But if we allow the Place of the fix'd Star to be as by the *Caroline* Tables, the Place of *Jupiter* will come out ♎ 13° 32′ 09″.

Having thus the Place and Latitude of the Planet, we ſhall thence endeavour to derive the Inclination of *Jupiter*'s Orbit to the Plain of the Earth's Orbit.

For finding of which the mutual Diſtances of *Jupiter* and the Earth, together with the Place of the Sun, are required; which we may ſafely take from any approved Tables. I generally uſe the *Caroline* Tables, which I have found to be more accurate and more eaſy than any others. From theſe I have taken at 8^h 16′ after Noon,

The true Place of the Sun ———— ♈	10° 40′ 18″
His Diſtance from the Earth ————	100034
The Diſtance of *Jupiter* from the Sun —	544921
———— from the Earth ———	444952

Now in the annexed Figure let S be the Sun, T the Earth, ♃ the Planet, Fig. 140.
SE a Radius of the Ecliptick continued to the Orbit of *Jupiter*, and the Angle ♃ T S the Latitude of the Planet ſeen from the Earth 1° 35′ 40½″.

In the Triangle ♃ S T, having given the Angle ♃ T S the Complement of the obſerved Latitude to a Circle, ♃ S and ♃ T the Diſtances of the Planet from the Sun and the Earth, being found above, the Angle ♃ S E will be had, the Latitude or Inclination of the Planet as ſeen from the Sun, 1° 18′ 7″.

The *Geocentrick* Place of *Jupiter* was ♎ 13° 35′ 33″. Therefore from the given Diſtances of *Jupiter* and the Earth from the Sun, the *Heliocentrick* Place of the Planet will be found ♎ 13° 3′ 33″; from whence ſeverally ſubtracting thoſe Places of the Node, which have been aſſumed by the Authors mention'd in the following Table, the Arguments of Latitude here annexed will come out. Whence it may be obſerved, that none of them makes *Jupiter* more advanced from his Limit than 6° 29′ 59″, or leſs than 3° 58′ 59″: Which how great ſoever the Difference may ſeem, in inveſtigating the greateſt Inclination of the Orbit, cannot produce a greater Error than 23 Seconds.

Authors.	*Places of* ☊				*Arguments of Latitude.*			
	s.	°	′	″	s.	°	′	″
Kepler.	3	6	33	37	3	6	29	56
Street.	3	6	33	47	3	6	29	46
Wing.	3	7	11	39	3	5	21	54
Ricciolus.	3	7	18	00	3	5	45	33
Caſſini.	3	8	45	00	3	4	18	33
Bullialdus.	3	9	4	34	3	3	58	59

That Place of the Node which Mr. *Caſſini* has made choice of, though it Fig. 141.
ſeems a little more advanced than it ought, yet for divers other Reaſons I like it beſt. Therefore in the Triangle ♃ A ☊, taking ☊ A the Argument of Latitude 94° 18′ 33″, and ♃ A the Inclination 1° 18′ 7″; the Angle of the Inclination

clination of the Plain of *Jupiter*'s Orbit to the Ecliptick will be found 1° 1' 20", which *Kepler* makes 1° 19' 0", *Street* 1° 20', *Bullialdus* and *Wing* 1° 21' 48"; all of them something greater than it should be.

And that the Inclination is so much, at least not greater than this, not only the Observations of last Night, but also those of the Months *February*, *March*, and *May* of last Year, prevail with me to believe. Yet in the mean time I must acknowledge, that it may be proved to be greater, that is, 1° 20' 20", from that Transit of *Jupiter* near the 8th of ♍, *May* 29 and 30 *An.* 1649, observed at St. *Julian*, at *Bologna* and *Majorca*, by those very learned Men *Ricciolus* and *Mutus*. Which seems to insinuate to us, that since the Inclinations of the Orbits are esteem'd as invariable by all, there must be some Error in those Latitudes assigned by *Tycho* to some of these fix'd Stars. And this will hinder us from exactly determining the Quantity of this Inclination, till those Latitudes are duly restored. Yet this I dare affirm, because those Latitudes of the fix'd Stars are also found to be immutable; that the Angle of the greatest Inclination of the Plain of *Jupiter*'s Orbit to the Ecliptick, is less than 26' 40"; or than the Latitude of the 9th Star of ♍ of the Fourth Magnitude, which by *Tycho* is call'd the last of the Four in the left Wing of *Virgo*. Whenever therefore this is given correct, the Inclination aforesaid will be known also.

Almag. Nov. Par. 1 p. 710.

The Conjunctions of Saturn and Jupiter, An. 1682, and 1683. at Greenwich; by Mr. Flamsteed, n. 149. p. 244.

LXXXVI. Examining our antient *Ephemerides* I do not find that three *Conjunctions* of *Saturn* and *Jupiter* have ever happened in one Year's Space, since they were first in use to this present. Those of *Moletius* calculated from the *Alphonsine Tables* indeed make three in the Space of eight Months betwixt *August* 1563, and *April* 1564, inclusive; but the *Ephemerides* of *Statius* calculated from the *Prutenick*, make only one, on the 26 of *August*, of which *Junctinus* gives us the following Observation in the Preface to his *Astronomical Tables*, *An.* 1563. *Aug.* 24. 14^h 36' *p. m.* On the Northern Side *Jupiter* in a manner cover'd *Saturn*, who was on the Southern Side; now each of these Stars was found to be at the end of the 28th Degree of *Cancer*. *Riccioli* hence concludes, that the Planet ♃ covered some part of *Saturn* at this Time. But without Reason, for the Words *quasi cooperiebat* intimate not that the one did corporally cover the other, but rather that there was some small Interval betwixt them. The *Caroline Tables* make the visible Latitude of *Saturn* now, 11' 45", of *Jupiter* 20' 10", both North; the Conjunction being some few Days past: but because their Latitudes alter slowly, we may hence conclude the Difference 8' 25", to have been nearly their Distance at that Time. These Tables being grounded on the *Tychonick* Observations, made within less than 40 Years after, and shewing the Latitudes of the *Planets* well at this Time near 100 Years later, we may conclude to have answer'd them as well then; and if we consider how small a Space the Distance of 8½ Minutes appears to the naked Eye in the Heavens, especially betwixt two such bright Planets as *Saturn* and *Jupiter* are, that the *Caroline* Distance agrees very well with the Words of *Junctinus*, and that *Riccioli* was mistaken.

Their next *Conjunction* according to *Maginus*'s *Ephemerides* founded on the *Prutenick* Numbers, was *April* 29, 1583. in 21 *Deg.* of ♓, the *Sun* being then

then in 17 *Deg.* of ♉, ſo that that Planet's Riſing before him in *Signs* of ſhort Aſcenſion and with South Latitude, this *Congreſs* could not be obſerved by the Noble *Tycho* who was mindful of it, as appears by this Note in Page 55 of his *Hiſtoria Cœleſtis.* *May* 30. before Noon. As ſoon as ever we ſaw *Saturn* after the Conjunction, theſe Diſtances between *Jupiter* and *Saturn* were taken by a Radius ;

1^h	47′	3°	24′
1	50	3	24

The ſame *Ephemerides* ſhew the next *Conjunction* of *Saturn* and *Jupiter* 1603, *December* 14 at Noon in 9° 36′ of ♐ ; but the ingenious *Kepler* and our Sir *Chr. Heydon* found it by Obſervation 7 Days ſooner, or the Day of the ſame Month in the Morning, in near 8 *Degrees* of ♐, the Planets being but then newly emerged from the Rays of the *Sun*.

The *Ephemerides* of the Learned *Kepler*, calculated from his own *Rudolphine Tables*, make the next *Conjunction* 1623, betwixt the 7th and 8th of *July* in 6° 46′, of ♌, the Planet of *Saturn* being then only 4 Minutes to the North of *Jupiter* ; but this firſt *Conjunction* in the *Fiery Trigon* happening under the *Sun*'s Beams was not obſervable.

By the ſame *Tables*, and *Ephemerides* of *Eichſtade* calculated from them, theſe *Planets* met again in the 25° of ♓, betwixt the 15 and 16 of *February* 1643, with a Degree Difference of Latitude.

By the joint Conſent of *Eichſtade*'s and our *Wing*'s *Ephemerides*, the ſame Planets were in *Conjunction* again 1663, on the 10 of *October* at Noon in 13° 30′ of ♐, with one Degree of Difference of Latitude ; this *Conjunction* was obſervable after *Sun-ſet* in our Latitude, but I hear not that any one obſerved it.

In every one of theſe Years there happened only one *Conjunction* of the two *Superiors*, nor is it poſſible that there ſhould be more, except the *Heliocentrical Conjunction* fall near the *Oppoſition* of the *Sun* ; for then there may be three, two *Direct*, and one *Retrograde*, as has been within the Space of 7 Months, betwixt *October* and *May* laſt incluſive, of which the true Times are determined from the following Obſervations.

An. 1682.					° ′ ″
d.					
Oct. 5	17	51	Betwixt the Centers of *Sat.* and *Jup.*		0 34 54
	17	54	——————————— rep.		0 34 48
12	13	49	Betwixt their Centers——————		0 16 2
	13	54	——————————— rep.		0 16 4
	14	3	Betwixt their next Limbs ———		0 15 22

Oct. 17.

	d.	h.	′		°	′	″
Oct.	17	14	10	Betwixt their Centers ———	0	20	9
		14	17	——— rep.	0	20	12
		14	21	——— again	0	20	14
		14	25	Betwixt their next Limbs ———	0	19	44
		14	33	Betwixt their remoter Limbs ———	0	20	37
		15	9	*Saturn* from the *Heel* of *Castor* ———	48	32	25
		15	14	*Jupiter* from the same *Star* ———	48	45	5
		15	17	———	48	45	20
		15	20	——— rep.	48	32	20
		15	50	*Saturn* from the same *Star* again ———	0	20	30
———	———	———		Betwixt their Centers again ———	———		
	19	15	41	Betwixt their remote Limbs ———	0	26	2
		15	45	Their Center ———	0	25	37
		15	47	Their next Limbs ———	0	25	11
	22	18	25	Betwixt their Centers ———	0	33	19
		18	29	——— rep.	0	33	26

The *Distances* betwixt the *Planets* were measured with the *Micrometer* and 16 Foot Glass, from the *Fixt Stars* with the *Sextant* : those of the 12th by my Assistant, the rest myself.

On the 22 Day, the Planet *Jupiter* was in Consequence of *Saturn* something less distant from him than he had been observed on the 5th Day near the same Hour. Hence the middle Time, betwixt these Observations is pointed out for the Time of their true *Conjunction* ; but to determine it more accurately, I shall examine the Observations made with the Sextant on the seventeenth Day, which being nearest the Time are most proper for this Purpose.

The correct Longitude of the *Heel* of *Castor* now ♋ 0° 50′ 42″, its Latitude 51′ 40″, South. The Latitude of *Saturn* by the *Caroline Tables* 96′ 20″, of *Jupiter* 41′ 30″, both North.

By the assumed Latitude of *Saturn* 56′ 20″, and his Distance from the *Heel* of *Castor* observed and corrected 48° 32′ 30″, I found their Difference of Longitude 48° 30′ 37″ ; therefore *Saturn* in ♌ 19° 21′ 19″.

By the Latitude of *Jupiter* assumed 41′ 30″, and his Distance from the *Star* 48° 45′ 20″, their Difference of Longitude 48° 43′ 56″, and *Jupiter*'s Place in ♌, 19° 34′ 39″.

Hence *Jupiter*'s Place in consequence of *Saturn*'s 13′ 20″, with which and the Distance of their Centers observed the same Night 20′ 12″, I find the true Difference of their Latitudes 15′ 20″ ; but half a Minute different from what I assumed it on the Authority of the *Tables*.

The apparent Motion of *Jupiter* from the 14 to the 18 Day of *October* by an *Ephemeris* exactly calculated and made agreeable to these Observations, is 29′ 16″, of *Saturn* 15′ 01″, both direct : hence the Motion of *Jupiter* from *Saturn* in four Days is 14′ 15″. I say therefore as 4 Days Motion, or 14′ 15″, is

15″, is to 4 Days, or 96 Hours; ſo is 13′ 20″ (which *Jupiter* is paſt the *Conjunction* of *Saturn*) to 90 Hours or 3 Days 18 Hours; the Time interlapſed ſince the *Conjunction*; which taken from the 17 Day 15 Hours, the Time of my Obſervation, gives the true Time of the *Conjunction* of the two Planets on the 13 Day, one and twenty Hours after Noon, or according to the common Account, the 14 Day, at 9 a Clock in the Morning.

At which Time *Saturn* is with *Jupiter* in ♌ 19° 07′$\frac{1}{5}$, with 15′$\frac{1}{3}$ more Northern Latitude.

The *Acta Eruditorum Lipſienſia*, p. 366. make this *Conjunction* to have happened the ſame Day in the ſame Longitude with the *Eleventh Star of Leo*; whoſe Place they ſtate in ♌ 19° 04′ Lat. 0′ 16″ N. with 14 Minutes Difference of Latitudes betwixt the two *Planets*. But their Obſervation ſeems to have been made only by the Judgment of the bare Eye without an Inſtrument, which conſider'd, I wonder not that it differs at all, but rather that the Difference is ſo ſmall from this Determination.

On the 19 of *January* following, viewing then the Planets both *Retrograde* with the 16 Foot Glaſs, I found them approached within a meaſurable Diſtance of each other.

An. 1683.			
d.	h. ′		° ′ ″
Jan. 19	6 41	Betwixt their Centers ——— ———	0 33 28
	6 45	——— ——— ——— ——— ——— rep.	0 33 24
	6 49	Betwixt their remote Limbs ——— —	0 33 52
26	6 3	Betwixt their Centers — — — —	0 15 8
Both being	6 7	— — — — — — — rep.	0 15 6
in ☍ to the	7 00	Betwixt their remote Limbs — ———	0 15 31
Sun.	7 8	By *T. Smith* — — — — rep.	0 15 9
	7 12	Betwixt their Centers ——— ———	0 15 5
	7 14	——— ——— ——— ——— ———	0 15 2
	7 17	Betwixt their next Limbs — — —	0 14 29
	7 20	— — — — — — — rep.	0 14 31
	7 21	——— ——— ——— again	0 14 26
	9 24	*Jupiter* from the *Heel* of *Caſtor* ———	46 18 10
	9 26	——— ——— ——— rep.	46 18 5
	9 28	*Saturn* from the ſaid *Heel* — ———	46 8 50
	9 30	——— ——— ——— rep.	46 9 55
	9 37	*Jupiter* from the bright Star of the *Lion's Head* E. — ——— ———	8 42 5
	9 39	——— ——— ——— rep.	8 42 5
	9 40$\frac{1}{2}$	*Saturn* from the ſame Star ——— ———	8 29 35
	9 42$\frac{1}{2}$	——— ——— ——— rep.	8 29 40

	h. '		° ' "
	9 48	*Jupiter* from the *Lion's Heart* ——	8 18 00
	9 50	—— —— rep.	8 17 55
	9 52	*Saturn* from the ſame —— ——	8 29 35
	9 54	—— —— —— rep.	8 29 35
	9 59	The *Lion's Heart* from *E* in the *Head* —— —— ——	12 58 50
	10 3	The *Heel* of *Caſtor* from the *Lion's Heart* —— —— ——	54 34 20
	10 8½	The *Heel* of *Caſtor* from *E* ♌ ——	46 24 45
Jan. 30.	5 28	Betwixt their Centers — ——	0 11 36
	5 30	—— —— — rep.	0 11 33
	5 34	Betwixt their remote Limbs ——	0 11 58
	5 36	—— —— —— rep.	0 12 1
	5 38	Betwixt their next Limbs —— ——	0 11 1
	5 41	—— —— — rep.	0 11 00
Feb. 7	7 37	Between their Centers —— ——	0 28 35
	7 40	—— —— —— rep.	0 28 34

From Obſervations formerly made, I have determined the true Places and Latitudes, to this preſent Time, of,

	s ° ' "		° ' "
The Heel of Caſtor ——	♋ 0 51 10	*Lat.*	0 51 40 *South.*
Bright * in the *Lion's Head*, E —	♌ 16 15 27		9 41 07 *North.*
Lion's Heart —— ——	♌ 25 24 45		0 26 20 *North.*

And from the above recited Meaſures, the true Diſtances of the *Planets* from theſe Stars, *January* the 26th at 9^h 40′ *p. m.* as follows,

	°	'	"
Saturn from the *Heel* of *Caſtor* ——	46	09	00
Jupiter from the ſame ——	46	18	10
Saturn from the *Lion's Heart* ——	8	29	40
Jupiter from the ſame —— — —	8	18	00
Saturn from the bright * in the *Lion's Head* E —	8	29	40
Jupiter from the ſame —— ——	8	42	10

Whence I collect the true Places at this Time.

	s ° ' "		° ' "
Of *Saturn*	♌ 16 57 10	*Lat.*	1 13 10
Of *Jupiter*	♌ 17 07 10	*Lat.*	1 01 30
Difference of *Long.* ——	10 00	of *Lat.*	11 40

The

The *Retrograde* Motion of *Jupiter* from *Saturn* in 4 Days, betwixt the Twenty-ſixth and thirtieth of this Month, by my correct *Ephemeris* is 12′ 15″. I ſay therefore as 12′ 15″, is to 4 Days or 96 Hours; ſo is 10′ 00″ (the Difference of the *Planets* preſent Longitudes) to 78 Hours, or three Days ſix Hours; which therefore added to the Time of that Obſervation *Jan.* 26^{d} $9^{h}\frac{2}{3}$, gives the true Time of the *Conjunction* Jan. 29^{d} 16^{h}, or according to the common Account *Jan.*. 30 at 4^{h} in the Morning. At which Time both the *Planets* are in ♌ 16° 51′$\frac{1}{3}$, with 11′$\frac{1}{2}$ Difference of Latitude or Diſtance from each other; which is further confirmed by the meaſured Diſtances of the *Planets* on the 30*th* at Night before recited.

On the 26*th* Day at 9^{h} 40′ the *Sun*'s true Place was by my Tables in ♒ 17° 21′$\frac{1}{2}$, ſo that he was now about $\frac{1}{3}$ of a Degree paſt their Oppoſition.

Towards the latter end of the following *April*, the Planet *Jupiter* began to approach *Saturn* again, both being now direct; the 28 at Night with the 16 Foot Glaſs and *Micrometer* I meaſured the Diſtances.

An. 1683.	d.	h.	′			°	′	″
Apr.	28.	10	21	Betwixt their Centers ——		0	32	35
		10	23	—— ——	rep.	0	32	33
		10	24	Betwixt their next Limbs ——		0	32	4
		10	26	—— ——	rep.	0	32	2
				Betwixt their remote Limbs but not accurately, Clouds interpoſing.				
		10	29	—— —— ——		0	33	22
May	7.	8	59	*Jupiter* from the *Lion's Heart* ——		10	59	00
		9	1	—— ——	rep.	10	59	00
		9	3½	*Saturn* from the *Lion's Heart* ——		10	58	50
		9	5	—— —— —	rep.	10	58	50
		9	11	*Jupiter* from *E* in the *Lion's Head*		8	55	35
		9	15	—— —— —	rep.	8	55	40
		9	17	*Saturn* from the ſame Star —		8	39	40
		9	18	— — —	rep.	8	39	40
		9	30	Betwixt their Centers —		0	15	38
		9	33	— — — —	rep	0	15	37
		9	35	Betwixt their next Limbs		0	15	3
		9	36	—— —	rep.	0	15	00
		9	40	Betwixt their remote Limbs		0	16	2
		9	42	—— ——	rep.	0	15	58
		10	20	*Jupiter* from *B* in ♍ —		38	11	45
		10	23	— —	rep.	38	11	45
		10	26	*Saturn* from the ſame Star —		38	10	55
		10	28	— —	rep.	38	10	45
May	11.	9	28	Betwixt their Centers —		00	20	2
			31	— —	rep.	00	20	2
	16.	9	22	Betwixt their Centers ——		00	34	4

From these Observations I state the Distances of the *Planets* from the fixed Stars *May* 7th at 9^h $5'$ *p. m.* as follows.

		°	′	″
Saturn from the *Lion's Heart*	——— ———	10	58	50
Jupiter from the same	——— ———	10	59	00
Saturn from E in the *Lion's Head*	———	8	39	40
Jupiter from the same	——— ———	8	55	35

	°	′	″		°	′	″	
Hence the true Longitude of *Sat.* ♌	14	27	42.	*Lat.*	1	12	46.	*North.*
of *Jupiter* ♌	14	26	37.	*Lat.*	0	56	43.	*North.*
Difference of Longitude		1	4.	*Lat.*	0	16	3.	

The Difference of Latitudes something exceeds the Distance measured with the *Micrometer*, by reason that the Wind then shaking the *Sextant* permitted us not to be so exact as usually; but the Difference, being less than half a Minute, I esteem inconsiderable.

The Diurnal Motion of *Jupiter* from *Saturn* was now $3'$ $15''$. It holds therefore, as $3'$ $15''$ (one day's Motion,) is to one Day or 24 Hours; so $1'$ $4''$ (the Distance of *Jupiter* from the ☌ with *Saturn*) to 8 Hours, the Interval betwixt the Observation and following *Conjunction*, which was therefore 17^h after Noon, or according to the vulgar reckoning, *May* 8. at 5 a Clock in the Morning: At which Time the true Place of the *Planet* is ♌ 14° $28'\frac{3}{4}$, the Difference of their Latitudes, $15'$ $40''$. *Saturn* being so much more Northerly than *Jupiter*.

