures, which just rely on daily data.

Although financial yields clearly exhibit leptokurtosis, the standardised yields by realised volatility are roughly normal. Furthermore, although the realised volatility distribution poses a clear asymmetry to the right, the distributions of the realised volatility logarithms are approximately Gaussian (Pong et al. (2004)). In addition, the long-term dynamics of the realised volatility logarithm can be inferred by a fractionally integrated long memory process. The theory suggests that realised volatility is a non-skewed estimator of the volatility yields and is highly efficient. The use of the realised volatility obtained from the high-frequency intra-daily yields allows for the use of traditional procedures of temporal times series to create patterns and forecasts.

One of the most representative realised volatility models is that proposed by Pong et al. (2004):

$$(1-\phi_1 L-\phi_2 L^2)(\ln RV_t-\mu)=(1-\delta_1 L)u_t \qquad (2.9)$$

As in the case of GARCH family models and stochastic volatility models, some extension of the standard RV model have been development in order to capture the leverage effect and long-range dependence of volatility. The former issue has been investigated by Bollerslev et al. (2011), Chen and Ghysels (2010) and Patton and Sheppard (2009), among others. With respect to the latter point, the autoregressive fractionally integrated model has been used by Andersen et al. (2001a, 2001b and 2003), Koopman et al. (2005) and Pong et al. (2004), among others.

a) Empirical results of volatility models in VaR

This section lists the results obtained from research on the comparison of volatility models in terms of VaR. The EWMA model provides inaccurate VaR estimates. In a comparison with other volatility models, the EWMA model scored the worst performance in forecasting VaR (see Chen et al. (2011), Abad and Benito (2012), Ñiguez (2008), Alonso and Arcos (2006), González-Rivera et al. (2004) and Huang and Lin (2004) among others). The performance of the GARCH models strongly depends on the assumption concerning returns distribution. Overall, under a normal distribution, the VaR estimates are not very accurate. However, when asymmetric and fat-tail distributions are considered, the results improve considerably.

There is scarce empirical evidence of the relative performance of the SV models against the GARCH models in terms of VaR (see Fleming and Kirby (2003), Lehar et al. (2002), González-Rivera et al. (2004) and Chen et al. (2011)). Fleming and Kirby (2003) compared a GARCH model with a SV model. They found that both models had comparable performances in terms of VaR. Lehar et al. (2002) compared option pricing models in terms of VaR using two family models: GARCH and SV. They found that as to their ability to forecast the VaR, there are no differences between the two. Chen et al. (2011) compared the performance of two SV models with a range wide of GARCH family volatility models. The comparison was conducted on two different samples. They found that the SV and EWMA models had the worst performances in estimating VaR. However, in a similar comparison, González-Rivera et al. (2004) found that the SV model had the best performance in estimating VaR. In general, with some exceptions, evidence suggests that SV models do not improve the results obtained GARCH model family.

The models based on RV work quite well to estimate VaR (see Asai et al. (2011), Brownlees and Gallo (2010), Clements et al. (2008), Giot and Laurent (2004) and Andersen et al. (2003)). Some papers show that an even simpler model, (such as an autoregressive) combined with the assumption of normal distribution for returns yields reasonable VaR estimates.

As for volatility forecasts, there are many papers in literature showing that the models based on RV are superior to the GARCH models. However, not many papers report comparisons on their ability to forecast VaR. Brownlees and Gallo (2011) compared several RV models with a GARCH and EWMA model and found that the models based on RV outperformed both EWMA and GARCH models. Along this same line, Giot and Laurent (2004) compared several volatility models: EWMA, an asymmetric GARCH and RV. The models are estimated with the assumption that returns follow either normal or skewed *t*-Student distributions. They found that under a normal distribution, the RV model performed best. However, under a skewed *t*-distribution, the asymmetric GARCH and RV models provided very similar results. These authors emphasised that the superiority of the models based on RV over the GARCH family is not as obvious when the estimation of the latter assumes the existence of asymmetric and leptokurtic distributions.

There is a lack of empirical evidence on the performance of fractional integrated volatility models to measure VaR. Examples of papers that report comparisons of these models are those by So and Yu (2006) and Beltratti and Morana (1999). The first paper compared, in terms of VaR, a FIGARCH model with a GARCH and an IGARCH model. It showed that the GARCH model provided more accurate VaR estimates. In a similar comparison that included the EWMA model, So and Yu (2006) found that FIGARCH did not outperform GARCH. The authors concluded that, although their correlation plots displayed some indication of long memory volatility, this feature is not very crucial in determining the proper value of VaR. However, in the context of the RV models, there is evidence that models that capture long memory in volatility provide accurate VaR estimates (see Andersen et al. (2003) and Asai et al. (2011)). The model proposed by Asai et al. (2011) captured long memory volatility and asymmetric features. Along this line, Ñíguez (2008) compared the ability to forecast VaR of different GARCH family models (GARCH, AGARCH, APARCH, FIGARCH and FIAPARCH, and EWMA) and found that the combination of asymmetric models with fractional integrated models provided the best results.

Although this evidence is somewhat ambiguous, the asymmetric GARCH models seem to provide better VaR estimations than the symmetric GARCH models. Evidence in favour of this hypothesis can be found in studies by Sener et al. (2012), Bali and Theodossiou (2007), Abad and Benito (2012), Chen et al. (2011), Mittnik and Paolella (2000), Huang and Lin (2004), Angelidis et al. (2007), and Giot and Laurent (2004). In the context of the models based on RV, the asymmetric models also provide better results (see Asai et al. (2011)). Some evidence against this hypothesis can be found in Angelidis et al. (2007).

Finally, some authors state that the assumption of distribution, not the volatility models, is actually the important factor for estimating VaR. Evidence supporting this issue is found in the study by Chen et al. (2011).

2.2.2.2. Density functions

As previously it has been mentioned, the empirical distribution of the financial return has been documented to be asymmetric and exhibits a significant excess of kurtosis (fat tail and peakness). Therefore, assuming a normal distribution for risk management and particularly for estimating the VaR of a portfolio does not produce good results and losses will be much higher.

As *t*-Student distribution has fatter tails than normal distribution, this distribution that has been commonly used in finance and risk management, particularly to model conditional asset return (Bollerslev (1987)). In the context of VaR methodology, some applications of this distribution can be found in studies by Cheng et al. (2011), Abad and Benito (2012), Polanski and Stoja (2010), Angelidis et al. (2007), Alonso and Arcos (2006), Guermat and Harris (2002), Billio and Pelizzon (2000), and Angelidis and Benos (2004). The empirical evidence of this distribution performance in estimating VaR is ambiguous. Some papers show that the *t*-Student distribution performs better than the normal distribution (see Abad and Benito (2012), Polanski and Stoja (2010), Alonso and Arcos (2006), and So and Yu (2006)[6]). However other papers, such as those by Angelidis et al. (2007), Guermat and Harris (2002), Billio and Pelizzon (2000), and Angelidis and Benos (2004), report that the *t*-Student distribution overestimates the proportion of exceptions.

The *t*-Student distribution can often account well for the excess kurtosis found in common asset returns, but this distribution does not capture the skewness of the return. Taking this into account, one direction for research in risk management involves searching for other distribution functions that capture these characteristics. In the context of VaR methodology, several density functions have been considered: the Skewness *t*-Student Distribution (SSD) of Hansen (1994); Exponential Generalized Beta of the Second Kind (EGB2) of McDonald and Xu (1995); Error Generalised Distribution (GED) of Nelson (1991);Skewness Error Generalised Distribution (SGED) of Theodossiou (2001); *t*-Generalised Distribution of McDonald and Newey (1988); Skewness *t*-Generalised distribution (SGT) of Theodossiou (1998) and Inverse Hyperbolic Sign (IHS) of Johnson (1949). In Table 2.2, we present the density functions of these distributions.

[6] This last paper shows that *t*-Student at 1% performs better in larger positions, although it does not in short positions.

Table 2.2. Density functions

Formulations		Restrictions
Skewness t-Student distribution (SSD) of Hansen (1994)		
$f(z_t\|v,\eta)=\begin{cases} bc\left[1+\frac{1}{\eta-2}\left(\frac{bz_t+\alpha}{1-\eta}\right)^2\right]^{-(\eta+1)/2} & if \quad z_t<-\left(\frac{a}{b}\right) \\ bc\left[1+\frac{1}{\eta-2}\left(\frac{bz_t+\alpha}{1+\eta}\right)^2\right]^{-(\eta+1)/2} & if \quad z_t\geq-\left(\frac{a}{b}\right)\end{cases}$	$a=4\lambda c\left(\frac{\eta-2}{\eta-1}\right)$ $b^2=1+3\lambda^2-a^2$ $c=\frac{\Gamma\left(\frac{\eta+1}{2}\right)}{\sqrt{\pi(\eta-2)}\Gamma\left(\frac{\eta}{2}\right)}$ $z_t=(r_t-\mu_t)/\sigma_t$	$\|\lambda\|<1$ $\eta>2$
Beta Exponential Generalised of the Second Kind (EGB2) of McDonald and Xu (1995)		
$EGB2(z_t;p;q)=C\frac{e^{p(z_t+\delta)/\theta)}}{\left(1+e^{p(z_t+\delta)/\theta)}\right)^{p+q}}$	$C=1/(B(p,q)\theta)$ $\delta=(\Psi(p)-\Psi(q))\theta$ $\theta=1/\sqrt{\Psi'(p)+\Psi'(q)}$ $z_t=(r_t-\mu_t)/\sigma_t$	p=q EGB2 symmetric p>q positively skewed p<q negatively skewed
Error Generalised (GED) of Nelson (1991)		
$f(z_t)=\frac{\eta}{\lambda\,2^{(1+1/\eta)}\Gamma(1/\eta)}exp\left\{-\left(\frac{1}{2}\right)\left\|\frac{z_t}{\lambda}\right\|^{\eta}\right\}$	$\lambda\equiv\left[2^{(-2/v)}\Gamma\left(\frac{1}{\eta}\right)/\Gamma\left(\frac{3}{\eta}\right)\right]^{0,5}$ $z_t=(r_t-\mu_t)/\sigma_t$	$-\infty<z_t<\infty$ $0<\eta<\infty$ (thickness of tail) $\eta<2$ Thinner tail than Normal $\eta=2$ Normal distribution $\eta>2$ Excess kurtosis
Skewness Generalised Error (SGED) of Theodossiou (2001)		
$f(z_t\|\lambda,k)=\frac{C}{\sigma}exp\left(-\frac{\|z_t+\delta\|^k}{\left(1+sign(z_t+\delta)\lambda\right)^k\theta^k}\right)$	$z_t=(r_t-\mu_t)/\sigma_t$ $C=k/\left(2\theta\Gamma(1/k)\right)$ $\delta=2\lambda AS(\lambda)^{-1}$ $\theta=\Gamma(1/k)^{0.5}\Gamma(3/k)^{-0.5}S(\lambda)^{-1}$ $\delta=2\lambda AS(\lambda)^{-1}$ $S(\lambda)=\sqrt{1+3\lambda^2-4A^2\lambda^2}$	$\|\lambda\|<1$ skewed parameter k=kurtosis parameter
t-Generalised Distribution (GT) McDonald and Newey (1988)		
$f(z_t\|\lambda,h,k)=\frac{k\Gamma(h)}{2\lambda\Gamma(1/k)\Gamma(h-1/k)}\left\{1+\left(\frac{\|z_t\|}{\lambda}\right)^k\right\}^{-h}$		$\lambda>0$, $k<0$, $h>0$ $-\infty<z_t<\infty$
Skewness t-Generalised Distribution (SGT) of Theodossiou (1998)		
$f(z_t\|\lambda,\eta,k)=C\left(1+\frac{\|z_t+\delta\|^k}{\left((\eta+1)/k\right)\left(1+sign(z_t+\delta)\lambda\right)^k\theta^k}\right)^{-\frac{\eta+1}{k}}$	$C=0,5k\left(\frac{\eta+1}{k}\right)^{-\frac{1}{k}}B\left(\frac{\eta}{k},\frac{1}{k}\right)^{-1}\theta^{-1}$ $\theta=\frac{1}{\sqrt{g-\rho^2}}$ $\rho=2\lambda B\left(\frac{\eta}{k},\frac{1}{k}\right)^{-1}\left(\frac{\eta+1}{k}\right)^{\frac{1}{k}}B\left(\frac{\eta-1}{k},\frac{2}{k}\right)$ $g=(1+3\lambda^2)B\left(\frac{\eta}{k},\frac{1}{k}\right)^{-1}\left(\frac{\eta+1}{k}\right)^{\frac{1}{k}}B\left(\frac{\eta-1}{k},\frac{3}{k}\right)$ $\delta=\rho\theta$	$\|\lambda\|<1$ skewed parameter $\eta>2$ tail-thickness parameter k>0 peakedness parameter δ Pearson's skewness $z_t=(r_t-\mu_t)/\sigma_t$
Inverse Hyperbolic sine (IHS) of Johnson (1949)		
$IHS(z_t\|\lambda,k)=-\frac{k}{\sqrt{2\pi\left(\theta^2+(z_t+\delta)^2\right)}}\times exp\left(-\frac{k^2}{2}\left(ln\left((z_t+\delta)+\sqrt{\theta^2+(z_t+\delta)^2}\right)-(\lambda+ln(\theta))\right)^2\right)$	$\theta=1/\sigma_w$ $\delta=\mu_w/\sigma_w$ $\sigma_w=0.5(e^{2\lambda+k^{-2}}+e^{-2\lambda+k^{-2}}+2)^{0.5}(e^{k^{-2}}-1)$	μ_w mean σ_w standard deviation $w=sinh(\lambda+x/k)$ x standard normal variable

In this line, some papers such as Duffie and Pan (1997) and Hull and White (1998) show that a mixture of normal distributions produces distributions with fatter tails than a normal distribution with the same variance.

Some applications to estimate the VaR of skewed distributions and a mixture of normal distributions can be found in Cheng et al. (2011), Polanski and Stoja (2010), Bali and Theodossiou (2008), Bali et al. (2008), Haas et al. (2004), Zhang and Cheng (2005), Haas (2009), Ausín and Galeano (2007), Xu and Wirjanto (2010) and Kuester et al. (2006).

These papers raise some important issues. First, regarding the normal and *t*-Student distributions, the skewed and fat-tail distributions seem to improve the fit of financial data (see Bali and Theodossiou (2008), Bali et al. (2008), and Bali and Theodossiou (2007)). Consistently, some studies found that the VaR estimate obtained under skewed and fat-tailed distributions provides a more accurate VaR than those obtained from a normal or *t*-Student distribution. For example, Cheng et al. (2011) compared the ability to forecast the VaR of a normal, *t*-Student, SSD and GED. In this comparison the SSD and GED distributions provide the best results. Polanski and Stoja (2010) compared the normal, *t*-Student, SGT and EGB2 distributions and found that just the latter two distributions provide accurate VaR estimates. Bali and Theodossiou (2007) compared a normal distribution with the SGT distribution. Again, they found that the SGT provided a more accurate VaR estimate. The main disadvantage of using some skewness distribution, such as SGT, is that the maximization of the likelihood function is very complicated so that it may take a lot of computational time (see Nieto and Ruiz (2008)).

Additionally, a mixture of normal distributions, *t*-Student distributions or GED distributions provided a better VaR estimate than the normal or *t*-Student distributions (see Hansen (1994), Zhang and Cheng (2005), Haas (2009), Ausín and Galeano (2007), Xu and Wirjanto (2010) and Kuester et al. (2006)). These studies showed that in the context of the

Parametric method, the VaR estimations obtained with models involving a mixture with normal distributions (and *t*-Student distributions) are generally quite precise.

Lastly, to handle the non normality of the financial return Hull and White (1998) develop a new model where the user is free to choose any probability distribution for the daily return and the parameters of the distribution are subject to an updating scheme such as GARCH. They propose transforming the daily return into a new variable that is normally distributed. The model is appealing in that the calculation of VaR is relatively straightforward and can make use of Riskmetrics or a similar database.

2.2.2.3. Higher-order conditional time-varying moments

The traditional parametric approach for conditional VaR assumes that the distribution of returns standardised by conditional means and conditional standard deviations is iid. However, there is substantial empirical evidence that the distribution of financial returns standardised by conditional means and volatility is not iid. Earlier studies also suggested that the process of negative extreme returns at different quantiles may differ from one to another (Engle and Manganelli (2004), Bali and Theodossiou (2007)). Thus, given the above, some studies developed a new approach to calculate conditional VaR. This new approach considered that the higher-order conditional moments are time-varying (see Bali et al. (2008), Polanski and Stoja (2010) and Ergun and Jun (2010)).

Bali et al. (2008) introduced a new method based on the SGT with time-varying parameters. They allowed higher-order conditional moment parameters of the SGT density to depend on the past information set and hence relax the conventional assumption in the conditional VaR calculation that the distribution of standardised returns is iid. Following Hansen (1994) and Jondeau and Rockinger (2003), they modelled the conditional high-order moment parameters of the SGT density as an autoregressive process. The maximum likelihood estimates show that the time-varying conditional volatility, skewness, tail-thickness, and peakedness parameters of the SGT density are statistically significant. In addition, they found that the conditional SGT-GARCH models with time-varying skewness and kurtosis provided a better fit or returns than the SGT-GARCH models with constant skewness and kurtosis. In their paper, they applied this new approach to calculate the VaR. The in-sample and out-of-sample performance results indicated that the conditional SGT-GARCH approach with autoregressive conditional skewness and kurtosis provided very accurate and robust estimates of the actual VaR thresholds.

In a similar study, Ergun and Jun (2010) considered the SSD distribution, which they called the ARCD model, with a time-varying skewness parameter. They found that the GARCH-based models that take conditional skewness and kurtosis into account provided an accurate VaR estimate. Along this same line, Polanski and Stoja (2010) proposed a simple approach to forecast a portfolio VaR. They employed the Gram-Charlier expansion (GCE) augmenting the standard normal distribution with the first four moments, which are allowed to vary over time. The key idea was to employ the GCE of the standard normal density to approximate the probability distribution of daily returns in terms of cumulants.[7] This approach provides a flexible tool for modelling the empirical distribution of financial data, which, in addition to volatility, exhibits time-varying skewness and leptokurtosis. This method provides accurate and robust estimates of the realised VaR. Despite its simplicity, their dataset outperformed other estimates that were generated by both constant and time-varying higher-moment models.

All previously mentioned papers compared their VaR estimates with the results obtained by assuming skewed and fat-tail distributions with constant asymmetric and kurtosis parameters. They found that the accuracy of the VaR estimates improved when time-varying asymmetric and

[7] Although in different contexts, approximating the distribution of asset returns via the GCE has been previously employed in the literature (e.g., Jarrow and Rudd (1982), Baillie and Bollerslev (1992), Jondeau and Rockinger (2001), Leon et al. (2005) and Christoffersen and Diebold (2006)).

kurtosis parameters are considered. These studies suggest that within the context of the Parametric method, techniques that model the dynamic performance of the high-order conditional moments (asymmetry and kurtosis) provide better results than those considering functions with constant high-order moments.

2.2.3. Semi-parametric methods

The Semi-parametric methods combine the Non-parametric approach with the Parametric approach. The most important Semi-parametric methods are Volatility-weight Historical Simulation, Filtered Historical Simulation (FHS), CAViaR method and the approach based on Extreme Value Theory.

2.2.3.1. Volatility-weight Historical Simulation

Traditional Historical Simulation does not take any recent changes in volatility into account. Thus, Hull and White (1998) proposed a new approach that combines the benefit of Historical Simulation with volatility models. The basic idea of this approach is to update the return information to take into account the recent changes in volatility.

Let $r_{t,i}$ be the historical return on asset *i* on day *t* in our historical sample, $\sigma_{t,i}$ be the forecast of the volatility[8] of the return on asset *i* for day *t* made at the end of *t-1*, and $\sigma_{T,i}$ be our most recent forecast of the volatility of asset *i*. Then, we replace the return in our data set, , $r_{t,i}$, with volatility-adjusted returns, as given by:

$$r^*_{t,i} = \sigma_{T,i} r_{t,i} / \sigma_{t,i} \tag{2.10}$$

According to this new approach, the $\mathrm{VaR}(\alpha)$ is the α quantile of the empirical distribution of the volatility adjusted return ($r^*_{t,i}$).

[8] To estimate the volatility of the returns, several volatility models can be used. Hull and White (1998) proposed a GARCH model and the EWMA model.

This approach directly takes into account the volatility changes, whereas the Historical Simulation approach ignores them. Furthermore, this method produces a risk estimate that is appropriately sensitive to current volatility estimates. The empirical evidence presented by Hull and White (1998) indicates that this approach produces a VaR estimate superior to that of the Historical Simulation approach.

2.2.3.2. Filtered Historical Simulation

Filtered Historical Simulation was proposed by Barone-Adesi et al. (1999). This method combines the benefits of Historical Simulation with the power and flexibility of conditional volatility models.

Suppose we use Filtered Historical Simulation to estimate the VaR of a single-asset portfolio over a 1-day holding period. In implementing this method, the first step is to fit a conditional volatility model to our portfolio return data. Barone-Adesi et al. (1999) recommended an asymmetric GARCH model. The realised returns are then standardised by dividing each one by the corresponding volatility, $z_t = (\varepsilon_t / \sigma_t)$. These standardised returns should be independent and identically distributed and therefore be suitable for Historical Simulation. The third step consists of bootstrapping a large number L of drawings from the above sample set of standardised returns.

Assuming a 1-day VaR holding period, the third stage involves bootstrapping from our data set of standardised returns: we take a large number of drawings from this data set, which we now treat as a sample, replacing each one after it has been drawn and multiplying each such random drawing by the volatility forecast 1 day ahead:

$$r_{t+1} = \mu + z^* \sigma_{t+1} \qquad (2.11)$$

where z^* is the simulated standardised return. If we take M drawings, we therefore obtain a sample of M simulated returns. With this approach, the VaR(α) is the α % quantile of the simulated return sample.

Recent empirical evidence shows that this approach works quite well in estimating VaR (see Barone-Adesi and Giannopolous (2001), Barone-Adesi et al. (2002), Zenti and Pallotta (2001), Pritsker (2001), and Giannopoulos and Tunaru (2005)). As for other methods, Zikovic and Aktan (2009), Angelidis et al. (2007), Kuester et al. (2006) and Marimoutou et al. (2009) provide evidence that this method is the best for estimating the VaR. However, other papers indicate that this approach is not better than any other (see Nozari et al. (2010) and Alonso and Arcos (2006)).

