Economic pipeline design. Optimal diameter in drives.

Diseño económico de tuberías. Diámetro óptimo en impulsiones.

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ABSTRACT / SUMMARY

For a flow fixed to raise, the greater the diameter of the pipeline the lower will be the power that is invested in overcoming the losses by friction, but higher is the cost of driving. On the other hand, a small diameter will lower the initial cost of installation but will raise the cost of pumping. In short, the most economic diameter of driving will be the one that will make the combined annual cost of tubing and pumping minimum. Concerning the sizing of the discharge pipes, it has been turning to different formulations such as Bresse, Weyrauch, Mendiluce, Forchheimer, Vibert-Koch, Melzer or Agüera, among others (Mougnie, Prevedello, Allasia, …). The author of this article is based on the proposal in its own formulations of exclusively dimensioned hydraulic, already published in 2005, for pipelines in service of different constituent materials, with six roughness categories. This article presents new formulations for the optimum economic dimensioning of such pipes, adapted to each category of roughness.

Key words: pipeline; formula; diameter; pumping; cost; drive; performance; price; depreciation.

RESUMEN

Para un caudal fijo a elevar, cuanto mayor sea el diámetro de la tubería de impulsión tanto menor será la potencia que se invierta en vencer las pérdidas por fricción, aunque mayor es el coste de la conducción. Por el contrario, un diámetro pequeño abaratará el coste inicial de la instalación pero elevará los gastos de bombeo. El diámetro más económico de la conducción, en definitiva, será aquel que haga que el costo combinado anual de la tubería y el del bombeo sea mínimo. En el dimensionamiento de las tuberías de impulsión se ha venido recurriendo a diversas formulaciones como las de Bresse, Weyrauch, Mendiluce, Forchheimer, Vibert-Koch, Melzer o Agüera, entre otras (Mougnie, Prevedello, Allasia, …). El autor de este artículo basa su propuesta en sus propias formulaciones de dimensionado exclusivamente hidráulico, ya publicadas en el año 2005, para tuberías en servicio de diferentes materiales constitutivos, con seis categorías de rugosidad. En el presente artículo se presentan formulaciones inéditas para el dimensionado económico óptimo de dichas tuberías, adaptadas a cada categoría de rugosidad.

Palabras clave: *tubería; fórmula; diámetro; bombeo; coste; impulsión; rendimiento; precio; amortización.*

INTRODUCTION

To perform the calculation of any pipe, it is necessary to know a series of data such as: flow to transport, transport speed, pipe material, geometric and piezometric difference between the starting and ending point, pressure drop, conduction profile, etc. With this, we will determine the most economical commercial diameter, the wall thickness, the nominal pressure (stamping) and the special parts and devices that are necessary for the proper operation of the installation.

When a flow of water has to be driven to a given height difference (Figure 1), the total or manometric height that the pump must generate is equal to the geometric height to more than overcome the existing head losses and the kinetic height, that is:

$$Hm = Hg + hr + V^2/2g.$$

The first sum (Hg) depends exclusively on the ground levels (tachymetric difference between the pump and the tank, including the suction and delivery pipes of the pumping group) and the residual or minimum pressure required at the end of the journey, by what is an energy that is independent of the diameter of the pipe, like this:

$$Hg = \Delta Z + \frac{P_B}{\gamma}.$$

However, for a given flow rate, the second sum (hr) depends exclusively on the adopted diameter, so that as the head losses, both in the suction and discharge pipes, decrease considerably with increasing diameter, it would be necessary less energy to transport water. Conversely, an increase in diameter results in a higher cost of installation.

In every installation there is a solution that minimizes the sum of the cost of the energy necessary to overcome the losses (calculated for an average year) plus the corresponding annual amortization of the pipeline.

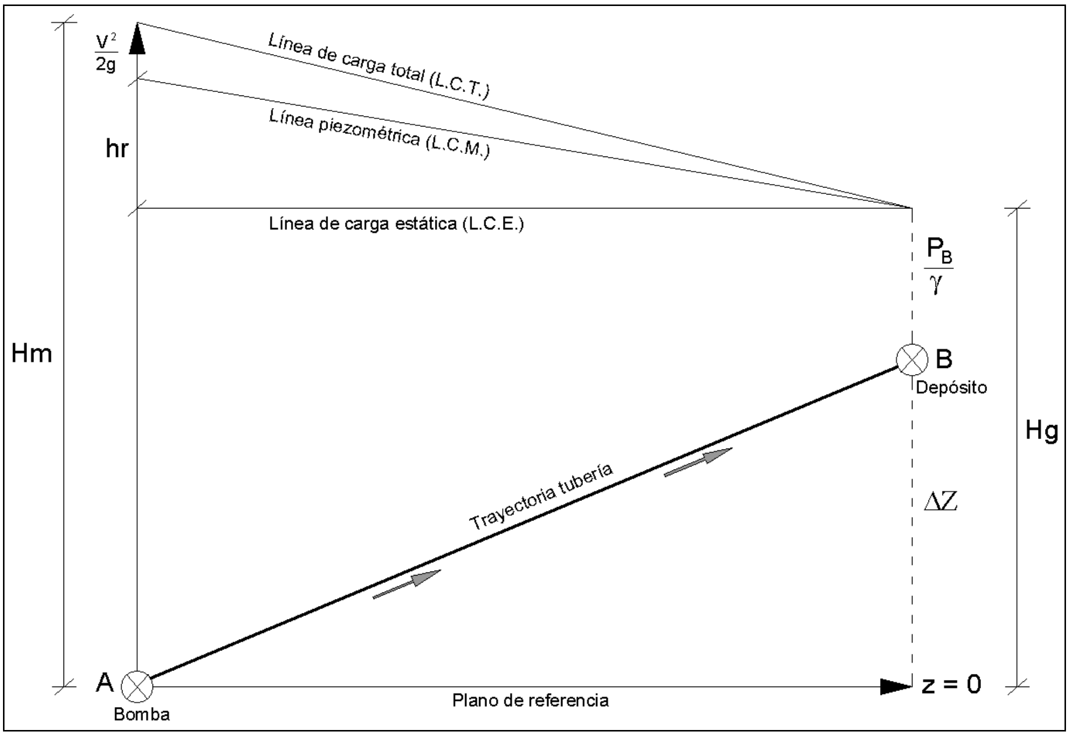

Figure 1 | Power lines in a drive system.

The purpose of the present work lies in the elaboration of practical formulations that allow the project engineer to obtain the optimum diameter of the impulsion pipes taking into account the concurrent economic and hydraulic factors.

EXISTING FORMULAS FOR THE ECONOMIC DIMENSION OF DRIVES

The most common formulations that have been used in the manuals used for this type of sizing are the following:

- Bresse formula (minimalist criteria)

It is the first formula that appears in the hydraulic bibliography on the economical dimensioning of pipes (Bresse, 1860). This is a very elementary and excessively conservative criterion, since it corresponds to a constant speed of 0.57 m/s, which turns out to be a speed well exceeded today. It is given by the expression:

$$D = 1.50\sqrt{Q} \tag{1}$$

- Weyrauch's formula (conservative criterion)

In this case, the expression of Weyrauch (1915), which continues the modus operandi of the previous formula, is:

$$D = 1.04\sqrt{Q} \tag{2}$$

what offers a speed of 1.18 m/s.