In all best esteemed *Astronomical Tables* extant, the Mean Motions of the *Planet Saturn* are too swift, of *Jupiter* too slow considerably: Hence it came to pass that they made the *Direct Conjunctions* some Days later, the *Retrograde* earlier than they were found by Observation.

At Dantzick; *by M.* Hevelius. *n.* 143. *p.* 18. & *n.* 151. *p.* 326.

2. *Octob.* 26. New Style, at 1^h $40'$ in the Morning, I happen'd to take the Situation of *Jupiter* and *Saturn* with my Tube and Micrometer, as well as I could wish. At the same time a certain conspicuous fix'd Star, (which is worth observing) was very near the said Planets. *Jupiter* then offer'd himself to me with three of his Companions; and perhaps the fourth was present, but because of the little Clouds was not to be seen. *Saturn* was distant from *Jupiter* $16'$ $44''$; *Jupiter* was distant from the Star (in the right Shoulder of ♌, if I am not mistaken,) $27'$ $55''$; and again *Saturn* was distant from the Star $38'$ $1''$. This Star was now according to our Catalogue in 19° $2'$ $9''$ of ♌, with Northern Latitude 0° $20'$ $45''$. On *Octob.* 30. at 5^h in the Morning, the Distance of *Saturn* and *Jupiter* was $25'$ $5''$; whence we may surely conclude, that the Conjunction was celebrated several Days sooner than *Nov.* 3. when the Ephemerides and the Calculation exhibit it.

This the following Observations will still farther evince. For if the Conjunction was near, the Distance of *Jupiter* and *Saturn* would daily decrease; whereas it continually increased. On the first Day of *November* at two in the Morning, by help of our Micrometer the aforesaid Distance was found to be $31'$ $35''$. And *Nov.* 2. I found the same Distance to be $35'$ $21''$. On *Nov.* 3. at One in the Morning, I found it now $39'$ $9''$. And *Nov.* 4. the Sky being very clear, the same Distance was still greater.

As

As to the other Conjunction, which according to the Writers of Ephemerides ought to happen by the Retrogressions of these Planets on the 26 Day of *January* of the current Year 1683; I shall here add some of the principal Observations.

An. 1683.	*Time correct. by Alt. in Evening.*			*Observations.*		*Distances.*		
	h.	′	″			°	′	″
Feb. 1	6	40	0	Dist. of *Sat.* and *Jup.* was found to be	3300	0	35	5
Feb. 2	9	30	0	Again, &c. ———	2900	0	22	3
Feb. 3	9	00	0	Again, &c. ———	2500	0	19	0
Feb. 4	10	00	0	Again, &c. ———	2300	0	17	29
Feb. 5	8	30	0	Again, &c. ———	2100	0	15	59
Feb. 6	7	51	0	Again, &c. ———	1850	0	14	6
Feb. 7	8	17	19	Again, &c. ———	1700	0	12	55
Feb. 8	6	10	0	Again, &c. ———	1600	0	12	10

Feb. 9. in the Evening, at 9ʰ 0′ 8″, I perceived the Planets in my Tube through the thick opening Clouds, and pursuing them with my Eye, I perceived the Conjunction had been celebrated the Night before between the 8 and 9 of *February*. For the said Distance now appear'd something larger: As also happen'd *Feb.* 11 in the Evening at 9ʰ 0′ 0″. For the Distance between *Saturn* and *Jupiter* was 2000, that is, 0° 15′ 12″ in the Micrometer, which on the 8 Day of *February* was found only 0° 12′ 0″.

Besides, that the Conjunction was now past most surely appeared from hence, that each Planet was no longer in a right Line with the Belly of *Ursa Major*, as it was on the 8th of *February*, and likewise that *Saturn* was no longer in a right Angle to the Orbit of *Jupiter*'s Satellites.

Now that this may become still more evident, I shall here add some Observations, which were continued on the following Days.

	h.	″			′	″
Feb. 12	9	0	Dist. of *Sat.* and *Jup.* was taken —	2200	17	6
13	7	15	———	2550	19	24
14	9	0	———	2900	22	3
17	6	0	———	3750	28	30
20	9	0	———	5250	30	12

From the Continuation of these Observations it is abundantly manifest, that since the Planets from day to day recede farther from one another, the Conjunction itself happen'd really between the 8 and 9 of *February*.

Lastly, for the Sake of the Lovers of Astronomy, I shall here add what I have observed concerning their third Conjunction in the Month of *May*.

An. 1683.

An. 1683.		Time by the Watch in Evening.		Observations.		Distances.	
		h.	'			'	"
May	8	9	6	The Distance of *Saturn* and *Jupiter* was found by the Micrometer ——	4300	32	41
	10	9	14	—— ——	3750	28	30
	11	9	10	—— ——	3450	26	13
	12	8	45	—— ——	3050	23	10
	13	9	15	—— ——	2800	21	17
	14	9	45	—— ——	2550	19	23
	15	9	30	—— ——	2400	18	15
	16	9	30	—— ——	2250	17	6
	17	9	40	—— ——	2150	16	10
	18	10	00	—— ——	2100	15	58
				On this day happen'd the true Conjunct.			
	20	11	45	—— ——	2450	18	37
	21	9	15	—— ——	2650	20	9
	22	9	20	—— ——	2900	22	3
	23	9	5	—— ——	3250	24	43
	24	10	6	—— ——	3600	27	22
	25	9	30	—— ——	4000	30	25
	26	11	0	—— ——	4453	33	50
	27	9	25	—— ——	4900	37	15
	28	9	56	—— ——	5325	40	29

From which Observations it now appears to every one, that since the Distance of *Saturn* and *Jupiter* continually decreased from day to day to *May* the 18th, and from this Day again increased, the Conjunction of these Planets happen'd on that same Day; and indeed, (as appears from the Observations of the 15 and 20 of *May*,) at the Hour of 10 in the Forenoon: Which according to the Compilers of the Ephemerides, ought to have come on not before the 26. So that this third and last great Conjunction of this Year has also much eluded the Tables, as having happen'd above 8 whole Days sooner than they predicted

Besides, it appears from hence, that this last Conjunction was celebrated on *May* 18 at 10 in the Evening, because *Saturn* at that time was no longer at right Angles to the Orbit of *Jupiter*'s Satellites; and further, (as may be seen from the Observations of the following days,) that from the 18 of *May* to the 28, the Distance of *Saturn* and *Jupiter* went on continually increasing, as far as I could determine that Distance by my Micrometer.

Lastly, it is also to be noted, that on the 21 of *May* in the Evening, when among other Observations I also obtain'd with my Sextant the Distance of each

each of the Planets from the upper Star in the anterior right Foot of the great Bear, and the aforesaid Star was then almost in the same right Line with each of the Planets; so that it is easy for an Astronomer to judge, whether the Observations obtain'd with my new Sextant agree with those taken by the Micrometer. By the Sextant the Distance of *Jupiter* from the said Star was 32° 38′ 40″, and of *Saturn* 32° 19′ 45″. So that the true Distance of each Planet was 18′ 55″. But the same Distance found by the Micrometer the same Day, that is, *May* 21, was 20′ 9″; so that it was 1′ 14″ less than that obtain'd by the Sextant; yet I would not have you think, my Friend, that I err'd either with this or that Instrument; by no means. For *Saturn* and *Jupiter* were not exactly in the same Line with the said Star, as may appear to any one either from the Globe, or by Calculation. Hence it necessarily follows, that the Distance derived by the Sextant should be something less.

LXXXVII. *Riccioli* in the Second Part of the first Tome of his *Almagest*, has given us a Table of all the mean *Conjunctions* of the two Superiors from the Creation till the *Year* of *Christ* 1358, but very coarse and incorrect. I have therefore made a new one for 43 *Revolutions*, which are completed in 853 *Julian Years*, and 235 Days, from their mean Motions, which I have corrected by very late Observations. This being the *Period* of the *greatest Conjunctions*, after which Space of Time they return to the same Place of the *Zodiack* within $\frac{1}{5}$ of a Degree.

The mean Conjunctions of Saturn and Jupiter; by Mr. Flamsteed. n. 149. p. 254. July An. 1683.

The *ordinary Conjunctions* happen once in twenty Years, or more precisely in 19 *Julian* Years, and 312 Days; in which Time *Saturn*'s mean Motion is 8ˢ 2° 48′$\frac{1}{2}$, *Jupiter*'s the same above one *Revolution.*

These are commonly term'd the *Lesser* of the *Great Conjunctions*, which continue in *Signs* of the same *Triplicity* for 10 *Revolutions* to each other, or 198 Years: each *Conjunction* according to the mean Motions being 8ˢ 2° 48′$\frac{1}{3}$ removed from the preceding; so that if any *Conjunction* was made upon the first of ♈, the next following shall be in 2° 48′ of ♐, and all the following for 198 Years shall fall in ♈, ♌ and ♐, *Signs* of the same *Triplicity.*

But the 11 *Conjunctions* after, shall happen in the first Degree of ♍, and the following ten *Conjunctions* in ♉, ♍, and ♑, *Signs* of the same *Triplicity*. Of these the first are called by our Astrologers the *Greater Conjunctions.*

But the *Greatest* is, when after 43 *Conjunctions* completed in 853 Years, 235 Days, the *mean Conjunctions*, having been made in all the *Signs*, return to that Point of the *Ecliptick* from whence they began: though I must confess, had I been to name them, I would have called those the *greatest* which happen in the *Signs*, ♋ and ♌, because then the *Planets* rise highest, and are longest visible in our *Horizon*; as also being near their North *Nodes*, they approach nearest, and if they have any extraordinary Influence (which *Naboyd* thinks either they have not, or if they have, we understand not) it must according to their Axioms be strongest.

Those which happen in ♑ and ♒ I should call the *Greater* or *Middle*, because the *Planets* being then near their South *Nodes*, may approach each other

other again very nearly, tho' they rise not high in our *Horizon*, being in Southern *Signs*; the rest might be accounted the *Lesser* or *Ordinary*.

The *Mean Conjunction* of *Saturn* and *Jupiter* this Year 1683, was on the 14*th* Day of *January* old Stile, at 12 Hours after Noon, in the Meridian of *London*; at which Time the mean Motions of both the *Planets* were 4ˢ 11° 45′. This may be the *Radix* of the following *Table*.

A Table of the *mean Conjunctions* of *Saturn* and *Jupiter*.

Intervals.						Intervals.					
Revolutions complete	*Time.*		*Motion.*			*Revolutions complete*	*Time.*		*Motion.*		
	y.	d.	s	°	′		y.	d.	s	°	′
1	19	312	8	2	48	23	456	219	6	4	31
2	39	258	4	5	37	24	476	165	2	7	19
3	59	204	0	8	25	25	496	111	10	10	8
4	79	150	8	11	13	26	516	57	6	12	56
5	99	96	4	14	1	27	536	3	2	15	44
6	119	42	0	16	50	28	555	315	10	18	32
7	138	353	8	19	38	29	575	261	6	21	21
8	158	299	4	22	26	30	595	207	2	24	9
9	178	245	0	25	15	31	615	153	10	26	57
10	198	191	8	28	3	32	635	99	6	29	45
11	218	137	5	00	51	33	655	45	3	2	34
12	238	83	1	3	40	34	674	356	11	5	22
13	258	29	9	6	28	35	694	302	7	8	10
14	277	340	5	9	16	36	714	248	3	10	59
15	297	286	1	12	4	37	734	194	11	13	47
16	317	232	9	14	53	38	754	140	7	16	35
17	337	178	5	17	41	39	774	86	3	19	24
18	357	124	1	20	29	40	794	32	11	22	12
19	377	70	9	23	18	41	813	343	7	25	00
20	397	16	5	26	6	42	833	289	3	27	49
21	416	327	1	28	54	43	853	235	0	00	37
22	436	273	10	1	43						

By

By this *Table* to find the Time of any *mean Conjunction*, paſt or future, neareſt to any Place of the *Zodiack*; for Times paſt, ſubtract the Longitude of the given Place from the Longitude of the *Radix* $4^s\ 11^\circ\ 45'$, the Reſidue ſeek in the laſt Column of the *Table*; if you find not the preciſe Number take the next to it, againſt this you have in the ſecond Column the Years and Days, in the firſt the Number of *Conjunctions* paſt ſince any was made in that Place; ſubtract the Years and Days from 1683. *Jan.* 14. and the Motion from $4^s\ 11^\circ\ 15'$, ſo have you the true Time of the *mean Conjunction*, and Longitudes of the *Planets* then.

But for Times to come ſubtract the *Radix* from the given Place, ſeek the Reſidue as before in the laſt Column; if you find it not, take that you find neareſt it; againſt which, as before, you have in the ſecond Column, the Years and Days; in the firſt, the Revolutions future: for Example,

If it were required to know when the laſt *Conjunction* happen'd in the firſt Deg. of ♒, Subtracting ♒ or Ten *Signs* from $4^s\ 11^\circ\ 15'$, the Reſidue is $6^s\ 11^\circ\ 15'$, which ſeeking I cannot find in the third Column of the *Table*, but I find $6^s\ 12^\circ\ 56'$, which is not two Degrees more, and againſt them 516 Years 57 Days, and in the firſt Column 26, for the Number of *Conjunctions* interlapſed. Subtracting 516 Years 57 Days from 1683. *Jan.* 14. there remains 1166 Years, 322 Days; which ſhews me that the *Conjunction* was in the *Year* 1166. *Nov.* 18. and ſubtracting the Motion $6^s\ 12^\circ\ 56'$, from $4^s\ 11^\circ\ 45'$, it points me to the Place in $9^s\ 28^\circ\ 49'$.

Or if the Time of the firſt *Conjunction* in ♎ to come were demanded, I ſubtract the *Radix* $4^s\ 11^\circ\ 45'$, from 6 *Signs*, the Reſidue $1^s\ 17^\circ\ 15'$, I ſeek in the *Table* but find it not, I take therefore the next to it $1^s\ 20^\circ\ 29'$, againſt which ſtands 357 Years, 124 Days, theſe added to 1683. *Jan.* 14, give me the Year 2040, and 138 Days, *May* 18, for the Time of this *Conjunction*, and adding the $1^s\ 20^\circ\ 29'$, to the *Radix* $4^s\ 11^\circ\ 45'$, it makes $6^s\ 2^\circ\ 14'$, for the true mean Longitude of this *Conjunction*.

From the *mean Conjunction* the apparent may be found by the Help of a Planetary Inſtrument, or the uſual Aſtronomical Tables.

LXXXVIII. 1. S. *Campani* affirms, that he hath remarked in the *Belts* of *Jupiter*, the *Shadows* of his *Satellites*, and followed them, and at length ſeen them emerge out of his *Diſk*.

The Shadows of Jupiter's *Satellites observed; by* S. Campani. *n.* 1. p. 3.

2. M. *Caſſini*, after he had diſcover'd (by the Means of thoſe excellent *Glaſſes* of 35 Foot made by *S. Campani*) the *Shadows* caſt by the *Satellites* of *Jupiter* upon his *Diſk* when they happen to be between the *Sun* and *Him*; and after he had alſo diſtinguiſh'd their Bodies upon the *Diſk* of *Jupiter*, made ſome Predictions when they ſhould appear, to the End that the Curious might be convinc'd of this Matter by their own Obſervations.

By M, Caſſini. *and others.* n. 8. p. 143. *n.* 10. p. 171. n. 82. p. 4039.

Some of theſe Predictions have been verified, not only at *Rome*, and in other Places of *Italy*, but alſo at *Paris* by M. *Auzout*, and in *Holland* by M. *Huygens*; particularly *Sep.* 26, 1665. at half an Hour after Seven a Clock, one of theſe *Shadows* was ſeen both in *France* and in *Holland*.

These *Spots* have this Peculiar, which diſtinguiſheth them from all others, that they are found preciſely in that Place of *Jupiter*, where ſome *Satellite* is ſeen by the *Sun*; that they go from the Oriental Limb to the Occidental of the Diſk of *Jupiter*, with a Motion always equal to that of the *Satellite*; that in reſpect to us they precede the *Satellite*, before the Oppoſition of *Jupiter* to the *Sun*, and follow him after the Oppoſition; that the further *Jupiter* is diſtant from the Oppoſition, the greater is the apparent Diſtance of the ſame *Satellite*; that at divers Times of the Year, this Diſtance changeth in proportion of the annual *Parallax* of the *Satellite*, according as he is differently ſeen by the *Sun*, and by the Earth; and that at one and the ſame Time of the Year, when divers *Satellites* happen to be between *Jupiter* and the *Sun*, the Spots correſpondent to them are diſtant from them in proportion of the Semidiameters of the Circles of the ſame *Satellites*.

By Dr. Hook. *n*, 14 *p*. 246. *Fig*. 139.

3. *An*. 1666. *Jan*. 26. about $3^h\ 15'$ in the Morning, I perceived (with a 60 Foot Glaſs) near the Middle of the Zone *d*, a very round Spot, like that repreſented at *g*, which was not to be perceived about half an Hour before; and I obſerv'd it in about 10′ Time to be gotten almoſt to *d*, keeping equal Diſtance from the *Satellite h*, which moved alſo weſtwardly, and was joined to the Diſk at *i*, at $3^h\ 25'$: So that it was ſufficiently evident that this black *Spot* was nothing elſe, ſave the Shadow of the *Satellite h*, eclipſing a Part of the Face of *Jupiter*. The other three *Satellites* in the Time of the *Eclipſe* were weſtwards of the Body of *Jupiter*.

The Elongations of Jupiter's Satellites; *by Mr.* Flamſteed *n*. 82. *p*. 4036, 4337. *n*. 94. *p*. 6033. *n*. 96. *p*. 6094.

LXXXIX. 1. 'Tis now above two Years ſince the learned *R. Townley* Eſq; communicated to me the greateſt Digreſſions, which he had obſerved, of *Jupiter*'s Satellites from the Center of *Jupiter*, as alſo the mean Motion of every one, and the Epochs of thoſe Motions, deduced from his Obſervations made at his Seat at *Townley*. Afterwards I obtain'd from him your Ephemerides, O moſt learned *Caſſini*, of the *Medicean* Stars, for the Year 1668; from which having found, that not only the Motions, but the Epochs of Motion, as alſo the greateſt Elongations determin'd by you, were ſomething different from thoſe of Mr. *Townley*; I thought it might be worth while to make ſome Obſervations as Opportunity ſhould preſent, being earneſtly deſired by him ſo to do. Therefore in the Month of March of the Year 1672, *Julian* Style, with a Micrometer and a Tube of 14 Feet, with as much Care as I was able, I made the following firſt Experiments, repeating my Obſervations every Night for greater Certainty.

* 9′ 37″. *n*. 28. *p*. 4637.

An. 1672. *Mar*. $19^d\ 7^h\ 11'$. The remoter Limb of *Jupiter* was diſtant from the fourth Satellite 1601 *$=9'-34''$

27. 8. The remoter Limb from the ſame fourth Satellite 1591 $=9-30$

28. 8. The ſame Diſtance 1598 $=9-33$

By ſeveral Obſervations the Diameter of *Jupiter* was found 128. Therefore its Semidiameter was 64; by which the obſerved Diſtances being divided, there

there will ariſe the apparent Diſtances of the Satellite from the remoter Limb of *Jupiter* in his Semidiameters.

	d.	ſd.	′		ſd.	′
Mar.	19.	25	1	The Semidiameter being taken away, it will be from the Center	24	1
	27.	24	51		23	51
	28.	24	58		23	58

Then the Motion of which from *Jupiter*, and the Diſtances from the Center, according to your Numbers were thus.

	d.	h.	′	s	°	′	ſd.	′
Mart.	19	7	11	8	25	33	22	56
	27	8	. .	2	19	35	22	37
	28	8	. .	3	11	13	22	34

Therefore in the firſt Obſervation the Satellite was diſtant from his extream Elongation only 4′; in the ſecond 23′; in the third 26′; ſexagenary Scruples of the Semidiameter. Therefore if we add theſe to the obſerved Elongations in a due manner, the greateſt Digreſſions of this fourth Satellite from the Center of *Jupiter* will be by the firſt Obſervation 24 Semidiameters, 5′; by the ſecond 24 Semidiameters, 14′; by the third 24 Semidiameters, 24′; which you fix at only 23 Semidiameters, and Mr. *Townley* at 24 Semidiameters, 72′.

Of theſe three Elongations I judge the two latter to be the more accurate, becauſe for their Determination I had very commodious Obſervations, which I perform'd with all poſſible care. Afterwards I found the former among ſeveral Notes which I had taken on the 19th Night, which I cannot affirm was made with the ſame Exactneſs. However I admitted the Obſervations, as not ſo far diſagreeing with the reſt, but that it may ſerve to confirm them; as alſo to ſhew, very little if at all leſs, that the Elongation of this Satellite from *Jupiter* was to the Left Hand rather than to the Right.