2.2.3.3. CAViaR Model

Engle and Manganelli (2004) proposed a conditional autoregressive specification for VaR. This approach is based on a quantile estimation. Instead of modelling the entire distribution, they proposed directly modelling the quantile. The empirical fact that the volatilities of stock market returns cluster over time may be translated quantitatively in that their distribution is autocorrelated. Consequently, the VaR, which is tightly linked to the standard deviation of the distribution, must exhibit similar behaviour. A natural way to formalise this characteristic is to use some type of autoregressive specification. Thus, they proposed a conditional autoregressive quantile specification that they called the CAViaR model.

Let r_t be a vector of time *t* observable financial return and β_α a p-vector of unknown parameters. Finally, let $VaR_t(\beta) \equiv VaR_t(r_{t-1}, \beta_\alpha)$ be the α quantile of the distribution of the portfolio return formed at time *t-1*, where we suppress the α subscript from β_α for notational convenience.

A generic CAViaR specification might be the following:

$$VaR_t(\beta) = \beta_0 + \sum_{i=1}^{q} \beta_i VaR_{t-i}(\beta) + \sum_{j=1}^{r} \beta_j l(x_{t-j}) \qquad (2.12)$$

where $p=q+r+1$ is the dimension of β and l is a function of a finite number of lagged observable values. The autoregressive terms $\beta_i VaR_{t-i}(\beta)$ $i=1,..,q$ ensure that the quantile changes "smoothly" over time. The role of $l(r_{t-1})$ is to link $VaR_t(\beta)$ to observable variables that belong to the information set. A natural choice for x_{t-1} is lagged returns. The advantage of this method is that it makes no specific distributional assumption on the return of the asset. They suggested the first order is sufficient for practical use:

$$VaR_t(\beta) = \beta_0 + \beta_1 VaR_{t-1}(\beta) + \beta_2 l(r_{t-i}, VaR_{t-1}) \qquad (2.13)$$

In the context of CAViaR model, different autorregresive specifications can be considered

- Symmetric absolute value (SAV):

$$VaR_t(\beta) = \beta_0 + \beta_1 VaR_{t-1}(\beta) + \beta_2 \left| r_{t-1} \right| \qquad (2.14)$$

- Indirect GARCH(1,1) (IG):

$$VaR_t(\beta) = (\beta_0 + \beta_1 VaR_{t-1}^2(\beta) + \beta_2 \left(r_{t-1}\right)^2)^{1/2} \qquad (2.15)$$

In these two models the effects of the extreme returns and the variance on the VaR measure are modeled symmetrically. To account for financial market asymmetry, via the leverage effect (Black, 1976), the SAV model was extended in Engle and Manganelli (2004) to asymmetric slope (AS):

$$VaR_t(\beta) = \beta_0 + \beta_1 VaR_{t-1}(\beta) + \beta_2 \left(r_{t-1}\right)^+ + \beta_3 \left(r_{t-1}\right)^- \qquad (2.16)$$

In this representation, $(r)^+ = \max(r_t,0)$ and $(r_t)^- = -\min(r_t,0)$ are used as the functions.

The parameters of the CAViaR models are estimated by regression quantiles, as introduced by Koenker and Basset (1978). They showed how to extend the notion of a sample quantile to a linear regression model. In order to capture leverage effects and other nonlinear characteristics of the financial return, some extensions of the CAViaR model have been proposed. Yu et al. (2010) extend the CAViaR model to include the Threshold GARCH (TGARCH) model (an extension of the double threshold ARCH (DTARCH) of Li and Li (1996)) and a mixture (an extension of Wong and Li (2001)'s mixture ARCH). Recently, Chen et al. (2011) proposed a non-linear dynamic quantile family as a natural extension of the AS model.

Although empirical literature on CAViaR method is not extensive, the results seem to indicate that the CAViaR model proposed by Engle and Manganelli (2004), fails to provide accurate VaR estimate although it may provide accurate VaR estimates in a stable period (see, Bao et al. (2006) and Polanski and Stoja (2009)). However, some recent extensions of the CAViaR model such as those proposed by Gerlach et al. (2011) and Yu et al. (2010) work pretty well in estimating VaR. As in the case of the parametric method, it appears that when use is made of an asymmetric version of the CAViaR model the VaR estimate notably improves. The paper of Sener et al. (2012) supports this hypothesis. In a comparison of several CAViaR models (asymmetric and symmetric) they find that the asymmetric CAViaR model outperforms the result from the standard CAViaR model. Gerlach et al. (2011) compared three CAViaR models (SAV, AS and Threshold CAViaR) with the Parametric method using different volatility GARCH family models (GARCH-Normal, GARCH-Student-*t*, GJR-GARCH, IGARCH, Riskmetric). At the 1% confidence level, the Threshold CAViaR model performs better than any other.

2.2.3.4. Extreme Value Theory

The Extreme Value Theory (EVT) approach focuses on the limiting distribution of extreme returns observed over a long time period, which is essentially independent of the distribution of the returns themselves. The two main models for EVT are (1) the block maxima models (BM) (McNeil (1998)) and (2) the peaks-over-threshold model (POT). The second one

is generally considered to be the most useful for practical applications due to the more efficient use of the data at extreme values. In the POT models, there are two types of analysis: the Semi-parametric models built around the Hill estimator and its relatives (Beirlant et al. (1996), Danielsson et al. (1998)) and the fully Parametric models based on the Generalised Pareto distribution (Embrechts et al. (1999)). In the coming sections each one of these approaches is described.

2.2.3.4.1. Block Maxima Models (BMM)

This approach involves splitting the temporal horizon into blocks or sections, taking into account the maximum values in each period. These selected observations form extreme events, also called a maximum block.

The fundamental BMM concept shows how to accurately choose the length period, n, and the data block within that length. For values greater than n, the BMM provides a sequence of maximum blocks $M_{n,1}, \ldots, M_{n,m}$ that can be adjusted by a generalised distribution of extreme values (GEV). The maximum loss within a group of n data is defined as $M_n = \max(X_1, X_2, \ldots, X_n)$.

For a group of identically distributed observations, the distribution function of M_n is represented as:

$$P(M_n \leq x) = P(X_1 \leq x, \ldots, X_n \leq x) = \prod_{i=1}^{n} F(x) = F^n(x) \tag{2.17}$$

The asymptotic approach for $F^n(x)$ is based on the maximum standardised value

$$Z_n = \frac{M_n - \mu_n}{\sigma_n} \qquad (2.18)$$

where μ_n and σ_n are the location and scale parameters, respectively. The theorem of Fisher and Tippet establishes that if Z_n converges to a non-degenerated distribution, this distribution is the generalised distribution of the extreme values (GEV). The algebraic expression for such a generalised distribution is as follows:

$$H_{\xi,\mu,\sigma}(x) = \begin{cases} \exp(-(1+\xi(x-\mu)/\sigma)^{-1/\xi} & \xi \neq 0 \; y \quad (1+\xi(x-\mu)/\sigma) > 0 \\ \exp(-e^{-x}) & \xi = 0 \end{cases} \qquad (2.19)$$

where $\sigma > 0$, $-\infty < \mu < \infty$, and $-\infty < \xi < \infty$. The parameter ξ is known as the shape parameter of the GEV distribution, and $\eta = \xi^{-1}$ is the index of the tail distribution, H. The prior distribution is actually a generalisation of the three types of distributions, depending on the value taken by ξ: Gumbel type I family (ξ =0), Fréchet type II family (ξ >0) and Weibull type III family (ξ <0). The ξ, σ and μ parameters are estimated using maximum likelihood. The VaR expression for the Gumbel and Fréchet distribution is as follows:

$$VaR = \begin{cases} \mu_n - \frac{\sigma_n}{\xi_n}(1-(-n\ln(\alpha))^{-\xi_n}) & to \quad \xi > 0 \quad (Fréchet) \\ \mu_n - \sigma_n \ln(-n\ln(\alpha)) & to \quad \xi = 0 \quad (Gumbel) \end{cases} \qquad (2.20)$$

In most situations, the blocks are selected in such a way that their length matches a year interval and n is the number of observations within that year period.

This method has been commonly used in hydrology and engineering applications but is not very suitable for financial time series due to the cluster phenomenon largely observed in financial returns.

2.2.3.4.2. Peaks over Threshold Models (POT)

The POT model is generally considered to be the most useful for practical applications due to the more efficient use of the data for the extreme values. In this model, we can distinguish between two types of analysis: (a) the fully Parametric models based on the Generalised Pareto distribution (GPD) and (b) the Semi-parametric models built around the Hill estimator.

2.2.3.4.2.a. Generalised Pareto Distribution

Among the random variables representing financial returns $(r_1, r_2, r_3, \ldots, r_n)$, we choose a low threshold u and examine all values (y) exceeding u: $(y_1, y_2, y_3, \ldots, y_{N_u})$, where $y_i = r_i - u$ and N_u are the number of sample data greater than u. The distribution of excess losses over the threshold u is defined as:

$$F_u(y) = P(r - u < y | r > u) = \frac{F(y+u) - F(u)}{1 - F(u)} \qquad (2.21)$$

Assuming that for a certain u, the distribution of excess losses above the threshold is a Generalised Pareto Distribution, $G_{k,\xi}(y) = 1 - \left[1 + \frac{k}{\xi} y\right]^{-1/k}$, the distribution function of returns is given by:

$$F(r) = F(y+u) = \left[1 - F(u)\right] G_{k,\sigma}(y) + F(u) \qquad (2.22)$$

To construct a tail estimator from this expression, the only additional element we need is an estimation of $F(u)$. For this purpose, we take the obvious empirical estimator $(u-N_u)/u$. We then use the historical simulation method. Introducing the historical simulation estimate of $F(u)$ and setting $r = y + u$ in the equation, we arrive at the tail estimator

$$F(r) = 1 - \frac{N_u}{n}\left[1 + \frac{k}{\xi}(r-u)\right]^{-1/k} \qquad r > u \qquad (2.23)$$

For a given probability $\alpha > F(u)$, the VaR estimate is calculated by inverting the tail estimation formula to obtain

$$VaR(\alpha) = u + \frac{\xi}{k}\left[\left[\frac{n}{N_u}(1-\alpha)\right]^{-k} - 1\right] \qquad (2.24)$$

None of the previous Extreme Value Theory-based methods for quantile estimation yield VaR estimates that reflect the current volatility background. These methods are called *Unconditional Extreme Value Theory* methods. Given the conditional heteroscedasticity characteristic of most financial data, McNeil and Frey (2000) proposed a new methodology to estimate the VaR that combines the Extreme Value Theory with volatility models, known as the *Conditional Extreme Value Theory*. These authors proposed GARCH models to estimate the current volatility and extreme value theory to estimate the distributions tails of the GARCH model shocks.

If the financial returns are a strictly stationary time series and ε follows a Generalised Pareto Distribution, denoted by $G_{k,\sigma}(\varepsilon)$, the conditional α quantile of the returns can be estimated as

$$VaR(\alpha) = \mu + \sigma_t G_{k,\sigma}^{-1}(\alpha) \qquad (2.25)$$

where σ_t^2 represents the conditional variance of the financial returns and $G_{k,\sigma}{}^{-1}(\alpha)$ is the α quantile of the GPD, which can be calculated as:

$$G_{k,\sigma}{}^{-1}(\alpha) = u + \frac{\xi}{k}\left[\left[\frac{n}{N_u}(1-\alpha)\right]^{-k} - 1\right] \qquad (2.26)$$

2.2.3.4.2.b. Hill estimator

The parameter that collects the features of the tail distribution is the tail index, $\eta = \xi^{-1}$. Hill proposed a definition of the tail index as follows:

$$\hat{\eta}_H = \left[\frac{1}{u}\left(\sum_{i=1}^{u} log(r_i) - log\, r_{u+1}\right)\right]^{-1} \qquad (2.27)$$

where r_u represents the threshold return and u is the number of observations equal to or less than the threshold return. Thus, the Hill estimator is the mean of the most extreme u observations

minus $u+1$ observations (r_{u+1}). Additionally, the associated quantile estimator is (see Danielsson and de Vries (2000)):

$$VaR(\alpha) = r_{u+1}\left(\frac{1-\alpha}{u/n}\right)^{-1/\eta} \qquad (2.28)$$

The problem posed by this estimator is the lack of any analytical means to choose the threshold value of u in an optimum manner. Hence, as an alternative, the procedure involves using the feature known as Hill graphics. Different values of the Hill index are calculated for different u values; the Hill estimator values become represented in a chart or graphic based on u, and the u value is selected from the region where the Hill estimators are relatively stable (Hill chart leaning almost horizontally). The underlying intuitive idea posed in the Hill chart is that as u increases, the estimator variance decreases, and thus, the bias is increased. Therefore, the ability to foresee a balance between both trends is likely. When this level is reached, the estimator remains constant.

Existing literature on EVT models to calculate VaR is abundant. Regarding BMM, Silva and Melo (2003) considered different maximum block widths, with results suggesting that the extreme value method of estimating the VaR is a more conservative approach for determining the capital requirements than traditional methods. Byström (2004) applied both unconditional and conditional EVT models to the management of extreme market risks in the stock market and found that conditional EVT models provided particularly accurate VaR measures. In addition, a comparison with traditional Parametric (GARCH) approaches to calculate the VaR demonstrated EVT as being the superior approach for both standard and more extreme VaR quantiles. Bekiros and Georgoutsos (2005) conducted a comparative evaluation of the predictive performance of various VaR models, with a special emphasis on two methodologies related to the EVT, POT and BM. Their results reinforced previous results and demonstrated that some "traditional" methods might yield similar results at conventional confidence levels but that the EVT methodology produces the most accurate forecasts of extreme losses at very high confidence levels. Tolikas et al. (2007) compared EVT with traditional measures (Parametric method, HS and Monte Carlo) and agreed with Bekiros and Georgoutsos (2005) on the outperformance of the EVT methods compared with the rest, especially at very high confidence levels. The only model that had a performance comparable with that of the EVT is the HS model.

Some papers showed that unconditional EVT works better than the traditional HS or Parametric approaches when a normal distribution for returns is assumed and a EWMA model is used to estimate the conditional volatility of the return (see Danielsson and de Vries (2000)).

However, the unconditional version of this approach has not been profusely used in the VaR estimation because such an approach has been overwhelmingly dominated by the conditional EVT (see McNeil and Frey (2000), Ze-To (2008), Velayoudoum et al. (2009), and Abad and Benito (2012)). Recent comparative studies of VaR models, such as Nozari et al. (2010), Zikovic and Aktan (2009) and Gençay and Selçuk (2004), show that conditional EVT approaches perform the best with respect to forecasting the VaR.

Within the POT models, an environment has emerged in which some studies have proposed some improvements on certain aspects. For example, Brooks et al. (2005) calculated the VaR by a semi-nonparametric bootstrap using unconditional density, a GARCH (1,1) model and EVT. They proposed a Semi-nonparametric approach using a GPD, and this method was shown to generate a more accurate VaR than any other method. Marimoutou et al. (2009) used different models and confirmed that the filtering process was important for obtaining better results. Ren and Giles (2007) introduced the media excessive function concept as a new way to choose the threshold. Ze-To (2008) developed a new conditional EVT-based model combined with the GARCH-Jump model to forecast extreme risks. He utilised the GARCH-Jump model to asymmetrically provide the past realisation of jump innovation to the future volatility of the return distribution as feedback and also used the EVT to model the tail distribution of the GARCH-Jump-processed residuals. The model is compared with unconditional EVT and conditional EVT-GARCH models under different distributions, normal and t-Student. He shows that the conditional EVT-GARCH-Jump model outperforms the GARCH and GARCH-*t* models. Chan and Gray (2006) proposed a model that accommodates autoregression and weekly seasonals in both the conditional mean and conditional volatility of the returns as well as leverage effects via an EGARCH specification. In addition, EVT is adopted to explicitly model the tails of the return distribution.

Finally, concerning the Hill index, some authors used the mentioned estimator, such as Bao et al. (2006), whereas others such as Bhattacharyya and Ritolia (2008) used a modified Hill estimator.

2.2.3.5. Monte Carlo

The simplest Monte Carlo procedure to estimate the VaR on date *t* on a one-day horizon at a 99% significance level consists of simulating *N* draws from the distribution of returns on date *t+1*. The VaR at a 99% level is estimated by reading off element *N*/100 after sorting the *N* different draws from the one-day returns, i.e., the VaR estimate is estimated empirically as the α quantile of the simulated distribution of returns.

However, applying simulations to a dynamic model of risk factor returns that capture path dependent behaviour, such as volatility clustering, and the essential non-normal features of their multivariate conditional distributions is important. With regard to the first of these, one of the most important features of high-frequency returns is that volatility tends to come in clusters. In this case, we can obtain the GARCH variance estimate at time *t* $\left(\hat{\sigma}_t\right)$ using the simulated returns in the previous simulation and set $r_t = \hat{\sigma}_t z_t$, where z_t is a simulation from a standard normal variable. With regard to the second item, we can model the interdependence using the standard multivariate normal or *t*-Student distribution or use copulas instead of correlation as the dependent metric.

Monte Carlo is an interesting technique that can be used to estimate the VaR for non-linear portfolios (see Estrella et al. (1994)) because it requires no simplifying assumptions about the joint distribution of the underlying data. However, it involves considerable computational expenses. This cost has been a barrier limiting its application into real-world risk containment problems. Srinivasan and Shah (2001) proposed alternative algorithms that require modest computational costs and, Antonelli and Iovino (2002) proposed a methodology that improves the computational efficiency of the Monte Carlo simulation approach to VaR estimates.

Finally, the evidence shown in the studies on the comparison of VaR methodologies agree with the greater accuracy of the VaR estimations achieved by methods other than Monte Carlo (see Abad and Benito (2012), Huang (2009), Tolikas et al. (2007) and Bao et al. (2006)).

To sum up, in this section we have reviewed some of the most important VaR methodologies, from the standard models to the more recent approaches. From a theoretical point of view, all of these approaches show advantages and disadvantages. In Table 2.3 we resume these advantages and disadvantages. In the next sections, we will review the results obtained for these methodologies from a practical point of view.

Table 2.3. Advantages and disadvantages of VaR approaches

<table>
<tr><th colspan="2"></th><th>Advantages</th><th>Disadvantages</th></tr>
<tr><td colspan="2">Non Parametric approach (HS)
Minimal assumptions made about the error distribution, nor the exact form of the dynamic specifications</td><td>• Not making strong assumptions about the distribution of the returns portfolio, they can accommodate wide tails, skewness and any other non-normal features.
• Very easy to implement.</td><td>• Results are completely dependent on the data set.
• Sometimes slow to reflect major events.
• Only allows VaR estimations at discrete confidence intervals determined by the size of our data set.</td></tr>
<tr><td colspan="2">Parametric approach
Makes a full parametric distributional and model form assumption. For example AGARCH model with Gaussian errors</td><td>• Ease of implementation when a normal or Student-t distributions is assumed.</td><td>• Ignores leptokurtosis and skewness when a normal distribution is assumed.
• Difficulties of implementation when a skewed distributions is assumed(*).</td></tr>
<tr><td colspan="2">Riskmetrics
A kind of Parametric Approach</td><td>• Ease of implementation can be calculated using a spreadsheet.</td><td>• Assumes normality of return ignoring fat tails, skewness, etc.
• Lacks non-linear property which is a significant of financial return.</td></tr>
<tr><td rowspan="4">Semi Parametric approach:
Some assumptions are made, either about the error distribution, its extremes, or the model dynamics</td><td>Filter Historical Simulation</td><td>• Approach retains the nonparametric advantage (HS) and at the same time addresses some of HS´s inherent problems, i.e. FHS take volatility background into account.</td><td>• Results slightly dependent on the data set.</td></tr>
<tr><td>ETV</td><td>• Capture curtosis and changes in volatility (conditional ETV).</td><td>• Depends on the extreme return distribution assumption.
• Results depend on the extreme data set.</td></tr>
<tr><td>CAViaR</td><td>• Makes no specific distributional assumption on the return of the asset.
• Captures non linear characteristics of the financial returns</td><td>• Difficulties of implementation.</td></tr>
<tr><td>Monte Carlo</td><td>• Large number of scenarios generated provide a more reliable and comprehensive measure of risk than analytical method.
• Captures convexity of non-linear instruments and changes in volatility and time.</td><td>• Reliance on the stochastic process specified or historical data selected to generate estimations of the final value of the portfolio and hence of the VaR
• Involves considerable computational expenses</td></tr>
</table>

(*) In addition, as the skewness distributions are not included in any statistical package, the user of this methodology have to program their code of estimation To do that, several program language can be used: MATLAB, R, GAUSS, etc. It is in this sense we say that the implementation is difficult. As the maximization of the likelihood based on several skewed distributions, such as, SGT is very complicated so that it can take a lot of computational time.

2.3. Back-testing VaR methodologies

Many authors are concerned about the adequacy of the VaR measures, especially when they compare several methods. Papers, which compare the VaR methodologies commonly use two alternative approaches: the basis of the statistical accuracy tests and/or loss functions. As to the first approach, several procedures based on statistical hypothesis testing have been proposed in the literature and authors usually select one or more tests to evaluate the accuracy of VaR models and compare them. The standard tests about ***the accuracy*** VaR models are: (i) unconditional and conditional coverage tests; (ii) the back-testing criterion and (iii) the dynamic quantile test. To implement all these tests an exception indicator must be defined. This indicator is calculated as follows:

$$I_{t+1} = \begin{cases} 1 \;\; if \;\; r_{t+1} < VaR(\alpha) \\ 0 \;\; if \;\; r_{t+1} > VaR(\alpha) \end{cases} \qquad (2.29)$$

Kupiec (1995) shows that assuming the probability of an exception is constant, then the number of exceptions ($x = \sum I_{t+1}$) follows a binomial distribution B(N, α), where N is the number of observations. An accurate VaR(α) measure should produce an *unconditional coverage* ($\hat{\alpha} = \sum I_{t+1} / N$) equal to α percent. The *unconditional coverage test* has as a null hypothesis $\hat{\alpha} = \alpha$, with a likelihood ratio statistic:

$$LR_{UC} = 2\left[log\left(\hat{\alpha}^x (1-\hat{\alpha})^{N-x}\right) - log\left(\alpha^x (1-\alpha)^{N-x}\right)\right] \qquad (2.30)$$

which follows an asymptotic $\chi^2(1)$ distribution.