- Weighted formula

In the same order of ideas, in this case, we have to: $D = 0.92\sqrt{Q}$, which offers a constant speed of: $V = \frac{4Q}{\pi D^2} = \frac{4}{\pi 0.8464} = 1.50\ m/s$, or it is required that it be fulfilled: $D \geq \sqrt{0.236Q}$, then D is expressed in mm and Q in liters/hour. A variant of this formula is that of Dacach (1979), widely used in the USA, in which:

$$D = 0.9Q^{0.45} \tag{3}$$

and then the speed varies depending on the flow, since:

$$V = \frac{4Q}{\pi D^2} = \frac{4Q}{\pi 0.81 Q^{0.9}} = 1.5719Q^{0.1} \tag{4}$$

- Mendiluce formula (1966)

It already introduces cost and performance factors of the installation. Namely:

$V_{opt.} = 0.348\left(\frac{ca\eta_g}{Kpn}\right)^{1/3}$, from which it follows that:

$$D = 1.913\left(\frac{Kpn}{ca\eta_g}\right)^{0.167}\sqrt{Q} \qquad (5)$$

being:

$$\begin{cases} c = \text{cost of the installed pipeline per meter diameter and per meter lenght } (€/m \cdot m). \\ \eta_g = \text{overall performance of the motor – pump group} = \eta_m \cdot \eta_b . \\ K = \text{coefficient of pressure loss in the pipeline, applying the Darcy and Bazin's formulation (1865).} \\ p = \text{kWh price.} \\ n = \text{number of hours of annual operation.} \\ a = \text{amortization factor or type.} \end{cases}$$

This formula shows the need to use moderate speeds when the values of the variables *c* and *n* are high. The approximation it offers is sufficient in practice, since in the vast majority of cases that arise it will not be possible to size the pipe exactly with the value found, and the commercial diameters closest to the theoretically necessary one must be adopted.

Among the six factors involved in determining the most profitable speed, some of them generally vary little (depreciation, group performance and cost of energy), while others may suffer considerable fluctuations (cost of installed piping, coefficient of load losses and annual number of hours worked).

- Forchheimer's formula (1914-1916)

A simplification of the methodology applied by this author leads to:

$V_{opt.} = \frac{0.5 \text{ a } 0.6}{\sqrt{A}}$, where: $A = \frac{n}{24 \cdot 365} = \frac{n}{8760}$, where *n* is the number of hours of annual operation,

whence an average value for the speed of: $V_{opt.} = \frac{52}{\sqrt{n}}$, which determines the diameter of the pipe to be installed from the expression:

$$D = 0.156\ Q^{0.5}\ n^{0.25} \qquad (6)$$

- Vibert-Koch formula (1948) in Agüera (1998)

Also here, as in the Mendiluce formula (5), cost factors are introduced. In a first approximation, the economically optimal internal diameter, in meters, is obtained according to the expression:

$$D = 1.456\left(\frac{ne}{c}\right)^{0.154} Q^{0.46} \qquad (7)$$

being:

$$\begin{cases} c = \text{cost of the installed pipeline } (€/kg). \\ e = \text{kWh price.} \\ n = \text{number of hours of daily operation divided by 24.} \end{cases}$$

The previous coefficient 1.456 takes into account an amortization rate of 8% over a period of 50 years (a = 0.08174), so it must be altered based on the data to be used. In any case, these writers, with regard to said coefficient, made the following distinction based on the degree of use of the pipeline: 1.547 (in continuous service) or 1.35 (in the case of nightly pumping of 10 hours a day, valley-hours).

In a more general case, the optimal diameter will be given by:

$$D = 1.71\left(\frac{Kpn}{ca\eta_g}\right)^{0.154} Q^{0.46} \tag{8}$$

- Formula of Melzer (1964) in Agüera (1998)

In a similar way to the previous formulations, and particularly to that of Vibert-Koch (8) of which only the coefficient and the exponents are changed, this is expressed as follows:

$$D = 1.579\left(\frac{Kpn}{ca\eta_g}\right)^{0.143} Q^{0.43} \tag{9}$$

- Agüera's formula

It is also similar to the previous formulations. The following expression can be seen in Agüera (1998), which takes into account cost, performance and amortization factors:

$$D = 1.165\left[\frac{f}{\eta_g}\left(0.5+\frac{p\cdot n}{c\cdot a}\right)\right]^{0.154} Q^{0.462} \tag{10}$$

Regarding its application, the following value of the deductible coefficient of friction of the general or universal expression of the unit load losses due to Darcy-Weisbach (Weisbach, 1843) must be taken into account:

$$J = K\frac{Q^2}{D^5} = \frac{f}{D}\frac{V^2}{2g} = \frac{f16Q^2}{D\pi^2 D^4 2g} = f\,0.0826\frac{Q^2}{D^5} \tag{11}$$

whence: f = K/0.0826.

In short, Figure 1 and Figure 2 of the annex show the differences between the exponential coefficients α that affect the three exposed formulations of Mendiluce, Vibert and Melzer (Agüera, 1998) in relation to the flow rate Q and the term: $S = \left(\frac{Kpn}{ca\eta_g}\right)$. There are more formulas proposed by different authors, such as the classic by Mougnie (1914), Prevedello (2000) or Allasia (2000), which also try to determine the optimal diameter for forced driving.

CALCULATION BASED ON REAL EVALUATION OF COSTS

For a fixed flow to raise Q, the larger the diameter of the pipe, the less power will be invested in overcoming friction losses, although the cost of conduction is greater. On the contrary, a small diameter will lower the initial cost of the installation but will increase the pumping costs with increasing head losses. The most economical diameter of the pipeline, in short, will be that which makes the combined annual cost of the pipeline and that of the pumping is minimal.

In this way, the most economical diameter is the one whose sum of the annual expenses due to the energy consumed plus the value of the annuity for the investment made is minimal (Figure 2). Therefore, the equation to be met, whose well-known graphical representation can be seen below, is:

$$G_{\text{total}} = G_{\text{amortization}} + G_{\text{energy}} = \text{Minimum, that is: } \frac{dG_{\text{total}}}{dD} = 0\ ; \quad \frac{d^2G_{\text{total}}}{dD^2} > 0.$$

These calculations normally require computer programs due to the large volume of data to be taken into account and the tedious and repetitive nature of their execution, prior to the analytical determination of the corresponding cost equations.

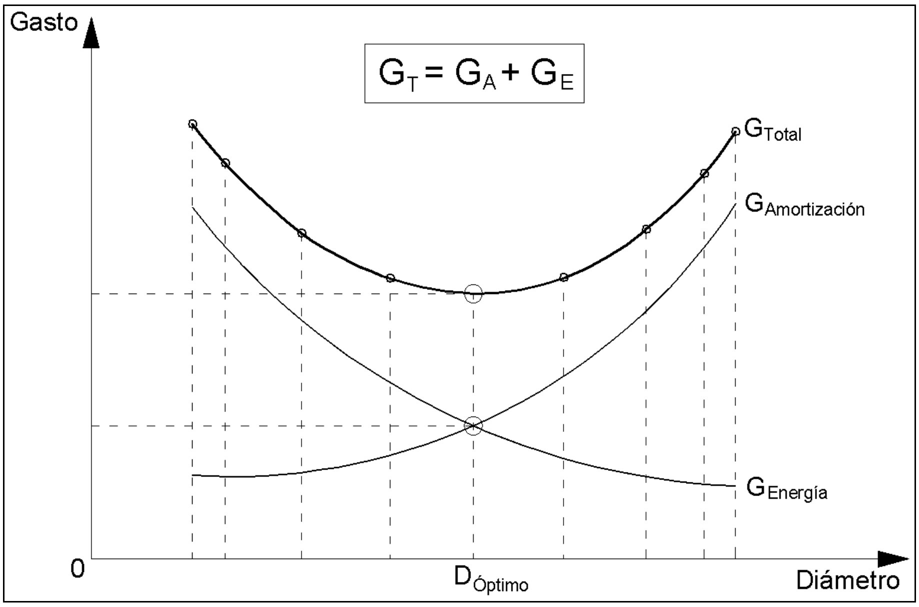

Figure 2 | Diameter-cost graph.