Yet in my Obſervation I felt the Motion of the Air and Wind, which by ſhaking and agitating the Tube, (hanging in the open Air at an upright Pole, by means of a Rope and Pulley,) made the Obſervation difficult; and often was the Occaſion, that I might take the Diſtances too narrow. Therefore I reſolved to forbear from making many ſuch Obſervations, which require the utmoſt Care and Preciſion, till I have fitted up a more convenient Place for performing them, which at laſt I have provided. I have took care to have a Wooden Machine, like a ſhort Ladder, fitted into my Window, upon which my Tube being layed, may be turned about all ways, nor can be agitated this way or that by the Winds, except they are very violent, as it was in the open Air. Laying my Tube upon this, the 4th in the Evening, *An.* 1683. I apply'd my ſelf to making Obſervations with the greateſt Diligence, nor without Succeſs. For the Sky being exceeding clear, I ſaw all the four Satellites thro' a Tube conſiſting of convex Lens's, and meaſured their Diſtances from the remoter Limb of *Jupiter*, as follows.

3 985 Again—988
2 628 ——— 636
1 425 ——— 427
4 272 ——— 272. The Altitude of *Jupiter* taken with a Quadrant near two Foot long 24°—00′ Therefore the apparent Hour at *Derby* was 8^h 26′ after Noon. And then the fourth Satellite appear'd below the Line drawn on both Sides through the outmoſt Satellites; but, if I miſtake not, hardly a whole Semidiameter.

The Diameter of *Jupiter* was found 133, by Obſervations ſeveral Times repeated. Therefore the Semidiameter was 66½, which being taken from the obſerved Diſtances, the Intervals between the Centers of *Jupiter* and of the Satellites will become, that of the firſt 360, of the ſecond 569, of the third 921, of the fourth 205; which being divided by 66½, there will ariſe the viſible Elongations from the Center of *Jupiter* in his Semidiameters.

	ſd. ′			s ° ′	ſd.
1	5 25	The Motions of the Satellites from *Jupiter*, and the apparent Diſtances, according to your Tables accommodated to *Derby*, were	1	9 4 52	4 59
2	8 33		2	2 22 47	7 57
3	13 51		3	2 20 26	12 48
4	3 5		4	4 23 49	2 29

Therefore the firſt Satellite was deficient only 1′, the ſecond 3′, the third 12′, ſexagenary Scruples of the Semidiameter, from the utmoſt Elongation; if therefore we add theſe to what were obſerved, the extream Digreſſions will become

	ſd. ′		ſd.		ſd. ′
Of the Firſt,	5 26	Which you make,	5	But Mr. *Townley*, as I find in ſome Papers,	5 31
Second,	8 37		8		8 47
Third,	14 2		13		13 28

It being foreſeen that a convenient Opportunity would be offer'd *Apr.* 11. in the Evening, and as I concluded that theſe Diſtances were not to be deduced from an Experiment or two, I reſolved with my ſelf to make a farther Inquiry, by Obſervations that might then be had. When I firſt began theſe, the Sky near *Jupiter* was ſo cover'd with thin Clouds, that ſometimes I could perceive the Satellites but obſcurely. Yet I took their Diſtances from the remoter Limb of *Jupiter*, as the Air would give me Leave, which were as follows.

At the Hour 7½ after Noon 3 947 Again 932
2 628 ——— 614
1 405. Afterwards when the Heaven became as clear as I could wiſh, I obſerved more accurately

3 947
2 622
1 405
4 942. Again 957. The Altitude of *Jupiter* was 24° 0′ Therefore the apparent Hour was 7^h 56′.

The

The fourth Satellite appear'd a little above the Line drawn through the firſt and ſecond ; the third below that Line, but ſometimes I thought it was in it. The Diameter of *Jupiter* was taken at 132, and therefore his Semidiameter is 66 ; which taken from the obſerved Diſtances, gives the Interval between the Center of *Jupiter* and of the firſt 339, of the ſecond 556, of the third 881, of the fourth 891 ; which being ſeverally divided by 66, the apparent Elongations of the Center of *Jupiter* come out in the ſame Semidiameters, of the firſt 5 Semidiameters, 8′ ; of the ſecond 8 Semidiameters, 25′ ; of the third 13 Semidiameters, 21′ ; of the fourth 13 Semidiameters, 30′.

The Mean Motions of the Satellites from the *Pleni-mediceans*, with their Diſtances from the Center of *Jupiter*, according to your Numbers were as is ſet down in this Table. Whence it is to be ſeen, that the firſt was diſtant from its greateſt Elongation 10′ Scruples of the Semidiameter, the ſecond 26′, the third 38′. Which therefore if we add to the obſerved Elongations, the greateſt Digreſſions to be deduced from hence will become

	s	°	′	ſd.	′
1	8	15	35	4	50
2	2	10	59	7	34
3	2	12	2	12	22
4	10	25	8	13	15

Of the firſt	5	18
the ſecond	8	51
the third	13	59

very little differing from thoſe which we have deduced from the Obſervations of the fourth Night.

Now at each Time the inmoſt Satellite appear'd to the Left Hand from *Jupiter*, and the ſecond and third to the Right Hand. But *Apr.* 15. in the Evening I foreſaw that the third would appear on the Left Hand in its greateſt Elongation, on which Phænomenon therefore I thought it worth while to attend, that I might be ſatisfied, whether the greateſt Elongation from the Center of *Jupiter*, of the ſame Satellite, were the ſame on either Hand. The Sky was clear for Obſervation that Night, ſo that about the Hour of $7\frac{1}{2}$ I obſerved according to my Wiſh, that the Diſtance of the third was 955, and the Diameter of *Jupiter* 131. Therefore his Semidiameter was $56\frac{1}{2}$, which being ſubducted from the obſerved Diſtance, made the Interval between the Center of *Jupiter* and the Satellite to be 889; which being divided by the ſame viſible Semidiameter, gives the Elongation of the Satellite from the Center of *Jupiter* in his Semidiameters 13 Semidiameters, 35′. The Mean Motion of the Satellite was $3^s\ 4^\circ\ 9'$. The true Place of *Jupiter* ♎ $10^\circ\ 27'$. Therefore the Planet was diſtant from the *Pleni-medicean* $9^s\ 3^\circ\ 42'$, and from the greateſt Elongation only 3 Scruples. Now if we add theſe to the obſerved Digreſſion 13 Semidiameters, 35′; we ſhall have the greateſt Digreſſion this Time to the Left Hand 13 Semidiameters, 38′. This is leſs by the third part of a Semidiameter than that on the Right Hand, as we have found by Obſervations twice repeated and agreeing together. This ſeems to inſinuate, that there is ſome Excentricity of the Center of the Orbit of this Planet from the Center of *Jupiter*.

2. The Little Circle in the Middle repreſents the Planet *Jupiter*, the four Concentrick Circles, the proper Orbits of his four *Satellites*, duly proportioned

An Instrument for finding the Distances of Jupiter's Satel-*lites from his Axis; by Mr.* Flamsteed. *n.* 178. *p.* 1262. Dec. An. 1685. *Fig.* 142.

ed to the Breadth of his Body; the Distances betwixt the parallel Lines intersecting them, being each equal to one of his Semidiameters.

The 4 divided Circles next without these, are distinguished into so many Parts as there are Days and Hours in each *Satellite*'s Revolution; the innermost of them serving for the *first* or *innermost Satellite*; that next it, for the 2^d; that next without this for the 3^d; and outermost for the 4^{th}; above which is a small divided Arch of 15 Degrees.

By this to find the Distances of the *Satellites* from ♃'s *Axis* to a proposed Time.

1. Find the *Parallex* of *Jupiter*'s Orb to the Time proposed, and note whether it be to be added or subtracted.

2 Extend the Thread from the Center of the *Instrument* over the *Parallax* numbred in the small Arch; it cuts off in the four divided Circles, so many Hours as each *Satellite* spends in passing from the *Axis* of the Shadow to the *Axis* of ♃ viewed from our Earth; these I call the simple *Sarallactick Intervals*, which if the Parallax was to be added, are also Additional, if to be subtracted, Subductive.

3. To these *parallactick Intervals* add the Times of half the Duration of the *Eclipse* of each *Satellite*, which for the *1st* may be assumed $1^h\ 10'$; for the *2d.* $1^h\ 30'$, (greater Exactness being needless;) but for the *3d* and *4th* when *Eclipsed*, (their *Immersion* into the Shadow and *Emersion* from it being commonly given in the *Catalogues*) take half the Difference of these Times at the next *Eclipse* to the Time proposed for half the Duration, and add them to the simple *parallactick Intervals*, so have you them augmented. But as often as the 4th *Satellite* is not *Eclipsed*, (which is two Years in every six), its Interval needs no Augmentation.

4. Find in the *Tables* the Times of the *Eclipses* of each *Satellite* next preceding the Time proposed, and when the 4th is not *eclipsed*, of its passing the *Axis* of the Shadow; to which if the *parallactick Intervals* augmented were Additional, add them to, if Subductive, subtract them from, each the Time of its proper *Satellite*'s *Eclipse*; so have you very near the apparent Times when each *Satellite* last past over the *Axis* of ♃ viewed from our Earth.

5. Subtract each of the Times thus got from the Time proposed, the Remainders are the Intervals of the Motion of each *Satellite* from ♃'s *Axis*.

6. Extend the Thread from the Center, over each of these Intervals of Motion numbred severally in the divided Circles, belonging each to its proper *Satellite*; where it cuts the proper *Orbit* of that *Satellite*, whose Interval was numbred in its peculiar Circle, it shews amongst the Parallels how many Semidiameters of ♃ that *Satellite* is distant from him, and on which side of him 'tis posited.

Note further, that the Thread as it lay extended over the *Parallax* of the *Orb* numbred in the small Arch, where it cut the several proper Orbits of each *Satellite*, shewed amongst the Parallels, how many Semidiameters of ♃ the Center of the Shadow was distant from the Center of ♃, viewed from our Earth:

Earth: And that if the *Parallax* of the *Orb* were Additional, the Shadow lies on the Right-hand from ♃, if Subductive, on the Left.

To explain these Precepts, I shall give two brief Examples. Let it be then proposed to know, how far each *Satellite* appears distant from ♃ on the 26th of *Dec.* this present Year 1685, at 6ʰ 52′ *p. m.* when the *Third Satellite* falls into the Shadow; also on *Jul.* 16. 1686. at 10ʰ 00′ *p. m.* when there is no *Eclipse*.

An. 1685. *December* 26ᵈ 16ʰ 52′ *p. m.* the *Parallax* of the *Orb* is 9° 20′ Additional; Therefore *Fig.* 143.

	1	2	3	4
	d. h. ′	d. h. ′	d. h. ′	d. h. ′
The *Simple Parallactick* Intervals *add.* ——	0 1 5	0 2 10	0 4 25	0 10 20
The half *Durations* of the *Eclipses* to be added —— ——	0 1 10	0 1 30	0 1 18	0 0 0
The *Parallactick Intervals* augmented ——	0 2 15	0 3 40	0 5 43	0 10 20
Last *Immersions*, and ☌. *Dec.* —— ——	25 9 37	25 5 47	19 12 58	10 00 30
Times of last passing *Jupiter's Axis*, *Dec.* —	25 11 52	25 9 27	12 18 41	10 10 50
Subtracted from the Time proposed. *Dec.*	26 16 52	26 16 52	26 16 52	26 16 52
Leaves the *Intervals* of *Motion.* ——	1 5 00	1 7 25	6 22 11	16 6 2
Over which, numbred in their peculiar Circles, the Thread being severally laid, cuts the proper *Orbit* of each at their visible *Distances* from *Jupiter* —— ——	ſd. 5. *dext.*	ſd. 6½ *ſin.*	ſd. 3. *dext.*	ſd. 4½ *dext.*

Again,

Again, *An.* 1686. *Jul.* 16, 10^h *p. m.* the *Parallax* of the *Orb* is 10° 46 Subductive. Hence

	1			2			3			4		
	d.	h.	′	d.	h.	′	d.	h.	′	d.	h.	′
The *Simple Parallactick Intervals* Sub.	0	1	12	0	2	35	0	5	10	0	12	00
Half *Duration* of the *Eclipses* add.	0	1	10	0	1	30	0	1	21	—	—	—
The *Parallactick Intervals* augmented.	0	2	22	0	4	5	0	6	31	0	12	00
The next last *Emersions*, and passing the *Axis* of the *Shadow*, *Jul.*	15	5	55	15	22	2	15	9	19	15	17	52
The Time of last passing the visible *Axis* of *Jupiter*	15	3	33	15	17	57	15	2	48	15	5	52
The Time proposed	16	10	00	16	10	00	16	10	00	16	10	00
The *Intervals* of *Motion*	1	6	27	0	16	3	1	7	12	1	4	8
Therefore *Distan.* from *Jupiter*'s *Axis*	sd. 5½ *dext.*			sd. 8½ *sin.*			sd. 13½ *sin.*			sd. 10½ *sin.*		

And the *Satellites* stand at the two proposed Times as in the two Figures.

In drawing of which, though I have considered their *Latitudes* from the Line of their utmost *Elongations* passing thro' *Jupiter*'s Center ; yet I give no Rules for determining it, the Contrivances and Directions necessary on that Account, being too many and troublesome to be inserted here : my design is only to shew the ingenious Observer, how to find at what Distance from ♃ each *Satellite* appears, that so he may not mistake one for another when he is to observe any of their *Eclipses*.

Eclipses and Places of the Satellites *of* Jupiter *observed at* Paris ; *by* . . . . . *n.* 44. *p.* 892.

XC. 1. *An.* 1688. The *French Astronomers* have made these Observations by a 14 Foot Telescope.

Octob. 7. 10^h 32′ *p. m.* The *First Satellite* (called *Pallas*) entred upon the Face of *Jupiter*.

Octob. 8.

Octob. 8. 8^h 11′. The *Second Satellite* (called *Juno*) went out behind *Jupiter.*
Octob. 9. 8^h 54′. The *Second Satellite* went out from the Face of *Jupiter.*
Octob. 16. 10^h 4′. The *Second Satellite* entered upon the Face of *Jupiter.*
Octob. 22. 10^h 41′ 33″. The *First Satellite* entred into the Shadow of *Jupiter.*
Octob. 23. 8^h 32′. The *First Satellite* entred upon the Face of *Jupiter.*
Nov. 12. 10^h 40′. The *Second Satellite* entred into the Shadow of *Jupiter.*
Nov. 20. 2^h 38′ 30″. After Midnight, the *Third Satellite* (called *Themis*) entred into the Shadow of *Jupiter.*

2. *An.* 1671, *Sept.* 7. New Style, in the Morning. It had been agreed on by Mr. *Cassini* at *Paris*, and Mr. *Picard* at *Uraniburg*, to observe the Occultation of the first *Satellite* of *Jupiter*; wherefore I also thought proper to attend diligently to the same Phænomenon. Therefore at 4^h 27′ as soon as *Jupiter* appeared, I found that all his *Satellites* were present also, three to the left Hand, and one to the Right. The two which were nearest him to the left seemed to be not far from his Limb, as also that on the right, which was the least of them all. All four appear'd distinctly almost to 5^h 7′, tho' now the Heavens began to grow blue. At 5^h 12′, beyond my Expectation that nearer *Satellite* on the left Hand (in respect of my Tube, which exhibits Objects in an inverted Order,) seem'd to me intirely to vanish, the three others remaining, though that on the right Hand also seem'd to approach nearer and nearer to *Jupiter.* Whether indeed that was the very Moment of the Immersion of that *Satellite*, I dare not certainly affirm; however that Occultation did not happen later. But I can readily grant that it might happen perhaps about a Minute sooner, or thereabouts. *At* Dantzick; *by* M. Hevelius. n. 78. p. 3029.

By the Watch in the Morn.	*Observations.*	*Distances and Altitudes.*	*The Time corrected.*
h. ′ ″		° ′ ″	h. ′ ″
4 36 25	*Jupiter* was first seen————	— — —	4 32 0
5 7 25	The Altitude of *Procyon*————	34 43 0	5 2 7
5 16 35	The first of *Jupiter*'s *Satellites* disappeared —	— — —	5 12 0
5 26 5	The Altitude of *Procyon* ————	36 39 0	5 23 27

3. *An.* 167½. *Feb.* 17. 7^h 25′ Afternoon, the Altitude of *Jupiter* was 15° 54′. But at 8^h 59′, Afternoon, or perhaps one Minute sooner, the first *Satellite* on the right Hand of *Jupiter* fell into his Shadow, and as he vanish'd his Distance from the Limb was so small, that I could not form a Judgment how much it was. *At* Derby; *by* Mr. Flamsteed. n. 82. p. 4036.

March 19^d 6^h 45′. The Altitude of *Jupiter* was 29° 35′. The first *Satellite* approach'd to the Limb of *Jupiter*, to which it was joyn'd at 7^h 51′. ib. 4037.

An. 1672. *Apr.* 15. 7^h 43′ in the Evening. The Altitude of *Jupiter* was 25° 0′. The first *Satellite* appeared to be about $\frac{1}{6}$ of his Diameter from his Limb, being about to disappear behind him. n. 96. p. 6099.

At 8h 6′ The Altitude of *Jupiter* was 27° 20′. He enter'd *Jupiter*, his height being 27° 26′; for certainly he could not be ſeen.

At Paris; *by M.* Caſſini. n. 1:7. *p.* 389.

4. *July* 6. New Style, 1675. before Midnight, the Hour being 11h and 16″ preciſely. *Jupiter*'s ſecond *Satellite* began to emerge out of the Shadow of this Planet, which had obſcur'd him.

Ph. Col. n. 1. p. 33. §. LXIV. 2.

5. An. 1679. *Jun.* 5. ſt. n. 3h *m.* I diſcovered 3 *Satellites* of *Jupiter:* The *Firſt* was diſtant Weſtward of the Limb of *Jupiter*, a little leſs than a Diameter; the *Second* was diſtant, on the Eaſt-ſide, a little more than a Diameter. The *Third* was more Eaſtward than the *Second*, by ſomewhat leſs than a Diameter of *Jupiter*.

The Equation of Light; by M. Romer. n. 136. *p.* 893. *Fig.* 145. June, An. 1677.

XCI. 1. Let A be the *Sun*, B *Jupiter*, C the *Firſt Satellite* of *Jupiter*, which enters into the *Shadow* of *Jupiter*, to come out of it at D; and let EFGHLK be the *Earth* placed at divers Diſtances from *Jupiter*.

Now ſuppoſe the *Earth*, being in L towards the 2d Quadrature of *Jupiter*, hath ſeen the *Firſt Satellite* at the Time of its *Emerſion*, or iſſuing out of the Shadow, in D; and that about 42½ Hours after (*viz.* after one Revolution of this *Satellite*) the *Earth* being in K, doth ſee it returned in D; it is manifeſt, that if the Light require time to traverſe the Internal LK, the *Satellite* will be ſeen returned later in D, than it would have been if the *Earth* had remained in L; ſo that the Revolution of this *Satellite* being thus obſerved by the *Emerſions*, will be retarded by ſo much time as the Light ſhall have taken in paſſing from L to K; and that on the contrary, in the other Quadrature FG, where the *Earth* by approaching goes to meet the light, the Revolutions of the *Emerſions* will appear to be ſhortened by ſo much, as thoſe of the *Emerſions* had appeared to be lengthened.

The Theory of Jupiter's Satellites; *by M.* Caſſini. n. 128. *p.* 681. Sept. An. 1676.

This new *Equation* of the Motion of Light, which hath been eſtabliſhed by the *Royal Academy*, and in the *Oſervatory* for the ſpace of 8 Years, was confirmed by the *Emerſion* of the *Firſt Satellite* obſerv'd at *Paris* 1676. *Nov.* 9. 5h 35′ 45″, at Night, 10′ later than it was expected by deducing it from thoſe that had been obſerved in the Month of *Auguſt*, when the Earth was much nearer to *Jupiter:*

M. Caſſini's *Tables, for the Eclipſes of the* Firſt Satellite of Jupiter, *Abridged, and Reduced to the Meridian of* London; *by Mr.* Edm. Halley. n. 21. *p.* 238. Nov. An. 1694.

XCII. 1. *M. Caſſini*, having formed a new Hypotheſis for the *Satellites* of *Jupiter*, different from that of *Galileo*, thinks that the Plane of their *Orbs* is inclined to the Plane of the *Ecliptick:* He ſettles their *Nodes* with the *Orbs* of *Jupiter* towards the 13° of *Leo* and *Aquarius:* and finds that the *Obliquity* of their Circles to the Orbit of *Jupiter* is almoſt double to the *Obliquity* of this Orbit to the *Ecliptick*.