Christoffersen (1998) developed a *conditional coverage test.* This jointly examines whether the percentage of exceptions is statistically equal to the one expected and the serial independence of I_{t+1}. He proposed an independence test, which aimed to reject VaR models with clustered violations. The likelihood ratio statistic of the conditional coverage test is

LR_{cc}=LR_{uc}+LR_{ind}, which is asymptotically distributed $\chi^2(2)$, and the LR_{ind} statistic is the likelihood ratio statistic for the hypothesis of serial independence against first-order Markov dependence. [9]

A similar test for the significance of the departure of $\hat{\alpha}$ from α is the *back-testing criterion* statistic:

$$Z = \left(N\hat{\alpha} - N\alpha\right) / \sqrt{N\alpha\left(1-\alpha\right)} \qquad (2.31)$$

which follows an asymptotic N(0,1) distribution.

Finally, the *Dynamic Quantile* (DQ) test proposed by Engle and Manganelli (2004) examines whether the exception indicator is uncorrelated with any variable that belongs to the information set Ω_{t-1} available when the VaR was calculated. This is a Wald test of the hypothesis that all slopes in the regression model

$$I_t = \beta_0 + \sum_{i=1}^{p} \beta_i I_{t-1} + \sum_{j=1}^{q} \mu_j X_j + \varepsilon_t \qquad (2.32)$$

are zero, where X_j are explanatory variables contained in Ω_{t-1}. VaR(α) is usually an explanatory variable to test if the probability of an exception depends on the level of the VaR.

The tests described above are based on the assumption that the parameters of the models fitted to estimate the VaR are known, although they are estimations. Escanciano and Olmo (2010) show that the use of standard unconditional and independence backtesting procedures can be misleading. They propose a correction of the standard backtesting procedures. Additionally, Escanciano and Pei (2012) propose correction when VaR is

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[9] The LR_{ind} statistic is $LR_{ind} = 2\left[\log L_A - \log L_0\right]$ and has an asymptotic $\chi^2(1)$ distribution. The likelihood function under the alternative hypothesis is $L_A = \left(1-\pi_{01}\right)^{N_{00}} \pi_{01}^{N_{01}} \left(1-\pi_{11}\right)^{N_{10}} \pi_{11}^{N_{11}}$ where N_{ij} denotes the number of observations in state $_j$ after having been in state $_i$ in the previous period, $\pi_{01} = N_{01}/(N_{00}+N_{01})$ and $\pi_{11} = N_{11}/(N_{10}+N_{11})$. The likelihood function under the null hypothesis is ($\pi_{01} = \pi_{11} = \pi = (N_{11}+N_{01})/N$) is $L_0 = \left(1-\pi\right)^{N_{00}+N_{01}} \pi^{N_{01}+N_{11}}$.

estimated with HS or FHS. On a different route, Baysal and Staum (2008) provide a test on the coverage of confidence regions and intervals involving VaR and Conditional VaR.

The second approach is based on the comparison of loss functions. Some authors compare the VaR methodologies by evaluating ***the magnitude of the losses experienced*** when an exception occurs in the models. The "magnitude loss function" that addresses the magnitude of the exceptions was developed by Lopez (1998, 1999). It deals with a specific concern expressed by the Basel Committee on Banking Supervision, which noted that the magnitude, as well as the number of VaR exceptions is a matter of regulatory concern. Furthermore, the loss function usually examines the distance between the observed returns and the forecasted VaR (α) values if an exception occurs.

Lopez (1999) proposed different loss functions:

$$lf_{t+1} = \begin{cases} z\left(r_{t+1}, VaR(\alpha)\right) & if \quad r_{t+1} < VaR(\alpha) \\ 0 & if \quad r_{t+1} > VaR(\alpha) \end{cases} \qquad (2.33)$$

where the VaR measure is penalized with the exception indicator $(z(.)=1)$, the exception indicator plus the square distance ($z(.)=1+(r_{t+1}-VaR(\alpha))^2$) or using weight ($z(r_{t+1}, VaR(\alpha), x)=k$, where x, being the number of exceptions, is divided into several zones and k is a constant which depends on zone) based on what regulators consider to be a minimum capital requirement reflecting their concerns regarding prudent capital standards and model accuracy.

More recently, other authors have proposed loss function alternatives, such as Abad and Benito (2012) who consider $z(.)=(r_{t+1}-VaR(\alpha))^2$ and $z(.)=\left|r_{t+1}-VaR(\alpha)\right|$ or Caporin (2008) who proposes $z(.)=\left|1-\left|\frac{r_{t+1}}{VaR(\alpha)}\right|\right|$ and $z(.)=\frac{(\left|r_{t+1}\right|-\left|VaR(\alpha)\right|)^2}{\left|VaR(\alpha)\right|}$. Caporin (2008) also designs a measure of the opportunity cost.

In this second approach, the best VaR model is that which minimizes the loss function. In order to know which approach provides minimum losses, different tests can be used. For instance Abad and Benito (2012) use a non-parametric test while Sener et al. (2012) use the Diebold and Mariano (1995) test as well as that of White (2000).

Another alternative to compare VaR models is to evaluate the loss in a set of hypothetical extreme market scenarios (stress testing). Linsmeier and Pearson (2000) discuss the advantages of stress testing.

2.4. Comparison of VaR methods

Empirical literature on VaR methodology is quite extensive. However, there are not many papers dedicated to comparing the performance of a large range of VaR methodologies. In Table 2.4, we resume 24 comparison papers. Basically, the methodologies compared in these papers are HS (16 papers), FHS (8 papers), the Parametric method under different distributions (22 papers included the normal, 13 papers include t-Student and just 5 papers include any kind of skewness distribution) and the EVT based approach (18 papers). Only a few of these studies include other methods, such as the Monte Carlo (5 papers), CAViaR (5 papers) and the Non-Parametric density estimation methods (2 papers) in their comparisons. For each article, we marked the methods included in the comparative exercise with a cross and shaded the method that provides the best VaR estimations.

Table 2.4. Overview of papers that compare VaR methodologies: What methodologies compare?

	HS	FHS	RM	Parametric Approaches					ETV	CF	CAViaR	MC	N-P
				N	T	SSD	MN	HOM					
Abad and Benito (2012)	x		x	x	x				x			x	
Gerlach et al. (2011)			x	x	x						x		
Sener et al. (2012)	x		x	x					x		x	X	
Ergun and Jun (2010)				x	x	X		x	x				
Nozari et al. (2010)		x							x				
Polansky and Stoja (2010)			x	x	x	X		x			x		
Brownlees and Gallo (2010)	x		x	x									x
Yu et al. (2010)			x	x	x						x		
Ozun et al. (2010)				x	x	X			x				
Huang (2009)	x			x								x	x
Marimoutou et al. (2009)	x	x		x					x				
Zikovic and Aktan (2009)	x	x		x					x				
Giamouridis and Ntoula (2009)	x	x		x					x	x			
Angelidis et al. (2007)		x		x	x	X			x				
Tolikas et al. (2007)	x			x					x			x	
Alonso and Arcos (2006)	x	x		x	x								
Bao et al. (2006)	x	x	x						x		x	x	x
Bhattacharyya and Ritolia (2008)	x			x					x				
Kuester et al. (2006)	x	x		x	x	X	x		x				
Bekeiros et al. (2005)	x		x	x	x				x				
Gençay and Selçuk (2004)	x			x	x				x				
Gençay et al. (2003)	x			x	x				x				
Darbha, G. (2001)	x			x					x				
Danielsson and de Vries (2000)			x	x	x				x				

Note: In this table, we present some empirical papers involving comparisons of VaR methodologies. The VaR methodologies are marked with a cross when they are included in a paper. A shaded cell indicates the best methodology to estimate the VaR in the paper. The VaR approaches included in these paper are the following: Historical Simulation (HS); Filtered Historical Simulation (FHS); Riskmetrics (RM); Parametric Approaches estimated under different distributions, including the normal distribution (N), t-Student distribution (T), skewed t-Student distribution (SSD), mixed normal distribution (MN) and high-order moment time-varying distribution (HOM); Extreme Value Theory (EVT); CAViaR method (CAViaR); Monte non-parametric estimation of the density function (N-P).

The approach based on the EVT is the best for estimating the VaR in 83.3% of the cases in which this method is included in the comparison, followed closely by FHS, with 62.5% of the cases. This percentage increases to 75.0% if we consider that the differences between EVT and FHS are almost imperceptible in the paper of Giamouridis and Ntoula (2009), as the authors underline. The CAViaR method ranks third. This approach is the best in 3 out of 5 comparison papers (it represents a percentage of success of 60%, which is quite high). However, we must remark that only in one of these 3 papers, ETV is included in the comparison and FHS is not included in any of them.

The worst results are obtained by HS, Monte Carlo and Riskmetrics. None of those methodologies rank best in the comparisons where they are included. Furthermore, in many of these papers HS and Riskmetrics perform worst in estimating VaR. A similar percentage of success is obtained by Parametric method under a normal distribution. Only in 2 out of 18 papers, does this methodology rank best in the comparison. It seems clear that the new proposals to estimate VaR have outperformed the traditional ones.

Taking this into account, we highlight the results obtained by Berkowitz and O´Brein (2002). In this paper the authors compare some internal VaR models used by banks with a parametric GARCH model estimated under normality. They find that the bank VaR models are not better than a simple parametric GARCH model. It reveals that internal models work very poorly in estimating VaR.

The results obtained by the Parametric method should take into account when the conditional high-order moments are time-varying. The two papers that include this method in the comparison obtained a 100% outcome success (see Ergun and Jun (2010) and Polanski and Stoja (2010)). However, only one of these papers included EVT in the comparison (Ergun and Jun (2010)).

Although not shown in the Table 2.4, the VaR estimations obtained by the Parametric method with asymmetric and leptokurtic distributions and in a mixed-distribution context are also quite accurate (see Abad and Benito (2012), Bali and Theodossiou (2007, 2008), Bali et al. (2008), Chen et al. (2011) and Polanski and Stoja (2010)). However this method does not seem to be superior to EVT and FHS (Kuester et al. (2006), Cifter and Özün (2007) and Angelidis et al. (2007)). Nevertheless, there are not many papers including these three methods in their comparison. In this line, some recent extensions of the CAViaR method, seem to perform quite well, such as those proposed by Yu et al. (2010) and Gerlach et al. (2011). This last paper compared three CAViaR models (SAV, AS and Threshold CAViaR) with the Parametric model under some distributions (GARCH-N, GARCH-*t*, GJR-GARCH, IGARCH, Riskmetric). They find that at 1% confidence level, the Threshold CAViaR model performs better than the Parametric models considered. Sener et al. (2012) carried out a comparison of a large set of VaR methodologies: HS, Monte Carlo, EVT, Riskmetrics, Parametric method under normal distribution and four CAViaR models (symmetric and asymmetric). They find that the asymmetric CAViaR model joined to the Parametric model with an EGARCH model for the volatility performs the best in estimating VaR. Abad and Benito (2012), in a comparison of a large range of VaR approaches that include EVT, find that the Parametric method under an asymmetric specification for conditional volatility and *t*-Student innovations performs the best in forecasting VaR. Both papers highlight the importance of capturing the asymmetry in volatility. Sener et al. (2012) state that the performance of VaR methods does not depend entirely on whether they are parametric, non-parametric, semi-parametric or hybrid but rather on whether they can model the asymmetry of the underlying data effectively or not.

In Table 2.5, we reconsider the papers of Table 2.4 to show which approach they use to compare VaR models. Most of the papers (62%) evaluate the performance of VaR models on the basis of the forecasting accuracy. To do that not all of them used a statistical test.

There is a significant percentage (25%) comparing the percentage of exceptions with that expected without using any statistical test. 38% of the papers in our sample consider that both the number of exceptions and their size are important and include both dimensions in their comparison.

Table 2.5. Overview of papers that compare VaR methodologies: How do they compare?

	The accuracy	Loss function
Abad and Benito (2012)	LRuc-ind-cc, BT, DQ	quadratic
Gerlach et al. (2011)	LRuc-cc, DQ	
Sener et al. (2012)	DQ	absolute
Ergun and Jun (2010)	LRuc-ind-cc	
Nozari et al. (2010)	LRuc	
Polansky and Stoja (2010)	LRuc-ind	
Brownlees and Gallo (2010)	LRuc-ind-cc, DQ	tick loss function
Yu et al. (2010)	%	
Ozun et al. (2010)	LRuc-ind-cc	quadratic's Lopez
Huang (2009)	LRuc	
Marimoutou et al. (2009)	LRuc-cc	quadratic's Lopez
Zikovic and Aktan (2009)	LRuc-ind-cc	Lopez
Giamouridis and Ntoula (2009)	LRuc-ind-cc	
Angelidis et al. (2007)	LRuc	quadratic
Tolikas et al. (2007)	LRcc	
Alonso and Arcos (2006)	BT	quadratic's Lopez
Bao et al. (2006)	%	predictive quantile loss
Bhattacharyya and Ritolia (2008)	LRuc	
Kuester et al. (2006)	LRuc-ind-cc, DQ	
Bekeiros et al. (2005)	LRuc-cc	
Gençay and Selçuk (2004)	%	
Gençay et al. (2003)	%	
Darbha, G. (2001)	%	
Danielsson and de Vries (2000)	%	

Note: In this table, we present some empirical papers involving comparisons of VaR methodologies. We indicate the test to evaluate the accuracy of VaR models and/or the loss function used in the comparative exercise. LRuc is the unconditional coverage test. LRind is the statistic for the serial independence. LRcc is the conditional coverage test. BT is the back-testing criterion. DQ is the Dynamic Quantile test. % denotes the comparison of the percentage of exceptions with the expected percentage without a statistical test.

Although there are not many articles dedicated to the comparison of a wide range of VaR methodologies, the existing offer quite conclusive results. These results show that the approach based on the EVT and FHS is the best method to estimate the VaR. We also note that VaR estimates obtained by some asymmetric extensions of CAViaR method and the Parametric method under the skewed and fat-tail distributions lead to promising results, especially when the assumption that the standardised returns is iid. is abandoned and the conditional high-order moments are considered to be time-varying.

2.5. Some important topics of VaR methodology

As we stated in the introduction, VaR is by far the leading measure of portfolio risk in use in major commercial banks and financial institutions. However, this measurement it is not exempt from criticism. Some researchers have remarked that VaR is not a *coherent* market measure (see Artzner et al. (1999)). These authors define a set of criteria necessary for what they call a "coherent" risk measurement. These criteria include homogeneity (larger positions are associated with greater risk), monotonicity (if a portfolio has systematically lower returns than another for all states of the world, its risk must be greater), subadditivity (the risk of the sum cannot be greater than the sum of the risk) and the risk free condition (as the proportion of the portfolio invested in the risk free asset increases, portfolio risk should decline). They show that VaR is not a coherent risk measure because it violates one of their axioms. In particular VaR does not satisfy the subadditivity condition and it may discourage diversification. On this point Artzner et al. (1999) proposed an alternative risk measure related to VaR which is called Tail Conditional Expection, also called Conditional Value at Risk (CVaR).The CVaR measures the expected loss in the α% worst cases and is given by

$$CVaR_t^{\alpha} = -\underset{t-1}{E}\left\{R_t \middle| R_t \le -VaR_t^{\alpha}\right\} \qquad (2.33)$$

The CVaR is a coherent measure of risk when it is restricted to continuous distributions. However, it can violate sub-additivity with non-continuous distributions. Consequently, Acerbi and Tasche (2002) proposed the Expected Shortfall (ES) as a coherent measure of risk. The ES is given by

$$ES_t^{\alpha} = CVaR_t^{\alpha} + (\lambda - 1)\left(CVaR_t^{\alpha} - VaR_t^{\alpha}\right) \qquad (2.34)$$

where $\lambda \equiv \frac{P_{t-1}\left[R_t \leq -VaR_t^{\alpha}\right]}{\alpha} \geq 1$. Note that CVaR=ES when the distribution of returns is continuous. However, it is still coherent when the distribution of returns is not continuous. The ES has also several advantages when compared with the more popular VaR. First of all, the ES is free of tail risk in the sense that it takes into account information about the tail of the underlying distribution. The use of a risk measure free of tail risk avoids extreme losses in the tail. Therefore, the ES is an excellent candidate for replacing VaR for financial risk management purposes.

Despite the advantages of ES, it is still less used than VaR. The principal reason for this pretermission is that the ES backtest is harder than VaR one. In that sense, in the last years some ES backtesting procedures have been developed. We can cite here the residual approach introduced by McNeil and Frey (2000), the censored Gaussian approach proposed by Berkowitz (2001), the functional delta approach of Kerkhof and Melenberg (2004), and the saddlepoint technique introduced by Wong (2008, 2010).

However, these approaches present some drawbacks. The backtests of McNeil and Frey (2000), Berkowitz (2001) and Kerkhof and Melenberg (2004) rely on asymptotic test statistics that might be inaccurate when the sample size is small. The test proposed by Wong (2008) is robust to these questions, nonetheless it has some disadvantages (as with the Gaussian distribution assumption).

Regardless of the sector in which a financial institution participates, all such institutions are subject to three types of risk: market, credit and operational. So, to calculate the total VaR of a portfolio it is necessary to combine these risks. There are different approximations to carry this out. First, an approximation that sums up the three types of risk (VaR). As VaR is not a subadditivity measure this approximation overestimates total risk or economic capital. Second, assuming joint normality of the risk factors, this approximation imposes tails than are thinner than the empirical estimates and significantly underestimates economic capital and the third approach to asses the risk aggregation is based on using copulas. To obtain the total VaR of a portfolio it is necessary to obtain the joint return distribution of the portfolio. Copulas allow us to solve this problem by combining the specific marginal distributions with a dependence function to create this joint distribution. The essential idea of the copula approach is that a joint distribution can be factored into the marginals and a dependence function, called copula. The term copula is based on the notion of coupling: the copula couples the marginal distributions together to form a joint distribution. The dependence relation is entirely determined by the copula, while location, scaling and shape are entirely determined by the marginals. Using a copula, marginal risk that is initially estimated separately can then be combined in a joint risk distribution preserving the marginals original characteristics. This is sometimes referred to as obtaining a joint density with predetermined marginals. The joint distribution can then be used to calculate the quantiles of the portfolio return distribution, since the portfolio returns are a weighted average on individual returns. Embrechts et al. (1999, 2002) were among the first to introduce this methodology in financial literature. Some applications of copulas focussing on cross-risk aggregation for financial institutions can be found in Alexander and Pezier (2003), Ward and Lee (2002) and Rosemberg and Schuermann (2006).

2.6. Conclusion

In this article we review the full range of methodologies developed to estimate the VaR, from standard models to the recently proposed and present their relative strengths and weaknesses from both theoretical and practical perspectives.

The performance of the parametric approach in estimating the VaR depends on the assumed distribution of the financial return and on the volatility model used to estimate the conditional volatility of the returns. As for the return distribution, empirical evidence suggests that when asymmetric and fat-tail distributions are considered, the VaR estimate improves considerably. Regardless of the volatility model used, the results obtained in the empirical literature indicate the following. (i) The EWMA model provides inaccurate VaR estimates. (ii) The performance of the GARCH models strongly depends on the assumption of returns distribution. Overall, under a normal distribution, the VaR estimates are not very accurate, but when asymmetric and fat-tail distributions are applied, the results improve considerably. (iii) Evidence suggests with some exceptions that SV models do not improve the results obtained by the family of GARCH models. (iv) The models based on the realised volatility work quite well to estimate VaR, outperforming the GARCH models estimated under a normal distribution. Additionally, Markov-Switching GARCH outperforms the GARCH models estimated under normality. In the case of the realised volatility models, some authors indicate that its superiority compared with the GARCH family is not as high when the GARCH models are estimated assuming asymmetric and fat-tail returns distributions. (v) In the GARCH family, the fractional-integrated GARCH models do not appear to be superior to the GARCH models. However, in the context of the realised volatility models, there is evidence that models, which capture long memory in volatility provide more accurate VaR estimates. (vi) Although evidence is somewhat ambiguous, asymmetric volatility models appear to provide a better VaR estimate than symmetric models.

Although there are not many works dedicated to the comparison of a wide range of VaR methodologies, the existing offer quite conclusive results. These results show that the approaches based on the EVT and FHS are the best method to estimate the VaR. We also note that VaR estimates obtained by some asymmetric extension of CAViaR method and the Parametric method under the skewed and fat-tail distributions lead to promising results, especially when the assumption that the standardised returns is iid is abandoned and that the conditional high-order moments are considered to be time-varying. It seems clear that the new proposals to estimate VaR have outperformed the traditional ones.

To further the research, it would be interesting to explore whether in the context of an approach based on the EVT and FHS considering asymmetric and fat-tail distributions to model the volatility of the returns could help to improve the results obtained by these methods. Along this line, results may be further improved by applying the realised volatility model and Markov-switching model.

Chapter 3*

Evaluating the performance of the skewed distributions to forecast Value at Risk in the Global Financial Crisis

3.1. Introduction

A primary tool for financial risk assessment is Value at Risk (VaR). It is defined as the maximum loss expected of a portfolio of assets over a certain holding period at a given confidence level. Since the Basel Committee on Bank Supervision at the Bank for International Settlements requires the financial institution to meet capital requirements on the basis of VaR estimates, allowing them to use internal models for VaR calculations, this measurement has become a basic market risk management tool for financial institutions.

Despite VaR´s conceptual simplicity, its calculation could be rather complex. Many approaches have been developed to forecast VaR: non parametric approaches, eg, Historical Simulation; semi-parametrics approaches, eg, Extreme Value Theory and the Dynamic quantile

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regression CAViaR model (Engle and Manganelli (2004)); and parametric approaches eg, RiskMetrics (J.P. Morgan (1996)).

The parametric approach is one of the most used by financial institutions. This approach usually assumes that the asset returns follow a normal distribution. This assumption simplifies the computation of VaR considerably. However, it is inconsistent with the empirical evidence of asset returns, which finds that the distribution of asset returns is skewed, fat-tailed, and peaked around the mean (see Bollerslev (1987)). This implies that extreme events are much more likely to occur in practice than would be predicted by the symmetric thinner-tailed normal distribution. Furthermore, the normality assumption can produce VaR estimates that are inappropriate measures of the true risk faced by financial institutions.