METHODOLOGY

When the forced hydraulic pipes are dimensioned, it turns out that the calculation differences obtained using the most commonly used classical formulas raise serious doubts for the resolution of ordinary cases that arise in engineering practice. Possibly, the revision of these formulas lost interest some time ago, apparently, as it was a solved problem. Of course, this article is not intended to question the validity of those formulations, which are universally recognized, although we do consider it necessary to develop our own formulations that statistically subsume the most relevant factors of the previous ones (Franquet, 2005).

At this point, let's see that identical formulations to those proposed by this author in his study for the case of free pipes (Franquet, 2003) can be applied, with the corresponding corrections, in the calculation and design of forced or pressure pipes. For this, the formulas corresponding to the first 6 categories of roughness have been used, and they are expressed in Table 1, depending on the material of the tube and for pipes used or in service.

These formulas, which have the advantage of being able to be applied independently of the hydraulic regime and the Reynolds number (Re) that characterizes the flow, will adopt the following general configuration: $V = K \cdot R^{\beta} \cdot J^{0.5}$, where the speed (m/s) as a function of the hydraulic radius (m), the unit head loss (m/ml) and the coefficients according to the different roughness categories.

Table 1 | Roughness categories and K and β coefficients corresponding to the different materials.

Roughness degree (k)	Material	K	β
1	Plastics, glass, brass	86.85	0.62150
2	Fiber cement, aluminum	78.29	0.63455
3	Steel, other metals	70.02	0.64760
4	Foundry	63.92	0.65560
5	Concrete	56.24	0.66540
6	Ceramics	49.51	0.67725

Source: self made.

The graphic representation of the different values obtained from the K coefficient (head loss) in relation to the 6 categories of roughness resulting from our proposal, is as follows Figure 3.

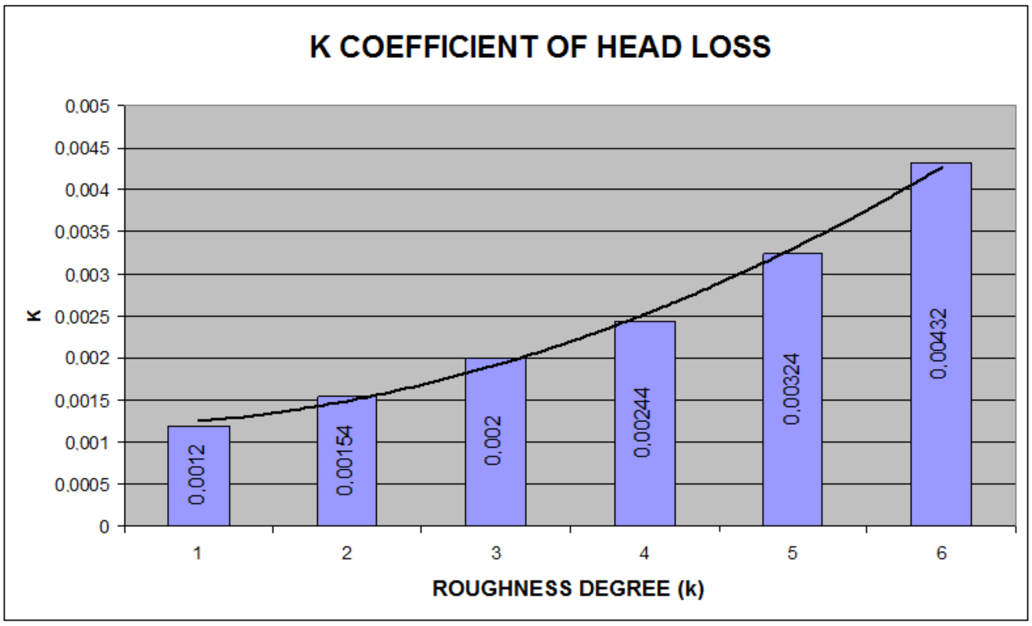

Figure 3 | K Coefficient of head loss.

Based on the formulas proposed by this author (Franquet, 2005) for the pipes in service, and according to the different categories of roughness, $\forall$ k $\in$ (1, 6), the expressions that appear in Table 2 would have, correlatively for the hydraulic dimensioning of the pipes in service, in which the unit head loss (m/m), the flow (m^3/s) and the speed (m/s) have also been cleared and the intermediate formulas have been included obtained by linear interpolation:

Table 2 | Proposed expressions of speed, flow and unit pressure drop for pipes in service.

Roughness (k)	V (m/s)	Q (m^3/s)	J (m/m)
1.0	**$36.69\ D^{0.6215}\ J^{0.5}$**	**$28.82\ D^{2.6215}\ J^{0.5}$**	**$0.000743\ V^2\ D^{-1.243}$**
1.5	$34.59\ D^{0.62802}\ J^{0.5}$	$27.16\ D^{2.62802}\ J^{0.5}$	$0.000845\ V^2\ D^{-1.256}$
2.0	**$32.48\ D^{0.63455}\ J^{0.5}$**	**$25.51\ D^{2.63455}\ J^{0.5}$**	**$0.000948\ V^2\ D^{-1.2691}$**
2.5	$30.51\ D^{0.6411}\ J^{0.5}$	$23.96\ D^{2.6411}\ J^{0.5}$	$0.001088\ V^2\ D^{-1.2821}$
3.0	**$28.53\ D^{0.6476}\ J^{0.5}$**	**$22.41\ D^{2.6476}\ J^{0.5}$**	**$0.001229\ V^2\ D^{-1.2952}$**
3.5	$27.14\ D^{0.6516}\ J^{0.5}$	$21.32\ D^{2.6516}\ J^{0.5}$	$0.001368\ V^2\ D^{-1.3032}$
4.0	**$25.76\ D^{0.6556}\ J^{0.5}$**	**$20.23\ D^{2.6556}\ J^{0.5}$**	**$0.001507\ V^2\ D^{-1.3112}$**
4.5	$24.06\ D^{0.6605}\ J^{0.5}$	$18.89\ D^{2.6605}\ J^{0.5}$	$0.001753\ V^2\ D^{-1.321}$
5.0	**$22.36\ D^{0.6654}\ J^{0.5}$**	**$17.56\ D^{2.6654}\ J^{0.5}$**	**$0.002\ V^2\ D^{-1.3308}$**
5.5	$20.86\ D^{0.6713}\ J^{0.5}$	$16.38\ D^{2.6713}\ J^{0.5}$	$0.002334\ V^2\ D^{-1.3426}$
6.0	**$19.36\ D^{0.67725}\ J^{0.5}$**	**$15.21\ D^{2.67725}\ J^{0.5}$**	**$0.002668\ V^2\ D^{-1.3545}$**

Source: self made.

Note that the formula that offers the speed has been based on the internal diameter of the pipe D instead of the hydraulic radius R that appears in the general expression, as it is more practical. These formulas are then combined with the relevant cost / benefit analysis in order to estimate the optimal economic diameter of the pipe. Let's see, in this sense, that the weight of the unit of length of a pipe of diameter D and thickness *e* will be:

$$P = \gamma_m\ S = \gamma_m\ \pi\ (D + e)\ e \tag{12}$$

since, in effect, the section of the circular crown of the tube is:

$$S = \frac{\pi(D+2e)^2}{4} - \frac{\pi D^2}{4} = \frac{\pi(D^2+4e^2+4De}{4} - \frac{\pi D^2}{4} = \pi e^2 + \pi De = \pi(D + e)e \tag{13}$$

where γ_m is the specific weight of the material constituting the pipe.

In practice, for not very large diameters, the factor $\frac{(D+e)e}{D}$ is approximately constant, then multiplying and dividing by D will obtain the expression:

$$P = \gamma_m \pi \frac{(D+e)e}{D} D \tag{14}$$

which, in turn, multiplied by the price of the unit of weight of the material, offers a cost per linear meter of pipe of: $C = \lambda \cdot D$, where λ is independent of the diameter. Thus, the annual amortization costs of the pipeline will be: $G_a = L \cdot \lambda \cdot D \cdot a$, where *L* (m.) Is the length of the pipeline and *a* the type of amortization given by the expression:

$$a = \frac{r(1+r)^t}{(1+r)^t - 1} \tag{15}$$

where t = number of years and r = interest rate.