2. *M. Caſſini*, in the laſt Treatiſe of a Book, entitul'd, *Recuil d'Obſervations faites en Pluſieurs Voyages*, &c. has employed his Skill, to make eaſy the Calculation of the *Eclipſes* of the *Firſt Satellite* of *Jupiter*, which is otherwiſe operoſe even to the ſkilful. The Tables have for Principles, that this *Satellite* revolves to the *Sun* in 1d 18h 28′ 36″, ſo preciſely, that in 100 Years the Difference is not ſenſible: That in the Time of the Revolution of *Jupiter* to his *Aphelion*, which he ſuppoſes in 4332d 14h 52′ 48″, this *Satellite*

lite makes exactly 2448 Months or Revolutions to the *Sun*; and dividing the Orbit of *Jupiter* into 2448 parts, he has in a large Table of *Equation* ſhewn what is the Inequality of the Motion of *Jupiter* in each Revolution reduced to time; aſſuming, *Thirdly*, the greateſt *Equation* of *Jupiter* 5° 32′ 40″, whence the hourly Motion of the *Satellite* from *Jupiter* being 8° $28\frac{1}{2}$′, it follows that the greateſt Inequality (*Jupiter* paſſing the Signs of *Cancer* and *Capricorn*) amounts to 39′ 8″ of Time, to be added in *Cancer*, ſubtracted in *Capricorn*. *Laſtly*, as to the *Epocha*, or beginning of this Series of Revolutions, he has determined the *Aphelion* of *Jupiter* about $1\frac{1}{2}$ Degree forwarder than *Aſtronomia Carolina*, and above two Degrees more than the *Rudolphine* Tables, *viz.* preciſely in 9° of *Libra*, in the beginning of this Century; which perhaps he finds the proper Motion of *Jupiter* about the *Sun* at this Time to require: and the Number of Revolutions, ſince *Jupiter* was laſt *in Perihelio*, is here ſtiled *Num.* I.

A ſecond Inequality is that which depends on the Diſtance of the Sun from *Jupiter*, which he ſays Mr. *Romer* did moſt ingeniouſly explain by the Hypotheſis of the Motion of Light; to which yet *Caſſini* by his manner of *Calculus* ſeems not to aſſent, tho' it be hard to imagine how the *Earth*'s Poſition in reſpect of *Jupiter* ſhould any way affect the Motion of the *Satellites*. This Inequality he makes to amount to two Degrees in the *Satellite*'s Motion, or 14′ 10″ of Time; wherein he ſuppoſes the *Eclipſes* to happen ſo much ſooner when *Jupiter* oppoſes the *Sun*, than when he is in Conjunction with him. The Diſtribution of this Inequality he makes wholly to depend on the Angle at the *Sun* between the *Earth* and *Jupiter*, without any Regard to the *Excentricity* of *Jupiter*, (who is ſometimes $\frac{1}{2}$ a Semidiameter of the *Earth*'s Orb farther from the *Sun* than at other times) which would occaſion a much greater Difference than the Inequality of *Jupiter* and the *Earth*'s Motion; both of which are accounted for in theſe Tables with great Skill and Addreſs. But what is moſt ſtrange, he affirms that the ſame Inequality of two Degrees in the Motion, is likewiſe found in the other *Satellites*, requiring a much greater Time; as above two Hours in the 4th *Satellite*: which if it appeared by Obſervation would overthrow M. *Romer*'s Hypotheſis entirely. Yet I doubt not herein to make it demonſtratively plain, that the Hypotheſis of the Progreſſive Motion of Light is found in all the other *Satellites* of *Jupiter* to be neceſſary, and that it is the ſame in all; there being nothing near ſo great an Annual Inequality as M. *Caſſini* ſuppoſes in their Motions, by his Table, *p.* 9. and his *Præcepta Calculi*. The Method however uſed to compute this, is very curious; for having found that whilſt the *Sun* revolves to *Jupiter* there paſs 398d 29h 13′, wherein are made $225\frac{3}{8}$ Revolutions of the *Satellite* of *Jupiter*, the Number of Revolutions ſince *Jupiter* was laſt in Oppoſition to the *Sun*, is what he calls *Num.* II. in which the Inequality of the *Earth*'s Motion is allowed for in the Months, and that of *Jupiter*'s Orb by a Table of the *Equation* of *Num.* II. amounting in all to $3\frac{1}{2}$ Revolutions of the *Satellite* to *Jupiter*. This in the Tables following I have thought fit to leave out, ſhewing how to find it by the help of the former Equation of *Num.* I. The Numbers are in effect the ſame with M. *Caſſini*'s, only reduced to our Style and Meridian, and the form of them abridged, and, it's hoped, amended.

Vid. Sup. §. xci.

Epochs of the Revolutions of the first Satellite *to the Shadow of* Jupiter, *under the Meridian of* London.

In the Julian *Year current.*	d.	h.	′	″	*Num.* I.	*Num.* II.	
1660	0	11	5	48	968	200	6
1661	0	1	17	24	1174	181	2
1662	1	9	57	36	1381	162	2
1663	1	00	9	12	1587	143	5
1664	1	8	49	24	1794	125	1
1665	0	23	1	00	2000	105	7
1666	0	13	12	36	2206	86	4
1667	0	3	24	12	2412	67	0
1668	0	12	4	24	171	48	6
1669	0	2	16	00	377	29	2
1670	1	10	56	12	584	10	9
1671	1	1	7	48	790	216	9
1672	1	9	48	00	997	198	5
1673	0	23	59	36	1203	179	1
1674	0	14	11	12	1409	159	7
1675	0	4	22	48	1615	140	3
1676	0	13	3	00	1822	121	9
1677	0	3	14	36	2028	102	5
1678	1	11	54	48	2235	84	1
1679	1	2	6	24	2441	64	7
1680	1	10	46	36	200	46	4
1681	1	00	58	12	406	27	0
1682	0	15	9	48	612	7	6
1683	0	5	21	24	818	213	6
1684	0	14	1	36	1025	195	3
1685	0	4	13	12	1231	175	9
1686	1	12	53	24	1438	157	5
1687	1	3	5	00	1644	138	1
1688	1	11	45	12	1851	119	7
1689	1	1	56	48	2057	100	4

Epochs of the Revolutions of the first Satellite *to the Shadow of* Jupiter, *under the Meridian of* London.

In the Julian *Year Cur.*	d.	h.	′	″	*Num.* I.	*Num.* II.
1690	0	16	8	24	2263	81 0
1691	0	6	20	00	31	61 6
1692	0	15	00	12	228	43 3
1693	0	5	11	48	434	23 9
1694	1	13	52	00	641	5 5
1695	1	4	3	36	847	211 5
1696	1	12	43	48	1054	193 1
1697	1	2	55	24	1260	173 7
1698	0	17	7	00	1466	154 4
1699	0	7	18	36	1672	135 0
1700	0	15	58	48	1879	116 6
1701	0	6	10	24	2085	97 3
1702	1	14	50	36	2292	78 9
1703	1	5	2	12	50	59 5
1704	1	13	42	24	257	41 1
1705	1	3	54	00	463	21 8
1706	0	18	5	36	669	2 4
1707	0	8	17	12	875	208 4
1708	0	16	57	24	1082	190 0
1709	0	7	9	00	1288	170 6
1710	1	15	49	12	1495	152 3
1711	1	6	00	48	1701	132 9
1712	1	14	41	00	1908	114 5
1713	1	4	52	36	2114	95 1
1714	0	19	4	12	2320	75 8
1715	0	9	15	48	78	56 4
1716	0	17	56	00	285	38 0
1717	0	8	7	36	491	18 6
1718	1	16	47	48	698	0 3
1719	1	6	59	24	904	206 3
1720	1	15	39	36	1111	187 9

A Table of the Revolutions of the first Satellite *of* Jupiter *in a Year.*

January.	N. I.	N.II	*March.*	N. I.	N.II.
d. h. ′ ″			d. h. ′ ″		
0 0 0 0	0	0 0	1 4 12 24	34	34 8
1 18 28 36	1	1 0	2 22 41 00	35	35 8
3 12 57 12	2	2 1	4 17 9 36	36	36 8
5 7 25 48	3	3 1	6 11 38 12	37	37 9
7 1 54 24	4	4 1	8 6 6 48	38	38 9
8 20 23 00	5	5 2	10 00 35 24	39	39 9
10 14 51 36	6	6 2	11 19 4 00	40	40 9
12 9 20 12	7	7 2	13 13 32 36	41	41 9
14 3 48 48	8	8 2	15 8 1 12	42	42 9
15 22 17 24	9	9 3	17 2 29 48	43	43 9
17 16 46 00	10	10 3	18 20 58 24	44	44 9
19 11 14 36	11	11 3	20 15 27 00	45	45 9
21 5 43 12	12	12 3	22 9 55 36	46	46 9
23 00 11 48	13	13 4	24 4 24 12	47	47 9
24 18 40 24	14	14 4	25 22 52 48	48	48 9
26 13 9 00	15	15 4	27 17 21 24	49	49 9
28 7 37 36	16	16 5	29 11 50 00	50	50 9
30 2 6 12	17	17 5	31 6 18 36	51	51 9
31 20 34 48	18	18 5			
February.			*April.*		
0 20 34 48	18	18 5	0 6 18 36	51	51 9
2 15 3 24	19	19 6	2 00 47 12	52	52 9
4 19 32 00	20	20 6	3 19 15 48	53	53 9
6 4 00 36	21	21 6	5 13 44 24	54	54 9
7 22 29 12	22	22 6	7 8 13 00	55	55 9
9 16 57 48	23	23 7	9 2 41 36	56	56 9
11 11 26 24	24	24 7	10 21 10 12	57	57 9
13 5 55 00	25	25 7	12 15 38 48	58	58 9
15 00 23 36	26	26 7	14 10 7 24	59	59 9
16 18 52 12	27	27 7	16 4 36 00	60	60 8
18 13 20 48	28	28 7	17 23 4 36	61	61 8
20 7 49 24	29	29 7	19 17 33 12	62	62 8
22 2 18 00	30	30 8	21 12 1 48	63	63 8
23 20 46 36	31	31 8	23 6 30 24	64	64 8
25 15 15 12	32	32 8	25 00 59 00	65	65 7
27 9 43 48	33	33 8	26 19 27 36	66	66 7
			28 13 56 12	67	67 7
			30 8 24 48	68	68 6

A Table of the Revolutions of the first Satellite *of Jupiter in a Year.*

May.				*N. I.*	*N. II.*	
d.	h.	′	″			
0	8	24	48	68	68	6
2	2	53	24	69	69	6
3	21	22	00	70	70	6
5	15	50	36	71	71	6
7	10	19	12	72	72	5
9	4	47	48	73	73	5
10	23	16	24	74	74	5
12	17	45	00	75	75	5
14	12	13	36	76	76	4
16	6	42	12	77	77	4
18	1	10	48	78	78	4
19	19	39	24	79	79	3
21	14	8	00	80	80	3
23	08	36	36	81	81	3
25	3	5	12	82	82	3
26	21	33	48	83	83	3
28	16	2	24	84	84	2
30	10	31	00	85	85	2
June.						
1	4	59	36	86	86	1
2	23	28	12	87	87	1
4	17	56	48	88	88	0
6	12	25	24	89	89	0
8	6	54	00	90	90	0
10	1	22	36	91	90	9
11	19	51	12	92	91	9
13	18	19	48	93	92	9
15	8	48	24	94	93	8
17	3	17	00	95	94	8
18	21	45	36	96	95	7
20	16	14	12	97	96	7
22	10	42	48	98	97	7
24	5	11	24	99	98	6
25	23	40	00	100	99	6
27	18	8	36	101	100	6
29	12	37	12	102	101	5

July.				*N. I.*	*N. II.*	
h.	d.	′	″			
1	7	5	48	103	102	5
3	1	34	24	104	103	5
4	20	3	00	105	104	4
6	14	31	36	106	105	4
8	9	00	12	107	106	5
10	3	28	48	108	107	3
11	31	57	24	109	108	3
13	16	26	00	110	109	3
15	10	54	36	111	110	2
17	5	23	12	112	111	2
18	23	51	48	113	112	2
20	18	20	24	114	113	1
22	12	49	00	115	114	1
24	7	17	36	116	115	1
26	1	46	12	117	116	0
27	20	14	48	118	117	0
29	14	43	24	119	118	0
31	9	12	00	120	119	0
August.						
0	9	12	00	120	119	0
2	3	40	36	121	119	9
3	22	9	12	122	120	9
5	16	37	48	123	121	9
7	11	6	24	124	122	9
9	5	35	00	125	123	8
11	00	3	36	126	124	8
12	18	32	12	127	125	8
14	13	00	48	128	126	8
16	7	29	24	129	127	7
18	1	58	00	130	128	7
19	20	26	36	131	129	7
21	14	55	12	132	130	7
23	9	23	48	133	131	7
25	3	52	24	134	132	7
26	22	21	00	135	133	6
28	16	49	36	136	134	6
30	11	18	12	137	135	6

A Table of the Revolutions of the first Satellite *of* Jupiter *in a Year.*

September.	N.I.	N. II.	*November.*	N.I.	N. II.
d. h. ′ ″			d. h. ′ ″		
1 5 46 48	138	136 6	0 9 59 12	172	170 7
3 00 15 24	139	137 6	2 4 27 48	173	171 8
4 18 44 00	140	138 6	3 22 56 24	174	172 8
6 13 12 36	141	139 6	5 17 25 00	175	173 8
8 7 41 12	142	140 6	7 11 53 36	176	174 8
10 2 9 48	143	141 5	9 6 22 12	177	175 9
11 20 38 24	144	142 5	11 00 50 48	178	176 9
13 15 7 00	145	143 5	12 19 19 24	179	177 9
15 9 35 36	146	144 5	14 13 48 00	180	178 9
			16 8 16 36	181	180 0
17 4 4 12	147	145 5	18 2 45 12	182	181 0
18 22 32 48	148	146 5	19 21 13 48	183	182 0
20 17 1 24	149	147 5	21 15 42 24	184	183 0
22 11 30 00	150	148 5	23 10 11 00	185	184 0
24 5 58 36	151	149 5	25 4 39 36	186	185 1
26 00 27 12	152	150 5	26 23 8 12	187	186 1
27 18 55 48	153	151 5	28 17 36 48	188	187 2
29 13 24 24	154	152 5	30 12 5 24	189	188 2
October.					
1 7 53 00	155	153 5	0 12 5 24	189	188 2
3 2 21 36	156	154 5	2 6 34 00	190	189 2
4 20 50 12	157	155 5	4 1 2 36	191	190 3
6 15 18 48	158	156 5	5 19 31 12	192	191 3
8 9 47 27	159	157 5	7 13 59 48	193	192 3
10 4 16 00	160	158 5	9 8 28 24	194	093 4
11 22 44 36	161	159 5	11 2 57 00	195	194 4
13 17 13 12	162	160 5	12 21 25 36	196	195 5
15 11 41 48	163	161 6	14 15 54 12	197	196 5
17 6 10 24	164	162 6	16 10 22 48	198	197 6
19 00 39 00	165	163 6	18 4 51 24	199	198 6
20 19 7 36	166	164 6	19 23 20 00	200	199 7
22 13 36 12	167	165 6	21 17 48 36	201	200 7
24 8 4 48	168	166 6	23 12 17 12	202	201 8
26 2 33 24	169	167 7	25 6 45 48	203	202 8
27 21 2 00	170	168 7	27 1 14 24	204	203 9
29 15 30 36	171	169 7	28 19 43 00	205	204 9
31 9 59 12	172	170 7	30 14 11 36	206	206 0

A Table of the first Equation of the Conjunctions of the first Satellite *with* Jupiter.

N. I.	*Æquat.* ′	″	N. I.	*Æquat.* ′	″	N. I.	*Æquat.* ′	″
0	0	0	400	34	20	810	33	21
10	1	3	410	34	51	820	32	50
20	2	5	420	35	21	830	32	17
30	3	8	430	35	47	840	31	44
40	4	12	440	36	6	850	31	10
50	5	15	450	36	26	860	30	32
60	6	16	460	36	47	870	29	56
70	7	19	470	37	8	880	29	19
80	8	20	480	37	29	890	28	40
90	9	23	490	37	44	900	27	59
100	10	25	500	37	59	910	27	19
110	11	25	510	38	16	920	26	37
120	12	25	520	38	29	930	25	53
130	13	25	530	38	39	940	25	8
140	14	25	540	38	49	950	24	23
150	15	22	550	38	55	960	23	37
						970	22	50
160	16	18	560	38	59	980	22	3
170	17	17	570	39	3	990	21	15
180	18	11	580	39	6	1000	20	26
190	19	9	590	39	8	1010	19	37
200	20	5	600	39	7	1020	18	47
210	20	56	610	39	5	1030	17	56
220	21	49	620	39	03	1040	17	5
230	22	41	630	38	58	1050	16	13
240	23	32	640	38	51	1060	15	19
250	24	20	650	38	44	1070	14	25
260	25	7	660	38	34	1080	13	32
270	25	57	670	38	24	1090	12	37
280	26	43	680	38	10	1100	11	42
290	27	27	690	37	56	1110	10	47
300	28	9	700	37	40	1120	9	52
310	28	54	710	37	24	1130	8	57
320	29	35	720	37	5	1140	8	00
330	30	11	730	36	45	1150	7	3
340	30	45	740	36	25	1160	6	7
350	31	28	750	36	4	1170	5	1
360	32	10	760	35	40	1180	4	13
370	32	44	770	35	15	1190	3	15
380	33	15	780	34	49	1200	2	19
390	33	49	790	34	19	1210	1	21
400	34	20	800	34	49	1220	0	20
			810	33	21	1224	0	0

A Table of the second Equation of the Conjunctions of the first Satellite *with* Jupiter.

N. II.	Æquat. add. ′	″	N. II.	Æquat. add. ′	″	N. II.	Æquat. add. ′	″	N. II.	Æquat. add. ′	″
0	0	0	28	2	4	56	7	0	84	12	0
1	0	0	29	2	13	57	7	12	85	12	9
2	0	1	30	2	21	58	7	24	86	12	16
3	0	2	31	2	30	59	7	36	87	12	24
4	0	3	32	2	39	60	7	47	88	12	32
5	0	4	33	2	48	61	7	59	89	12	40
6	0	6	34	2	58	62	8	11	90	12	47
7	0	8	35	3	8	63	8	22	91	12	53
8	0	10	36	3	17	64	8	34	92	13	00
9	0	14	37	3	27	65	8	46	93	13	6
10	0	17	38	3	37	66	8	57	94	13	13
11	0	20	39	3	48	67	9	8	95	13	19
12	0	23	40	3	59	68	9	20	96	13	24
13	0	27	41	4	9	69	9	32	97	13	30
14	0	32	42	4	20	70	9	44	98	13	35
15	0	37	43	4	31	71	9	54	99	13	39
16	0	42	44	4	41	72	10	3	100	13	45
17	0	47	45	4	53	73	10	14	101	13	48
18	0	53	46	5	4	74	10	25	102	13	51
19	0	58	47	5	15	75	10	35	103	13	54
20	1	4	48	5	27	76	10	45	104	13	57
21	1	11	49	5	39	77	10	55	105	14	00
22	1	18	50	5	50	78	11	5	106	14	3
23	1	25	51	6	2	79	11	15	107	14	5
24	1	3	52	6	14	80	11	25	108	14	7
25	1	40	53	6	25	81	11	34	109	14	8
26	1	47	54	6	37	82	11	43	110	14	9
27	1	56	55	6	49	83	11	52	111	14	10
28	2	4	56	7	00	84	12	00	112	14	10

A Table

A Table of half the Moræ *of the first* Satellite *in the Shadow of* Jupiter.

N. I.	h.	′	″	*N. I.*	h.	′	″
0	1	4	56	1200	1	5	6
40	1	4	33	1240	1	4	48
80	1	4	12	1280	1	4	26
120	1	3	59	1320	1	4	7
160	1	3	48	1360	1	3	54
200	1	3	39	1400	1	3	38
240	1	3	38	1440	1	3	38
280	1	3	48	1480	1	3	44
320	1	4	1	1520	1	3	52
360	1	4	16	1560	1	4	7
400	1	4	36	1600	1	4	24
440	1	4	56	1640	1	4	42
480	1	5	18	1680	1	5	00
520	1	5	41	1720	1	5	22
560	1	6	1	1760	1	5	46
600	1	6	21	1800	1	6	10
640	1	6	39	1840	1	6	28
680	1	6	53	1880	1	6	45
720	1	7	3	1920	1	6	57
760	1	7	11	1960	1	7	7
800	1	7	15	2000	1	7	13
840	1	7	13	2040	1	7	14
880	1	7	9	2080	1	7	15
920	1	7	2	2120	1	7	15
960	1	6	54	2160	1	7	10
1000	1	6	39	2200	1	6	49
1040	1	6	22	2240	1	6	32
1080	1	6	5	2280	1	6	15
1120	1	5	45	2320	1	5	58
1160	1	5	26	2360	1	5	38
1200	1	5	6	2400	1	5	18
				2440	1	5	2

From

From these Tables, to any given Year, Month and Day, to find the next Eclipse of the First Satellite *of* Jupiter, *proceed thus.*

1. In the Table of the *Epochæ* find the Year of our Lord, and ſet down the Day, Hours, Minutes and Seconds, with *Num.* I. and *Num.* II. thereto annex'd; and in the Table of Revolutions, ſeek the Month, and Day of the Month, with the Hours and Minutes, and *Num.* I. and *Num.* II. affix'd, and add them together; and the reſpective Sums ſhall ſhew the mean Time of the middle of the *Eclipſe* ſought, with *Num.* I. and *Num.* II. required. But it muſt be obſerved, that in *Jan.* and *Feb.* in the *Leap Year*, one Day is to be added to the Day thus found.

2. If *Num.* I. be found leſs than 1224, with *Num.* I. or if greater than 2448, ſubtracting 2448, therefrom, with the Reſidue enter the Table, and you will have the firſt Æquation to be added to the mean Time before found. But if *Num.* I. be leſs than 2448, but greater than 1224, ſubtract it from 2448, and entring the ſame Table with the Remainder, you ſhall have the firſt Æquation to be ſubtracted from the mean Time. Then divide the Minutes of the ſaid firſt Æquation by 11, or rather $\frac{34}{3}$, and the *Quote* ſhall be the Æquation of *Num.* II. (anſwering to the Eccentrick Motion of *Jupiter*) to be added thereto when the firſt Æquation ſubtracts, and *è contra* ſubtracted when that adds.