Since the *t*-Student distribution (ST) has fatter tails than the normal one, this distribution has been commonly used in finance and risk management, particularly to model conditional asset returns (Bollerslev (1987)). The empirical evidence of this distribution performance in estimating VaR is ambiguous. Some papers show that the ST distribution performs better than the normal distribution (see Abad and Benito (2013), Orhan and Köksal (2012) and Polanski and Stoja (2010)) while other papers report that the ST distribution overestimates the proportion of exceptions (see Angelidis et al. (2007) and Guermat and Harris (2002)).

The ST distribution can often account well for the excess kurtosis found in common asset returns, but this distribution does not capture the skewness of the returns. Taking this into account, one direction for research in risk management involves searching for other distribution functions that capture this characteristic. The skewness Student-t distribution (SSD) of Hansen (1994), the exponential generalized beta of the second kind (EGB2) of McDonald and Xu (1995), the generalised error distribution (GED) of Nelson (1991), the skewness generalised-t distribution (SGT) of Theodossiou (1998), the skewness error generalised distribution (SGED) of Theodossiou (2001) and the inverse hyperbolic sign (IHS) of Johnson (1949) are the most used in VaR literature. Some applications of skewness distributions to forecast the VaR can be found in Chen et al. (2012), Polanski and Stoja (2010), Bali and Theodossiou (2008), Bali et al. (2008), Haas et al. (2004), Zhang and Cheng (2005), Haas (2009), Ausín and Galeano (2007), Xu and Wirjanto (2010) and Kuester et al. (2006). Chen et al. (2012) compared the ability to forecast the VaR of a normal, ST, SSD and GED. In this comparison the SSD and GED distributions provide the best results. Polanski and Stoja (2010) compared the normal, ST, SGT and EGB2 distributions and found that just the latter two distributions provide accurate VaR estimates. Bali and Theodossiou (2008) compared a normal distribution with the SGT distribution and showed that the SGT provided a more accurate VaR estimate.

In this paper we carry out a comprehensive comparison of the skewed distributions aforementioned: SSD, SGT, SGED and IHS. Besides, we include both the normal and the ST distribution. The comparative is performed following two directions. First, we compare the distributions in statistical terms to determine which it is the best for fitting financial returns. Then, we compare the distributions in terms of VaR, in order to select which is best for forecasting VaR.

The main differences with the previous literature are as follows: (1) we consider a larger number of skewed distributions; (2) the comparison in statistical terms is made using a large battery of tests: Likelihood ratio, Chi-square (Chi2) and Kolmogorov-Smirnov (KS) test; the papers aforementioned only used the likelihood ratio test; 3) to carry out the comparison in terms of VaR we evaluate the results on the basis of two criteria: (i) the accuracy of VaR and (ii) the minimization of two loss functions which reflect the concerns of the financial regulator and the firm (Sarma et al. (2003)).

In the next section, we present the methodology used to estimate the VaR and summarize the statistical tests and the loss functions that we have used to evaluate the VaR estimates. In section 3.3, we present the data. The results of the comparison in statistical terms and in terms of VaR are presented in sections 3.4 and 3.5 respectively. The last section includes the main conclusions.

3.2. Methodology

According to Jorion (2001), VaR measure is defined as the worst expected loss over a given horizon under normal market conditions at a given level of confidence. The VaR is thus a conditional quantile of the asset return distribution. Let $r_1, r_2, r_3,..., r_n$ be identically distributed independent random variables representing the financial returns. Use $F(r)$ to denote the cumulative distribution function, $F(r)=Pr(r_t<r|\Omega_{t-1})$, conditionally on the information set Ω_{t-1} that is available at time t-1. Assume that r_t follows the stochastic process $r_t=\mu+\varepsilon_t$ where $\varepsilon_t = z_t\sigma_t \quad z_t \sim iid(0,1)$, μ is the conditional mean, σ_t the conditional standard deviation of returns. The VaR with a given probability $\alpha \in (0,1)$, denoted by VaR(α), is defined as the α quantile of the probability distribution of financial returns: $F(VaR(\alpha)) = Pr(r_t < VaR(\alpha)) = \alpha$.

Under the framework of the parametric techniques (see Jorion (2001)), the conditional VaR estimate can be calculated as $VaR_t = \mu_t + \hat{\sigma}_t k_\alpha$, where μ_t represents the conditional mean, which we assume is zero, $\hat{\sigma}_t$ sigma is the conditional standard deviation and k_α denotes the corresponding quantile of the distribution of the standardized returns at a given confidence level 1-α .[10]

Having obtained significant evidence from the Engle and Ng (1993) test on the fact that good and bad news have a different impact on conditional volatilities of asset returns, we use the Exponential GARCH model of Nelson (1991) to estimate σ_t needed for conditional VaR analysis[11,12]. Finally, once the variance has been calculated, we estimate the distributions of the

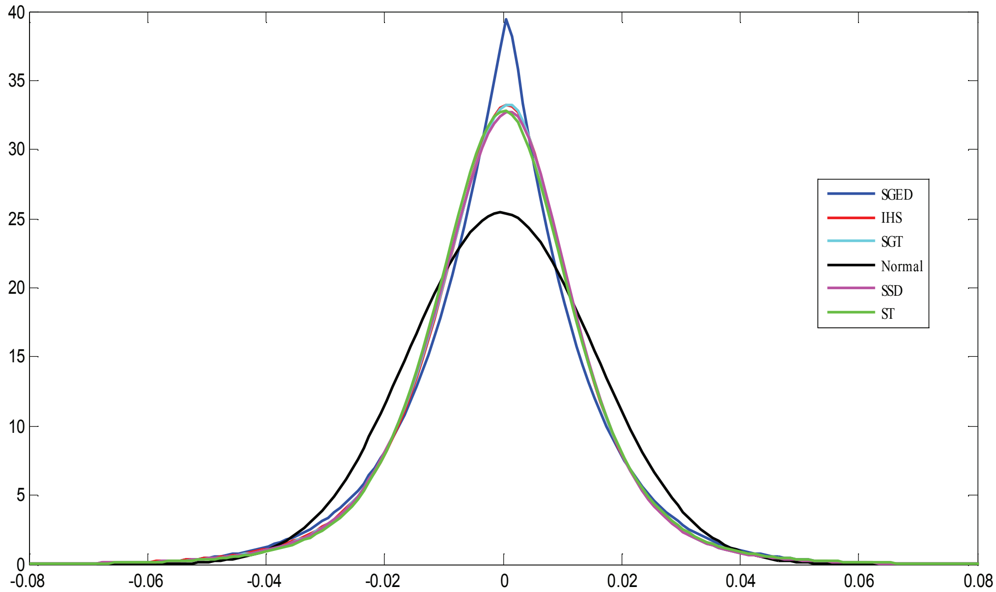
[10] In case of the skewed distributions the k_α value is a function of the skewness and kurtosis parameters.

[11] The EGARCH models have been estimated below a ST distribution.

standardized returns under each of the considered distribution functions: normal, ST, SGT, SGED, SSD and IHS.

Table 3.1 shows the density function of these skewed distributions and their graphs for Nikkei index are shown in Figure 3.1.

Figure 3.1. Density functions



Note: This figure depicts the considered distributions. The data used in the graphs are those obtained from the Nikkei Index and the sample spans from January 3, 2000 to November 30, 2012.

[12] The squared daily returns, as employed by GARCH models are not the most efficient measure of the daily volatility. The recent literature has focused on the realized volatility and daily stock ranges. Latest are also known to be more efficient measures of return volatility than daily returns, since they employ all price changes during the day. Chou et al. (2009) present a thorough review of range-based models. Lin, Chen and Gerlach (201) propose a nonlinear smooth transition conditional autoregressive range model for capturing smooth volatility asymmetries in international financial stock markets. However, we consider only GARCH models because the database is daily based.

Table 3.1. Density functions

	Formulations		Restrictions
SSD of Hansen (1994)	$f(z_t\|v,\eta)=\begin{cases} bc\left[1+\frac{1}{\eta-2}\left(\frac{bz_t+\alpha}{1-\eta}\right)^2\right]^{-(\eta+1)/2} & if \quad z_t<-\left(\frac{a}{b}\right) \\ bc\left[1+\frac{1}{\eta-2}\left(\frac{bz_t+\alpha}{1+\eta}\right)^2\right]^{-(\eta+1)/2} & if \quad z_t\geq-\left(\frac{a}{b}\right)\end{cases}$	$a=4\lambda c\left(\frac{\eta-2}{\eta-1}\right)$ $b^2=1+3\lambda^2-a^2$ $c=\frac{\Gamma\left(\frac{\eta+1}{2}\right)}{\sqrt{\pi(\eta-2)\Gamma\left(\frac{\eta}{2}\right)}}$ $z_t=(r_t-\mu_t)/\sigma_t$	$\|\lambda\|<1$ $\eta>2$
SGED of Theodossiou (2001)	$f(z_t\|\lambda,k)=\frac{C}{\sigma}exp\left(-\frac{\|z_t+\delta\|^k}{\left(1+sign(z_t+\delta)\lambda\right)^k\theta^k}\right)$	$z_t=(r_t-\mu_t)/\sigma_t$ $C=k/\left(2\theta\Gamma(1/k)\right)$ $\delta=2\lambda AS(\lambda)^{-1}$ $\theta=\Gamma(1/k)^{0.5}\Gamma(3/k)^{-0.5}S(\lambda)^{-1}$ $\delta=2\lambda AS(\lambda)^{-1}$ $S(\lambda)=\sqrt{1+3\lambda^2-4A^2\lambda^2}$	$\|\lambda\|<1$ *skewed parameter* *k=kurtosis parameter*
SGT of Theodossiou (1998)	$f(z_t\|\lambda,\eta,k)=C\left(1+\frac{\|z_t+\delta\|^k}{\left((\eta+1)/k\right)\left(1+sign(z_t+\delta)\lambda\right)^k\theta^k}\right)^{-\frac{\eta+1}{k}}$	$C=0{,}5k\left(\frac{\eta+1}{k}\right)^{-\frac{1}{k}}B\left(\frac{\eta}{k},\frac{1}{k}\right)^{-1}\theta^{-1}$ $\theta=\frac{1}{\sqrt{g-\rho^2}}$ $\rho=2\lambda B\left(\frac{\eta}{k},\frac{1}{k}\right)^{-1}\left(\frac{\eta+1}{k}\right)^{\frac{1}{k}}B\left(\frac{\eta-1}{k},\frac{2}{k}\right)$ $g=(1+3\lambda^2)B\left(\frac{\eta}{k},\frac{1}{k}\right)^{-1}\left(\frac{\eta+1}{k}\right)^{\frac{1}{k}}B\left(\frac{\eta-1}{k},\frac{3}{k}\right)$ $\delta=\rho\theta$ $z_t=(r_t-\mu_t)/\sigma_t$	$\|\lambda\|<1$ skewed parameter $\eta>2$ tail-thickness parameter $\kappa>0$ peakedness parameter δ Pearson's parameter
IHS of Johnson (1949)	$IHS(z_t\|\lambda,k)=-\frac{k}{\sqrt{2\pi\left(\theta^2+(z_t+\delta)^2\right)}}\times exp\left(-\frac{k^2}{2}\left(ln\left((z_t+\delta)+\sqrt{\theta^2+(z_t+\delta)^2}\right)-(\lambda+ln(\theta))\right)^2\right)$	$\theta=1/\sigma_w$ $\delta=\mu_w/\sigma_w$ $\sigma_w=0.5(e^{2\lambda+k^{-2}}+e^{-2\lambda+k^{-2}}+2)^{0.5}(e^{k^{-2}}-1)$	μ_w *mean* σ_w *standard deviation* $w=sinh(\lambda+x/k)$ *x standard normal variable*

Note: In all these distributions *z* represents the standardized returns.

In the first stage, before the calculation of the VaR, we compare the distributions in statistical terms. To do this, we use a likelihood test (to compare the fit of two models) and two goodness of fit tests, Chi-square of Pearson (1900) and Kolmogorov-Smirnov test (Kolmogorov (1933), Smirnov (1939) and Massey (1951)) (to determine whether a sample can be considered as a draw sample from a given specified distribution). The KS test is based on the maximum difference between an empirical and a hypothetical cumulative distribution function. The Chi2 test is based on the probability distribution function and performs by grouping the data into bins, calculating the observed and expected counts for those bins.

In the second stage, we calculate the VaR and evaluate *the accuracy* of the VaR estimate under these distributions. We have an exception when $r_{t+1} < VaR(\alpha)$ and then the exception indicator variable (I_{t+1}) is equal one (zero in other cases).

A common non based on test criterion to compare VaR models is the rate of violation (VRate), defined as the proportion of exception, over the forecast period. The ratio VRate/α should be close to one. Thus, models with VRate/α ≈ 1 are better.

When VRate/α < 1 (VRate < α), risk and potential loss estimates are conservative, while alternatively, when VRate/α > 1 (VRate > α), financial institutions may not allocate sufficient capital to cover likely future losses. As in Gerlach, Chen, and Chan (2011), conservative rates are preferred for models where VRate/α is equidistant from 1. Following Gerlach et al. 2011, we evaluate the accuracy of the VaR estimates focusing in those ratios.

Besides, we test formally the accuracy of the VaR estimates to which we use four standard tests: unconditional and conditional coverage tests, the Back-Testing criterion (BTC) and the Dynamic Quantile test (DQ). Kupiec (1995) shows that the *unconditional coverage test* has as a null hypothesis $\hat{\alpha} = \alpha$, with a likelihood ratio statistic $LR_{UC} = 2\left[log\left(\hat{\alpha}^{x}\left(1-\hat{\alpha}\right)^{N-x}\right) - log\left(\alpha^{x}\left(1-\alpha\right)^{N-x}\right)\right]$ which follows an asymptotic $\chi^2(1)$ distribution.

A similar test for the significance of the deviation of $\hat{\alpha}$ from α is the *back-testing criterion* statistic $Z=\left(N\hat{\alpha}-N\alpha\right)/\sqrt{N\alpha\left(1-\alpha\right)}$ which follows an asymptotic N(0,1) distribution. The *conditional coverage test* (Christoffersen (1998)) jointly examines if the percentage of exceptions is statistically equal to the expected one and the serial independence of I_{t+1}. The likelihood ratio statistic of the conditional coverage test is $LR_{cc}=LR_{uc}+LR_{ind}$, which is asymptotically distributed $\chi^2(2)$, and the LR_{ind} statistic is the likelihood ratio statistic for the hypothesis of serial independence against first-order Markov dependence. Finally, the dynamic quantile test proposed by Engle and Manganelli (2004) examines if the exception indicator is uncorrelated with any variable that belongs to the information set Ω_{t-1} available when the VaR was calculated. This is a Wald test of the hypothesis that all slopes are zero in a regression of the exception indicator variable on a constant, 5 lags and the VaR.

Additionally, we evaluate the magnitude of the losses experienced. The model that minimizes the total loss is preferred to the other models. For this purpose, we have considered two loss functions: the regulator loss function and the firm's loss function.[13] Lopez (1998, 1999) proposed a loss function, which reflects the utility function of a regulator. In this specification, the magnitude loss function assigns a quadratic specification when the observed portfolio losses exceed the VaR estimate. Thus, we penalize only when an exception occurs according to the following quadratic specification:

$$RLF_t=\begin{cases}\left(VaR_t-r_t\right)^2 & if \quad r_t<VaR_t\\ 0 & otherwise\end{cases} \qquad (3.1)$$

[13] One strand of the literature has proposed a lot of loss functions. However, Abad et al. (2014) show that the VaR model that minimises the total losses is robust within groups of loss function but differs across firm's and supervisor's loss functions. Therefore, we consider only two loss functions: one function designed by regulators and other designed by risk managers.

This loss function gives higher scores when failures take place and considers the magnitude of the failure. In addition, the quadratic term ensures that large failures are penalized more than small failures.

But firms use VaR in internal risk management and, in this case, there is a conflict between the goal of safety and the goal of profit maximization. A too high VaR forces the firm to hold too much capital, imposing the opportunity cost of capital upon the firm. Taking this into account, Sarma et al. (2003) define the firm's loss function as follows:

$$FLF_t = \begin{cases} (VaR_t - r_t)^2 & if \quad r_t < VaR_t \\ -\beta.VaR_t & otherwise \end{cases} \tag{3.2}$$

β being the opportunity cost of capital.

3.3. Data

The data consist of closing daily returns on nine composite indexes from 1/1/2000 to 11/30/2012 (around 3250 observations). The indexes are: Japanese Nikkei, Hong Kong Hang Seng, Israeli Tel Aviv (100), Argentine Merval, US S&P 500 and Dow Jones, UK FTSE100, the French CAC40 and the Spanish IBEX-35. The data were extracted from the Bloomberg database. The computation of the indexes returns (r_t) is based on the formula, r_t=ln(I_t)-ln(I_{t-1}) where I_t is the value of the stock market index for period t.

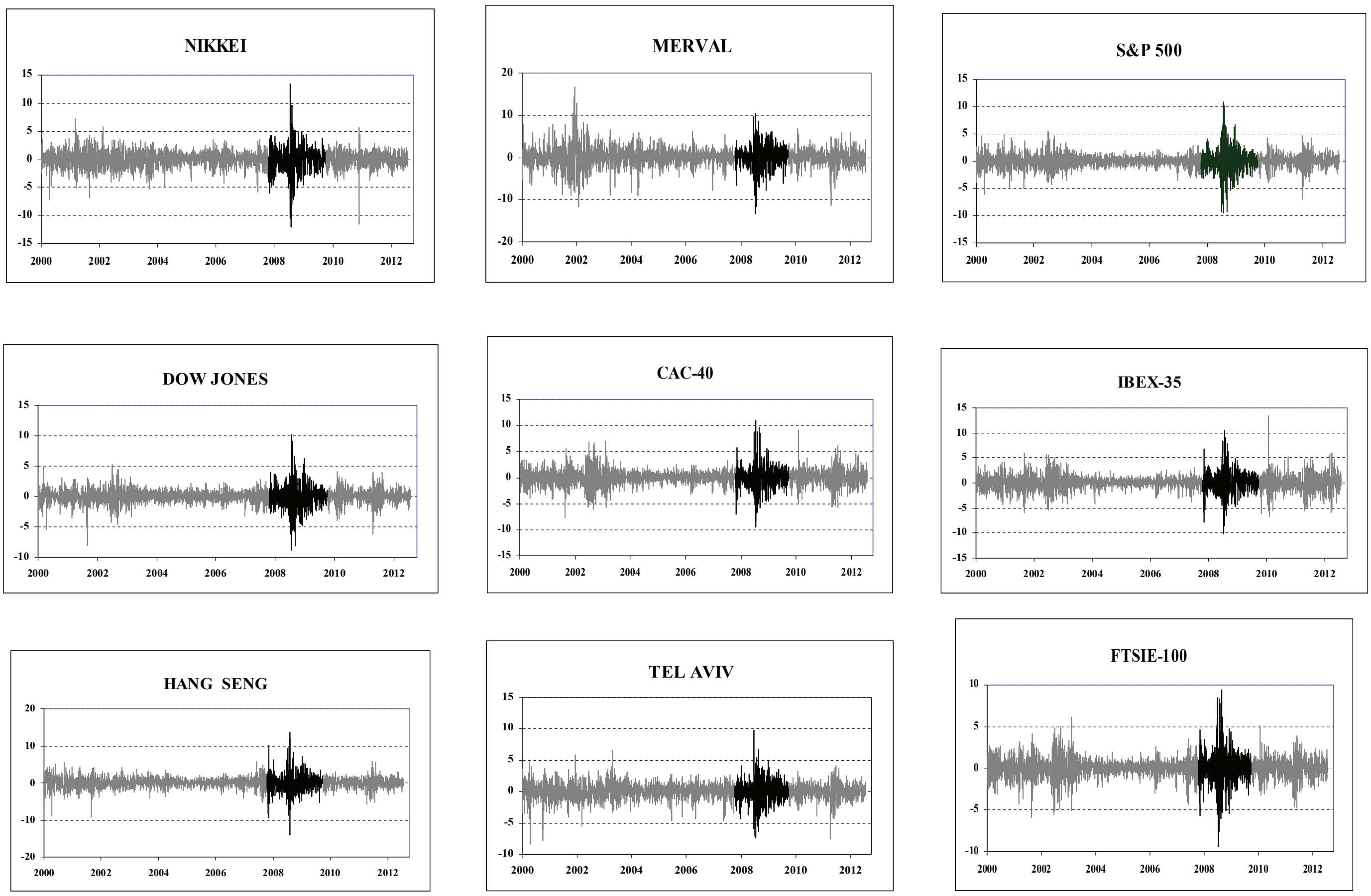
Figure 3.2 shows the daily returns and Table 3.2 provides basic descriptive statistics of them.

Table 3.2. Descriptive Statistics

Indexes	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis	Jarque Bera
Nikkei	-0.022	0.004	13.234	-12.111	1.568	-0.393** (0.044)	9.686** (0.087)	5996 (0.001)
Hang Seng	0.008	0.044	13.407	-13.582	1.632	-0.065 (0.043)	10.386** (0.087)	7253 (0.001)
Tel Aviv	0.024	0.055	9.782	-8.425	1.338	-0.311** (0.044)	6.945** (0.087)	2107 (0.001)
Merval	0.047	0.090	16.117	-12.952	2.140	-0.093* (0.043)	7.944** (0.087)	3243 (0.001)
S&P 500	-0.001	0.050	10.957	-9.47	1.354	-0.158** (0.043)	10.293** (0.086)	7212 (0.001)
Dow Jones	0.010	0.049	10.089	-8.7	1.265	-0.185** (0.043)	9.372** (0.086)	5515 (0.001)
FTSE100	-0.004	0.025	9.384	-9.266	1.301	-0.135** (0.043)	8.692** (0.086)	4416 (0.001)
CAC40	-0.015	0.019	10.595	-9.472	1.572	0.038 (0.043)	7.494** (0.085)	2782 (0.001)
IBEX35	-0.012	0.060	13.484	-9.5858	1.576	0.1227** (0.043)	7.8219** (0.086)	3177 (0.001)

Note: This table presents the descriptive statistics of the daily percentage returns of Nikkei, Hang Seng, Tel Aviv 100, Merval, S&P 500, Dow Jones, FTSE 100, CAC-40 and IBEX-35. The sample period is from January 2nd, 2000 to November 30th, 2012. The index return is calculated as Rt=100(ln(It)-ln(It-1)) where It is the index level for period t. Standard errors of the skewness and excess kurtosis are calculated as $\sqrt{6/n}$ and $\sqrt{24/n}$ respectively. The JB statistic is distributed as the Chi-square with two degrees of freedom. *, ** denote significance at the 5% and 1% level respectively.