On the other hand, the annual expenses of electrical energy consumed in the pumping are:

$$G_e = \frac{1000\,Q\,H}{\eta_g}0.00981\,n\,c = \frac{9.81\,Q\,H\,n\,c}{\eta_g} \tag{16}$$

where *c* is the cost of kWh in € and *n* is the number of annual hours of operation of the group in question. With this, the total annual expenses will be:

$$G = G_e + G_a = \frac{9.81}{\eta_g}(HQnc) + L\lambda Da \tag{17}$$

the total manometric height H given, as it is known, by the expression:

$$H = H_g + K\,Q^2\,D^{-5.243}\,L + V^2/2g \tag{18}$$

for a roughness category k = 1, in which the term of the kinetic height $V^2/2g$ can generally be neglected due to its low amount.

RESULTS

In order to obtain the optimal diameter of the conduction, the classic methodology of cost minimization has been followed, as is the case with the Mendiluce, Vibert-Koch, Melzer and Agüera formulations. If we now minimize the function (17) having previously substituted in it the value given by the expression (18), for the necessary or first degree condition it will result, respectively, for the first category of roughness (k = 1):

$$\frac{dG}{dD} = -5.243\frac{9.81}{\eta_g}\cdot LKQ^3ncD^{-6.243} + L\lambda a = 0 \tag{19}$$

whence it follows that:

$$\lambda a\eta_g = 5.243\cdot 9.81KcnQ^3D^{-6.243} \tag{20}$$

and doing in the previous expression:

$$Q = \frac{\pi D^2 V}{4} \tag{21}$$

it will turn out that:

$$\lambda a\eta_g = 5.243\cdot 9.81Kcn\frac{\pi^3D^6}{64}V^3D^{-6.243} = 24.92KcnV^3D^{-0.243} \tag{22}$$

whence, for K = 0.0012, corresponding to k = 1, we will have to:

$$V = 3.22\sqrt[3]{\frac{\lambda a\eta_g D^{0.243}}{cn}} \tag{23}$$

and the optimal diameter will be: $D^{6.243} = \frac{5.243 \cdot 9.81 \cdot 0.0012 cnQ^3}{\lambda a \eta_g}$; that is:

$$\boxed{D = \left(\frac{0.0617 cnQ^3}{\lambda a \eta_g} \right)^{0.1602}} \tag{24}$$

which constitutes the proposed formulation for determining the optimal diameter when k = 1.
In any case, the sufficient or second degree condition of relative extreme requires, again deriving in (19), that:

$$\frac{d^2G}{dD^2} = \frac{321.1}{\eta_g} LKQ^3 ncD^{-7.243} > 0 \tag{25}$$

then indeed it is a minimum.

In the formulations that follow for the determination of the optimal economic diameter for each category, the original formulations of this author have been used, which can be seen in Table 2, taking into account the following meanings:

J = unit pressure drop (m/ml).
Q = flow in m^3/s.
D = internal diameter on the pipe in m.
c = cost of electrical energy, in €/kWh.
n = number of hours of annual group operation.
λ = cost of the installed pipeline per meter diameter and per meter lenght (€/m·m).
a = amortization rate.
η_g = overall performance of the pumping group = $\eta_m \cdot \eta_b$

It is operated in the same way as in the previous case for the five remaining roughness categories. Table 3 is a summary of the results obtained for each degree of roughness (k) and can be seen below, in which the corresponding value of the known Darcy-Weisbach coefficient of friction has also been included, that is: f = K /0.0826.

Table 3 | Optimal diameters according to the roughness categories.

k	K	f	J (m/m)	D (m)	D_{opt}
1	0.0012	0.0145	$0.0012\ Q^2\ D^{-5.243}$	$\left(\frac{0.0617cnQ^3}{\lambda a\eta_g}\right)^{0.1602}$	$0.640T^{0.1602}$
2	0.00154	0.0186	$0.00154\ Q^2\ D^{-5.2691}$	$\left(\frac{0.0796cnQ^3}{\lambda a\eta_g}\right)^{0.1595}$	$0.668T^{0.1595}$
3	0.002	0.0242	$0.002\ Q^2\ D^{-5.2952}$	$\left(\frac{0.1039cnQ^3}{\lambda a\eta_g}\right)^{0.1589}$	$0.698T^{0.1589}$
4	0.00244	0.0295	$0.00244\ Q^2\ D^{-5.3112}$	$\left(\frac{0.1271cnQ^3}{\lambda a\eta_g}\right)^{0.1584}$	$0.721T^{0.1584}$
5	0.00324	0.0392	$0.00324\ Q^2\ D^{-5.3308}$	$\left(\frac{0.1694cnQ^3}{\lambda a\eta_g}\right)^{0.158}$	$0.755T^{0.158}$
6	0.00432	0.0523	$0.00432\ Q^2\ D^{-5.3545}$	$\left(\frac{0.2269cnQ^3}{\lambda a\eta_g}\right)^{0.1574}$	$0.792T^{0.1574}$

Source: self made.

In Table 3 above, for simplification purposes, the term that is repeated in all formulations has been considered:

$$T = \left(\frac{cnQ^3}{\lambda a\eta_g}\right) \tag{26}$$

therefore, for each roughness category, the corresponding reduced expressions of the economic optimum diameter of the impulsion pipe depending on this term appear in the last column of the roughness (D_{opt}).

In the supplementary material annex, the 6 graphic representations are offered up to a value T = 1000, where the necessary increase in diameter is observed with the increase in T influenced by the flow rate and the other cost factors, for the same roughness category . Its unified presentation on logarithmic axes has been dispensed with in order to increase the precision of its practical use.

EXAMPLE OF APLICATION

The central prismatic water distribution tank of a given Irrigation Community, with internal dimensions in plan: 50.00 × 40.00 m, is filled once a week in the irrigation period (March-October), on Sundays, taking advantage of the hours- valley (8 h) and flat-hours (8 h), with water from two wells of 300,000 and 200,000 liters/hour and lengths of the PVC delivery pipes of 158 ml. and 210 ml, respectively (the existence of two elevator groups will allow the sensitivity of the proposed formulation to be analyzed and its comparison with the rest).

In the case of the well and riser group 1, for a working pressure of 6 bar, with values Q = 300000 l/h = 0.083 m^3/s and c = λ = 250 €/m·m, the different formulations used have been applied, both those that take into account cost and economic performance factors (Mendiluce, Vibert-Koch, Melzer, Agüera, Franquet) and those that do not (Bresse, Weyrauch, Dacach, Forchheimer). As regards lift group 2, for a working pressure of 4 bar, only the following parameters will vary: Q = 200000 l/h = 0.056 m^3/s, and c = λ = € 200 / m·m.

All the corresponding calculations have been arranged in the complementary material and the results obtained can be seen in Table 4.

Let's see that the term T, whose expression can be seen in (26), reaches the value:

$$T = \frac{1 \cdot 560 \cdot 0.056^3}{200 \cdot 0.06646 \cdot 0.688} \cong 0.011 \tag{27}$$

that, for this category of roughness, it can be verified graphically (see Figure 4 of the annex) that also assumes a D_{opt} = 0.31 m. From this it is deduced the importance or influence of the degree of roughness of the pipe in fixing the optimal diameter of the pipeline under study, since with k = 6, in the same graph, it can be seen that the forecast of a D_{op}t = 0.39 m to ensure the fluid circulation requirements, substantially greater (and, consequently, more expensive) than calculated.