3. If *Num.* II. thus æquated exceed 225,4 ſubtract 225,4 therefrom; and if the Remainder or *Num.* II. be leſs than 113, with the ſaid Remainder or Number; or if greater than 113, with the Complement thereof to 225,4, ſeek in the Table the ſecond Æquation; which being added to the Time before found, gives the true Time of the middle of the *Eclipſe.*

4. With *Num.* I. ſeek the half Continuance of the Total *Eclipſe*, which is to be added for the *Emerſion*, when the æquated *Num.* II. is leſs than 113, or if more than 225,4, it be leſs than 338. But if it exceed 113, or 338, then is the *Semimora* to be ſubtracted for the *Immerſion.*

Vid. Sup. § XXII.
5. Laſtly, With the *Sun*'s true *Place* take out the Æquation of Natural Days, which added or ſubtracted according to the Title, gives the Time of the *Immerſion* or *Emerſion* ſought.

Now how few Figures ſerve for this Computation, will beſt appear by an Example.

An. 1677.

An. 1677. 17. 8^h 9′ 40″ at *Greenwich*, Mr. *Flamsteed* observed the *first Satellite* to begin to *emerge*; that is 8^h 9′ 26″, at *London*.

	d.	h.	′	″	N. I.	N. II.
1677.	0	3	14	36	2028	102, 5
Sept.	17	4	4	12	147	145, 5
Sept	17	7	18	48	2175	248, 0
Æquat. 1.	—		26	11	2448	2, 3
Æquat. 2.	17	6	52	37	273	250, 3
	+	0	1	39		225, 4
Semimora	+	1	7	00		
						24, 9
Equal Time	17	8	1	16	11)	26, 2 (2, 3
Equation	+	0	9	25	⊙ in ♎	5°, 00′
Apparent. T.	17	8	10	41		
Observ.	00	8	9	20		
Error.			1	21		

An *Immersion* of this *Satellite* being computed after the same manner according to these Tables, ought to have happened, *An.* 1683. *Nov.* 30. 16^h 52′ 7″; but I observ'd it at 16^h 48′ 40″, so that the Error was, — 3′ 27″.

Again, M. *Cassini* observed an *Emersion* at *Paris*, *An.* 1693, *Jan.* 14^d 10^h 40′ 28″; that is, at *London* 10^h 30′ 48″, which these Tables give at 10^h 30′ 39″; and therefore the Error was no more than +9″.

After this manner I have compared these Tables with many good and certain Observations, and scarce ever find them err above 3 or 4 Minutes of Time; which Errors are exceeding small in comparison of the short Time that the *Satellites* have been discover'd.

In the Construction of the Table, which shews the half Continuance of these *Eclipses*, the Semidiameter of the Shadow of *Jupiter* is made by *Cassini* just 10 Deg. and that of the *Satellite* 30′; and the *Satellite's ascending Node* being supposed in 15° of *Aquarius*, at the end of this Century (that is 55° 20′ before the *Perihelion* of *Jupiter*) it will thence follow, that *Num.* I. being 116, or 2102, *Jupiter* passes the *Nodes* of the *Satellite*'s Orb, and consequently these *Eclipses* are Central, and of the greatest Duration. But *Num.* I. being 215, or 1481, the *Satellite* passes the Shadow with the greatest Obliquity, *viz.* 2° 55′ from the Center; whence the *Semimora* becomes of all the shortest.

3. The Tables of the other three *Satellites* not being so perfect or exact as those of the first, are here given in another Form. The Periods of their Revolutions to *Jupiter*'s Shade are as follow. *Of the other 3 Satellites* *Ib. p.* 253.

2

Period.

	d.	h.	′	″	‴	
Period. Secundi	3	13	17	54	3	*ſive* $2\frac{1}{123}$ *Rev.* Primi.
Period. Tertii	7	3	59	39	22	*ſive* $4\frac{3}{61}$ *Rev.* Primi.
Period. Quarti	16	18	5	6	50	*ſive* $9\frac{7}{15}$ *Rev.* Primi.

Whence the Table of the Firſt Æquation of the *Firſt Satellite*, or M. *Caſſini*'s larger Table, may by an eaſy Reduction ſerve the other three; the Æquation of the *2d* being $2\frac{1}{123}$ or twice the Minutes with half ſo many Seconds as there are Minutes in the Æquation of the *Firſt*, and the greateſt Æquation thereof $1^h\ 18'\ 35''$. Æquation of the *3d* is $4\frac{1}{20}$ times greater than that of the *Firſt*, and when greateſt amounteth to $2^h\ 29'$. And the Æquation of the *4th* being $9\frac{7}{15}$ times that of the *Firſt*, is had by ſubtracting $\frac{1}{2}$ and $\frac{1}{3}$ from 10 times the Æquation of the *Firſt*, whence the greateſt becomes $6^h\ 10'\ 28''$; ſo that *Num.* I. and *Num.* II. as here collected for the *Firſt*, may indifferently ſerve all the reſt.

M Romer's *Æquation of Light Defended. Ibid. p.* 254. *Id. Sup.*

4. As to the Second Æquation of the other *Satellites*, M. *Caſſini* has, by his *Præcepta Calculi* (as is before mentioned) ſuppoſed the Minutes thereof to be increaſed in the ſame Proportion, as inſtead of 14′ 10″ in the *Firſt* to be 28′ 27″ in the *Second*, 57′ 22″ in the *Third*, and no leſs than $2^h\ 14'\ 7''$ in the *Fourth*; whereas if this Second Inequality did proceed from the ſucceſſive Propagation of Light, this Æquation ought to be the ſame in all of them, which M. *Caſſini* ſays, was wanting to be ſhewn, to perfect M. *Romer*'s Demonſtration: wherefore he has rejected it as ill founded But there is a good Cauſe to believe, that his Motive thereto is what he has thought not proper to diſcover. And the following Obſervations do ſufficiently ſupply the Defect complained of in the making out of that Hypotheſis.

An. 1676, Oct. ſt. n. $6^h\ 10'\ 37''$, *App.* but $5^h\ 59'\ 37''$, *Equal Time*; M. *Caſſini* at *Paris* obſerved the *Emerſion* of the *3d Satellite* from *Jupiter*'s Shadow. And again *Nov.* 14 following $6^h\ 20'\ 55''$. *App.* Time, but $6^h\ 5'\ 55''$, *Eq. Time*, he obſerved the like Emerſion of the ſame *Satellite*. The obſerved Interval of Time between theſe *Emerſions* was $43^d\ 0^h\ 6'\ 18''$, which is 8′ 22″ more than 6 Mean Revolutions of this *Satellite*, of which 4′ 27″ ariſe from the Difference of the Firſt Equation, and the greater Continuance of the Latter *Eclipſe*; ſo that the other 4 Minutes is all that is left to anſwer for the Difference of the *2d* Æquations and *Num.* II. in that Time increaſing from 48 to 72, gives 4′ 36″ for the Difference of the *2d* Æquations of the *Firſt Satellite*. So that here the *2d* Æquation of the *Third* is found rather leſs than that of the *Firſt*; but the Difference is ſo ſmall that it may rather be attributed to the Uncertainty of Obſervation: Whereas according to M. *Caſſini*'s Method of Calculating, inſtead of four Minutes it ought to be 18′ 38″, and the Interval of theſe two *Emerſions* $43^d\ 0^h\ 21'$, exceeding the Time obſerved by a whole Quarter of an Hour; which that curious Obſerver could not be deceived in.

The like appears yet more evidently in the *Fourth Satellite*. By the Obſervations of Mr. *Flamſteed* at *Greenwich*, *An.* 1682. *Sept.* 24. $17^h\ 45'$, *T. App.* but $17^h\ 32'$, *T. Eq.* the *Fourth Satellite* was ſeen newly come out of the Shadow;

Shadow; so that about $17^h\ 30'$, *T. Eq.* the first beginning of *Emersion* was conjectured; and after 5 Revolutions, *viz.* *Decemb.* $17^d\ 11^h\ 16'$ or $11^h\ 18'$, *T. Eq.* he again observed the first Appearance of the *Satellite* beginning to *Emerge*, that is, after an Interval of $83^d\ 17^h\ 48'$; whereas this *Satellite* makes five mean Revolutions in $83^d\ 18^h\ 25'\frac{1}{2}$. Here we have $37'\frac{1}{2}$ to be accounted for by the several Inequalities. Of this $21'$ is due to the first Æquations, which is reduced to $19'$ by the Greater Continuance of the latter *Eclipse*, *Jupiter* then approaching to his *Descending Node*: So that there remains only $18'\frac{1}{2}$ for the Difference of the 2^d Æquations, whilst the Earth approached *Jupiter*, by more than the *Radius* of its own Orb: and the Difference of the 2^d Æquations of the *First Satellite*, being according to *Cassini* $8'\ 30''$, the said Difference in the *Fourth* ought to be $1^h\ 20'\frac{1}{2}$, instead of $18'\frac{1}{2}$; whence the Interval of these two *Emersions* would be, according to his Precepts, but $83^d\ 16^h\ 46'$, instead of $83^d\ 17^h\ 48'$ observed. And whereas $18'\frac{1}{2}$, may seem too great a Difference; it must be noted, first, that M. *Romer* had stated the whole 2^d Æquation $22'\ 00''$, which M. *Cassini* has diminished to $14'\ 10''$; so that instead of $8'\frac{1}{2}$. M. *Romer* allows $13'$; and secondly, that in the first of these Observations, being about half an Hour before Sun-rise, the Brightness of the Morning might well hinder the seeing of this smallest and slowest *Satellite*, till such time as a good part thereof was *Emerged*.

XCIII. Having a great desire to observe the Body of *Mars*, whilst *Acronycal* and *Retrograde* (having formerly with a Glass about 12 Foot long, observ'd some kind of Spots in the Phase of it) tho' it was not in the *Perihelium* of its Orb, but nearer its *Aphelium*; yet I found that the Face of it, when near its opposition to the Sun (with a Charge, the 36 Foot Glass I made use of, would well bear) appeared very near as big, as that of the Moon to the naked Eye.

The Phases and Revolutions of Mars *about his Axis; by Dr.* Hook.
n. 11 p. 198.
n. 14. p. 239.
Mar. An. 1666.

But such had been the ill Disposition of the Air for several Nights, that from more than 20 Observations of it, which I had made since its being *Retrograde*, I could find nothing of Satisfaction, tho' I often imagined I saw *Spots*; yet the inflective Veins of the Air (if I may so call those parts, which being interspers'd up and down in it, have a greater or less Refractive Power than the Air next adjoining, with which they are mix'd) did make it so confused and glaring, that I could not conclude upon any thing.

On the 3^d of *Mar.* $0^h\ 30'$, $166\frac{5}{6}$, in the Morning, tho' the Air was still bad enough, yet I could see now and then the Body of *Mars*, which I described by the Scheme B, as exactly representing what I saw thro' the Glass, as I could. *Fig.* 146.

An. $166\frac{5}{6}$, *Mar.* 10. $0^h\ 20'$, in the Morning, finding the Air very bad, I made use of a very shallow Eye-Glass, as finding nothing distinct with the greater Charge; and saw the Appearance of it as in C, which I imagined might be the Representation of the former *Spots* by a lesser Charge. About 3 of the Clock the same Morning, the Air being very bad (tho' to appearance exceeding clear, and causing all the Stars to twinkle, and the minute Stars to *Fig.* 147.

appear

Fig. 148. appear very thick) the Body ſeemed like D; which I ſtill ſuppoſed to be the Repreſentation of the ſame *Spots* thro' a more confuſed and glaring Air.

But obſerving *Mar.* 21. I was ſurprized to find the Air (tho' not ſo clear, as to the appearance of ſmall Stars) ſo exceeding tranſparent, and the Face of *Mars* ſo very well defined, and round and diſtinct, that I could manifeſtly ſee
Fig. 149. it of the Shape in E, about half an Hour after 9 at Night. The Triangular *Spot* on the Right-ſide (as it was inverted by the Teleſcope) according to the appearances, thro' which all the preceding Figures are drawn, appeared very black and diſtinct, and the other towards the left more dim; but both of them ſufficiently plain and defined. About a Quarter before 12 of the Clock the ſame Night, I obſerved it again with the ſame Glaſs, and found the Ap-
Fig. 150. pearance exactly, as in F; which I imagined to ſhew me a Motion of the former Triangular *Spot*.

Mar. 22. about half an Hour after 8 at Night, finding the ſame *Spots* in
Fig. 151. the ſame poſture (as in G) I concluded that the preceding Obſervation was only the Appearance of the ſame *Spots* at another Height and Thickneſs of the Air; and thought myſelf confirmed in this Opinion, by finding them in much the ſame Poſture, *Mar.* 23. about half an Hour after 9 (as in H) tho' the Air was
Fig. 152. nothing ſo good as before.

Mar. 28. about 3 of the Clock, the Air being light (in Weight) tho' moiſt
Fig. 153. and a little hazy, I plainly ſaw it to have the Form repreſented in I; which is not reconcileable with the other Appearances, unleſs we allow a Turbinated Motion of *Mars* upon its Center: Which, if ſuch there be, from the Obſervations made *Mar.* 21, 22, and 23. we may gueſs it to be once or twice in about 24 Hours, unleſs it may have ſome kind of Librating Motion; which ſeems not ſo likely.

The Parallax of Mars; *by Mr.* Flamſteed. n. 89. p. 5118. n. 96. p. 6100.

XCIV. *An.* 1672. *Sept.* I meaſured with a Micrometer and a Tube of 14 feet, the Diſtance of *Mars* from two fixt Stars the ſame Night. Whence I learn'd that his Acronic and Perigean Parallax is never greater than 25 ſecond Scruples. Whence it follows, that at moſt the *Sun*'s is 10″, and his diſtance is 21000 of the *Earth*'s Semidiameters.

Places of Mars *Obſerved at* Darby; *by Mr.* Flamſteed. n. 86. p. 5039.

XCV. 1. *An.* 1672. *May* 14 in the Morning. The Planet *Mars* paſſed near the *Star* called that at the Buttocks of *Aquarius*, whoſe Latitude is 2° 0′ 0″. Its Place at that Time according to me was 24° 12′ 9″ ♒; according to *Street* 24° 9′ 0″. From whence I obſerved

	° ′		° ′ ″
The true Altitude of the fixt Star being	9 40	At 2h 29′ in the Morning, the diſtance of *Mars* exactly in the ſame *Azimuth*	0 24 17
	11 12	The ſame diſtance again — —	0 24 24
	12 00	At 2h 51′. The *Planet* had departed from the ſame *Azimuth*, and was to the Eaſt of the *Azimuth* Line; the Diff. being	0 00 55

	° ′ ″
At 2^h 29′. in the Morning the *Parallactical Angle* was ——— —	82 26 00
Therefore *Mars* is in Consequence of the fixt *Star* ——— —	00 3 11
With greater Southern Latitude ——— ——— ———	00 24 4
Therefore the Latitude of *Mars* ——— ——— ———	2 24 4
His true Place according to —— —— { *Street* ♒.	24 12 17
{ Me— ♒.	24 15 04

2. *An.* 1683. *May* 3^d. 9^h 12′. *Mars* celebrated a Conjunction with the last Star but one of the Southern Wing of *Virgo*, which now is in 13° 49′ 14″. of ♎, and its Latitude 1° 48′ 33″ North; so that at the Time of the Conjunction *Mars* pass'd hardly more than 40″ below the said Star. *At* Dantzick; *by M.* Hevelius *n.* 151. *p.* 332.

XCVI. 1. *M. Burattini* hath signified from *Poland*, that he hath observed Inequalities in *Venus* as in the *Moon*. *Spots in* Venus; *by M.* Burattini. *n.* 10. *p.* 173.

2. I have at last discovered towards the Middle of the Body of *Venus* a part clearer than the rest, by which one may judge of the Motion, or the Rest of this Planet. *The Rotation of* Venus; *by M.* Cassini. *n.* 32. *p.* 615. *n.* 35. *p.* 687.

The first time I saw it, was *Octob.* 14. 1666. 5^h 45′, *p. m.* and then this bright part was very near the Center, on the North-side. And at the same time I observ'd Westward two obscure *Spots*, somewhat oblong; but I could not then see that resplendent part long enough to conclude any thing from thence, nor was I able to see any thing well of those parts till *April* 28. 1667. on which Day a quarter of an Hour before Sun-rising, I saw again a bright part, situated near the Section, and distant from the Southern Horn a little more than $\frac{1}{4}$ of its Diameter. And near the Eastern Ring I saw a dark and somewhat oblong *Spot*, which was nearer to the Northern than the Southern Horn. At the Rising of the *Sun*, I perceived, that this bright part was then no more so near the Southern Horn, but distant from it $\frac{1}{3}$ of its Diameter. This gave me great Satisfaction. But I was surprized at the same time to find, that the same Motion, which was made from South to North in the inferior part, of the Disk, was on the contrary made from North to South in the Superior part whence the Determination of the Motion may be better taken: For we have no Example of the like Motion, except it be that of the Libration of the *Moon*.

The next Day, at the Rising of the *Sun*, the said bright part was not far from the Section, and distant from the Southern Horn $\frac{1}{4}$ of the Diameter. When the *Sun* was 4 Degrees high, the same was situated near the Section, remote from the Southern Horn $\frac{2}{5}$ of the Diameter. The *Sun* being high 6° 10′, it seemed to have passed the Center, and that the Section of the Disk did cut the same. The *Sun* being 7° high, it appeared yet more advanced Northward, together with two obscure *Spots* seated between the Section and the Circumference, and equally distant from one another, and from each Horn on both Sides, And the Sky being very clear, I observed the Motion of the bright part for $1\frac{1}{3}^h$. which then seemed to be exactly made from South to North without auy sensible Inclination Eastward or Westward. Mean time I perceived in the Motion of the dark *Spots* so great a Variation, that it cannot be ascribed to any Reason in *Opticks*.

May 10 and 13. Before *Sun-Rising*, I saw still the bright part near the Center Northward.

Lastly, *June* 5 and 6. Before the Rising of the *Sun*, I saw the same between the Northern Horn and the Center of this Planet, and I noted the same irregular Variation in the obscure *Spots*. But when *Venus* began to be farther removed from the *Earth*, it was more difficult to observe the *Phænomena*.

It is very difficult to determine any thing of the Motion of these *Spots*: Yet this I can say (supposing that this bright part of *Venus*, which I have observ'd, especially this Year 1667, hath always been the same) that in less than one Day it absolves its Motion, whether of Revolution or Libration, so as in near 23 Hours it returns to the same Situation in this *Planet*, which yet happens not without some Irregularity.

A Place of Venus, observ'd at Dantzick; by M. Hevelius. *n.* 154. *p.* 419.

XCVII. *An.* 1683. *Aug.* 4. at almost 2^h in the Afternoon. *Venus* was distant towards the North only 16′ from a fixt Star of the third Magnitude, called the *Belly of Pollux*; which I found exactly by the Micrometer.

XCVIII. Mercury *observed in the Sun*, 1690. *at* Nuremberg; *by M.* Jo. Phil. Wurtzelbaur. *n.* 192. *p.* 483.

Time by the Clock.	*Observations.*		*Time by Calculation*
h. ′ ″	*Octob.* 30. Afternoon, *An.* 1690.		h. ′ ″
6 32 00	*Os Pegasi* culminates — — — —		6 28 45
9 00 00	*Andromeda*'s Head culminates — — —		8 52 17
9 4 30	*Jupiter* culminates — — — —		.
	Octob. 31. Morning.		
8 30 00	The *Sun* emerged out of the Clouds; above his Disk, in the Table of Observation, at the Distance of more than half a Digit from the Vertical to the right Hand, (tho' really to the left,) *Mercury* appear'd going out of his Limb.		
8 36 00	After *Mercury* had adhered a Minute of Time to the undulating Limb of the *Sun*, he went out at 14° from the *Zenith* towards the North.	° ′	h. ′ ″
8 49 00	The Altitude of the *Sun* — — —	10 5	8 38 38
8 59 45	— — — —	11 10	8 47 48
9 7 10	— — — —	12 10	8 56 24
9 50 00	— — — —	16 28	9 38 7
11 1 30	— — — —	21 31	10 56 32

The Ratio of the Diameters of the *Sun*, and of the *Nucleus* of *Mercury*, while he continued in the lucid Disk of the *Sun*, as far as could be guess'd through the Air that was pretty thick, was as 1000 to 8½. After he had arrived

rived at the Limb of the *Sun*, and had adhered almoſt a Minute to the undulating Limb, and had recovered his genuine Roundneſs; (for he had aſſumed as it were an Elliptical Form before, from the Light of the *Sun*'s Diſk,) the proportion was as 1000 to $12\frac{1}{3}$.

XCIX. *An.* 1697. *Nov.* 3. New Style; at $7^h\ 25'$. After the *Sun* had emerged out of the Clouds, directing my Teleſcope to him, the Difference of the right Aſcenſion of the Center of *Mercury* to the Weſt, and the Center of the *Sun*, was obſerved by the Clock ——— ——— ——— $0^h\ 11'\ 52''$

Mercury obſerved in the Sun. Nov. 3. ſt. n. 1697; by M. Caſini. n. 245. p. 372.