Figure 3.2. Stock index returns



Note: This figure illustrates the daily evolution of returns of nine indexes (Nikkei, Merval, S&P 500, Dow Jones Industrial Average, CAC40, IBEX35, Hang Seng, Telaviv and FTSE-100.) from January 3rd 2000 to November 30th, 2012. Source: Bloomberg

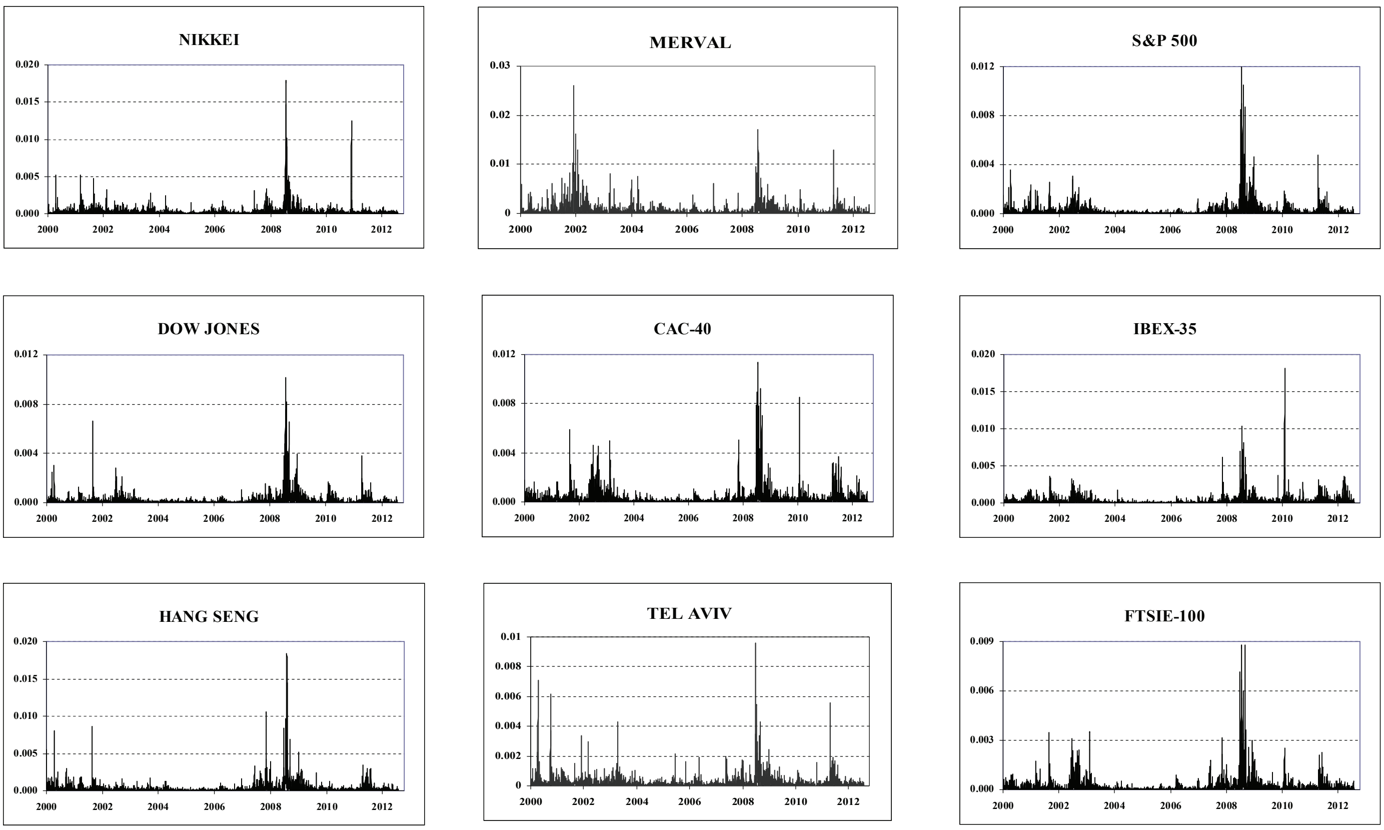
For each index, the unconditional mean of daily returns is very close to zero. The unconditional standard deviation is especially high for Merval (2.14). For the rest of stock index returns the standard deviation moves between 1.27 (Dow Jones) and 1.63 (Hang Seng). Going back to Figure 3.2, we can see that the range fluctuation of the returns is not constant, which means that the variance of these returns changes over time.

In order to gain some intuition, we adopt the volatility measure proposed by Franses and van Dijk (2000), wherein the volatility of returns is defined as:

$$V_t = \left(r_t - E\left(r_t^2 \middle| \Omega_{t-1} \right) \right)^2 \qquad (3.3)$$

where Ω_{t-1} is the information set at time *t*-1. Figure 3.3 presents V_t as “volatilities”.

Figure 3.3. Volatility of the returns



Note: This figure illustrates the conditional volatility of daily returns. The volatility was estimated using the approach proposed by Franses and van Dijk (1999). Sample runs from January 3rd 2000 to November 30th, 2012. Source: Bloomberg.

The volatility of the series was high during the early 2000s, especially in the Merval index. From 2001 to 2002 the conditional volatility of Merval was almost 1 point higher than the whole period, even greater than those showed from 2008 to 2009. This corresponds to the Argentine economic crisis (1999–2002) which was the major downturn in Argentine´s economy[14]. The period from 2003 to early 2007 was very quiet. In August 2007 the financial market tensions started and they were followed by a global financial and economic crisis leading to a significant rise in the volatility of returns. This increase was especially important after August 2008 coinciding with the fall of Lehman Brothers. From 2008 to 2009, the volatility of the S&P500, Nikkei and IBEX35, measured by the standard deviation of returns was 2.42, 2.20, and 2.10 respectively. In the case of S&P500, the standard deviation was almost 1 point higher than the standard deviation of the whole period 2000-2012 (1.57). A similar increase is observed in all indexes. In the last two years of the sample, we observe a more stable period than during the financial crisis.

The skewness statistic is negative and significant for all the indexes considered except in the case of the CAC40 and the IBEX35. This means that the distribution of those returns is skewed to the left. When considering the CAC40 and the IBEX35 the skewness statistic is positive, implying that these distributions are skewed to the right but only in the case of IBEX35 this statistic is significant at 1% level.

For all the indexes considered, the excess kurtosis statistic is very large and significant at 1% level implying that the distributions of those returns have much thicker tails than the normal distribution. Similarly, the Jarque-Bera statistic is significant rejecting the assumption of normality. These results are in line with those obtained by Bollerslev (1987), Bali and Theodossiou (2007), and Bali et al (2008), among others.

[14]It began in 1999 with a decrease of the real Gross Domestic Product. The crisis caused the fall of the government, default on the country's foreign debt, widespread unemployment, riots, the rise of alternative currencies and the end of the peso's fixed exchange rate to the US dollar.

All of them find evidence that the empirical distribution of the financial return is asymmetric and exhibits a significantly excess of kurtosis (fat tails and peakness).

In order to capture the non-normal characteristics observed in our data set, we fit several skewed distributions: SGT, SGED, SSD and IHS. In this comparison we also include the normal and symmetric ST distributions. In Table 3.3 we present the estimated parameters of these distributions.

Table 3.3. Maximum likelihood estimates of alternative distribution functions

Nikkei	***u***	**SE**	***σ***	**SE**	***λ***	**SE**	***H***	**SE**	***κ***	**SE**
SGT	0.000	(0.000)	0.016**	(0.001)	-0.047*	(0.021)	4.766**	(0.282)	1.896**	(0.078)
SGED	0.000	(0.000)	0.015**	(0.000)	-0.041**	(0.004)			1.133**	(0.033)
SSD	0.000	(0.000)	0.016**	(0.000)	-0.048*	(0.021)	4.442**	(0.236)		
IHS	0.000	(0.000)	0.015**	(0.000)	-0.086	(0.032)			1.472**	(0.054)
ST	0.000	(0.000)	0.016**	(0.001)			4.404**	(0.232)		
Normal	0.000	(0.000)	0.016**	(0.000)						
Hang Seng	***u***	**SE**	***σ***	**SE**	***λ***	**SE**	***H***	**SE**	***κ***	**SE**
SGT	0.000	(0.000)	0.016**	(0.001)	-0.034**	(0.014)	6.328**	(0.547)	1.338**	(0.044)
SGED	0.000	(0.000)	0.016**	(0.000)	-0.031	(--)			0.977**	(0.028)
SSD	0.000	(0.000)	0.017**	(0.000)	-0.041*	(0.018)	3.314**	(0.100)		
IHS	0.000	(0.000)	0.016**	(0.000)	-0.067*	(0.027)			1.21	(0.033)
ST	0.000	(0.000)	0.017**	(0.001)			3.297**	(0.100)		
Normal	0.000	(0.000)	0.016**	(0.000)						
Tel Aviv	***u***	**SE**	***σ***	**SE**	***λ***	**SE**	***H***	**SE**	***κ***	**SE**
SGT	0.000	(0.000)	0.013**	(0.001)	-0.060**	(0.021)	5.247**	(0.365)	1.785**	(0.068)
SGED	0.000	(0.000)	0.013**	(0.000)	-0.052**	(0.016)			1.175**	(0.035)
SSD	0.000	(0.000)	0.014**	(0.000)	-0.062**	(0.021)	4.381**	(0.232)		
IHS	0.000	(0.000)	0.013**	(0.000)	-0.102**	(0.032)			1.463**	(0.054)
ST	0.001**	(0.000)	0.014**	(0.001)			4.331**	(0.228)		
Normal	0.000	(0.000)	0.013**	(0.000)						
Merval	***u***	**SE**	***σ***	**SE**	***λ***	**SE**	***H***	**SE**	***κ***	**SE**
SGT	0.000	(0.000)	0.022**	(0.001)	-0.043*	(0.018)	4.456**	(0.241)	1.531**	(0.051)
SGED	0.000	(0.000)	0.021**	(0.000)	-0.033**	(0.002)			0.998**	(0.028)
SSD	0.000	(0.000)	0.023**	(0.000)	-0.047**	(0.018)	3.083**	(0.075)		
IHS	0.000	(0.000)	0.022**	(0.000)	-0.068*	(0.027)			1.171**	(0.029)
ST	0.001*	(0.000)	0.023**	(0.001)			3.088**	(0.078)		
Normal	0.000	(0.000)	0.021**	(0.000)						
S&P 500	***u***	**SE**	***σ***	**SE**	***λ***	**SE**	***H***	**SE**	***κ***	**SE**
SGT	0.000	(0.000)	0.014**	(0.001)	-0.064**	(0.013)	5.735**	(0.430)	1.239**	(0.038)
SGED	0.000	(0.000)	0.013**	(0.000)	-0.062	(--)			0.902**	(0.008)
SSD	0.000	(0.000)	0.016**	(0.000)	-0.069**	(0.016)	2.760**	(0.046)		
IHS	0.000	(0.000)	0.014**	(0.000)	-0.087**	(0.024)			1.079**	(0.023)
ST	0.000	(0.000)	0.015**	(0.001)			2.770**	(0.049)		
Normal	0.000	(0.000)	0.014**	(0.000)						
Dow Jones	***u***	**SE**	***σ***	**SE**	***λ***	**SE**	***H***	**SE**	***κ***	**SE**
SGT	0.000	(0.000)	0.013**	(0.001)	-0.058**	(0.017)	4.496**	(0.241)	1.524**	(0.051)
SGED	0.000	(0.000)	0.012**	(0.000)	-0.057**	(0.002)			0.983**	(0.027)
SSD	0.000	(0.000)	0.014**	(0.000)	-0.059**	(0.018)	3.122**	(0.078)		
IHS	0.000	(0.000)	0.013**	(0.000)	-0.088**	(0.026)			1.178**	(0.029)
ST	0.000	(0.000)	0.014**	(0.001)			3.122**	(0.080)		
Normal	0.000	(0.000)	0.013**	(0.000)						
FTSE100	***u***	**SE**	***σ***	**SE**	***λ***	**SE**	***H***	**SE**	***κ***	**SE**
SGT	0.000	(0.000)	0.013**	(0.001)	-0.054**	(0.018)	4.273**	(0.212)	1.623**	(0.055)
SGED	0.000	(0.000)	0.013**	(0.000)	-0.049**	(0.003)			1.015**	(0.028)
SSD	0.000	(0.000)	0.014**	(0.000)	-0.056**	(0.018)	3.237**	(0.089)		
IHS	0.000	(0.000)	0.013**	(0.000)	-0.083**	(0.027)			1.208**	(0.031)
ST	0.000	(0.000)	0.014**	(0.001)			3.231**	(0.091)		
Normal	0.000	(0.000)	0.013**	(0.000)						
CAC40	***u***	**SE**	***σ***	**SE**	***λ***	**SE**	***H***	**SE**	***κ***	**SE**
SGT	0.000	(0.000)	0.016**	(0.001)	-0.062**	(0.018)	4.545**	(0.249)	1.673**	(0.059)
SGED	0.000	(0.000)	0.015**	(0.000)	-0.044*	(0.021)			1.065**	(0.030)
SSD	0.000	(0.000)	0.016**	(0.000)	-0.066**	(0.019)	3.540**	(0.120)		
IHS	0.000	(0.000)	0.016**	(0.000)	-0.094**	(0.028)			1.277**	(0.036)
ST	0.000	(0.000)	0.016**	(0.001)			3.533**	(0.122)		
Normal	0.000	(0.000)	0.016**	(0.000)						
IBEX35	***u***	**SE**	***σ***	**SE**	***λ***	**SE**	***H***	**SE**	***κ***	**SE**
SGT	0.000	(0.000)	0.016**	(0.001)	-0.073**	(0.017)	7.127**	(0.717)	1.380**	(0.045)
SGED	0.000	(0.000)	0.016**	(0.000)	-0.068	(--)			1.050**	(0.030)
SSD	0.000	(0.000)	0.017**	(0.000)	-0.069**	(0.018)	3.548**	(0.125)		
IHS	0.000	(0.000)	0.016**	(0.000)	-0.092**	(0.028)			1.270**	(0.037)
ST	0.000	(0.000)	0.016**	(0.001)			3.584**	(0.132)		
Normal	0.000	(0.000)	0.016**	(0.000)						

Note: Parameter estimates of the Normal, SGT, SGED, SSD, IHS and ST. S.E. denotes standard errors (in parentheses). Nine stock market returns in the period 1/1/2000-11/30/2012. μ, σ, λ and η are the estimated mean, standard deviation, skewness parameter, and tail-tickness parameter; κ represents the peakness parameter. An *(**) denotes significance at the 5% (1%) level.

This table provides the estimates for the mean (μ) and the standard deviation (σ) of log-returns and its standard errors in brackets. As expected, these estimates are quite similar across distributions and do not differ much from the simple arithmetic means and standard deviations of log-returns (see Table 3.2). The unconditional mean is close to zero for all the indexes and the unconditional standard deviation moves around 1.5 (in percentage) except Merval (2.14). As expected from the previous analysis, the Merval index is the most volatile index.

The skewness parameter λ, for all indexes, is negative and significant at the 1% level, which means that the distributions of these returns are skewed to the left. This result is in opposition to the preliminary evidence that suggested a symmetric distribution for CAC40 and a skewed distribution to the right for IBEX35.

In the case of SGT, the parameter κ controls mainly the peakness of the distribution around the mode, while the parameter η controls mainly the tails of the distribution (adjusting the tails to the extreme values). The parameter η has the degrees of freedom interpretation as in ST. For all the series and all distributions considered, the kurtosis parameters (η and κ) are highly significant. For the SGT, the value of κ is around 1.5, except for Nikkei and Tel Aviv which are 1.89 and 1.78 respectively. The value of η is around 4.5 for Nikkei, Merval, Dow Jones, Footsie-100 and CAC40. For the rest of the indexes it is a little bit higher. These estimates are quite different from those of the normal distribution ($\kappa = 2$ and $\eta = \infty$), which indicates that this set of returns is characterized by excess kurtosis.

3.4. Comparison of the distributions in statistical terms

In this section we want to answer the following question: Which distribution is the best one for fitting asset returns? The above results provide strong support to the hypothesis that stock returns are not normal. As the normal distribution is nested within

the SGT, SGED and SSD distributions we can use the log-likelihood ratio for testing the null hypothesis of normality against that of SGT, SGED or SSD. For all the indexes considered, this statistic is quite large and significant at the 1% level, providing evidence against the normality hypothesis (see Table 3.4).

Table 3.4. Goodness-of-fit tests

		Log-L	LR_Normal	LR_SGT	Chi2		KS	
Nikkei	SGT	8920.4	463.2**	--	5.239	(0.022)**	0.031	(0.004)
	SGED	8897.4	417.2**	46.0**	7.715	(0.006)	0.027	(0.021)**
	SSD	8920.3	463.0**	0.2	13.448	(0.001)	0.034	(0.001)
	IHS	8918.6	--	--	3.453	(0.063)*	0.029	(0.011)**
	ST	8918.2	--	4.4	20.958	(0.000)	0.029	(0.008)
	Normal	8688.8		--	124.218	(0.000)	0.058	(0.000)
Merval	SGT	8016.9	612.6**	--	8.164	(0.017)**	0.019	(0.197)*
	SGED	8003	584.8**	27.8**	12.318	(0.002)	0.027	(0.021)**
	SSD	8012.5	603.8**	8.8*	15.965	(0.003)	0.020	(0.147)*
	IHS	8017	--	--	6.005	(0.111)*	0.018	(0.260)*
	ST	8010.4	--	13.0**	18.687	(0.000)	0.024	(0.053)*
	Normal	7710.6	--	--	253.700	(0.000)	0.072	(0.000)
S&P 500	SGT	9777.7	824.2**	--	14.092	(0.001)	0.028	(0.013)*
	SGED	9769.2	807.2**	17.0**	8.761	(0.013)**	0.033	(0.002)
	SSD	9762.2	793.2**	31.0**	35.861	(0.000)	0.038	(0.000)
	IHS	9769.2	--	--	22.316	(0.000)	0.035	(0.001)
	ST	9757.1	--	41.2**	33.963	(0.000)	0.037	(0.000)
	Normal	9365.6	--	--	266.854	(0.000)	0.080	(0.000)
Dow Jones	SGT	9929.7	682.6**	--	6.333	(0.042)**	0.028	(0.011)**
	SGED	9914.2	651.6**	31.0**	24.553	(0.000)	0.032	(0.002)
	SSD	9925.1	673.4**	9.2**	21.875	(0.000)	0.034	(0.001)
	IHS	9928.4	--	--	8.647	(0.034)**	0.029	(0.007)
	ST	9921.6	--	16..2**	30.360	(0.000)	0.030	(0.007)
	Normal	9588.4	--	--	256.272	(0.000)	0.071	(0.000)
CAC40	SGT	9297.4	523.6**	--	3.209	(0.201)*	0.023	(0.067)*
	SGED	9281	490.8**	32.8**	17.858	(0.000)	0.033	(0.002)
	SSD	9295.3	519.4**	4.2*	7.248	(0.027)**	0.027	(0.018)**
	IHS	9297.4	--	--	2.761	(0.430)*	0.022	(0.079)*
	ST	9291.1	--	12.6**	38.232	(0.000)	0.025	(0.030)**
	Normal	9035.6	--	--	191.314	(0.000)	0.064	(0.000)
IBEX35	SGT	9176.8	484.2**	--	3.767	(0.052)*	0.027	(0.018)**
	SGED	9169.8	470.2**	14.0**	11.509	(0.001)	0.028	(0.011)**
	SSD	9167.1	464.8**	19.4**	13.293	(0.001)	0.028	(0.011)**
	IHS	9170.9	--	--	7.174	(0.067)*	0.029	(0.010)**
	ST	9162.4	--	28.8**	25.413	(0.000)	0.034	(0.001)
	Normal	8934.7	--	--	118.562	(0.000)	0.065	(0.000)
Hang Seng	SGT	8927.5	649.0**	--	1.543	(0.214)*	0.027	(0.020)**
	SGED	8918.4	630.8**	18.2**	5.519	(0.063)*	0.029	(0.010)**
	SSD	8916.3	626.6**	22.4**	9.290	(0.002)	0.037	(0.000)
	IHS	8920.4	--	--	1.873	(0.392)*	0.034	(0.001)
	ST	8914.6	--	25.8**	15.599	(0.000)	0.035	(0.001)
	Normal	8603	--	--	23.434	(0.000)	0.072	(0.000)
Tel Aviv	SGT	9358.2	316.8**	--	5.721	(0.057)*	0.027	(0.023)**
	SGED	9343.6	332.6**	29.2**	4.288	(0.039)**	0.034	(0.002)
	SSD	9357.3	360.0**	1.8	11.097	(0.004)	0.029	(0.008)
	IHS	9358.6	--	--	5.878	(0.053)*	0.026	(0.024)**
	ST	9354	--	8.4*	33.459	(0.000)	0.025	(0.041)**
	Normal	9177.3	--	--	106.813	(0.000)	0.058	(0.000)
FTSE100	SGT	9857	628.2**	--	3.311	(0.191)*	0.025	(0.037)**
	SGED	9839.1	592.4**	35.8**	10.540	(0.005)	0.034	(0.001)
	SSD	9854.2	622.6**	5.6*	16.291	(0.000)	0.027	(0.018)**
	IHS	9857.3	--	--	4.518	(0.211)*	0.027	(0.015)**
	ST	9851.2	--	11.6**	25.173	(0.000)	0.029	(0.007)
	Normal	9542.9	--	--	203.848	(0.000)	0.072	(0.000)

Note: Log-L is the maximum likelihood value. LR_{Normal} is the LR statistic from testing the null hypothesis that the daily returns are distributed as Normal against they are distributed as SGT, SGED or SSD. LR_{SGT} is the LR statistic from testing the null hypothesis of alternative distribution against the SGT. Chi2 and KS denote Chi-square and Kolmogorov Smirnov tests. Figures in brackets denote p-value. An *(**) denotes significance at the 5% (1%) level.