In short, the set of 9 determinations carried out lead us to the elaboration of the following Table 4 comparing the theoretical optimal economic diameters and the resulting effective speeds of water circulation, as well as the projected commercial pipes, including the classic formulations that do not they take into account the various incident economic parameters. So:

Table 4 | Comparison of the calculation of theoretical optimal diameters, standard commercial diameters and effective speeds of water circulation.

FORMULATION	**GROUP 1**		**GROUP 2**	
	$D_{opt.}$ (mm)	**V (m/s)**	**$D_{opt.}$ (mm)**	**V (m/s)**
Bresse	432 (PVC 500)	0.47	355 (PVC 400)	0.48
Weyrauch	300 (PVC 315)	1.18	246 (PVC 250)	1.24
Dacach-ponderada	294-265 (PVC 315)	1.18	273-218 (PVC 250)	1.24
Forchheimer	219 (PVC 250)	1.87	180 (PVC 200)	1.93
Mendiluce	344 (PVC 400)	0.75	293 (PVC 315)	0.78
Vibert-Koch	352 (PVC 400)	0.75	304 (PVC 315)	0.78
Melzer	362 (PVC 400)	0.75	315 (PVC 355)	0.78
Agüera	353 (PVC 400)	0.75	304 (PVC 315)	0.78
Franquet	**362 (PVC 400)**	**0.75**	**310 (PVC 315)**	**0.78**

Source: self made.

Note, likewise, that saving Bress's original proposal, the determination of the economic optimum diameter by the application of the methodologies outlined induces, at least in the proposed example, the requirement of larger diameters than that which is simply deduced from the application of fast formulations, such as those of Weyrauch, Dacach or Forchheimer, which lack a stricter economic parameterization.

Likewise, a sensitivity analysis of the proposed formulas has been carried out considering other types of drives (with pipes of different diameters and materials) and the results obtained are similar to those set forth in the previous example with regard to its comparison with the other formulations of economic dimensioning to use.

On the other hand, according to Pérez (2004), the simplest form of diameter normalization is to replace the theoretical diameter with the closest normalized diameter in size, either the immediate superior (supra-normalization) or the immediate inferior (infra-normalization). Supra-normalization generates a lower pressure drop and under-normalization a higher one, both with respect to the theoretical diameter. The most convenient will be to replace the theoretical diameter with two sections of different normalized diameters, whose sum of pressure losses is equivalent to that obtained by the theoretical diameter under the same conditions (López, 2012). According to Fujiwara and Dey (1987) it can be verified that with the conventional structure of prices for pipes, the most economical combination is formed by two adjacent standardized diameters D_1 and D_2 whose values include the theoretical diameter.

CONCLUSIONS

There are various formulations for the optimal economic dimensioning of delivery pipes in pumping installations, the description of which is carried out. Said formulations provide acceptable initial results and can be used interchangeably, since the uncertainty in the data overcomes the discrepancy between the diameters obtained. However, in the calculations made with these expressions, the pressures to which the pipes will be subjected are not taken into account.

In the present article, its author, based on his own published and contrasted general formulations for the hydraulic dimensioning of pipes in service of different constituent materials, presents here a specific applicable proposal of new formulations based on the real evaluation of costs, with the particularity of adapting them, in each case, according to six different categories of wall roughness. Six graphic representations are included in the supplementary material annex to visually facilitate the determination of the optimal economic diameters based on the various concurrent factors.

In any case, the goodness of the formulations proposed here is demonstrated in view of the other formulations exposed, since they offer intermediate or compatible results with the others usually used that take into account cost factors, which is evident by solving an application example that is only a sample of the broader sensitivity analysis performed with pipes of different materials.

The diameters obtained with the aforementioned equations are theoretical diameters that must be normalized to commercial diameters. Therefore, there are two levels in the formulation of diameters, one of continuous theoretical diameters and the other of commercially available diameters, to which the final solutions should be adapted as far as possible. This indicates the need for a mechanism to normalize the theoretical diameters.

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LIST OF TABLES AND FIGURES:

SUPPLEMENTARY MATERIAL SCHEDULE

1. COMPLEMENTARY FIGURES

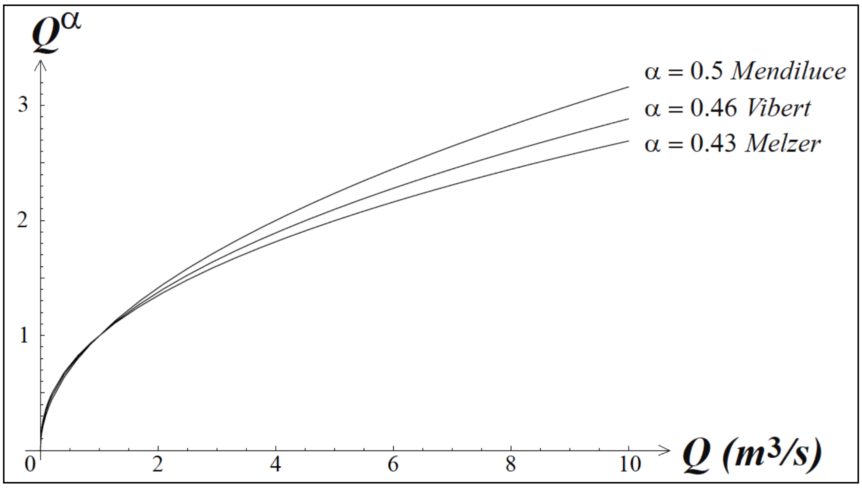

Figure 1 | Mendiluce, Vibert and Melzer formulas. Parameter α in relation to Q.

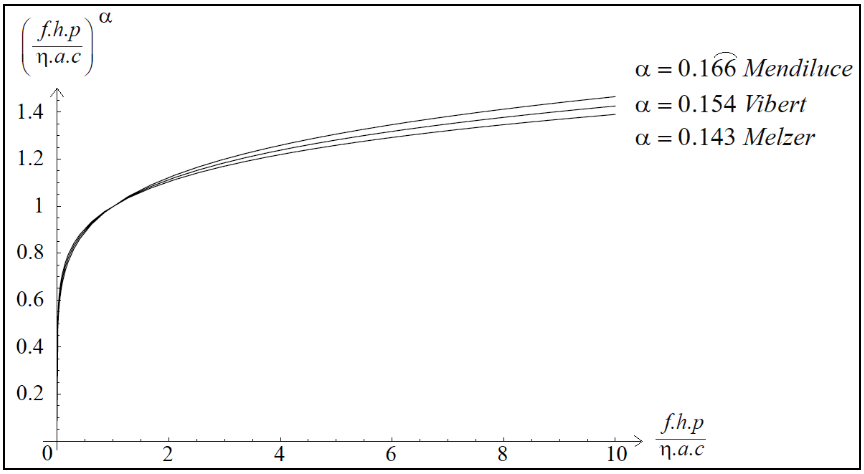

Figure 2 | Mendiluce, Vibert and Melzer formulas. Parameter α in relation to the term S.