The Difference of the Declination of *Mercury* to the South was 0 6 20

At $8^h\ 3'$. The Difference of Right Aſcenſion of the Centers of *Mercury* to the Weſt and of the *Sun* was, Hours ——— } 0 15 30

The Difference of Declination was, Degrees ——— ——— 0 4 42

At $8^h\ 8'\ 38''$. The preceding Limb of *Mercury* arrived at the preceding Limb of the *Sun*.

At $8^h\ 10'\ 24''$. *Mercury* wholly emerged out of the *Sun*'s Diſk, being obſerved with a Teleſcope of 18 Feet.

From theſe Obſervations compared together, as far as can be collected from ſo ſhort an Interval, I found by Trigonometry that *Mercury* came to the Middle of his Path in the *Sun*'s Diſk at $6^h\ 11'\ 18''$ in the Afternoon.

And that the Aſcending Node of *Mercury* was in ♉ 14° 42'. ſtill more advanced than by the Obſervations of the Year 1677.

Alſo I found by compariſon of the laſt Obſervations, that the Inclination of *Mercury*'s Orbit to the Ecliptick was 6° 23'; which however, becauſe of the ſhort Interval of theſe Obſervations, I dare not prefer to that which I derived from the *Chineſe* Obſervations of the Reverend Father *Fontenay*, which were diſtant by a much greater Interval. This was 6° 40'. which approaches nearer to the Rudolphine Tables.

C. It has been long ago diſcover'd, not only from the Principles of true Aſtronomy, but upon the Credit of the beſt Obſervers, that the Planets *Mercury* and *Venus* ſometimes enter the Diſk of the *Sun*, and are ſeen within his bright Orb to paſs along like blackiſh Spots. But by what Law and on what Conditions, alſo in what Intervals of Years, theſe Phenomena are preſented to our View, has not yet (I think) been duly determin'd by our modern Aſtronomers. Sure I am, that about this Matter I have ſeen nothing yet in Print. Therefore I thought it might not be unacceptable, if I ſeriouſly apply'd myſelf to this Inquiry; and I truſt that in this Diſſertation I ſhall fully unfold the whole Affair, which is perplexed enough, and underſtood but by few.

The Viſible Conjunctions of the Interior Planets with the Sun; by Mr. Halley. n. 193. p. 511. Mar. An. 1691

It is ſufficiently manifeſt, that theſe Phaſes of theſe Planets always happen when they are retrograde and in their Conjunctions with the *Sun*; that is, when the *Sun* is ſo near their Nodes, that the Latitude of the Planet in Conjunction with the *Sun*, may not exceed the *Sun*'s Semidiameter. Now that I may the more eaſily inveſtigate the Limits and Conditions of theſe Conjunctions, and as the Elements of Calculation for each of theſe Planets are very different, I ſhall manage them ſeparately; and firſt I will begin with *Mercury*.

 We

We are well assured, that the Ascending Node of this Planet, according to the latest and best Observations, is about 15 Degrees in *Taurus*, or rather at 0ˢ 15° 44′ from the first Star in *Aries*, in the present Age. And that the Descending or Opposite Node is at 6ˢ 15° 44′ from the first Star of *Aries*. And the Angle in which the Plain of *Mercury*'s Orbit is inclined to the *Ecliptic*, is pretty well determin'd by *Kepler* to be 6° 54′. Now by the most approved Hypotheses it appears, that when *Mercury* is in his Ascending Node his distance from the *Sun* is 31365 of such Parts, as the mean Distance of the *Sun* from the Earth is 100000. But when he is in his other Node, his Distance then measured in the same Parts is 45308. But the *Sun* when opposite to the Ascending Node, is distant from the Earth in Conjunction to the same 98955 of the same Parts; but at the other Node the same Distance is 101007. And therefore *Mercury* in Conjunction with the *Sun* at the Ascending Node is distant from the *Earth* 67591 Parts, and at the Descending Node 55699 Parts. As these differ very widely, the Conjunctions made at different Nodes must be considered separately; and for brevity sake we shall give the Elements of Calculation in a Synoptical Manner.

Mercury *being Retrograde is in central Conjunction with the* Sun, *at the Ascending Node, in the Month of* October; *and from the aforesaid Hypotheses we shall have,*

		s	°	′	″
The *Sun*'s Longitude from the first Star of *Aries*		6	15	44	00
The Longitude of *Mercury* seen from the *Sun*		0	15	44	00
Mercury's Distance from the *Sun*	31365				
Mercury's Distance from the *Earth*	67591				
The Angle of the Inclination of *Mercury*'s Orbit		0	6	54	00
Six Hours Motions of *Mercury* seen from the *Sun*		0	1	30	58
The *Sun*'s Motion in the same six Hours		0	0	15	5
Hence *Mercury*'s Motion from the Sun in six Hours		0	1	15	53
And his Motion in 6 Hours from the *Sun* seen from the *Earth*		0	0	35	12
And the Angle of *Mercury*'s Path seen within the *Sun* with the *Ecliptic*		0	8	15	00
Hence *Mercury*'s Motion in 6 Hours in his visible Orbit		0	0	35	40
Then *Mercury*'s Motion in a *Sidereal* Year beyond 4 Revolutions		1	24	45	8
Therefore in 13 Years		11	21	46	44
Therefore wanting to 54 intire Revolutions		0	8	13	16
			d.	h.	′
Which Space *Mercury* passes over in			2	00	11
			°	′	″
In which time the *Sun*'s Place is advanced, and the Situation of the Conjunction in the Node is as much distant from the Conjunction of the *Earth*			2	1	00
Now that Arch view'd from the *Earth* is			0	56	10
Whence from the given Angle of the Path seen 8° 15′. the Base may be found, or the Distance from the visible Conjunction			0	55	34

	h. ″
Which Arch is paſſed over by *Mercury* according to the given horary Motion in	9 21
But 13 ſidereal Years exceed ſo many *Julian* with three Intercalations by	8 0
	d. h. ′
Therefore *Mercury* returns to the *Sun* after 13 *Julian* Years, and moreover	2 17 34
Or with four Intercalations, if the foregoing Year be the third from the *Biſſextile*	1 17 34
	′ ″
But from the Arch 56′ 10″. and the given Angle, the Perpendicular, or the neareſt Diſtance of the Conjunction from the *Sun*, is	0 8 3
Therefore the Conjunction conſpicuous within the *Sun* after 13 Years proceeds more Northerly 8′ 3″.	
	s. ° ′ ″
By a like Reaſoning in 46 *Sidereal* Years the Conjunction moves	11 28 36 8
Therefore to 191 intire Revolutions are wanting	0 1 23 52
	h. ′
That is in Time	8 12
	° ′ ″
In which the *Sun* advances	0 20 41
This Arch view'd from the *Earth* becomes	0 9 36
And the Baſe belonging to it is	0 9 30
	h. ′ ″
But the Time in which *Mercury* deſcribes the Baſe is	1 30 00
But 46 *Sidereal* Years exceed ſo many *Julian* Years, with 11 Intercalations, by	19 3 00
	d. h. ′
And *Mercury* returns to the *Sun* after 46 *Julian* Years, and moreover	1 4 51
Or with 12 Intercalations, as it happens when the foregoing Year in the ſecond or third from the *Biſſextile*	0 4 51
	° ′ ″
But the Perpendicular, by which *Mercury* advances Northerly, is	0 1 22
	h. ′ ″
Now the moſt accurate Period of *Mercury* to the *Sun* is finiſh'd in 263 *Sidereal* Years, and moreover	1 11 30
But theſe *Sidereal* Years exceed ſo many *Julian*, with 66 Intercalations, by	10 20 0
Whence after 263 *Julian* Years *Mercury* returns to the *Sun*, but later by	11 33 30
	d.
Now if the foregoing Year is *Biſſextile*, let there be added	1 11 31 30
Then after this Interval he goes more to the North by	0 0 0 10

Now the other more extenſive Periods are very eaſily derived from theſe already found, and are of 6 or 7 Years.

That

That which is compleated in 7 Years depreſſes *Mercury* 22′ 47″, towards the South, and comes ſooner by 7 whole Days, excepting 9 Minutes, if there are two Intercalations. But with one Intercalation, that is when the foregoing Year is *Biſſextile*, 6 Days are to be ſubtracted, adding only 9 Minutes as before.

But that Vagabond Planet is ſeldom again to be found in the *Sun*'s Diſk after 6 Years; for having this Period, he paſſes 30′ 50″ more to the North: And this later by 9^d 17^h 25′, if the foregoing Year be the ſecond or third from the *Biſſextile*, otherwiſe 8^d 17^h 25′. are to be added.

In like manner if the Conjunction *happen at the* Deſcending Node, *in the Month of* April.

		s.	°	′	″
The *Sun*'s Longitude from the firſt Star of *Aries* ——— —		0	15	44	00
The Longitude of *Mercury* ſeen from the *Sun* ——— ———		6	15	44	00
The Diſtance of the Planet from the *Sun* as before —	45308				
His Diſtance from the *Earth* ——— ——— ———	55099				
The Motion of *Mercury* ſeen from the *Sun*, in ſix Hours		0	0	14	29
The *Sun*'s Motion in the ſame Time ——— ——— ———		0	0	43	21
The Motion of *Mercury* from the *Sun* ——— ——— ———		0	0	28	52
Hence the Angle of the Path of *Mercury* ſeen within the *Sun*'s Diſk made with the *Ecliptic* becomes ——— ——— —		0	10	18	00
But his Motion ſeen from the *Earth*, in ſix Hours is ———		0	0	23	52

Whence if we follow the Method of the foregoing Calculation, it is found that after 13 Years, and moreover 3^d 7^h 37′. *Mercury* will return to his Conjunction with the *Sun*. And if the foregoing Year be the third from the *Biſſextile*, then only 2^d 7^h 37′. are to be added, then *Mercury* will be found to move 16′ 55″ more to the South.

But after 46 Years, with 12 Intercalations muſt be added 0^d 7^h 14′. and there will be had *Mercury* in Conjunction with the *Sun* in a more Southern Path by 2′ 23″. But if the foregoing Year be *Biſſextile*, or the firſt from it, there muſt be added 1^d 7^h 14′, that the Conjunction may be had accurately. In like manner after 263 Years, in which *Mercury* will decline towards the South 0′ 22″, there muſt be added 1^d 11^h 49′ or 11^h 49′. according to the Rule preſcribed in the former Caſe.

But in 6 or 7 Years, becauſe of the Vicinity of the *Earth* and the Planet, and therefore becauſe of the Arches enlarged at this Node, he does not return to the *Sun* ſo as to appear within his Diſk. But after 33 Years he paſſes the *Sun* at a more northerly Path of 14′ 2″. And the Moment of Conjunction will be had by ſubtracting 3^d 0^h 25′, from the time of the former, if it be in the third Year from the *Biſſextile*. Otherwiſe only 2^d 0^h 23′ muſt be ſubtracted.

Theſe being found it will be eaſy to continue the Calculation for all theſe Conjunctions of *Mercury* with the *Sun*, and that with the greateſt certainty, and without any doubt whether or no all that are poſſible are included. By Addition alone the Moments of the Conjunctions will be obtain'd, and the Diſtances of the Planet from the Center of the *Sun*; whence alſo by help of the Table the Durations of these *Eclipses*, as they may be called, may be taken out, so that nothing may be wanting to compleat this Affair.

As to the *Epochs*, they may be found with more safety by the Industry of Observers, than derived by any subtile Calculation. Therefore in the first Case we have made choice of that notable Transit of *Mercury*, which I myself made a most compleat Observation of, in the Island of St. *Helena*, *Octob.* 28. *An.* 1677, Old Style, and whose Middle I determined from the Beginning and End, to be in the said Island at 0^h 4′ Afternoon, but at *London* 0^h 28′ Afternoon. Now the Path which the Planet seem'd to observe, was 4′ 40″ more northerly than the *Sun*'s Center. In the other Case, that is, when *Mercury* is in Conjunction with the *Sun* in the Month of *April*, I have thought fit to take the *Epoch* from the learned *Hevelius's Treatise of* Mercury *seen in the* Sun, *p.* 72, 75, which is, that *April* 23. *An.* 1662, Old Style, at 6^h 8′ Afternoon at *Dantzick*, that is, at 4^h 52′ at *London*, *Mercury* appeared very near the Center of the *Sun*, as being in the Middle of his Transit, and at the same Time was distant from that Center 4′ 27″ to the North. Hence by the foregoing Precepts it will be a Work of but little Trouble, to exhibit in order all the visible Conjunctions of *Mercury* with the *Sun* at the same Time. And for an Example that any one may imitate hereafter, here are all the Phenomena of this kind, that have ever appeared this present Age since the Invention of the Telescope, or that will appear to Posterity in the Age following.

A Series of the Moments at which Mercury *is seen in Conjunction with the* Sun, *and within his Disk, for the present and the next Age, with the Distances of the same Planet from the* Sun's *Center.*

In *APRIL.*

Years	*Times of the Conjunction.*				*Distances from the* Sun's *Center.*		
	d.	h.	′		′	″	
1615	22	21	38	*	7	20	N.
1628	25	5	15	*	9	35	S.
1661	23	4	52	*	4	27	N.
1674	26	12	29		12	28	S.
1707	24	12	6		1	34	N.
1720	26	19	43	*	15	21	S.
1740	21	11	43		15	36	N.
1753	24	19	20	*	1	19	S.
1786	22	18	57	*	12	43	N.
1799	26	2	34	*	4	12	S.

In

In *OCTOBER.*

Years	*Times of the Conjunction.*				*Distances from the* Sun's *Center.*		
	d.	h.	′		′	″	
1605	22	8	29		12	48	South
1618	25	2	4	*	4	45	S.
1631	27	19	37	*	3	18	North
1644	30	13	11		11	21	N.
1651	23	13	20		11	26	S.
1664	25	6	54	*	3	23	S.
1677	28	0	28	* *	4	40	N.
1690	30	18	2	*	12	43	N.
1697	23	18	11	*	10	4	S.
1710	26	11	45		2	1	S.
1723	29	5	19	*	6	2	N.
1730	22	5	28		16	45	S.
1736	30	22	53	* *	13	5	N.
1743	24	23	2	* *	8	42	S.
1756	26	16	36		0	39	S.
1769	29	10	10		7	24	N.
1776	22	10	19		15	23	S.
		Nov.					
1782	1	3	44	*	15	27	N.
		Oct.					
1789	25	3	53	*	7	20	S.

Those Transits which have the Mark *, are but partly visible at *London*; but those which are marked * *, are totally visible.

Now it is to be observed, that at the Ascending Node of *Mercury* in the Month of *October*, the Diameter of the *Sun* takes up 32′ 34″, and therefore the greatest Duration of a Central Transit is $5^h\ 29'$. But in the Month of *April* the Diameter of the *Sun* is 31′ 54″, whence by reason of the slower Motion of the Planet, there arises the greatest Duration $8^h\ 1'$. Now if *Mercury* approaches obliquely, these Durations become shorter on account of the Distance from the Center of the *Sun*. And that the Calculation may be made more perfect, I have added the following Tables, in which are exhibited the half Durations of these *Eclipses*, to every Minute of the Distance seen from the Center of the *Sun*. These added to or subtracted from the Moment of Conjunction found by the foregoing Table mark out the Beginning and End of the whole Phenomenon.

In

OCTOBER.

The Dist. in Min.	The half duration
′	h. ′
0	2 44½
1	2 44
2	2 43
3	2 41½
4	2 39½
5	2 36½
6	2 33
7	2 28½
8	2 23
9	2 17
10	2 10
11	2 1
12	1 51
13	1 39
14	1 24
15	1 4
15½	0 50
16	0 30

APRIL.

The Dist. in Min.	The half duration
′	h′ ′
0	4 0½
1	4 0
2	3 58½
3	3 56
4	3 53
5	3 48½
6	3 43
7	3 36
8	3 28
9	3 18½
10	3 7
11	2 54
12	2 38
13	2 19
14	1 55
15	1 21½
15½	0 56

These Numbers truly represent all the Observations hitherto made, nor have I any reason to doubt of the future, since of all the Planets *Mercury* being nearest the *Sun*, approaches so near to his Center, that it is least disturb'd by the Intervention of other Centers; nor is it sensibly affected by those Deviations which arise from the System of the rest, to which the superior Planets, and especially *Saturn*, is much exposed.

I have purposely omitted the Parallaxes as very small, and which being different in different Places, ought not to be admitted in a general Calculation. And likewise because it is not yet certain how much they are, but rather may be very safely derived from such Observations. Neither have I taken any Account of *Mercury*'s Diameter, because being incredibly small, he seems to adhere to the *Sun*'s Limb only a very few Minutes. From a most accurate Observation I have found, that hardly two Minutes were over whilst he wholly came out of the *Sun*, *Octob.* 28. 1677. whence I concluded his Diameter to be

0′ 11″; and according to the ratio of the Diſtances from the *Earth*, at the other *Node* it was almoſt 0′ 13″½. Therefore then 3½ Minutes of Time were ſpent, whilſt the whole Planet directly paſs'd through the Limb of the *Sun*. But when he paſſes obliquely, he continues ſomething longer, according as the Secants of the Angles of Incidence are encreaſed. Alſo there is hardly any need that we ſhould make any Eſtimate of the Equations of Time; becauſe a great many Days they continue conſtant, and as it were invariable on both ſides in each Month.

Of the viſible Conjunction of Venus *with the* Sun.

Tho' *Venus* is the moſt beautiful of all the Stars, yet like the reſt of her Sex, ſhe does not care to appear in ſight without her borrowed Ornaments, and her aſſumed Splendor. For the confined Laws of Motion envy this Spectacle to the Mortals of a whole Age, like the Secular Games of the Ancients; tho' it be far the moſt noble among all thoſe that Aſtronomy can pretend to ſhow. Now it ſhall be declared hereafter, that by this one Obſervation alone, the Diſtance of the *Sun* from the *Earth* may be determined with the greateſt Certainty, which hitherto has been included within wide Limits, becauſe of the Parallax which is otherwiſe inſenſible. But as to the Periods, they cannot be deſcribed ſo accurately as thoſe of *Mercury*, ſince *Venus* has been obſerved within the *Sun*'s Diſk but once ſince the Beginning of the World, and that by our *Horrox*. Take here the Sum of the Calculation, the Motions being corrected, as far as was poſſible by the rude Obſervations of the Ancients.

		s.	°	′	″
The Longitude of the *Aſcending Node* of *Venus* from the firſt Star of *Aries*		1	15	16	00
Therefore the *Sun* is in Conjunction with it in the oppoſite Point, that is, for ſome Ages about the End of *November*		7	15	16	00
The Diſtance of *Venus* from the *Sun* in Parts	71997				
The Diſtance of *Venus* from the *Earth*	26438				
The Inclination of the Orbit of *Venus* to the *Ecliptic*		0	3	23	0
The Motion of *Venus* in 8 *Sidereal* Years, above 13 Revolutions		0	1	30	28½
The Motion of *Venus* in 235 *Sidereal* Years, above 381 Revolutions		11	29	17	39
The Motion of *Venus* in 243 *Sidereal* Years, above 395 Revolutions		0	0	48	8

From theſe Principles a Calculation being made, according to the Method explain'd in *Mercury*, the Intervals of Times and Diſtances will come out as follows.

After 18 Years *Venus* returns to the *Sun*, taking away $2^d\ 10^h\ 52'\frac{1}{2}$; from the Moment of the foregoing Tranſit; and the Planet proceeds in a Path which is 24′ 41″. more to the South than the former.

After 235 Years adding $2^d\ 10^h\ 9'$, *Venus* may again enter the *Sun*, but in a more

more northern Path by 11′ 33″. But if the foregoing Year is *Bissextile*, 3^d 10^h 9′ must be added.

After 243 Years *Venus* may also pass the *Sun*, only taking away 0^h 43′ from the Time of the former; but procceeds more southerly by 13′ 8″. Now if the foregoing Year be *Bissextile*, add 23^h 17′.

And in all these Appulses of *Venus* to the *Sun*, in the Month of *November*, the Angle of her Path with the *Ecliptic* is 9° 5′, and her Horary Motion within the *Sun* is 4′ 7″. And since the Semidiameter of the *Sun* is 16′ 21″, the greatest Duration of the Transit of the Center of *Venus* comes out 7^h 56′.

Then let the *Sun* and *Venus* be in Conjunction at the *Descending Node* in the Month of *May*; and by the same Numbers the same Intervals may be computed. After 8 Years let there be taken away 2^d 6^h 55′. And *Venus* will make her Transit in a more northern Path by 19′ 58″.

After 235 Years add 2^d 8^h 18′, or if the foregoing Year be *Bissextile* 3^d 8^h 18′, and you will have *Venus* more to the South by 9′ 21″.

Lastly after 243 Years add 0^d 1^h 23′, or if the foregoing Year be *Bissextile*, 1^d 1^h 23′, and *Venus* will be found again in Conjunction with the *Sun*, but in a more northerly Path by 10′ 37″.