To evaluate which is the most adequate, we perform several kinds of tests. First, as the SGT nets all the distributions considered in this paper (except IHS), we use the likelihood ratio test to evaluate which distribution is best for fitting the data[15]. Overall, for all the indexes considered, the likelihood statistics indicate rejection of the SGED, the SSD, and the ST in favour of the SGT (see Table 3.4). As the IHS is not nested in the SGT distribution, we cannot conclude that the SGT distribution is the best. So, to ensure the robustness of these results, several alternative tests have been used: Chi2 and KS tests. Unlike the likelihood ratio test used to compare two distributions, the Chi2 and the KS tests are used to examine if the asset returns' empirical distribution follows a particular theoretical distribution. The theoretical distributions considered are: normal, ST, SGT, SSD, SGED and IHS. The Chi2 statistic (see Table 3.4) suggests that the empirical distributions of the returns can be adequately characterized using two distributions: SGT and IHS. Both distributions seem to fit the data well in 8 of the 9 indexes considered. For the Hang Seng, Tel Aviv and S&P 500 indexes, the SGED distribution cannot be refused either. On the other hand, the ST and the normal distributions do not fit any index. The KS test provides similar results (see Table 3.4).

According to this test, the empirical distribution of all the indexes (except Nikkei) follows a SGT distribution. The IHS fits the data well in only five of the indexes (Merval, CAC40, IBEX35, Tel Aviv and Nikkei). The SSD distribution fits the data well in four indexes (Merval, CAC40, IBEX35 and Footsie) and the SGED distribution fits the data well in four indexes (Nikkei, Merval, IBEX35 and Hang Seng). The ST distribution only fits well in three of the nine indexes while the normal distribution does not do well in any index.

Taking into account the results described in this section, we can conclude that the symmetric distributions (normal and ST) do not fit financial returns well. This is in line with the previous results shown in the above sections. Among the set of skewed distributions

[15] Specifically, it gives for $\eta = \infty$ the SGED, for $\kappa = 2$ the SSD, for $\lambda=0$ and $\kappa = 2$ the ST and for $\lambda=0$, $n = \infty$ and $k = 2$ the normal distribution (see Hansen, McDonald and Theodossiou (2001) for a comprehensive survey on the skewed fat-tailed distributions).

considered in this paper, the SGT distribution seems to be the best in fitting the data, followed closely by the IHS distribution.

3.5. Evaluating the performance in terms of VaR

In this section we compare the normal, the ST and the skewed distributions in terms of VaR. The comparison is carried out evaluating (i) the accuracy of the VaR estimates and (ii) the losses that VaR produces. For each distribution, we use parametric approaches to forecast the VaR out-of-the-sample one-step-ahead at 1% and 0.25% confidence level.

The data period is divided into a learning sample from January 1, 2000 to December 31, 2007; and a forecast sample from January 1, 2008 to the end of December 2009. We choose this forecast period because it is characterized by a high volatility all over the world so that it is known in financial literature as the Financial Global Crisis period. In Figure 3.2, we highlight in black the period analyzed.

3.5.1 Back Testing

As Gerlach et al. (2011), Table 3.5 shows the ratio VRate/α for α=0.01 and 0.0025 across all 6 models and 9 indexes.

Table 3.5. Ratio of VRate/α at α = 1%, 0.25% for each VaR model across the 9 stock indexes

Model	**Nikkei**	**Merval**	**S&P500**	**Dow Jones**	**CAC40**	**IBEX35**	**Hang Seng**	**Tel Aviv**	**FTSE 100**
					α = 1%				
Normal	**2.9**	**2.2**	**3.6**	**2.8**	**2.3**	**2.2**	**1.6**	**2.6**	**3.6**
ST	1.6	0.6	1.2	1.0	1.2	1.2	0.6	0.6	2.2
SGT	1.8	1.4	1.8	1.4	1.2	1.6	1.0	1.0	1.8
IHS	1.8	1.4	1.8	1.2	1.2	1.6	1.0	1.0	1.6
SSD	1.8	1.8	2.2	1.4	1.2	1.6	1.2	1.6	2.0
SGED	1.8	1.4	1.8	1.4	1.2	1.6	1.0	1.2	2.0
					α = 0.25%				
Normal	**3.3**	**3.2**	**4.8**	**2.4**	**3.9**	**4.7**	**2.4**	**2.4**	**4.7**
ST	0.8	0.0	1.6	0.8	2.4	0.8	0.0	0.0	4.0
SGT	0.8	0.8	2.4	1.6	2.4	0.8	1.6	0.8	4.0
IHS	0.8	0.8	2.4	1.6	2.4	0.8	0.0	0.8	3.2
SSD	0.8	2.4	2.4	1.6	2.4	0.8	1.6	1.6	4.0
SGED	0.8	1.6	2.4	1.6	2.4	0.8	1.6	0.8	4.0

Note: Shaded cells indicate closest to 1 in that index. Bold figures indicate the least favored model.

The best model´s ratio in each index is shaded, while bolding indicate that the LR_{UC} test rejects the model at the 5% level. The results are similar for both levels, α=1% and 0.25%. Most of the normal distribution ratios are far above 1 across the 9 indexes. This distribution consistently under-estimate risk. The ST, SGT and IHS distribution provide the closest ratios to 1. We must highlight that SGT and IHS are the only distributions that provide ratios equal to 1 in some indexes. At 1% level the SSD distribution performance is the worst one although slightly improves at 0.25%.

Following Gerlach et al. (2011), Table 3.6 displays summary statistics for the VRate/α ratio for each model across the 9 indexes, using the results of Table 3.5.

Table 3.6. Summary statistics for the ratio of VRate/α at α = 1%, 0.25% for each VaR model

	α = 1%					α = 0.25%				
	Mean	**Median**	**Std (1)**	**1st**	**In top 3**	**Mean**	**Median**	**Std (1)**	**1st**	**In top 3**
Normal	**2.64**	**2.6**	**3.43**	**0**	**0**	**3.54**	**3.3**	**8.32**	**0**	**0**
ST	1.07	1.1	0.29	5	5	1.15	0.8	1.76	5	6
SGT	1.44	1.4	0.33	4	9	1.68	1.6	1.67	6	9
IHS	1.40	1.4	0.28	5	9	1.41	0.8	1.24	6	8
SSD	1.64	1.6	0.58	1	5	1.95	1.6	1.97	4	7
SGED	1.49	1.4	0.37	4	9	1.77	1.6	1.71	5	9

Note: Shaded cells indicate the favored model and bold figures indicate the least favored model, in each column. Std (1) is the standard deviation in ratios from an expected value of 1. 1st indicates the number of markets where that model's VRate/α ratio ranked closest to 1. In top 3 counts the number of markets where the model's VRate/α ratio ranked in the top 3 models.

The Std (1) column shows the standard deviation from expected ratio of 1 (not the mean sample), while 1 st column counts the indexes where the model had VRate/α

ratio closest to 1 and In top3 column counts the indexes where the model ranked in top 3 models by VRate/α ratio. The results confirm the above conclusion. The normal distribution shows a very poor performance. For both level, 1% and 0.25%, the mean VRate/α ratio is far above 1 and this distribution never rank in the top three models. The ST, SGT and IHS distribution are most favored across all criteria. For both levels, 1% and 0.25%, the ST distribution provides the mean VRate/α ratio closest to 1. However, the IHS which provides a mean ratio close to 1 has the smallest standard deviation from the expected ratio of 1, followed by the SGT and ST distribution. It seems that the performance of the ST distribution is more volatile. The data presented in columns four and five corroborate this idea. The ST distribution ranks first in 5 indexes, similar to the IHS distributions which rank first in 5 and 6 indexes, depending on the level considered (1% and 0.25%). For both levels, this last distribution in conjuntion with the SGT distribution ranks in the top 3 distributions for all 9 indexes.

To help distinguishing between the better models at each quantile level's Table 3.7 shows summary statistics for each distribution´s rank, in terms of how close its Vrate/α ratio is to 1 across the indexes. For ratios that are equidistant from 1, conservative ratios (less than 1) are preferred.

Table 3.7. Summary statistics for model ranks, in terms of VRate/α at α = 1%, 0.25% across the 9 stock indexes

	α = 1%				α = 0.25%			
	Mean	**Median**	**Std (1)**	**Range**	**Mean**	**Median**	**Std (1)**	**Range**
Normal	**6.00**	**6.00**	**5.00**	0	**6.00**	**6.00**	**5.00**	0
ST	2.44	1.00	2.24	4	3.11	3.00	2.48	3.5
SGT	2.83	3.00	1.99	2.5	2.78	3.00	1.91	2
IHS	2.50	3.00	1.72	2.5	2.78	3.00	2.05	3.5
SSD	4.06	4.00	3.14	2	3.39	3.50	2.51	3
SGED	3.17	3.00	2.23	2	2.94	3.00	2.02	1.5

Note: Shaded cells indicate the favored model and bold figures indicate the least favored model, in each column. Std (1) is the standard deviation in ranks from the value of 1.

Table 3.7 displays the average, median, standard deviation (from 1) and range of the forecast ranks for each model over the 9 indexes. For both levels, 1% and 0.25%, normal distribution has by far the highest mean rank, equal highest median rank, and by far the highest deviation in ranks, away from 1 across de distributions. SSD and SGED distributions display the following worst performance. At 1% level, the ST distribution has the lowest mean rank, follow close by IHS and SGT distributions. However, the IHS and SGT distribution have the lowest standard deviation in ranks away from 1 and lower range than the ST distribution. At 0.25% level, the SGT and IHS distributions have the lowest mean rank and equal lowest standard deviation in ranks away from 1. For this level the SGT distribution has the lowest range. In fact, the SGT distribution joins the IHS ranks in top three in each index, and thus has the smaller range in ranks than the ST distributions.

Finally, Table 3.8 counts the number of rejections for each distribution, over the 9 indexes at 5% level for each of the five tests considered (LR_{UC}, BTC, LR_{ind}, LR_{CC} and DQ).

Table 3.8. Counts of model rejections for 5 tests, across the 9 stock indexes

	α = 1%					α = 0.25%				
	UC P	**BTC**	**CC P**	**UC IND**	**DQ P**	**UC P**	**BTC**	**CC P**	**UC IND**	**DQ P**
Normal	**5**	**8**	**2**	0	0	**1**	**6**	0	0	**1**
ST	0	1	0	0	**1**	0	1	0	0	0
SGT	0	0	0	0	**1**	0	1	0	0	0
IHS	0	0	0	0	**1**	0	1	0	0	0
SSD	0	2	0	0	**1**	0	1	0	0	0
SGED	0	1	0	0	**1**	0	1	0	0	0

Note: Shaded cells indicate the favored model and bold figures indicate the least favored model, in each column.

The accuracy tests corroborate the conclusion from Tables 3.5 and 3.6. At both levels, the normal distribution is rejected by many tests. For the ST distribution and all the skewness distributions the number of rejections is minimum, just one or two.

Overall, we can conclude that i) normal distribution performs very poor in estimating VaR (this distribution underestimate risk in almost all indexes); ii) after the normal distribution the SSD and SGED are the worst; iii) the ST distribution performs very well in estimating VaR but shows a volatile behavior (this distribution works well in some indexes and poorly in others) and iv) however, the IHS and SGT distributions outpace the ST distribution in many indexes and perform well in the cases in which ST is the best one.

3.5.2 Loss Functions

In this section we evaluate the VaR estimate in terms of the regulator loss function (Table 3.9) and the firm's loss function (Table 3.10). The results in Table 3.9 have been multiplied by 1000 given the small value obtained. The shaded cell represents the minimum value for this function in each case (index and confidence level).

From the regulator loss function (see Table 3.9), we find that the parametric approach under a normal distribution provide the highest losses while the ST distribution provides the lowest losses followed by the IHS and the SGT distributions. Among the skewed distributions, the SSD gives the worst outcome in all cases. According to this result, we can conclude that from the point of view of the regulator the best distribution is the ST, as this distribution is the most conservative.

Table 3.9. Magnitude of the regulatory loss function

	Level	NORMAL	ST	SGT	IHS	SSD	SGED
Nikkei	1.00%	0.00338	0.00134	0.00186	**0.00176**	0.00212	0.00186
	0.25%	0.00065	0.00004	0.00015	**0.00008**	0.00020	0.00015
Merval	1.00%	0.00667	0.00053	0.00256	**0.00244**	0.00340	0.00251
	0.25%	0.00191	0.00000	0.00013	**0.00009**	0.00039	0.00022
S&P 500	1.00%	0.00617	0.00337	0.00352	0.00362	0.00393	**0.00349**
	0.25%	0.00293	0.00121	0.00133	**0.00130**	0.00167	0.00137
Dow Jones	1.00%	0.00220	0.00056	0.00073	**0.00065**	0.00080	0.00067
	0.25%	0.00044	0.00003	**0.00004**	0.00003	0.00008	0.00006
CAC40	1.00%	0.00568	0.00462	0.00445	0.00427	0.00445	**0.00443**
	0.25%	0.00282	0.00185	**0.00158**	0.00148	0.00175	0.00178
IBEX35	1.00%	0.00554	0.00308	0.00355	**0.00336**	0.00366	0.00350
	0.25%	0.00274	0.00152	0.00161	**0.00158**	0.00186	0.00182
Hang Seng	1.00%	0.00333	0.00048	**0.00124**	0.00127	0.00165	0.00125
	0.25%	0.00062	0.00000	**0.00001**	0.00000	0.00006	0.00001
Tel Aviv	1.00%	0.00150	0.00024	0.00060	**0.00054**	0.00069	0.00062
	0.25%	0.00030	0.00000	**0.00000**	**0.00000**	0.00004	0.00003
FTSE100	1.00%	0.00376	0.00254	**0.00227**	0.00205	0.00228	0.00228
	0.25%	0.00126	0.00056	**0.00036**	0.00029	0.00047	0.00048

Note: This table reports the average of the loss function of each VaR model in both confidence levels. The average was multiplied by 1,000. Shaded cell the minimum value for the average of the loss function for each index, following by boldface figures.

Table 3.10. Magnitude of the firm's loss function

	Level	NORMAL	ST	SGT	IHS	SSD	SGED
Nikkei	1.00%	0.00054	0.00062	0.00059	0.00059	**0.00058**	0.00059
	0.25%	0.00066	0.00080	0.00076	0.00077	**0.00074**	0.00075
Merval	1.00%	0.00056	0.00079	0.00065	0.00066	**0.00062**	0.00066
	0.25%	0.00068	0.00112	0.00090	0.00092	**0.00081**	0.00085
S&P 500	1.00%	0.00044	0.00052	0.00051	0.00050	**0.00049**	0.00051
	0.25%	0.00054	0.00066	0.00065	0.00065	**0.00062**	0.00064
Dow Jones	1.00%	0.00040	0.00048	0.00046	0.00047	**0.00045**	0.00046
	0.25%	0.00050	0.00062	0.00060	0.00061	**0.00058**	0.00059
CAC40	1.00%	0.00111	**0.00121**	0.00122	0.00123	0.00122	0.00122
	0.25%	0.00136	0.00150	0.00153	0.00154	0.00150	**0.00150**
IBEX35	1.00%	0.00109	0.00132	0.00125	0.00127	**0.00124**	0.00125
	0.25%	0.00132	0.00173	0.00167	0.00168	**0.00158**	0.00159
Hang Seng	1.00%	0.00062	0.00080	0.00072	0.00071	**0.00069**	0.00071
	0.25%	0.00077	0.00107	0.00092	0.00096	**0.00089**	0.00092
Tel Aviv	1.00%	0.00040	0.00052	0.00046	0.00047	**0.00045**	0.00046
	0.25%	0.00050	0.00069	0.00062	0.00062	**0.00058**	0.00059
FTSE100	1.00%	0.00099	**0.00108**	0.00111	0.00113	0.00110	0.00110
	0.25%	0.00122	**0.00135**	0.00140	0.00143	0.00137	0.00136

Note: This table reports the average of the loss function of each VaR model in both confidence levels. Shaded cell denote the minimum value for the average of the loss function for each index, following by boldface figures.

The problem associated with the regulator loss function is that this function does not take into account the firms' opportunity cost. So that one model that overestimates the risk, as the ST distribution does in three of the cases, may be considered the most appropriate[16]. Taking this into account we calculate the losses from a firm´s point of view.[17]

In terms of the firm's loss function (see Table 3.10), the normal distribution provides the lowest losses while the ST distribution shows the highest losses. This result is coherent since it is well known that the normal distribution underestimates risk providing the lowest capital opportunity cost. Since the ST distribution tends to overestimate risk, the capital opportunity cost with this distribution is the highest. The magnitudes of losses obtained by all the skewed distribution are very similar. In terms of this loss function, the best skewed distribution is the SSD. This distribution obtains the lowest losses in seven of the nine cases. The SGT distribution, although it is not the best, works out well giving lower losses than the ST does.

Overall, following this selection process in two stages, where first we ensure that the distributions provide accurate VaR estimate and then focusing in the firm's loss function, we can conclude that the skewed and fat tail distributions outperformed the normal and the ST distribution. From a point of view of the regulator, the superiority of the skewed distributions related to the ST is not so clear.

[16] For Merval, Hang Seng and Tel Aviv, and for both levels (1% and 0.25%), the ST distribution by far overestimates risk compare to IHS and SGT distribution (see Table 3.9).

[17] In order to calculate the firm's loss function we need to know the cost of capital. For this purpose, we have used the daily data of the interest rate of the Eurosystem monetary policy operations for the European indexes. For the rest of the indexes, we took the interest rate of the open market operations used by the Federal Reserve in the implementation of its monetary policy.

3.6. Conclusion

This paper evaluates the performance of several skewed and symmetric distributions in modeling the tail behavior of daily returns and in forecasting VaR. The skewed distributions considered are: (i) the skewed Student-*t* distribution of Hansen (1994); (ii) the skewed error generalised distribution of Theodossiou (2001); (iii) the skewed generalised-*t* distribution of Theodossiou (1998) and (iv) the inverse hyperbolic sign of Johnson (1949). The symmetric distributions are the normal and the Student-*t* ones.

For this study we have used daily returns on nine composite indexes: the Japanese Nikkei, Hong Kong Hang Seng, Israeli Tel Aviv (100), Argentine Merval, US S&P 500 and Dow Jones, UK's FTSE100, the French CAC40 and the Spanish IBEX-35. The sample used for the statistical analysis runs from January 2000 to the end of November 2012. The analysis period for forecasting VaR runs from 2008 to 2009, which is known as the Global Financial Crisis period.

From the results presented in the paper, we can conclude that the skewness and fat tail distributions outperform the normal one in fitting financial returns and forecasting VaR. Among all the skewed distributions considered in this paper, the skewed generalised-*t* distribution of Theodossiou (1998) is the best one in fitting data. In terms of their ability to forecast the VaR, the inverse hyperbolic sign and skewed generalised-*t* distribution provide the more accurate VaR estimates across the indexes.

Therefore, we find evidence in favor of the skewed distributions compared to the Student-*t* distribution. In statistical terms, most of them fit the data better than the Student-*t*. According to the accuracy VaR estimates, the inverse hyperbolic sign and the skewed generalised-*t* distribution outperformance the Student-*t* distribution as those distributions provide less volatile results. The Student-*t* distribution performs very well in estimating VaR but shows a more volatile behavior across indexes.

On the other hand, regards to the loss function, the result depends on the kind of function used to measure the losses. From a point of view of the regulator, Student-*t* distribution is the best in forecasting VaR as this distribution provides the more conservative VaR estimate. However, from the point of view of the firm, the skewed distributions outperform the ST distribution, since the latter distribution tends to raise the firm´s capital cost. As companies are free to choose the VaR model they use to forecast VaR, it is clear that they will prefer the skewed distributions.

Chapter 4*

Role of the loss function in the VaR comparison

4.1. Introduction

The global financial crisis suffered in the last years has taught us the importance of measuring risk accurately. Because the Basel Committee on Banking Supervision (BCBS) at the Bank for International Settlements requires that financial institutions meet capital requirements for the base Value at Risk (VaR), this methodology has become a basic market risk management tool. Consequently, the last decade has witnessed the growth of literature proposing new models to estimate the VaR. To know which is the best of these models has been and still is a primary aim of the empirical

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This chapter has been published as a working paper. The complete reference is Abad P, Benito S, López C. (2014). Role of the Loss Function in the VaR Comparison. WP nº 756/2014. Fundación de Cajas de Ahorros (FUNCAS).

Several studies dedicated to comparing VaR models have used a standard backtesting procedure (see Bhattacharyya and Ritolia (2008), Yu et al. (2010), Nozari et al. (2010), Bao et al. (2006), and Mittnik and Paolella (2000), among others).

The standard backtesting is based on calculating the number of times that losses exceed the VaR and comparing this value with the expected number using statistical tests.

Jorion (2001) defines backtesting as an ex-post comparison of a risk measure generated by a risk model against actual changes in the portfolio value over a given period. The Basel Committee on Banking Supervision (1996a) and the amendments of the Basel Committee on Banking Supervision (1996b) developed several statistical tests to evaluate the accuracy of the VaR estimates. More recently, in Basel III (2010), the committee pointed out the necessity of verifying the model's accuracy through frequent backtesting, although no particular backtesting technique is recommended.

A different perspective is given by Lopez (1998, 1999) who indicates that it is also important to know the size of the non-covered losses. To calculate the uncovered losses, he proposes using a loss function. The loss function is based not on a hypothesis-testing framework such as the statistical test but on examining the distance between the observed returns and the forecasted VaR(α) when the losses are uncovered. Some papers dedicated to comparing VaR models use both backtesting procedures: statistical tests and loss function (see Abad and Benito (2013), Orhan and Köksal (2012), Marimoutou et al. (2009) and Angelidis and Degiannakis (2007), among others).

There is a trade-off between the regulators and the financial enterprises regarding the aims in the market risk management tool. Supervisors are concerned about how many times losses exceed the VaR and the size of the non-covered losses. However, the risk managers have a conflict between the goal of safety and the goal of profit maximisation. An excessively high VaR forces them to hold too much capital, imposing large opportunity costs of capital upon the firm. Considering this factor, Sarma et al. (2003) propose a firm's loss function.