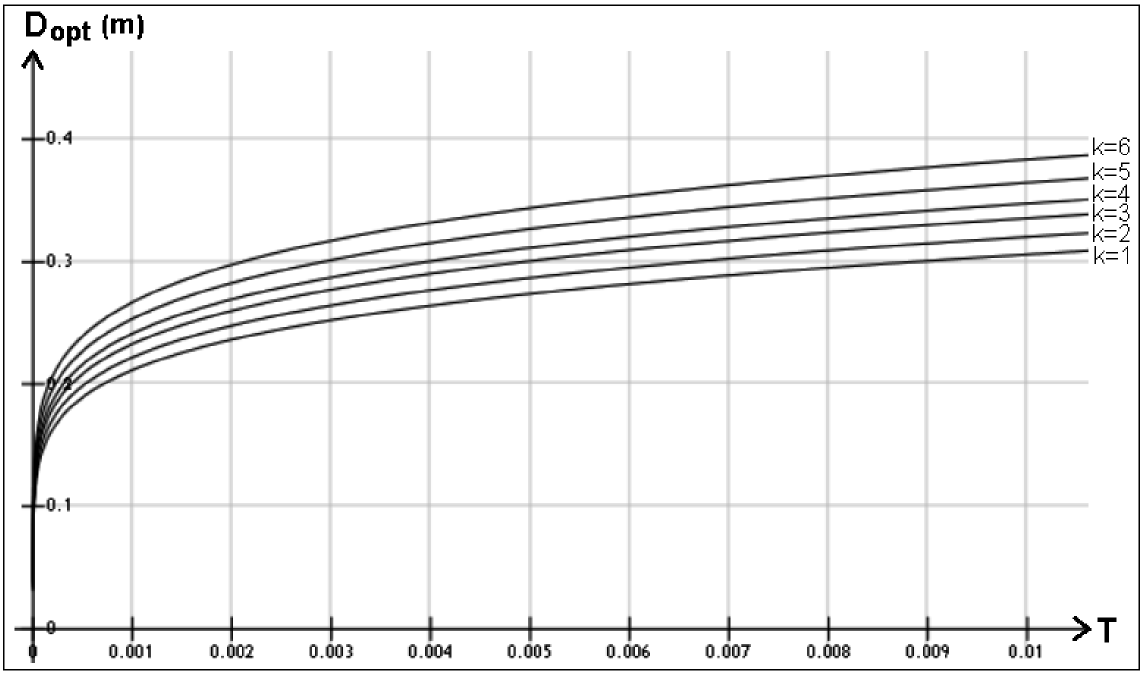

Figure 3 | Optimum diameter as a function of the term T ∈ [0.0.01].

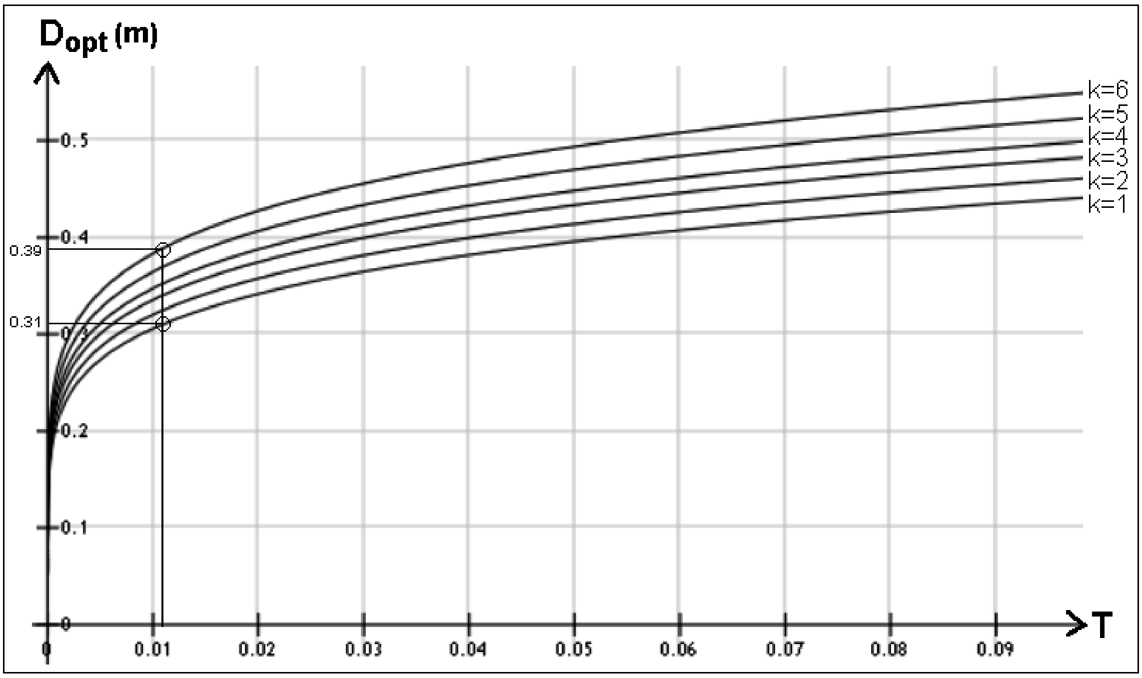

Figure 4 | Optimum diameter as a function of the term T ∈ [0.0.1].

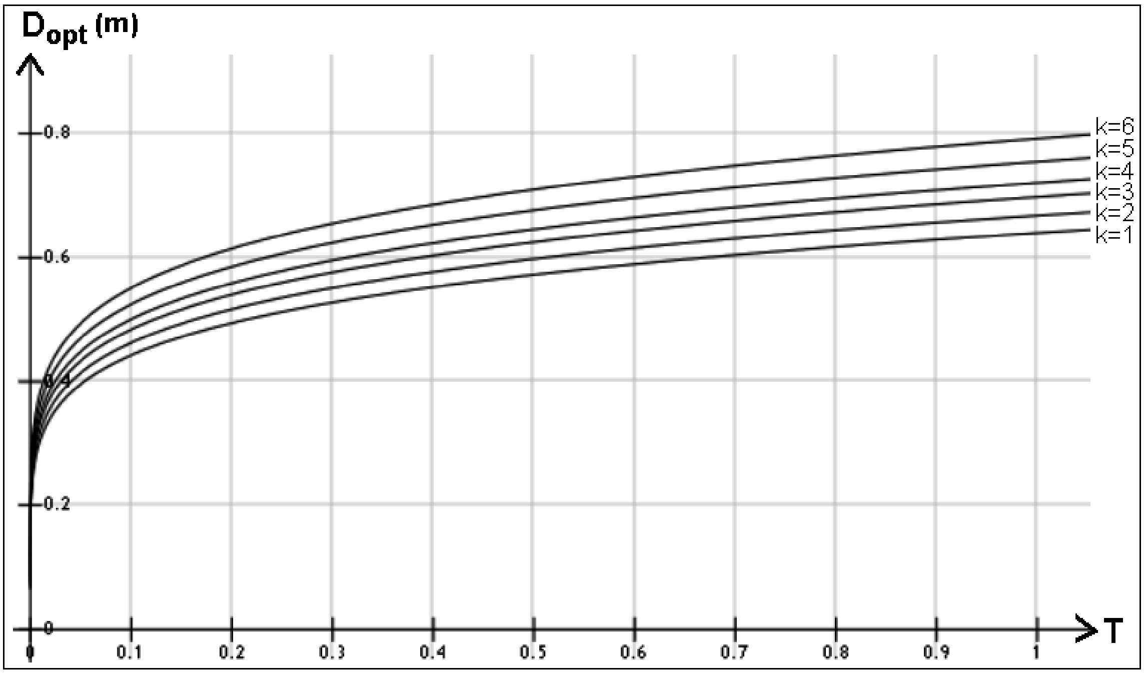

Figure 5 | Optimal diameter as a function of the term T ∈ [0.1].