In every Transit within the *Sun* at this *Node*, the Angle of *Venus*'s Path with the *Ecliptic* is 8° 28′, and her Horary Motion is 4′ 0″ and the Semidiameter of the *Sun* subtending 15′ 51″, the greatest Duration of the central Transit comes out also 7^h 56′ exactly the same as at the other *Node*.

As to the *Epochs*, from that only Ingress which *Horrox* observed, the *Sun* being then just ready to set, it is concluded, that *Venus* was in Conjunction with the *Sun* at *London* in the Year 1639, *Nov.* 24^d 6^h 37′, and that she declined towards the South 8′ 30″. But in the Month of *May* no Mortal has seen her as yet within the *Sun*. But from my Numbers, which I judge to be not very different from the Heavens, it appears that *Venus* for the next Time will enter the *Sun An.* 1761. *May* 25^d 17^h 55′, that being the Middle of the *Eclipse*, and then will be distant from his Center 4′ 15″, towards the South. Hence and from the foregoing Revolutions all the Phenomena of this kind will be easily exhibited for a whole Millennium, as I have computed them in the following Table.

In *NOVEMBER.*

Years.	*Times of the Conjunction.*			*Dist. from the Cent. of the* Sun		
	d.	h.	′	′	″	
918	20	21	53	6	12	N
1161	20	21	10	6	55	S.
1396	23	7	20	4	38	N.
1631	26	17	29	16	11	N.
1639	24	6	37	8	30	S.
1874	26	16	46	3	3	N
2109	29	2	56	14	36	N.
2117	26	16	3	10	5	S.

In *MAY.*

Years.	*Times of the Conjunction.*			*Dist. from the Cent. of the* Sun.		
	d.	h.	′	′	″	
1048	24	13	45	3	50	N.
1283	23	8	14	5	31	S.
1291	25	15	9	14	27	N.
1518	25	16	32	14	52	S.
1526	23	9	37	5	6	N.
1761	25	17	55	4	15	S.
1769	23	11	00	15	43	N.
1996	28	2	13	13	36	S.
2004	25	19	18	6	22	N.

As for the Durations of theſe *Eclipſes* of *Venus*, they may be computed after the ſame Manner as thoſe of *Mercury* in reſpect of the Center. But ſince *Venus*'s Diameter is pretty large, and ſince the Parallaxes alſo may bring a very notable Difference as to Time, a particular Calculation muſt neceſſarily be made for every Place.

Now the Diameter of *Venus* is ſo great, that while ſhe adheres to the *Sun*'s Limb almoſt 20 Minutes of Time will be elapſed, that is, when ſhe applies directly to the *Sun*. But when ſhe is incident obliquely, ſhe continues longer in the Limb. Now that Diameter, according to *Horrox*'s Obſervation. takes up 1′ 18″, when ſhe is in Conjunction with the *Sun* at the *Aſcending Node*, and 1′ 12″ at the other *Node*. Now the chief uſe of theſe Conjunctions is accurately to determine the *Sun*'s Diſtance from the *Earth*, or his Parallax, which *Aſtronomers* have in vain attempted to find by various other Methods; for the Minuteneſs of the Angles required eaſily elude the niceſt Inſtruments. But in obſerving the Ingreſs of *Venus* into the *Sun*, and her Egreſs from the ſame, the Space of Time between the Moments of the internal Contacts, obſerved to a Second of Time, that is, to $\frac{1}{10}$ of a Second or 4‴ of an Arch, may be obtain'd by the Aſſiſtance of a moderate Teleſcope and a Pendulum Clock, that is conſiſtent with itſelf exactly for 6 or 8 Hours. Now from two ſuch Obſervations rightly made in proper Places, the Diſtance of the *Sun* within a five hundredth Part may be certainly concluded, as I ſhall ſhew at another Opportunity.

Fig. 154, 155. Left any thing ſhould ſeem obſcure to a Reader not much verſed in *Aſtronomical* Matters, I have delineated Schemes for the Tranſit of each Planet, by which I have endeavour'd to repreſent every thing to view.

The Motion of the Comet, A. 1664. *Predicted; by M.* Auzout. n. 1. p. 3. n. 2. p. 18.

CI. 1. M. *Auzout*, after he had ſeen the *Comet* (which was firſt obſerved in *Holland Decemb.* 2 1664.) 4 or 5 times, made the *Ephemerides* of its Motion upon an *Hypotheſis* that it moveth juſtly enough in the Plane of a great Circle, which

which inclineth to the *Equinoctical* about 30°, and to the *Ecliptick* 49° or 49°½, cutting the *Æquator* at about 45°½, and the *Ecliptick* at 28° of *Aries* or a little more.

He takes notice, that more *Comets* enter into our Syſtem by the Sign of *Libra* and about *Spica Virginis* than by all the other parts of the Heavens: For, both the preſent *Comet* and many others regiſtred in *Hiſtory* have entered that way, and conſequently paſs'd out of it by the Sign of *Aries*; by which alſo many have entered. n. 2. p. 19.

2. Till the *6th* of *Feb.* this *Comet* always advanced: But after that Day, I found that it returned in augmenting always its Latitude. I left it *Mar.* 8. at the 18. of the *Horn* of *Aries*, almoſt in the ſame Latitude; and I am apt to believe it will be *Eclipſed* this Evening. *Obſerved; by M.* Auzout. *ib.*

I ſhall only add, that on *Feb.* 3. we were ſurprized, to ſee the *Comet* again much brighter than ordinary, and with a conſiderable *Train.* Some did believe that it approached again to us. But having beheld it with a Teleſcope, I ſoon ſaid, that it was joined with two ſmall Stars, whereof one was pretty bright, and that this *Conjunction* gave the *Comet* that Brightneſs. Hence it was, that I aſſur'd my Friends here, that we ſhould no more ſee it ſo bright.

M. *Auzout* alſo ſtrongly conceives, that this *Comet* could not be *Feb.* 18 *ſt. n.* where M. *Hevelius*, in his *Prodromus Cometicus*, hath placed it, viz. *in prima Arietis*; unleſs it be ſaid that it viſited that Star of *Aries* on the 18th, and returned thence the 19th, into its ordinary Courſe: For, according to his, and his ſeveral Correſpondents Obſervations, the *Comet* on *Feb.* 17. was diſtant from that *Firſt Star of Aries* at leaſt 1° 17; and on *Feb.* 19. (He having miſſed, as well as his other Friends, the Obſervation on *Feb.* 18.) was advanced in its way 12′ or 13′, but yet diſtant from the ſaid *Star* ſome Minutes above a whole Degree, and conſequently far from having then paſſed it. After which time M. *Auzout* affirms to have ſeen it as well as ſeveral others, for many Days, and that until *Mar.* 17. obſerving, that about *Feb.* 26 or 27, when the *Comet* was neareſt to the often mentioned *Firſt of Aries*, it approached not nearer than 50′. n. 6. p. 107.

3. Some Eminent *Engliſh Aſtronomers*, who have attentively obſerved the Poſition of this *Comet*, do jointly conclude, that whatever that Appearance was, which was ſeen near the *Firſt Star of Aries* by M. *Hevelius* (the Truth of whoſe Relation concerning the ſame they do in no wiſe queſtion) the ſaid *Comet* did not come near that Star in the *left Ear of Aries*, where the ſaid M. *Hevelius* ſuppoſes it to have paſſed, but took its Courſe near the *Bright Star* in its *Left Horn*, according to *Bayer*'s Tables. *By ſome* Engliſh Aſtronomers. p. 108. n. 9. p. 150.

4. I have eaſily found the Principle of M. *Auzout's Ephemerides:* and 'tis this, that this *Comet* moves about a Centre, in a ſtraight Line drawn from the Earth thro' the *Great Dog*, in ſo great a Circle, that that Portion which is deſcribed, is exceeding ſmall in reſpect of the whole Circumference thereof, and hardly diſtinguiſhable by us from a ſtraight Line. *The Principle of M.* Auzout's *Hypotheſis; and the Motion of that* Comet *obſerved by M.* Caſſini. n. 2. p. 17.

Concerning the *New Comet* you mention, I obſerv'd it *Feb.* 11, about the 24° of *Aries*, with a Northern Latitude of 24° 40′.

CII.

The Motion of the Comet A. 1665. *predicted; by M.* Auzout, n. 3. p. 36.

CII. 1. M. *Auzout*, after 3 or 4 Obſervations, hath publiſhed another *Ephemerides* concerning the Motion of the *Comet*, which he firſt began to obſerve *Apr.* 2. 1665. He affirms that the Line deſcribed by this *Star* reſembles hitherto a great Circle, as it is found in all other *Comets* in the midſt of their Courſe. He finds the ſaid Circle inclined to the *Ecliptick* about 26°. 30′. and the *Nodes* where it cuts it, towards the beginning of *Gemini* and *Sagittary*: that it declines from the *Æquator* about 26°, and cuts it towards the 11°; and conſequently that its greateſt Latitude hath been towards *Piſces*, and its greateſt Declination towards the 25° of the *Æquator*. He puts it in its *Perigee* about 15° of *Piſces*, a little more Weſterly than *Marchab*, or the *Wing of Pegaſus*.

Obſerved; by M. Auzout. *ib.* p. 37.

2. He obſerves in General, that this *ſecond Comet* is contrary to the precedent almoſt in all Particulars: Seeing that the former moved very ſwift, this pretty ſlow; that againſt the Order of the *Signs* from Eaſt to Weſt, this, following them, from Weſt to Eaſt; that, from South to North, this, from North to South, as far as it hath been hitherto, that we hear of, obſerved: that, on the ſide oppoſite to the *Sun*, this on the ſame ſide; that having been in its *Perigee* at the Time of its Oppoſition, this having been there, out of the time of its Conjunction. He taketh alſo notice, that this *Comet* differs in Brightneſs from the other, as well in its *Body*, which is far more vivid and diſtinct, as in its *Train*, whoſe Splendor is much greater, ſince it may be ſeen even with great Teleſcopes, which were uſeleſs in the former by reaſon of its Dimneſs.

A Comet. An. 1668. *at* Bononia; *by M.* Caſſini. n. 35. p. 683.

CIII. 1. *Ann.* 1668. *Mar.* $10^{d}\ 1^{h}$ of the following night (after the *Italian* way of counting) I obſerved a Path of Light extended from the *Whale* thro' *Eridanus*; which I judged to be the *Train* of a *Comet* both by the Figure and Colour, as alſo becauſe that the Direction of it was to the part oppoſite to the *Sun*, like other *Comets*. By its extreme Point it reached to that Star in *Eridanus*, which is called the 14 by *Bayerus*: But it iſſued out of the Horizontal Clouds, ſo that I thought the *Head* of the *Comet* was either veiled by them, or hid under the *Horizon*. *Mar* 11. there was ſeen a Brightneſs in the *Whale*, amongſt the thin Clouds, at leaſt for half an Hour, which was very like the Splendor of *Venus*, likewiſe veiled with thin Clouds.

Mar. 12. When the *Great Dog* was in the *Mid-heaven*, the ſame *Tail* appeared again. It paſſed thro' the Star in *Eridanus*, which *Bayerus* calls the 15, and left to the Southward the 14, where it did terminate *Mar.* 10. Being by the Imagination drawn out to about 3°, and further, it tended to that Southern Star which precedes the *Ear* of *Lepus*. It was therefore more Northerly than the Day before Yeſterday, and more Eaſterly. We were doubtful whether its Head was hid by the Clouds or under the Horizon. But the Line from *Jupiter* to the Extremity of the ſaid *Tail* in the Clouds was perpendicular to that *Tail*; ſo that it was in the *Whale*, and the apparent part of the *Train* reached out in Length about 32°.

At Liſbon; *by* ... p. 684.

2. *Mar* 5. *ſt. n.* The *Comet* was firſt diſcovered: but for as much as it ſet few hours after the *Sun*, there could hitherto be taken no conſiderable Obſervations of it. The *Body* thereof is not ſeen, becauſe it remains hid in the

the Horizon. Its *Train* is of a ſtupendious Length, extended in Appearance over almoſt the 4th part of the viſible Heaven, from Weſt to Eaſt; its apparent Breadth is of a good *Palm*, and its Splendour very great, but it laſts but a few Hours.

At St. *Salvador Mar.* 5. (*ſt. n.*) at 7 a Clock at Night, *F. Eſtancel* began to ſee this *Comet* a little above the Horizon from Weſt to E.S.E. The beginning of its *Tail* was a little under the two lucid Stars, the 15 and 16th of the *Whale's Back*, over which it then paſſed, its Point being as 'twere at the 8 and 9th which are at the bottom of the *Whale's Belly*; and thus the whole Length thereof was about 23°. The *Globe* or *Head* of it was ſo ſmall and thin, that very few could diſcern it with the naked Eye.

In Braſil; by P Valentine Eſtancel. n. 15. p. 91.

Mar. 7. The former Brightneſs was ſomewhat leſs, and become ſo thin, that the Eye could eaſily ſee the Stars that were behind it, which by Conjecture were the 14 and 20th.

The *Tail* was always directly oppoſite to the *Sun*; and when it appear'd the firſt Time almoſt Horizontal, it was ſeen in the form of a Pillar, the *Head* ſtanding a little under, and on the ſide of the Star of the *Whale*, which is in the Lat. of 15° 46′. and the Long. of 12° 42′. of *Aries*. And the *Point* did ſhave the 14th North of the three that are in the *Belly*, in the Lat. of 20°. 30′. and Long. of 15° 57′. of *Aries*.

This *Comet* was at firſt very ſplendid, and caſt itſelf with that Vividneſs upon the Sea, that the Rays thereof were reverberated unto the Shoar, where the Obſervers ſtood. But this Brightneſs laſted only for three Days, after which it did conſiderably decay. But that which ſeemed ſomewhat ſtrange was, that having loſt ſo much of its Light, yet its Bulk was not diminiſhed, but continued rather increaſing until the *Comet* diſappear'd. It paſs'd more ſwiftly than *Venus*, whence he infers that it was under *Venus*: yet the Anticipation was not ſo great, that it could be believ'd to be under the *Moon*, as he would have it.

4. *P. Pietro Suſarte*, Rector of *Macao*, in the *Eaſt-Indies*, well verſed in Matters *Aſtronomical*, writes to have ſeen the ſame all along the Coaſt of *Bona Speranza*.

In Africa; by P. Pietro Suſarte. ib.

CIV. There hath been ſeen here a New *Comet* from the 2d of *Mar. ſt. n.* 1671. It is but little, having a *Train* not above a Degree or a Degree and an half long. It is now (*Mar.* 9.) about the Stars in the *Right Arm* of *Andromeda* on her Shoulder Blade. As far as I can collect from one or two Obſervations, it tends towards the *Lucida* of *Andromeda*'s *Girdle*, and that with a direct diurnal Motion of about 2 Degrees in its Courſe.

A Comet. An. 167½. at Dantzick; by M. Hevelius. n. 81. p. 40 17.

The 6th of *March* in the Evening 7^h 40′ it was in 7° ♈. and in 35° of Northern Latitude; as I gueſs'd by the haſty Inſpection of a Globe.

Mar. 7. in the Morning 3^h 30′, its Longitude was about 8° ♈. with a ſomewhat leſſer Latitude than before: in the Evening of the ſame Day it's Longitude was 10° ♈. and Lat. 34° *ferè*.

Mar. 8. In the Morning 4^h, the Long. was 12° ♈. and the Lat. 33° Which yet I would not have taken preciſely, becauſe I cannot yet reduce my Obſervations to a *Calculus*.

2. Mr.

At . . . . . by Mr. Newton. ib. p. 4018.

2. Mr. *Isaac Newton* about the 16 of *March. st. V.* saw a dull Star South-West of *Perseus*, which he now takes to have been that *Comet* It was very very small, and had not any visible *Tail*, which made him regard it no further.

At Paris by M. Cassini. n. 81. p. 4018. n. 82. p. 4042.

3. The *Mathematicians* of *La Flesche* perceived him from the 16 of *March*, *st. n.* and gave us here at *Paris* the first notice of it. Those of the *College* of *Clermont* being advertised of it, saw him the 25 of the same Month.

Mar. 26. 7^h 30′ in the Evening, M. *Cassini* saw him between the *Head* of *Medusa* and the *Pleiades*; without a Telescope he appeared no otherwise than a Star of the 3[d] Magnitude; his *Head* seen with a Telescope of 17 Foot, appear'd almost round; but it was well defined, and distinguish'd from the Mistiness, which formed a kind of *Chevelure*, wherewith it was encompassed; and even the middle was a little confused, and seemed to have Inequalities, as are seen in Clouds.

The *Tail* was almost imperceptible; yet by the Telescope it was seen turned opposite to the *Sun*, and it appeared of the Length of two Diameters of the *Head* or thereabout: For it was not easy to measure it precisely, because being thinner according as it was farther from the *Head*, its Extremity was insensibly lost, And so the whole *Comet*, *Head*, *Tail*, and *Chevelure*, taken all together, took up no more than 3 or 4 Minutes of a Degree. At 7^h 48′, he was in a straight Line with the *Lucida* in the *Head* of *Medusa*, and with the most *Occidental* one of the *Pleiades*, and above the two clearest Stars of the *Southern Foot* of *Perseus*; so that a straight Line drawn thro' these two Stars, did almost touch the Southern Extremity of his *Chevelure*. This Place of the *Comet*, transferred upon the Map of the fix'd Stars, fell precisely enough upon 23° 25′ of *Taurus*, in 15° Northern Latitude.

With a Telescope 3 Foot, we saw near the *Comet* two small Stars distant one Diameter of the *Sun* from one another, which Stars are not in the Catalogues. The *Comet* was in a Straight Line, drawn from one of those two Stars to the other percisely at 9^h 15′. but a little nigher to that which was Westward: But 9^h 33′, he was equally distant from them both. It was taken notice of, that from 8^h 5′ till 10^h 26′, *He* made, in respect of these two Stars, an oblique Motion sensible enough, going from North to South in the same time that he advanced from West to East.

Mar. 28. 7^h 42′. in the Evening, the *Comet* was distant from the less bright Star of the *Southern Foot* of *Perseus*, no more than about 24′ Westward. He had almost the same Latitude with this Star; so that he was precisely enough at 26° 8′ ♉, and in the Lat. 12° 8′. At 8^h 14′ we took, as well as we could, the Distance of the *Comet* to the Star in the Eye of *Taurus*, called *Aldebaran*, 19° 38′. and 8^h 29′. The Distance of the *Comet* to the Star called *Cappella*, was found to be of 22° 32′.

Mar. 30. 9^h 35′, at Night, the *Comet*, seen without a Telescope, appeared no otherwise than a Star of the fourth Magnitude: thro' the Telescope he exceeded even those of the First; but he was very dark, and in what manner soever we look'd upon him, we could observe almost no *Tail* at all of him. He had passed one Degree and an half beneath the *Lucida* of the *Southern Foot*

 of

of *Perseus*; so that this Star was exactly in the midst of the *Comet* and the little Star of the *Leg* of *Perseus*, marked η by *Bayerus*, which then we saw not but by a Telescope. A straight Line drawn from one of these Stars to the other, did almost touch the *Southern Limb* of the *Comet*, which being transferred upon the Map of the *Fix'd Stars*, fell upon 28° 45′ of *Taurus*, in the Northern Latit. of 9° 56′. At 9ʰ 45′, the Western Limb of the *Comet* touched a straight Line, drawn thro' this less bright Star of *Perseus*'s *Southern Foot*, and thro' the most Northern of the Head of *Taurus*; but that he was already got somewhat nearer to the Latter.

Mar. 31. 8ʰ in the Evening, the *Comet* was in a direct Line with the *Lucida* in the *Foot* of *Perseus*, and with the most Northern in the *Head* of *Taurus*; but he was more than twice as much remoter from the first than the other, and being transferred upon the Map of the *Fix'd Stars*, he was found at 15′ from *Gemini*, in the Latit. of 8° 49′. During the whole time that we could observe him this Night (which was till 10 a Clock) he quitted not this straight Line, which was almost parallel to the Horizon: notwithstanding that his own particular motion should raise him a little above it; as the Parallax, on the contrary, should sink him beneath it in approaching to the Horizon. It may be, there was a compensation made of these two contrary Motions: possibly also the Effect of both was not sensible.

April. 1. The *Comet* could not be seen without a Telescope, because the *Moon*, being very near it, hid him from our sight. But with a Telescope only of one Foot we discerned him easily enough, and found that he had passed 45′ beyond the most Northern Star of the *Head* of *Taurus*, and that he must have touch'd it by his Southern Limb; as also that he was distant 1° 43′, from the Star that was nearest to that toward the South; which is equally bright, yet not marked by *Bayerus*. This place being transferred upon the Map of the *Fix'd Stars*, we found that he was at 1° 30′ of *Gemini*, in the Northern Latit. of 7° 44′.

April. 2. 8ʰ in Even. M. *Cassini*, having observed the *Comet* with a Telescope of one Foot, which discovered 5°, found that he was two Deg. and half distant from the most Northern Star of *Taurus*; and one Deg. from the Star of the *Ear* marked φ by *Bayerus*, and by *Tycho* called *Sequentis Lateris Borei*.

Two Lines drawn from the most Northern Star of *Taurus*, one to the *Comet*, the other to the Star that is wanting in *Bayerus*, made a Right Angle; and the Distance of the *Comet* to this Angle, was double to that which is between these two Stars. This place transferred upon the Map of the *Fix'd Stars* fell on 2° 48′ of *Gemini*, in the Northern Latit. of 6° 40′.