This paper focuses on loss functions. We examine whether the results of comparing the VaR models depend on the loss function used. In a comparison of a large set of VaR models, we compare these models using several loss functions proposed in the literature from the point of view of the regulator and from the point of view of the firm. Additionally, we propose a new firm's loss function, in line with Sarma et al. (2003). This function has the advantage of precisely computing the opportunity cost of the firm when the losses are covered.

The relevance of this study is twofold. First, it fills a gap in the literature regarding the comparison of VaR models, as this is the first paper to analyse whether the results of the VaR model comparison are robust with the loss function used. Second, we propose a new loss function that better captures the aim of the firm. Our results can help market participants, supervisors and risk managers to select the best VaR models, taking into account the different utility functions that each one has to face.

The rest of the paper is organised as follows: in the next section, we describe the backtesting procedure, focusing mainly on the role of the loss function. In section 3, we present the data we have used in the paper and the results of the empirical application. The last section includes the main conclusions.

4.2. Loss Functions

Since the late 1990s, a wide variety of tests have been proposed for evaluating the performance of VaR models. The backtesting procedures used in the literature can be classified into two groups: backtesting based on any statistical test and backtesting based on the loss function.[18]

The unconditional coverage test (Kupiec (1995)), the conditional coverage test and the independence test of Christoffersen (1998) and the Backtesting Criterion Statistic are the most usual backtesting procedures based on any statistical test. To implement all these tests, the exception indicator (I*t*) must be defined. If r_t represents the returns and $VaR(\alpha)$ is the VaR obtained with a given probability $\alpha \in (0,1)$, we have an exception when $r_{.t} < VaR_t(\alpha)$, and then I*t* is equal one (zero otherwise).

The unconditional coverage test assumes that an accurate VaR(α) measure provides an *unconditional coverage*; i.e., the percentage of exceptions observed ($\hat{\alpha}$) should be consistent with the theoretical proportion of failures (α). Thus, the null hypothesis of this test is $\hat{\alpha} = \alpha$. A similar test for the significance of the departure of $\hat{\alpha}$ from α is the *backtesting criterion* statistic.

The *conditional coverage test* proposed by Christoffersen (1998) jointly examines whether the model generates a correct proportion of failures and whether the exceptions are statistically independent from one another. The independence property of exception is an essential property because the measures of risk must reply automatically to any new information; a model that does not consider this factor would provoke clustering of exceptions.

The backtesting procedures based on certain statistical tests present a drawback; they only show whether the VaR estimates are accurate, so this toolbox does not allow us to rank the models.

Backtesting based on the loss function pays attention to the magnitude of the failure when an exception occurs. Lopez (1998, 1999), who is a pioneer in this area,

[18] There is no general agreement in the literature addressing what backtesting really comprises.

proposes to examine the distance between the observed returns and the forecasted VaR(α). This difference represents the loss that has not been covered. The loss function enables the financial manager to rank the models. The model that minimises the total loss will be preferred to the other models.

Lopez (1999) proposed a general form of the loss function:

$$L_t = \begin{cases} f(r_t, \text{VaR}) & \text{if} \quad r_t < \text{VaR} \\ g(r_t, \text{VaR}) & \text{if} \quad r_t \geq \text{VaR} \end{cases} \qquad (4.1)$$

where $f(r_t, \text{VaR})$ and $g(r_t, \text{VaR})$ are functions such that $f(r_t, \text{VaR}) \geq g(r_t, \text{VaR})$, thereby penalising to a greater extent those cases where the real returns fall bellow the VaR estimations. He considers three loss functions: (i) the Binomial loss function that assigns the value 1 when the VaR estimate is exceeded by its loss and 0 otherwise, (ii)the Zone loss function based on the adjustments to the multiplication factor used in market risk amendment (see Sajjad et al. (2008), Hass (2001) and Lopez (1998) among others), and (iii) the magnitude loss function, which assigns a quadratic numerical score when a VaR estimate is exceeded by its loss and 0 otherwise. Subsequently, not only the VaR exception but also the magnitude of the losses is incorporated. Depending on the form adopted by $f(r_t, \text{VaR})$ and $g(r_t, \text{VaR})$, we can speak of two types of functions: regulator's loss functions and firm's loss functions.

The regulator's loss functions pay attention to the magnitude of the non-covered losses only when they occur. Thus, the Lopez's Magnitude loss function has the following quadratic specification:

$$RQL = \begin{cases} 1 + (VaR_t - r_t)^2 & if \quad r_t < VaR_t \\ 0 \quad if \quad r_t \geq VaR_t \end{cases} \qquad (4.2)$$

In this loss function, the quadratic term ensures that large failures are penalised more than small failures. This function was built mainly for regulatory purposes for evaluating the bank internal models. Applications of this loss function are numerous (see Ozun et al. (2010), Campell (2005), Marimoutou et al. (2009), Zatul (2011), Osiewalski and Pajor (2012) and Orhan and Köksal (2012), among others).

Since Lopez (1998, 1999), many authors have proposed other alternative functions with the same goal, to measure the distance between returns and VaR estimates when an exception occurs. In column 1 of Table 4.1, we report some of these functions.

Sarma et al. (2003) defined the regulator's loss function as follows:

$$RQ = \begin{cases} (VaR_t - r_t)^2 & if \quad r_t < VaR_t \\ 0 & otherwise \end{cases} \qquad (4.3)$$

Applications of this function can be found in Angelidis et al. (2007) and Abad and Benito (2013), among others. Caporin (2008) notes that there is an open issue with the function aforementioned. At a parity exception, we may reject a correctly specified model only because it provides higher losses. For this author, what is important is not the losses uncovered but their relative size. To solve this point, he divides $f\left(r_t, \text{VaR}\right)$ by VaR. The mathematical expression of these functions can be found in the first column of Table 4.1.

The aforementioned loss function only takes into account the magnitude of the failure but does not consider the cases in which the returns exceed the VaR estimates. This is an important point because a too high VaR overestimation would lead firms to hold much more capital than necessary, thus imposing an opportunity cost of capital above. Firms must resolve the conflict related to safety, in the same way that a regulator does, but they also have the objective of maximising their profits. For this purpose, Sarma et al. (2003) define a firm's loss function (FS), where the non-exception days are penalised according to the opportunity cost of the reserved capital held by the firm for risk management purposes:

$$FS = \begin{cases} \left(\mathrm{VaR}_t - r_t\right)^2 & \text{if} \quad r_t < \mathrm{VaR}_t \\ -\beta.\mathrm{VaR}_t & \text{if} \quad r_t \geq \mathrm{VaR}_t \end{cases} \qquad (4.4)$$

where β is the cost of capital for the firm. Thus, a model that may be adequate because it provides few exceptions becomes inadequate if the opportunity capital cost is high. Caporin (2008) suggests applying the same loss function not only to the exceptions but also to the entire sample, (an exception occurs and does not), i.e., he suggests applying a function such as $f\left(r_t, \mathrm{VaR}\right) = g\left(r_t, \mathrm{VaR}\right)$.

In line with Sarma et al. (2003), we propose a new loss function to capture the aim of the firm. The expression of our function is as follows:

$$FABL = \begin{cases} \left(\mathrm{VaR} - r_t\right)^2 & \text{if} \quad r_t < \mathrm{VaR} \\ \beta\left(r_t - \mathrm{VaR}\right) & \text{if} \quad r_t \geq \mathrm{VaR} \end{cases} \qquad (4.5)$$

As can be determined in this function, the exceptions are penalised as usual in the literature, following the instructions of the regulator. When there are no exceptions, the loss function penalises the difference between the VaR and returns weighted by a factor β that represents an interest rate. This product is exactly the opportunity cost of the capital, i.e., the excess capital held by the firm.

Sarma et al. (2003) penalises the cases in which there are no exceptions for multiplying the VaR estimate by a factor β. From our point of view, this product does not precisely capture the opportunity cost of the capital. Unlike Sarma et al. (2003), we are committed to measuring the real cost of opportunity, rather than the cost of security imposed by Basel. On the other hand, Sarma et al. (2003) do not identify the factor β. We propose the price of the capital opportunity cost to be an interest rate. These firm´s loss functions are presented in the second column of Table 4.1.

Table 4.1. Loss functions

Regulator's loss function (RLF)		**Firm's loss functions (FLF)**	
Lopez's quadratic (RQL)	$\begin{cases} 1+\left(VaR_t - r_t\right)^2 & \text{if } r_t < VaR_t \\ 0 & \text{if } r_t \geq VaR_t \end{cases}$	**Sarma (FS)**	$\begin{cases} \left(VaR_t - r_t\right)^2 & \text{if } r_t < VaR_t \\ -\beta.VaR_t & \text{if } r_t \geq VaR_t \end{cases}$
Lineal (RL)	$\begin{cases} \left(VaR_t - r_t\right) & \text{if } r_t < VaR_t \\ 0 & \text{if } r_t \geq VaR_t \end{cases}$	**Caporin_1 (FC_1)**	$\left\lvert 1 - \left\lvert \frac{r_t}{VaR} \right\rvert \right\rvert \quad \forall r_t$
Quadratic (RQ)	$\begin{cases} \left(VaR_t - r_t\right)^2 & \text{if } r_t < VaR_t \\ 0 & \text{if } r_t \geq VaR_t \end{cases}$	**Caporin_2 (FC_2)**	$\frac{\left(\lvert r_t \rvert - \lvert VaR \rvert\right)^2}{\lvert VaR \rvert} \quad \forall r_t$
Caporin_1 (RC_1)	$\begin{cases} \left\lvert 1 - \left\lvert \frac{r_t}{VaR} \right\rvert \right\rvert & \text{if } r_t < VaR \\ 0 & \text{if } r_t \geq VaR \end{cases}$	**Caporin_3 (FC_3)**	$\lvert VaR - r_t \rvert \quad \forall r_t$
Caporin_2 (RC_2)	$\begin{cases} \frac{\left(\lvert r_t \rvert - \lvert VaR \rvert\right)^2}{\lvert VaR \rvert} & \text{if } r_t < VaR \\ 0 & \text{if } r_t \geq VaR \end{cases}$	**Abad_Benito_López (FABL)**	$\begin{cases} \left(VaR_t - r_t\right)^2 & \text{if } r_t < VaR_t \\ \beta\left(r_t - VaR\right) & \text{if } r_t \geq VaR_t \end{cases}$
Caporin_3 (RC_3)	$\begin{cases} \lvert VaR - r_t \rvert & \text{if } r_t < VaR \\ 0 & \text{if } r_t \geq VaR \end{cases}$		

Note: This table presents the different loss functions used in this paper. In the first column, we show the regulator's loss functions (Lopez' magnitude loss function (RQL), lineal regulatory function (RL), Sarma et al. (2003) quadratic loss function (RQ) and the three loss function suggested by Caporin (2008) ((RC_1), (RC_2), and (RC_3)). The second column lists the firm's loss functions (Sarma et al. (2003) (FS), the three loss function suggested by Caporin (2008) (FC_1), (FC_2), and (FC_3) from the viewpoint of the firms, and our new loss function (FABL)).

4.3. Empirical results.

The purpose of this paper is to check whether the comparison of different VaR models is independent of the loss function used. With this aim, we compare several VaR models using a two-stage selection approach. In the first stage, Kupiec and Christoffersen's tests are applied. In the second stage, and only for the remaining models, we calculate the loss functions of Table 4.1. The VaR models included in the comparison are as follows: Historical Simulation (HS), Filtered Historical Simulation (FHS), Conditional and Unconditional Extreme Value Theory (CGPD and GPD) and Parametric approach based on a normal and *t*-Student distribution. Next, we consider the conditional and unconditional volatility measure: Normal, Conditional Normal (CN) and Conditional *t*-Student (CS). Table 4.2 shows the expressions for these VaR models.

Table 4.2. Statistical approaches for estimating the Value at Risk

Normal	$VaR_{t+1} = \phi^{-1}(p)$
Historical Simulation (HS)	$VaR_{t+1} = Quantile\left\{\{r_t\}_{t=1}^{n}\right\}$
Filtered Historical Simulation (FHS)	$VaR_{t+1} = \mu_{t+1} + \sigma_{t+1} Quantile\left\{\{r_t\}_{t=1}^{n}\right\}$
Unconditional GPD (GPD)	$VaR_{t+1} = \hat{u} + \frac{\hat{\sigma}}{\hat{\xi}}\left[\left(\frac{n}{N_u}(1-p)\right)^{-\hat{\xi}} - 1\right]$
Conditional Normal (CN)	$VaR_{t+1} = \mu_{t+1} + \sigma_{t+1}\phi^{-1}(p)$
Conditional Student (CS)	$VaR_{t+1} = \mu_{t+1} + \sigma_{t+1}\sqrt{\frac{\nu-2}{\nu}}F^{-1}(p)$
Conditional GPD (CGPD)	$VaR_{t+1} = \mu_t + \sigma_{t+1} VaR_t(Z)$

Note: $\phi^{-1}(p)$ is the quantile of the standard normal distribution, $\hat{u}$ is the threshold, $\hat{\sigma}$ is the estimated scale parameter, $\hat{\xi}$ is the estimated shape parameter, N_u is the number of exceedances over the threshold, μ_{t+1} and σ_{t+1} are the conditional forecasts of the mean and the standard deviation, $F^{-1}(p)$ and v are the quantile the *t*-distribution and the degrees of freedom, respectively, and *Z* is the standardised residual series.

The study has been performed using the spot price for crude oil (the Brent) and a stock index (Dow Jones), offering robustness to dataset.

4.3.1. Data

The data consist of a spot crude oil price (Brent) and a stock index (Dow Jones Industrials) extracted from DataStream database. The prices of crude oil Brent are measured in US $/Barrel. These prices are transformed into returns by taking logarithmic differences of the closing daily price. We use daily data for the period May 20, 1987 through November 30, 2014, thus our sample covers the recent years of turmoil.

The full data period is divided into a learning sample (May 20, 1987 to December 31, 2007) and a forecast sample (January 1, 2008 to November 30, 2014). Thus, we work with 7183 observations and generate 1803 out-of-sample VaR forecasts.

Figure 4.1 presents the daily prices of crude oil Brent and of Dow Jones index. Both of them show an increasing trend. The daily returns are too presented in Figure 4.1 and their basic descriptive statistics are provided in Table 4.3.

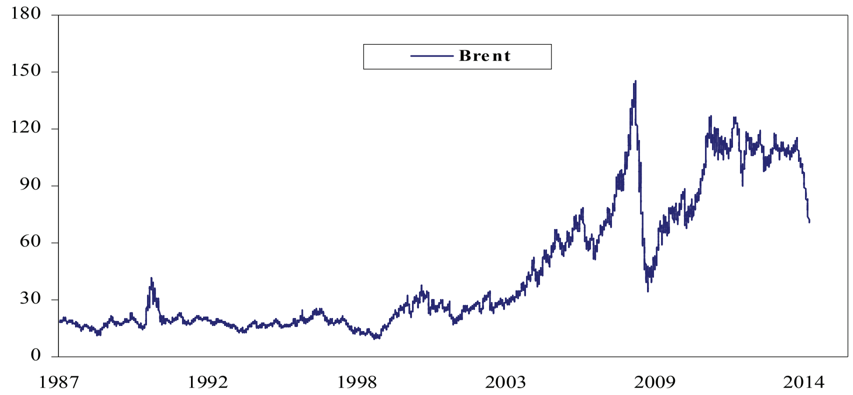
Table 4.3. Descriptive statistics of the daily returns

	Mean (%)	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis	Jarque-Bera
BRENT	0.0189* (0.0002)	0.000	0.135	-0.439	0.022	-1.143* (0.035)	28.144* (0.07)	190758 (0.001)
DOW JONES	0.029* (0.0001)	0.000	0.105	-0.256	0.011	-1.791* (0.029)	47.755* (0.058)	603238 (0.001)

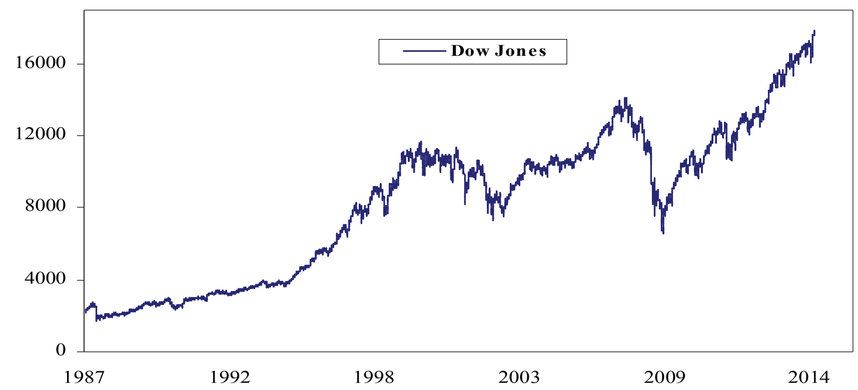
Note: This table presents the descriptive statistics of the daily returns of the Brent crude oil and Dow Jones index. The data set cover from May 20th, 1987, to November 30th, 2014. The return is calculated as $r_t=\ln(P_t)/(P_{t-1})$, where P_t is the price level for period *t*. The standard errors of the skewness, the mean and excess kurtosis are calculated as $\sqrt{6/n}$, $\sigma/\sqrt{n}$ and $\sqrt{24/n}$, respectively. The Jarque-Bera statistic is distributed as the Chi-square with two degrees of freedom. (*) denotes significance at the 5% and 1% levels, respectively

Figure 4.1: Crude oil Brent and Dow Jones index

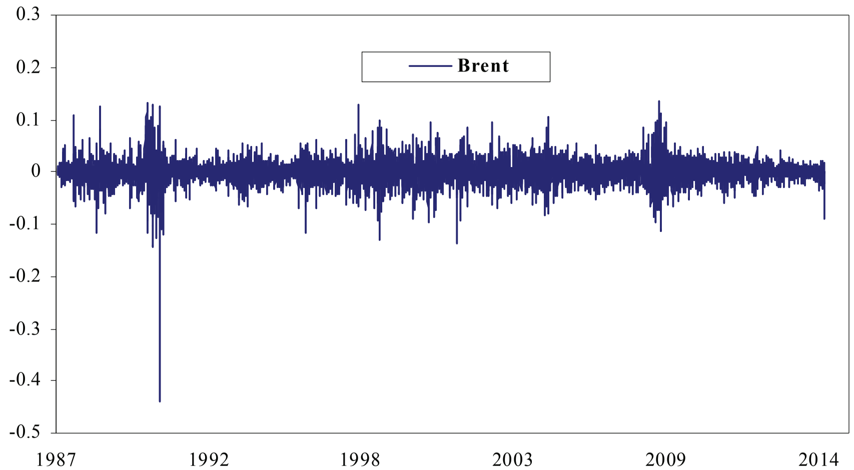
a) Daily prices of crude oil Brent



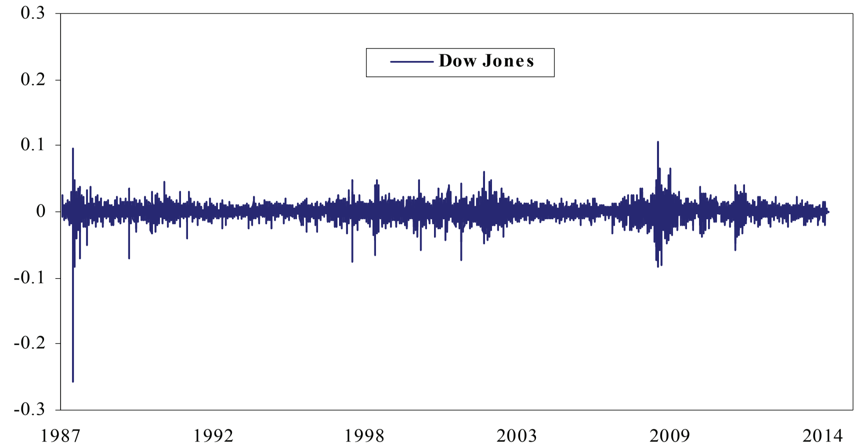
b) Dow Jones index



c) Daily returns of crude oil Brent



d) Daily returns of Dow Jones index



The unconditional mean daily return is very close to zero (0.02% and 0.03% respectively), which is significant. The skewness statistics are negative, implying that distributions of both daily returns are skewed to the left. The kurtosis coefficients imply that both distributions have much thicker tails than the normal distribution does. Similarly, the Jarque-Bera statistics are statistically significant, rejecting the assumption of normality. All of this evidence shows that the empirical distribution of both daily returns cannot be fit by a normal distribution, as they exhibit a significantly excess of kurtosis and asymmetry (fat tails and peakness).

Going back to Figure 4.1, we can see that the range fluctuation of both daily returns are not constant, which means that the variance of these returns changes over time. The volatility of Brent was high during the early 1900s, coinciding with the beginning of the Gulf War. Furthermore, we observe high volatility since middle 2008 to the end 2009 in both daily returns. In the last years of the sample, we observe a period more stable. We estimate an AR(1)-GARCH(1,1) specification. The VaR is calculated the forthcoming day for four confidence levels: 95%, 99%, 99.5% and 99.9%.

For evaluating the performance of each model in terms of the VaR, we also use a backtesting procedure in two stages. In the first stage, we use four accuracy tests (the unconditional coverage test (LR_{uc}), Backtesting criterion (BTC), the conditional coverage test (LR_{cc}) and the independence test (LR_{ind})). If a model passes all tests, the model is accurate. In the second stage, we evaluate the magnitude of the loss functions for the first stage overcoming models. Eleven loss functions have been used in our comparative: six loss functions which reflect the utility function of the regulator/supervisor and five loss functions from the viewpoint of the firm/risk manager.

Overall, we design an exhaustive comparison. We estimate the VaR by using seven different models for four levels of confidence ensued by two-stage backtesting procedure to assess the forecasting power of each model: four accuracy tests and eleven loss functions.

4.3.2. Results

The results of the accurate tests are presented in Table 4.4. In this table we show the percentage of exception obtained with each VaR model at the 99.9%, 99.5%, 99% and 95% confidence levels. The table reports the *p*-value obtained for the accurate tests for each confidence scenario. When the null hypothesis that "the VaR estimate is accurate" has not been rejected by any test, we have shaded the percentage of exceptions.

According to Table 4.4, we can assert that the VaR estimates obtained by the unconditional and conditional normal distribution are very poor, especially for the Dow Jones. For Brent, the unconditional normal distribution provide accurate VaR estimate at 0.5% and 1% probability level while conditional normal distribution yield good VaR estimate at 0.1% and 5% probability level. For the Dow Jones, these distributions provide unaccurate VaR estimates for whatever probability level.