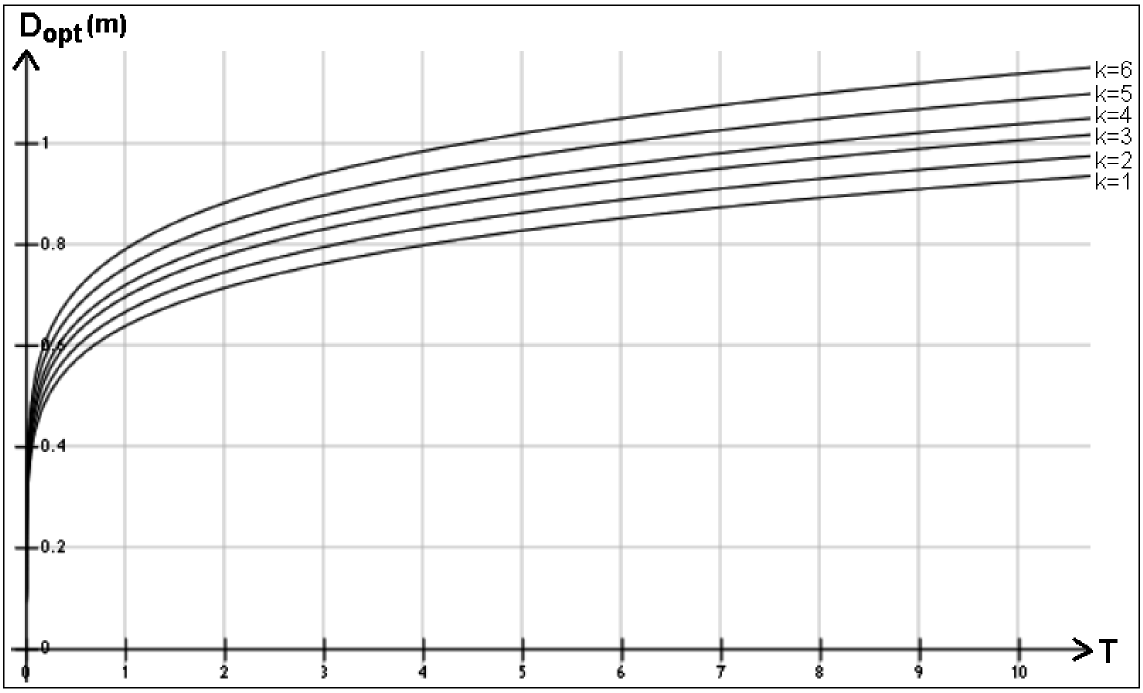

Figure 6 | Optimum diameter as a function of the term T ∈ [0.10].

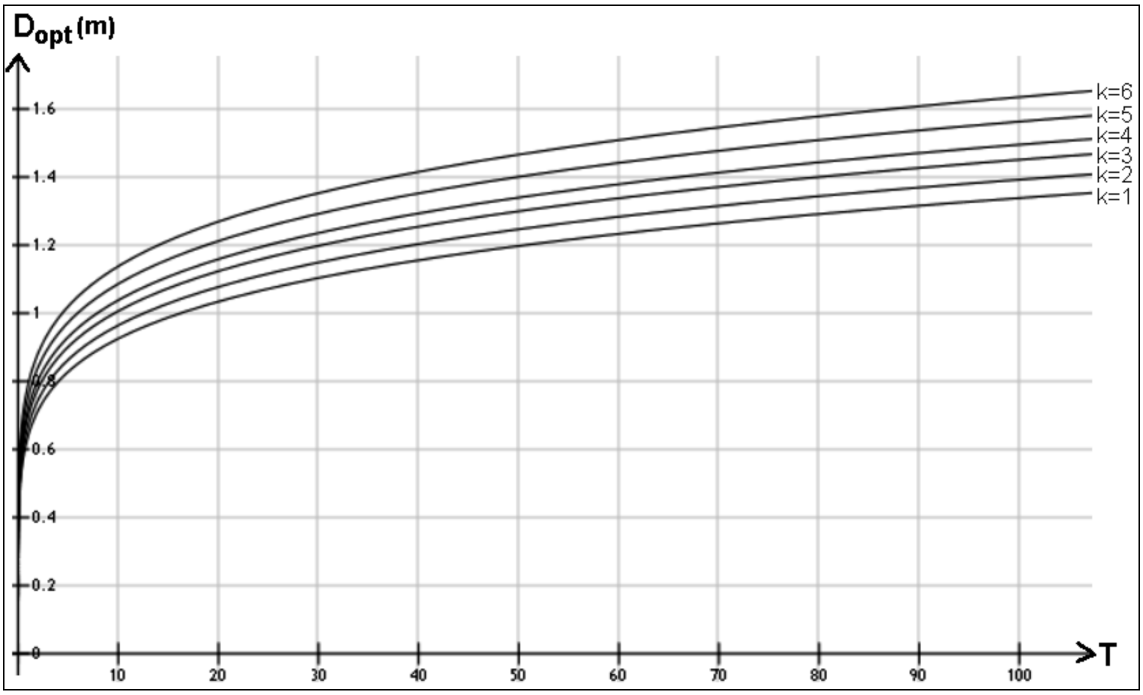

Figure 7 | Optimum diameter as a function of the term T ∈ [0.100].

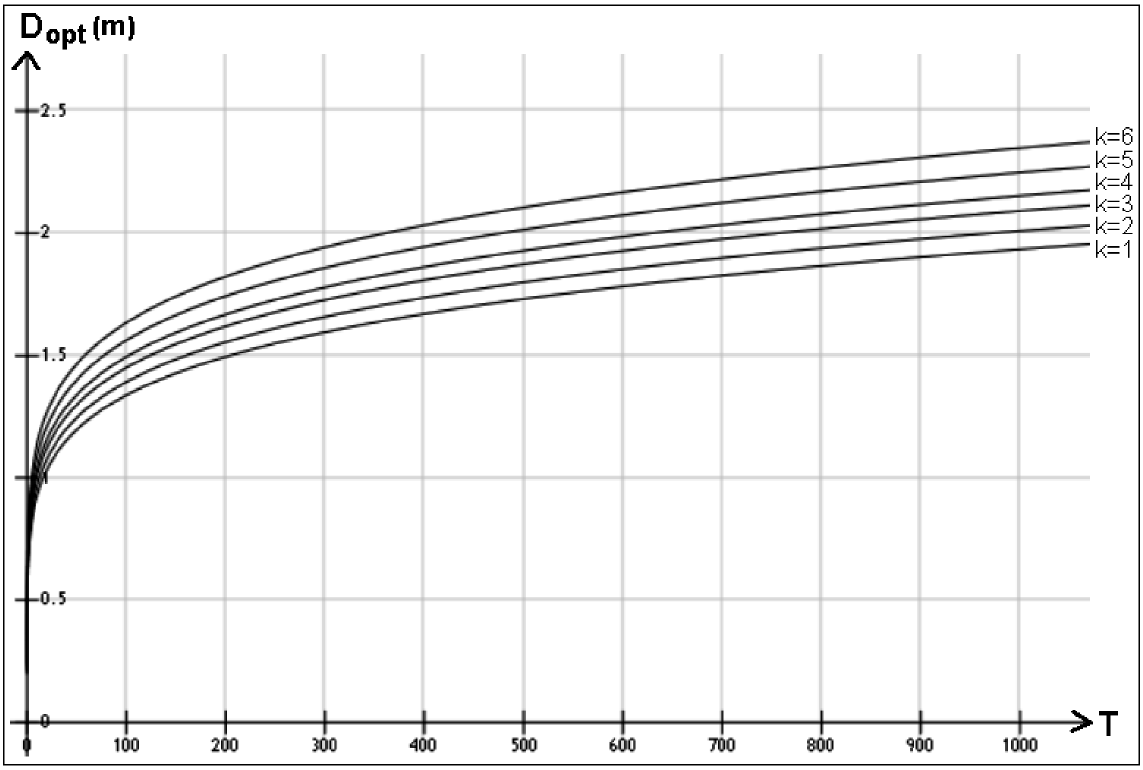

Figure 8 | Optimum diameter as a function of the term T ∈ [0.1000].

2. APPLICATION EXAMPLE CALCULATIONS

(*The numbering of the formulas that appear in the main body of the Article has been followed, starting from 28*).

In the case of the well and elevator group 1, we will have:

- **Bresse**: With a flow of Q = 300000 l/h = 0.083 m^3/s, it is necessary to:

$D = 1.5\sqrt{Q} = 1.5\sqrt{0.083} = 0.432\,m$, which requires a PVC pipe 500 · 12.3 mm (6 bar), with an internal diameter: D_i = 500 – 2 · 12.3 = 475.4 mm > 432 mm.

- **Weyrauch**: In this case, $D = 1.04\sqrt{Q} = 1.04\sqrt{0.083} = 0.300\,m$, which requires a PVC pipe 315 · 7.7 mm (6 bar), with an internal diameter: D_i = 315 – 2 · 7.7 = 299.6 mm ≅ 300 mm.

- **Dacach**: In this case, D = 0.9 $Q^{0.45}$ = 0.9 · $0.083^{0.45}$ = 0.294 m, or the weighted formula:

$D = 0.92\sqrt{Q} = 0.92\sqrt{0.083} = 0.265\,m$, which requires, in both cases, the same pipeline as in Weyrauch's previous assumption.

- **Forchheimer**: In this case, by application of expression (6), it will be necessary to:
D = 0.156 $Q^{0.5}$ $n^{0.25}$ = 0.156 · $0.083^{0.5}$ · $560^{0.25}$ = 0.219 m, which requires a PVC pipe 250 · 6.2 mm (6 bar), with an internal diameter: D_i = 250 – 2 · 6.2 = 237.6 mm > 219 mm.

- **Mendiluce**: With a flow of Q = 300000 l/h = 0.083 m^3/s, we have for the first category of roughness
k = 1 ⇒ K = 0.0012; p = 1 €/kWh;
n = 35 weeks · 16 h/week = 560 h; c = λ = 250 €/m·m; $\eta_g = \eta_m\,\eta_b$ = 0.86 · 0.80 = 0.688;
a (6% a 40 años) = 0.06646. That is, substituting in (5), we have to:

$$D = 1.913 \cdot \left(\frac{0.0012 \cdot 1 \cdot 560}{250 \cdot 0.06646 \cdot 0.688}\right)^{0.167} \cdot 0.083^{0.5} = 0.344\,m \tag{28}$$

which requires a PVC pipe 400·11.7 mm (6 bar), with an internal diameter: D_i = 400 – 2 · 11.7 = 376.6 mm > 344 mm.

- **Vibert-Koch**: With the same conditions as in the previous case, it will have, substituting in (8):

$$D = 1.71\left(\frac{0.0012 \cdot 1 \cdot 560}{250 \cdot 0.06646 \cdot 0.688}\right)^{0.154} \cdot 0.083^{0.46} = 0.352\,m < 0.3766\,m \tag{29}$$

which requires the installation of the same PVC 400·11.7 mm (6 bar) pipe as in the previous case.

- **Melzer**: With the same conditions as in the previous case, it will have, substituting values in (9):

$$D = 1.579\left(\frac{0.0012 \cdot 1 \cdot 560}{250 \cdot 0.06646 \cdot 0.688}\right)^{0.143} \cdot 0.083^{0.43} = 0.362\,m < 0.3766\,m \tag{30}$$

which requires the same 400·11.7 mm (6 bar) PVC pipe as in the previous case.

- **Agüera**: Also with the same conditions as in the previous case. Take into account, regarding the coefficient of friction, that:

f = K / 0.0826 = 0.0012 / 0.0826 = 0.0145 ≈ 0.015, thus substituting values in (10):

$$D = 1.165 \cdot [\frac{0.015}{0.688}\left(0.5 + \frac{1 \cdot 560}{250 \cdot 0.06646}\right)]^{0.154} \cdot 0.083^{0.462} = 0.353\,m < 0.3766\,m \tag{31}$$

which requires the same PVC pipe 400×11.7 mm (6 bar) as in the previous case.

- Franquet: With the same conditions as in the previous case, we can go directly to the formula proposed in our studies that corresponds to the category of roughness k = 1 (PVC), with which we will have, substituting in (24):

$$D=\left(\frac{0.0617\cdot 1\cdot 560\cdot 0.083^{3}}{250\cdot 0.06646\cdot 0.688}\right)^{0.1602}=0.362\,\mathrm{m}<0.3766\,\mathrm{m} \tag{32}$$

which is substantially in agreement with the previous determinations and exactly the same with that of Melzer.

Regarding well and riser group 2, there will be:

- Bresse: With a flow of Q = 200000 l/h = 0.056 m^3/s, it is necessary to:

$D=1.5\sqrt{Q}=1.5\sqrt{0.056}=0.355\,\mathrm{m}$, which requires a PVC pipe 400·7.9 mm (4 bar), with an internal diameter: D_i = 400 – 2 · 7.9 = 384.2 mm > 355 mm.

- Weyrauch: In this case, $D=1.04\sqrt{Q}=1.04\sqrt{0.056}=0.246\,\mathrm{m}$, which requires a PVC pipe 250·4.9 mm (4 bar), with an internal diameter: D_i = 250 – 2 · 4.9 = 240.2 mm ≅ 246 mm.

- Dacach: In this case, D = 0.9 $Q^{0.45}$ = 0.9 · $0.056^{0.45}$ = 0.273 m, or the weighted formula:

$D=0.92\cdot\sqrt{Q}=0.92\sqrt{0.056}=0.218\,\mathrm{m}$, which required, in both cases, the same pipeline as in Weyrauch's previous assumption.

- Forchheimer: In this case, D = 0.156 $Q^{0.5}$ $n^{0.25}$ = 0.156 · $0.056^{0.5}$ · $560^{0.25}$ = 0.180 m, which requires a PVC pipe 200·4.0 mm (4 bar), with an internal diameter: D_i = 200 – 2 · 4.0 = 192 mm > 180 mm.

- Mendiluce:

$$D=1.913\left(\frac{0.0012\cdot 1\cdot 560}{200\cdot 0.06646\cdot 0.688}\right)^{0.167}0.056^{0.5}=0.293\,\mathrm{m} \tag{33}$$

which requires a PVC pipe 315·6.2 mm (4 bar), with an internal diameter: D_i = 315 – 2 · 6.2 = 302.6 mm > 293 mm.

- Vibert-Koch: With the same conditions as in the previous case, you will have:

$$D=1.71\left(\frac{0.0012\cdot 1\cdot 560}{200\cdot 0.06646\cdot 0.688}\right)^{0.154}0.056^{0.46}=0.304\,\mathrm{m} \tag{34}$$

which requires the same pipeline as in the previous case.

- Melzer: With the same conditions as in the previous case, you will have:

$$D=1.579\left(\frac{0.0012\cdot 1\cdot 560}{200\cdot 0.06646\cdot 0.688}\right)^{0.143}0.056^{0.43}=0.315\,\mathrm{m} \tag{35}$$

which requires a PVC 355·7 mm (4 bar) or PVC 355·10.4 mm (6 bar) pipe, in both cases with a sufficient D_i to more than meet the requirement.

- Agüera: Con las mismas condiciones que en el caso anterior, y teniendo en cuenta, por lo que se refiere al coeficiente de fricción, que: f = K/0.0826 = 0.0012/0.0826 = 0.0145 ≈ 0.015, se tendrá:

- Agüera: With the same conditions as in the previous case, and taking into account, regarding the coefficient of friction, that: f = K/0.0826 = 0.0012/0.0826 = 0.0145 ≈ 0.015, we will have:

$$D = 1.165[\frac{0.015}{0.688}\left(0.5 + \frac{1 \cdot 560}{200 \cdot 0.06646}\right)]^{0.154} \cdot 0.056^{0.462} = 0.304 \text{ m} \tag{36}$$

which requires a 315·6.2 mm (4 bar) PVC pipe, coinciding, in this specific case, with the previous Vibert-Koch formulation.

- Franquet: With the same conditions as in the previous cases, we can go directly to the proposed formula that corresponds to the roughness category k = 1 (PVC), which will have:

$$D = \left(\frac{0.0617 \cdot 1 \cdot 560 \cdot 0.056^3}{200 \cdot 0.06646 \cdot 0.688}\right)^{0.1602} = 0.310\text{m} \tag{37}$$

which requires a 315·6.2 mm (4 bar) PVC pipe.