For almost all probability level, HS, FHS and the conditional GPD yield good VaR estimations for both series of data. For CS and unconditional GPD some differences are observed between the two series. For the Dow Jones, the CS distribution provides accurate VaR estimates at any probability level while for Brent just only at 0.1% and 0.5% probability. The opposite is observed for unconditional GPD. For Brent, this method yields accurate VaR estimates at three probability level while for Dow Jones this method provides inaccurate VaR estimates for all probability levels.

Table 4.4. Accuracy tests

	Brent				Dow Jones			
	0.10%	**0.50%**	**1%**	**5%**	**0.10%**	**0.50%**	**1%**	**5%**
Normal	**0.50%**	**0.78%**	**1.00%**	**3.33%**	**1.89%**	**2.44%**	**3.22%**	**6.38%**
LR_{UC}	0.0119	0.3105	0.9963	0.0226	0	0	0	0.0891
BTC	0	0.0998	0.3989	0.002	0	0	0	0.0108
LR_{IND}	0.3677	0.2726	0.3677	0.0554	0.096	0.1375	0.096	0.0754
LRcc	0.0281	0.3275	0.6665	0.0119	0	0	0	0.0485
HS	**0.22%**	**0.50%**	**0.94%**	**4.27%**	**0.22%**	**0.83%**	**1.44%**	**5.44%**
LR_{UC}	0.3534	0.9974	0.8711	0.3373	0.3534	0.2295	0.2436	0.5812
BTC	0.1045	0.3989	0.3873	0.1454	0.1045	0.0542	0.0673	0.2784
LR_{IND}	0.0849	0.1618	0.0849	0.0689	0.0659	0.2958	0.0659	0.2027
LRcc	0.1474	0.3759	0.2238	0.1206	0.1199	0.2813	0.0934	0.3815
FHS	**0.06%**	**0.28%**	**0.89%**	**4.60%**	**0.06%**	**0.44%**	**1.22%**	**6.32%**
LR_{UC}	0.6666	0.3345	0.7468	0.606	0.6666	0.8199	0.5494	0.1022
BTC	0.3336	0.1624	0.3554	0.296	0.3336	0.3767	0.2566	0.0144
LR_{IND}	0.7242	0.9125	0.7242	0.3094	0.627	0.8603	0.627	0.5862
LRcc	0.8564	0.6239	0.8919	0.5223	0.8098	0.9594	0.7429	0.2268
GPD	**0.00%**	**0.22%**	**0.72%**	**3.72%**	**0.33%**	**1.39%**	**2.16%**	**6.99%**
LR_{UC}	0.2107	0.2148	0.409	0.0847	0.1052	0.0039	0.0046	0.0157
BTC	0.1618	0.0982	0.1964	0.0175	0.003	0	0	0.0002
LR_{IND}	0.2496	0.9299	0.2496	0.0255	0.2145	0.0586	0.2145	0.0771
LRcc	0.2355	0.4615	0.3666	0.0187	0.1246	0.0026	0.0084	0.0114
CN	**0.22%**	**1.00%**	**1.50%**	**5.16%**	**0.67%**	**1.50%**	**2.55%**	**6.32%**
LR_{UC}	0.3534	0.0819	0.1925	0.84	0.0009	0.0014	0.0003	0.1022
BTC	0.1045	0.0044	0.0419	0.3805	0	0	0	0.0144
LR_{IND}	0.5963	0.6912	0.5963	0.3495	0.3063	0.5503	0.3063	0.8421
LRcc	0.565	0.2035	0.3718	0.6326	0.0025	0.0051	0.0008	0.2578
CS	**0.06%**	**0.17%**	**0.28%**	**3.00%**	**0.00%**	**0.39%**	**0.78%**	**4.22%**
LR_{UC}	0.6666	0.124	0.0161	0.0056	0.2107	0.6444	0.5129	0.3009
BTC	0.3336	0.0531	0.0034	0.0002	0.1618	0.3181	0.2531	0.1239
LR_{IND}	0.9125	0.9474	0.9125	0.2789	0.7576	0.8776	0.7576	0.3266
LRcc	0.9059	0.3057	0.055	0.0119	0.4356	0.8884	0.7698	0.3619
CGPD	**0.11%**	**0.33%**	**0.89%**	**5.05%**	**0.00%**	**0.44%**	**1.05%**	**6.16%**
LR_{UC}	0.9243	0.4798	0.7468	0.9518	0.2107	0.8199	0.8808	0.1512
BTC	0.3947	0.2404	0.3554	0.3973	0.1618	0.3767	0.3886	0.0315
LR_{IND}	0.7242	0.895	0.7242	0.4664	0.675	0.8603	0.675	0.442
LRcc	0.9354	0.7724	0.8919	0.7656	0.4184	0.9594	0.9056	0.2657

Note: VaR violation ratios of the daily returns (%) are boldfaced. The table reports the *p*-values of the following tests: (i) the unconditional coverage test (LRuc); (ii) the backtesting criterion (BTC); (iii) statistics for serial independence (LRind) and (iv) the conditional coverage test (LRcc). A *p*-value greater than 5% indicates that the forecasting ability of the VaR model is accurate. The shaded cells indicate that the null hypothesis that the VaR estimate is accurate is not rejected by any test.

Thus, only those VaR models which are more accurate remain. For Dow Jones only four of the seven models remain valid (HS, FHS, CS and CGPD), while all VaR models remain for the Brent.

For the remaining models, we calculate the loss functions[19]. The model that provides the lowest loss function value is the best. Tables 4.5 and 4.6 show the results obtained by the regulator´s loss functions and the firm's loss functions for Dow Jones and Brent, respectively. For each loss function, we marked in bold the model that provides the lowest losses. To calculate the FS and FABL firm's loss functions, we proxy the price of capital with the interest rate of the Eurosystem monetary policy operations of the European Central Bank for Brent oil and the interest rate of the open market operations used by the Federal Reserve in the implementation of its monetary policy for Dow Jones. Regarding the regulator's loss function, the results are as follow: (i) For the Dow Jones, the best model is the conditional Student (see panel a) of Table 4.5).

[19] We calculate the loss function only when the VaR model at this confidence levels is accurate according to tests of Table 4.4.

Table 4.5. Magnitude of the loss functions: Dow Jones

	Level	HS	FHS	CS	CGPD
Panel (a): Regulator´s loss function					
RQL	**0.10%**	4.0009	1	**0**	**0**
	0.50%	15.0064	8.0004	**7**	8.0003
	1%	26.0108	22.0012	**14.0004**	19.0011
	5%	98.035		**76.0061**	
RL	**0.10%**	0.0485	0.0022	**0**	**0**
	0.50%	0.2271	0.0416	**0.0102**	0.0312
	1%	0.391	0.1207	0.0546	**0.0986**
	5%	1.2646		**0.5002**	
RQ	**0.10%**	0.0009	0	**0**	**0**
	0.50%	0.0064	0.0004	**0**	0.0003
	1%	0.0108	0.0012	**0.0004**	0.0011
	5%	0.035		**0.0061**	
RC_1	**0.10%**	1.0799	0.1281	**0**	**0**
	0.50%	6.2797	1.4925	**0.4107**	0.8165
	1%	12.731	4.8116	**2.1122**	3.5973
	5%	73.2401		**27.1116**	
RC_2	**0.10%**	0.0211	0.0003	**0**	**0**
	0.50%	0.1734	0.011	**0.0006**	0.0071
	1%	0.3464	0.0377	**0.0111**	0.0327
	5%	1.9921		**0.2624**	
RC_3	**0.10%**	0.0485	0.0022	**0**	**0**
	0.50%	0.2271	0.0416	**0.0102**	0.0312
	1%	0.391	0.1207	**0.0546**	0.0986
	5%	1.2646		**0.5002**	
Panel (b): Firm´s loss function					
FS	**0.10%**	**0.0003**	0.0004	0.0003	0.0003
	0.50%	**0.0002**	0.0002	0.0002	0.0002
	1%	**0.0002**	0.0002	0.0002	0.0002
	5%	**0.0001**		0.0001	
FC_1	**0.10%**	1575.5669	1547.0887	1528.2154	**1517.1347**
	0.50%	1497.2591	**1387.6669**	1422.4349	1394.9188
	1%	1445.5347	**1311.6891**	1359.3426	1321.8967
	5%	1288.31		**1154.8907**	
FC_2	**0.10%**	99.3183	85.3595	69.1726	**67.3703**
	0.50%	63.8263	**38.4371**	44.2028	40.7233
	1%	50.3828	**28.9933**	35.0165	31.2904
	5%	23.1432		**17.0768**	
FC_3	**0.10%**	124.7218	111.4768	95.0095	**93.0125**
	0.50%	87.505	**62.0102**	68.4697	64.5222
	1%	72.844	**51.4048**	58.3761	53.9864
	5%	39.5071		**37.2432**	
FABL	**0.10%**	**0.0003**	0.0004	0.0003	0.0003
	0.50%	**0.0002**	0.0002	0.0002	0.0002
	1%	**0.0002**	0.0002	0.0002	0.0002
	5%	**0.0001**		0.0001	

Note: The table reports the values of the sum of the different loss functions of each VaR model at all confidence levels. The boldface figures denote the minimum value of the function.

Table 4.6. Magnitude of the loss functions: Brent

	Level	Normal	HS	FHS	GPD	CN	CS	CGPD
Panel (a): Regulator´s loss function								
RQL	**0.10%**		4.0015	1.0019	**0.0000**	4.0022	1.0004	2.0012
	0.50%	14.0072	9.006	5.0024	4.0018		**3.0015**	6.0024
	1%	18.0107	17.0112	16.0032	**13.0054**			16.0037
	5%		**77.0372**	83.017		93.0191		91.0185
RL	**0.10%**		0.0662	0.0431	**0.0000**	0.0667	0.0195	0.0352
	0.50%	0.2537	0.1929	0.0606	0.0786		**0.0459**	0.0727
	1%	0.3482	0.346	**0.1247**	0.1984			0.1392
	5%		1.1299	**0.7951**		0.9004		0.8895
RQ	**0.10%**		0.0015	0.0019	**0.0000**	0.0022	0.0004	0.0012
	0.50%	0.0072	0.006	0.0024	0.0018		**0.0015**	0.0024
	1%	0.0107	0.0112	**0.0032**	0.0054			0.0037
	5%		0.0372	**0.017**		0.0191		0.0185
RC_1	**0.10%**		0.9383	0.8899	**0.0000**	1.5423	0.2702	0.6244
	0.50%	4.3029	3.3443	1.4659	1.0386		**0.9089**	1.7678
	1%	6.5256	6.8983	**3.0948**	3.1515			3.61
	5%		35.0959	**26.9359**		32.42		32.1989
RC_2	**0.10%**		0.0232	0.0383	**0.0000**	0.0491	0.0053	0.0202
	0.50%	0.1232	0.108	0.0554	**0.0238**		0.0294	0.0562
	1%	0.201	0.2312	**0.0785**	0.0853			0.0961
	5%		1.1833	**0.5535**		0.6485		0.6447
RC_3	**0.10%**		0.0662	0.0431	**0.0000**	0.0667	0.0195	0.0352
	0.50%	0.2537	0.1929	0.0606	0.0786		**0.0459**	0.0727
	1%	0.3482	0.346	**0.1247**	0.1984			0.1392
	5%		1.1299	**0.7951**		0.9004		0.8895
Panel (b): Firm´s loss function								
FS	**0.10%**		0.0005	0.0006	0.0006	0.0009	0.0015	**0.0005**
	0.50%	**0.0003**	0.0003	0.0003	0.0004		0.0011	0.0003
	1%	**0.0003**	0.0003	0.0003	0.0003			0.0003
	5%		0.0002	0.0002		0.0005		**0.0002**
FC_1	**0.10%**		1543.4875	1528.4207	1585.2465	**1386.463**	1547.0633	1487.0991
	0.50%	1409.9798	1450.33	**1385.2812**	1489.3883		1446.8982	1385.8997
	1%	1374.7132	1404.4418	1323.2754	1430.9376			**1321.4011**
	5%		1211.841	1125.4982		1110.886		**1109.2768**
FC_2	**0.10%**		126.2354	137.0146	152.1202	**64.2721**	127.6408	96.5676
	0.50%	67.3055	79.9697	64.5803	94.4141		81.106	**64.3532**
	1%	58.4595	66.2946	51.7016	73.437			**51.2515**
	5%		31.8777	26.3528		24.8093		**24.6552**
FC_3	**0.10%**		166.9571	178.5091	194.0341	**102.3374**	169.62	137.1252
	0.50%	104.0549	118.127	102.6672	133.541		120.7028	**102.4565**
	1%	94.1364	102.9299	88.3235	110.8294			**87.8005**
	5%		61.9775	57.587		55.563		**55.3022**
FABL	**0.10%**		0.0005	0.0006	0.0006	0.0009	0.0015	**0.0004**
	0.50%	**0.0003**	0.0003	0.0003	0.0004		0.0011	0.0003
	1%	**0.0003**	0.0003	0.0003	0.0003			0.0003
	5%		0.0002	0.0002		0.0005		**0.0002**

Note: The table reports the values of the sum of the different loss functions of each VaR model at all confidence levels. The boldface figures denote the minimum value of the function.

This result is independently of the probability level and the regulator´s loss function used to evaluate losses; (ii) For Brent, the best model depends on the probability level we are considering (see panel a) of Table 4.6). However, once we have fixed the probability level, the results are robust to the regulator´s loss function used to evaluate the losses. At 0.1% probability the best model is unconditional GPD. At 0.5% the best model is CS. To last, at 1% and 5% the best model is FHS. Thus, the results of the comparison seem to indicate that the best model is robust to the regulator´s loss function although depend on the probability level.

Regarding the loss function from the viewpoint of the firm, the best model depends on the probability level and the loss function used to evaluate losses (see panel b) of Tables 4.5 and 4.6). However, the differences among the loss function are not arbitrary. The firm´s loss function we propose in this paper and the Sarma firm´s loss function move always together, while the three Caporin firm´s loss functions provide almost always the same results. We find two groups of firm's loss functions. Sarma firm´s loss function and the function which we propose in this paper capture the opportunity capital cost of the firm, while the Caporin firm´s loss functions do not capture these cost since them penalize in the same way the covered and non-covered losses.

For Dow Jones, according to firm´s loss function we and Sarma propose that the best model is HS for any probability level, while according to the three Caporin firm´s loss function the best model is FHS at 0.5% and 1% probability, CS at 5% and CGPD at 0.1%.

For Brent, we also find some differences depending on the probability level and the type of firm´s loss function used to evaluate losses. According to the loss functions which measure the opportunity capital cost of the firm, the best model at 0.5% and 1%

is unconditional Normal. For these probability levels the Caporin firm´s loss function point to CGPD as the best model. At 0.1% of probability, the Caporin firm's loss function point to CN, while the loss functions which proxy the opportunity capital cost indicate that CGPD is the best VaR model. Again, we find differences with regard to the regulator´s loss function.

Thus, we find some differences depending on the kind of firm´s loss function chosen. We distinguish between the firm's loss function which proxy the opportunity capital cost of the firm and the firm's loss function which penalize in the same way the covered and non-covered losses. According to these results, it seems that the risk manager shouldn´t be indifferent to the firm´s loss function chosen. To this respect, we recommend to use the firm´s loss function we propose in this paper as this loss function captures in the most reliable way the opportunity capital cost of the companies.

As a summary, we conclude that from the regulator point of view, the best VaR model is robust to the function. However, from the viewpoint of the companies, the best VaR model depends on the firm´s loss function. In any case, what is clear is that the best VaR model depends on the family functions used: regulator´s loss functions and/or firm´s loss function. [20]

[20] We find the same result for the Brent price in another forecast sample (January 1, 1991 to January, 24, 2006). They are available for any interested reader upon request to the authors.

4.4. Conclusions

This chapter investigates whether the results of the comparison VaR models depend on the loss function used for such purpose. Furthermore, we propose a loss function that captures the opportunity capital cost of the firm in the case in which losses have been covered.

For this study, we use daily returns of the Brent price and Dow Jones index from May 20, 1987 to November 30, 2014.

The VaR models that we have included in the comparison are Historical Simulation (HS), Filtered Historical Simulation (FHS), Conditional and Unconditional Normal, Conditional Student and Conditional and Unconditional Extreme Value (GPD).

The best model is selected by a two-stage selection approach. First, we apply a backtesting procedure based on four statistical tests. For the accurate models that remain at the first stage, we compute the losses using several loss functions from two groups: firm´s loss functions and regulator/supervisor's loss functions. Our results indicate that in terms of their ability to forecast the VaR, the best model is robust to the regulator's loss function. However, from the viewpoint of the companies, the best VaR model depends on the type of firm's loss function. We find two subgroup of firm's loss function: the firm's loss functions which proxy the opportunity capital cost of the firm when the losses are covered and the firm's loss functions which penalize in the same way the covered and non-covered losses. The best VaR model is robust to the firm's loss function within the subgroup. In any case, what is clear is that the best VaR model depends on the family functions used: regulator´s loss functions and/or firm´s loss function as these families provide different results.

Our results can help market participants make effective selections between VaR models. The market participants (supervisors and risk managers) must consider that they have specific loss functions, and the final result depends on who is the end-user of the VaR model. Finally, our results can also help researchers to understand the different results presented in the literature.

Chapter 5

Conclusions

In this dissertation, three specific issues have been developed: i) a deep analysis of the State of the Art, from standard approaches for measuring VaR to the more evolved, while highlighting their relative strengths and weaknesses; ii) the performance of several skewed and symmetric distributions in modeling the tail behavior of daily returns and forecasting Value at Risk (VaR); iii) and the analysis of the role that the loss function plays in the comparison of VaR models. This Chapter aims to conclude from the main results obtained in each section.

In the first part of the dissertation, corresponding to Chapter two, a theoretical review of the existing literature on Value at Risk (VaR) has been shown including the back-testing procedures used to assess VaR approach performance. Furthermore, some empirical studies dedicated to compare several VaR methodologies are reviewed. From this review, it can be concluded that the approach based on the EVT and FHS are the best methods to estimate the VaR. It is also shown that VaR estimates obtained by some asymmetric extensions of CAViaR method and the Parametric method under the skewed and fat-tail distributions lead to promising results, especially when the assumption that the standardised returns are independent and identically distributed is abandoned and that the conditional high-order moments are considered to be time-varying.. From the study, it seems clear that the new proposals to estimate VaR have outperformed the traditional ones.

According to the results shown in Chapter two, where the Parametric method under the skewed and fat-tail distributions lead to promising results, in Chapter three it is evaluated the performance of several skewed and symmetric distributions in modeling the tail behavior of daily returns and in forecasting VaR. For this, the skewed distributions considered are: (i) the skewed Student-*t* distribution of Hansen (1994); (ii) the skewed error generalised distribution of Theodossiou (2001); (iii) the skewed generalised-*t* distribution of Theodossiou (1998) and (iv) the inverse hyperbolic sign of Johnson (1949). The symmetric distributions are the normal and the Student-*t* ones.

For this study, it has been used daily returns on nine composite indexes: the Japanese Nikkei, Hong Kong Hang Seng, Israeli Tel Aviv (100), Argentine Merval, US S&P 500 and Dow Jones, UK's FTSE100, the French CAC40 and the Spanish IBEX35. The sample used for the statistical analysis runs from January 2000 to the end of November 2012. The analysis period covered to forecast VaR runs from 2008 to 2009, which is known as the Global Financial Crisis period.

Based on the submitted results, it can be concluded that the skewness and fat tail distributions outperform the normal one in fitting financial returns and forecasting VaR. Among all the skewed distributions considered, the skewed generalised-*t* distribution of Theodossiou (1998) is the best one in fitting data. However, in terms of their ability to forecast the VaR, it does not find significant differences as all of them provide accurate VaR estimates for a high number of indexes and produce similar losses. Finally, it is found evidence in favor of the skewed distributions compared to the Student distribution. In statistical terms, most of them fit the data better than the Student ones. In terms of VaR, the accuracy test indicates that the skewed distributions outperform the Student ones. On the other hand, with regards to the loss function, the result depends on the kind of function used to measure the losses. From the regulator point of view, Student distribution is the best one in forecasting VaR, as this distribution provides the more conservative VaR estimate. However, from the firm point of view, the skewed distributions outperform the Student distribution, since the latter distribution tends to raise the firm´s capital cost. As companies are free to choose the VaR model they use to forecast VaR, it is clear that they will prefer the skewed distributions.

The fourth Chapter of this dissertation examines whether the comparison of VaR models depends on the loss function used for such purpose. In this Chapter, a detailed comparison for several VaR models for two groups of loss functions (designed for regulators and for risk managers) is shown. The VaR models included in the comparison are as follows: Historical Simulation, Filtered Historical Simulation, Conditional and Unconditional Extreme Value Theory and Parametric approach based on a normal and *t*-Student distribution. In addition, conditional and unconditional volatility measures have been considered: Normal, Conditional Normal and Conditional *t*-Student. For this study, closing daily data of a spot crude oil price (Brent) and a stock index (Dow Jones Industrials) have been used. The data set covers the period May 20, 1987 through November 30, 2014, in order to include the economic shocks that occurred during the nineties (Gulf War and Asian Financial Crises)and the recent years of global crisis. Additionally, a firm’s loss function that exactly measures the opportunity cost of the firm when the losses are covered has been proposed. The best model is selected by a two-stage selection approach. First, a backtesting procedure based on four statistical tests has been applied. For the models that survive the first stage, the losses have been calculated using several loss functions from two groups: firm´s loss functions and regulator/supervisor's loss functions.

The results indicate that in terms of their ability to forecast the VaR, the best model is robust to the regulator's loss function. According to the firm's loss function, the results indicate that it must distinguish between two subgroups of firm's loss function: i) functions which proxy the opportunity cost of the firm when losses are covered and ii) functions which penalize in the same way when losses are covered or non-covered. The best VaR model is robust to these subgroups. In any case, what it is clear is that the best VaR model depends on the family functions used: regulator´s loss functions and/or firm´s loss function, as these families provide different results. These results can help market participants make effective selections between VaR models. The market participants (supervisors and risk managers) must consider that they have specific loss functions, and the final result depends on who is the end-user of the VaR model. Finally, the results can also help researchers understand the different results presented in the compared literature.

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