April 3. 9ʰ, we saw him with the one Foot Telescope. He had passed over the upper Star of the *Ear* of *Taurus*, and he made with this Star the *Basis* of an *Isosceles* Triangle, on the Top whereof was the inferior Star of the *Ear*. The two Sides of this Triangle were two times and an half bigger than the *Basis*; so that the *Comet* was 4° of *Gemini*, in the Northern Latit. of 5° 38′.

Apr. 5. 8ʰ at Even, the *Comet* had passed the Northern *Ear* of *Taurus*, and was equally distant from the Upper Star of the Northern *Ear* and from that which was on the *Front* of *Taurus*. He was also as distant from the Inferior

Star of the *Ear* of *Taurus*, as this Star is from the next Weſtward, by *Tycho* called *Inferior præcedentis Lateris Quadrilateri*; and a ſtraight Line, drawn thro' the *Comet* and the upper Star of the *Ear*, made an almoſt Right Angle with another Line, drawn from the *Comet* to the Inferior of the two Stars, that are above the *Eye* of *Taurus*. This Place being carried over to the Map of the *Fix'd Stars*, the *Comet* was found at 6° 18′ of *Gemini*, in the Northern Latit. of 3° 41′. He was ſo confuſed this Night, that even with the 17 Foot Teleſcope we could not exactly diſtinguiſh the *Head* from the *Chevelure* which environed him. The whole appeared a little bigger than the Diſk of *Jupiter*, ſeen by the ſame Teleſcope.

Apr. 6. 8^h at Even, a ſtraight Line drawn from the *Comet* to the Star that is in the *Front* of *Taurus*, made a Right Angle with another ſtraight Line drawn from this ſame Star to the Inferior of the two that are above the *Eye*: and the Diſtance of this latter Star to that of the *Front* of *Taurus* was twice the Diſtance of the ſame Star of the *Front* of *Taurus* to the *Comet*. This Place being transferred upon the Map of the *Fix'd Stars*, the *Comet* was found at 7° 25′ of *Gemini*, in the Northern Latit. of 2° 45′. At 9^h 6′ we ſaw on the ſide of the *Comet* a Star ſufficiently clear, which was not farther diſtant from him than a little more than the Diameter of the *Comet*, and that was at the ſame Height of the Horizon.

Apr. 7. 9^h in the Evening, the *Comet* was equally diſtant from the Inferior Star of the *Northern Ear* of *Taurus*, and from the Superior of the Root of the *Northern Horn*. He was alſo as far diſtant from this latter Star, as this Star is from that of the *Front*. This Place being carried over to the Map of the *Fix'd Stars*, fell on 8° 30′ of *Gemini*, in the Northern Latit. of 1° 56′.

All the Places of the *Comet*, that we have obſerved till now, fall into a Line little differing from an Arch of a great Circle, which cuts the *Ecliptick* in 10° 45′ of *Gemini*, and which conſequently hath its greateſt Latitude in 10° 45′ of *Piſces*; which Latitude is between 39° and 40° Northward. The ſame Circle cuts the *Æquator* at 101 degrees of the *Vernal Section* Eaſtward; and its greateſt *Declination* from the *Æquator* Northward is of $38°\frac{1}{2}$.

Having choſen two of our Firſt Obſervations (becauſe the latter are not ſo proper for this Purpoſe) and having taken a Mean between the firſt Obſervations of the *Mathematicians* of *La Fleſche*, we found, by our *Method* explained in the Theory of the *Comet* of 1665, that this *Comet* had been in his *Perigee* the 12 of *March* at 8 a Clock in the Morning: that in that time, which is that of his greateſt apparent Celerity, he made about 2° 32′ a day in the great Circle of his apparent Motion, and $\frac{444}{10000}$ of his *Perigee* Diſtance in the Line of his Equal Motion: that he was in his greateſt Declination the 11*th* and 12*th* of *March*; and that at that time, he paſſed thro' the Inferior Meridian at about two a Clock after Midnight.

If we have rightly determined his *Perigee*, and that the *Hypotheſis* of the Equality of his Motion be juſt for that time, he hath been viſible ſince the Middle of *February*, at which time he was as far diſtant from his *Perigee* by Approaching to the *Earth*, as he is at preſent by Receding from it. He muſt then have been at the extremity of the *Southern Wing* of the *Swan*, and arrived

 at

at the *Southern Foot* of *Pegasus* on the 23 of *Febr.* of the same bigness that he was seen to be of, *Mar.* 28. He must have arrived at the Stars of the *Northern Arm* of *Andromeda*, *Mar.* 9. at those of her *Girdle*, 12. when he was in his *Perigee*, and in his greatest Declination; to her *Southern Leg*, *Mar.* 15. between her *Southern Leg* and the *Triangle*, *Mar.* 18. very near as he was observed at *La Flesche*; and under the *Head* of *Medusa*, *Mar.* 25. The Days ensuing he must have arrived at the Places marked in our First Observations: But in the last he hath been swifter than this *Hypothesis* will bear. To represent these latter Observations, the Line of the Motion ought to have been made Curve, as we did for the end of the Apparent Motion of the *Comet* 1665. with this difference, that instead of that Line's being Convex in regard of the *Earth*, because the Motion was *Retrograde*, this was to be Concave towards the *Earth*, because that the Motion of this *Comet* is *Direct*.

It's a Thing worth observing, that this *Comet* keeps his Course almost like that of the 2 *Comet* of 1665, and another of 1577, observed by *Tycho*. For they have passed thro' almost the same *Constellations*; tho' this be more inclined Northward, and cut the *Ecliptick* 5 or 6 Degrees more forward than that of 1665. So that it seems that in this Place of the Heavens there is, as 'twere, a *Zodiack* for *Comets*.

A Comet, An. 1677. *at* Paris; *by M.* Cassini n. 135. p. 868.

CV. 1. Mr. *Romer* first took notice of the new *Comet*, *Apr.* 28. New Style, 1677. and I being presently inform'd of this appearance, at 4^h 6′ 31″, after Midnight we took its Altitude 12° 22′ 10″. I judged it to be in a Vertical declining from the East towards the North about 33°. On the 29 in the Morning it was seen by Mr. *Picard* for a Moment through the Clouds at 3^h 9′ 31″ after Midnight in the Altitude of 4° 39′.

On *May* 2^d in the Morning, the Right Ascension of the Midheaven by the fixt Stars being 267°. the Altitude of the Comet was 4° 5′. The Distance of the Vertical from the North towards the East was about 42° 8′. On the fourth Day in the Morning at 3^h 30′. after Midnight, the Altitude of the *Comet* was 5° 33′. The *Azimuth* Distance from the North to the East was about 42° 32′.

On the fifth Day at 3^h 32′. The Altitude of the *Comet* was 5° 10′. The *Azimuth* Distance from North to East was about 44° 10′.

By these Observations the *Comet* at first was in the Triangle, afterwards near the *Head* of *Medusa*, and shew that it proceeded according to the Series of the Signs, by a Line that was very near and almost parallel to that, which was described by the *Comet* in the Year 1590, in the Month of *February*. The bigness of its *Head* seen through the Telescope was almost equal to the Disk of *Jupiter*, or something less. Nor did it appear perfectly round, but of an Oval Figure, the longer Diameter being parallel to the Horizon; which seem'd to be owing to the Horizontal Refraction.

Its *Capillitium* seen through the Telescope was something wide, and nearly Parabolical; but to the naked Eye it seem'd narrow, and was a little inflected towards the West.

At Dantzick, *by* M. Hevelius *ib* p. 869.

2. A *Comet* has appear'd lately, which was first observed here at *Dantzick*, *Apr.* 27. in the Morning. On the 29th it arose, or rather appear'd to sight, at 1^h 52′ at North-East and by North. It had no large *Head*, yet it was

bright enough, consisting of one shining Nucleus, like that which was seen *An.* 1665. It stretch'd out a Tail pretty luminous, with Rays divaricating upwards, of almost two Degrees. The Line of Direction of the Tail being continued, proceeded between *Alamac* the bright foot of *Andromeda*, and her *Girdle*, and divided the Distance of these Stars as it were into two equal Parts. It was at that time above the *Head* of *Aries*, in the Triangle between the Apex and the Northern Star in its Base, that is, in 5° of *Taurus*, and in 12° North Latitude. At this time it was distant from the *Sun* only 5° according to Longitude, but in its own great Circle 20°. Since then this *Comet* was so near the *Sun*, it could not shew its Tail any longer, tho' in my Opinion it had one much longer, nay, I think in a few Days it will shew it much shorter.

Apr. 30. it was found in 9° of *Taurus*, with 18° of North Latitude, and almost as far from the *Sun*, being in 12° of ♉. It spread its Tail again two Degrees or more, to the Northern Star in the Base of the Triangle, which Star might plainly be seen by good Tubes at the Point of the Tail.

May 1. at 2^h 32′. in the Morning it was found in 11° of *Taurus* with North Latitude 18° almost in Conjunction with the *Sun*, and being distant almost as many Degrees from the *Sun*. Its Tail was still bright, but something shorter, tho' wider, which it stretched out to the bright Foot of *Andromeda.*

From *Apr.* 29. when first I observed it, to this Day *May* 1. it compleated nearly 5° 30′, by its own proper Motion.

As far as I can collect from these Observations, it moves with a direct Motion to the left Foot of *Perseus*, above *Taurus*, to the Feet of *Gemini*, if it will continue long enough. The Descending *Node* is about 20° of *Gemini*, (but this is only a rude Conjecture,) and thus it will pass the *Ecliptic* in that Place, and then will become Southward, with an Inclination of its Orbit of almost 27°.

May 2. in the Evening, at 8^h 45′ altho' no Stars shined out in that Part of the Heavens, and there was an intense Twilight, yet I presently found the *Comet* with my Optical Tube. A little after I found him with 3° 30′ of Altitude. His Tail on account of the Twilight was very thin, which he stretched out between each *Knee* of *Cassiopea*, but nearer to the left. It sat that Evening at North-north-west.

On *May* 3. in the Morning, the *Comet* arose at N. N. E. at 1^h 23′, tho' the Tail was discovered by us something sooner at 1^h 18′. It was at 14° in ♉, almost in *Conjunction* with the *Sun*, having 17° of Latitude, and almost the same Distance from the *Sun*. This day it shew'd its Tail much longer and better defined, and very bright, of near 2 or 3 Degrees. Thus it was found by myself and other quick-sighted Spectators with the naked Eye, at 3^h 34′, and with the Telescope at 3^h 40′, at the height of 11° 30′; so that the *Sun* at that time was depressed only 6° below the Horizon; nay, we had seen it longer, if some litte Clouds had not intervened. His daily Motion seem'd to decrease, as far as I could judge by Conjecture without any Calculation. For between 29 and 30 of *April* it was near 2° 45′, between *Apr.* 30 and *May* 1. it was 2° 15′, between 1 and 2 of *May* it was 1° 55′, between 2 and 3 *May* it was 1° 40′. But the Observations themselves and the Calculation will shew this plainer.

On *May* 4. in the Evening, the Air being very pure, at 8^h 53′ the *Comet* was

was again diſcover'd, but was a little obſcurer than the foregoing Days, and its Tail was ſhorter. On *May* 5 in the Morning at 1^h 41′, projecting its Tail towards the right Knee of *Caſſiopea*, it was in 17° of ♉, with 16° of North Latitude, and at the ſame Diſtance from the *Sun*. Its proper Motion from the 3^d to the 5^{th} of *May* was almoſt 2° 40′, the Latitude decreaſing, that is, from the Beginning almoſt to 3°, ſo as from the 29 of *April*. The proper Motion of the *Comet* from *May* 5. was almoſt 12°. On *May* 6, in the Morning its Place was in 18° of ♉, with North Latitude 15° 30′. The *Sun* being then in 17° of ♉. The daily Motion was about 50′. As to its Head, it ſeem'd thinner and weaker than the Tail, becauſe the *Sun* was diſtant not above $16\frac{1}{2}$ Degrees. On *May* 6, in the Evening it was ſeen with the Optic Tube at 8^h 35′. with its Tail ſtill ſhorter and more dilute; but as it was in a lower Situation, and an intenſe Twilight, it could not at all be perceived by the naked Eye.

On *May* 7. it was firſt perceived at 2^h 22′, at the Altitude of 3°, ſo that it ſeem'd to be very thin. It was at that time in 19° of ♉. Its proper Motion decreaſed more and more, as far as could be known without Calculation. On *May* 8. from one in the Morning it was carefully ſought for with the naked Eye, but did not appear. Yet it was found with a 12 foot Teleſcope, having a Tail ſtill, but a very ſhort one, extended a little from the Vertical Circle to the left Hand. As far as I could gueſs it was in 20° of ♉, at the Diſtance of 15° from the *Sun*, which was then in 19° of ♉. At this time it was nearly in a right Line with the right Shoulder of *Perſeus*, and *Algol* of *Meduſa*. The Diameter of the *Comet* compared with that of *Jupiter* hardly came up to half. As to the reſt, it was ſtill conſpicuous enough by help of the Tube, ſo that at 3^h 45′. I could ſee it diſtinctly, at the Altitude almoſt of 9°, whence we may collect, that the Arch of Viſion was then hardly 5°. For then the *Sun* was hardly 5° below the Horizon, at what time all the Stars except *Jupiter* were vaniſhed. On *May* 8 in the Evening, the *Comet* was no longer to be diſcovered, either with the naked Eye, or with any Teleſcope.

3. The firſt certain notice I had of this *Comet* was on *April* 21. The 22 of *Apr*. at about 2 a Clock after the Midnight following, I ſaw the *Tail* raiſed almoſt perpendicular to the Horizon; ſoon after the *Head* appeared thro' a thin Vapour, from which the *Tail* pointed, as near as I could gueſs, upon the ✱ in the *Knee* of *Caſſiopeia*, its Length being about 6 *Deg*. and Breadth at the Top of about 7 or 8 *Min*. Viewing the *Head* with a Teleſcope of 16 Foot, I found it was not perfectly round, but indented, and not near one *Min*. Diameter. Afterwards I haſtened to meaſure its Diſtances from ſeveral fixed Stars; which were as follow.

At Greenwich; *by Mr.* Flamſteed, *ib. p.* 173.

2^h

h. ′ ″		° ′ ″
2 44 00	Its *Head* and the *Foot* of *Androm. Alamech.* ——— —	11 26 0
2 47 15	That Diſtance repeated ——— ——— — ———	11 26 50
2 55 3	Its *Head* from *Capella* ——— ——— ——— ———	31 1 15
2 59 10	——— ——— ——— ——— ——— repeated	31 1 24
3 12 2	Its *Head* from *Algol* in *Meduſa*'s ——— ——— ———	18 16 54
3 21 22	——— from *Mirach* ——— ——— ——— ———	19 35 0
3 27 54	——— from *Alamech* again ——— ——— ——— ———	11 33 30
3 36 20	——— from *Capella* again ——— ——— ——— ———	30 59 45

At 4^h $21'\frac{1}{2}$ *p. m.* the height of the *Comet* was about $5°\frac{1}{2}$, therefore the Diſtance of the *Head* of the *Comet* from *Algol* corrected by Refraction, 8° 19′.
from *Mirach* ——— 19 37

And admitting with *Hevelius* the Place of *Mirach* now in ♈. 21° 40′ 34″ with North Latitude 25° 57′, its Diſtance from *Algol* will be 23° 42′ 40″, and the Place of the *Head* of the *Comet* in ♉ 14° $48'\frac{1}{6}$, with North Latitude 17° 8′.

At 3^h 28′, I ſtate the correct Diſtance of the *Comet's Head* from *Capella* 31° 00′; from *Alamech* 11° 40′; and therefore its true Place in ♉ 14° $05'\frac{1}{2}$, with North Latitude 17° 6′ 25″: agreeing very well with the Place derived from the former Diſtances from two other and different Stars.

The *Tail* was not, it ſeems, directly oppoſite to the *Sun:* for the *Sun*'s Place was now ♉ 13° 7′; but the *Comet* being in 14° 47′ of the ſame *Sign*, that is 1° 40′ in the Conſequence of the *Sun*, the *Tail* ought, if it had been exactly oppoſite to the *Sun*, to have lain in Conſequence of the *Head*; but the *Knee* of *Caſſiopeia* is now in ♉, 13° 24′, in Antecedence of the *Comet*, whoſe *Tail* lay not therefore in Conſequence, but in Antecedence of the Line paſſing thro' its *Head* and the *Sun*, at about an Angle of 10°.

Next Night, being that following the 23 of *April*, about $\frac{3}{4}$ of an Hour after two, its *Tail* appeared much ſhorter than laſt Morning: At 2^h 51′, its *Head* was from *Mirach* 21° 9′. Hence, and from a Courſe of Obſervation of it ſent me by an ingenions Friend, I found its Motion was direct, and its Latitude decreaſing.

A Comet, An. 1682. *at* Dantzick; *by M.* Hevelius. *Ph. Col. n.* 3. *p.* 65.

CVI. I obſerved the late *Comet* firſt in the Morning before Sun-riſing, from the 2^d to the 4^{th} of *December*, *An.* 1680. Then in the Evening, from *December* 24. to the *Vernal Equinox*. In the Morning it was in ♎ and ♏, with Southern Latitude. In the Evening in ♑. ♒, ♓, ♈, and ♉, with Northern Latitude.

A Comet, An. 1682. *at* Dantzick; *by M.* Hevelius. *n.* 143. *p.* 16.

CVII. I have obtain'd many Diſtances from the fixt Stars, as alſo Meridian Altitudes, of the late *Comet*. It would be too long to mention them all, nor have I time to ſubmit them to a ſtrict Calculation. It may ſuffice to ſay at this time, that this *Comet* was firſt ſeen here at *Dantzick* on *Aug*. 25. New Style, 1682. And that from *Aug*. 26. to *Sept*. 7. it was duly obſerved by me. But what

what was its Path, what its Velocity, what was the Angle that its Orbit made with the *Ecliptic*, may appear from the following Table. But this I would have you know, that it was not compoſed by any accurate Calculation, but drawn from a conſideration of the Globe, by a looſer way of reaſoning.

Day of the Month.		*Longitude of the Comet.*	*Latitude of the Comet.*	*Motion in its own Orbit.*	*Daily Motion ſomething more exactly.*
	h. ′	° ′ s.	° ′	° ′	° ′
Aug. 26	3 0 *Morn.*	23 30 ♋	21 0 North.	——	——
Aug. 27	11 0 *Even.*	5 0 ♌	23 30 North.	10 0 nearly	5 28
Aug. 28	—— *Even.*	——	——	——	5 35
Aug. 29	—— *Even.*	——	——	——	5 41
Aug. 30	3 30 *Morn.*	18 0 ♌	25 20 North.	13 20 ——	——
Aug. 30	9 0 *Even.*	22 0 ♌	25 40 North	3 30 ——	5 46
Aug. 31	3 30 *Morn.*	24 30 ♌	26 0 North	2 20 ——	5 50
Sept. 1	3 30 *Morn.*	1 0 ♍	26 0 Nearly	5 45	
Sept. 1	9 0 *Even.*	6 0 nearly ♍	25 40 North.	4 45 ——	5 46
Sept. 2	——	——	——	——	5 43
Sept. 3	8 30 *Even.*	20 0 nearly ♍	24 30 North	11 30 ——	5 40
Sept. 4	—— *Even.*	——	——	——	5 34
Sept. 5	—— *Even.*	——	——	——	5 24
Sept. 6	9 0 *Even.*	5 0 ♎	20 30 North.	15 0 ——	5 00
Sept. 7	—— *Even.*	——	——	——	4 30
Sept. 8	8 0 *Even.*	12 0 ♎	18 15 North.	8 0 nearly	4 00
Sept. 9	8 30 *Even.*	15 30 ♎	17 15 North.	3 30 ——	3 30
Sept. 10	8 0 *Even.*	18 30 ♎	15 45 North.	3 0 nearly	3 00
Sept. 11	—— *Even.*	——	——	——	2 40
Sept. 12	8 0 *Even.*	23 0 ♎	14 0 North.	5 0 ——	2 20
Sept. 13	7 30 *Even.*	25 0 ♎	13 30 North.	2 0 ——	2 00

So as by its own Motion in its own Orbit from *Aug.* 26. to *Sept.* 13. it has moved 83° 27′. And in the *Ecliptic* 91° 30′. But its Northern Latitude has increaſed to 26°, and again has decreaſed to 12° 30′.

Obſerve, that its Northern *Node* is in 24° ♉, and its Southern *Node* in 24° ♍. But its *Limits* were in 24° ♌ and ♒. The *Angle* of the *Orbit* and the *Ecliptic* was nearly 26°. But whether for its whole Duration it was intirely conſiſtent with its *Nodes*, or whether it varied and how much, will appear by the Calculation.

For its whole Continuance the Head was ſomething brighter and greater than that of the Year 1681. But on the contrary it had much a ſhorter Tail. In the Head, by means of a long Teleſcope, we could obſerve but one *Nucleus*, and that always of an *Oval* and *Gibbous Figure*; except that on *Sept.* 8. eſpecially, a very bright Ray proceeded from the *Nucleus*, which was partly crooked, ed, and paſſed into the Tail. This deſerves notice, for as I remember I have not ſeen the like appearance in any other *Comet*.

Beſides it may be obſerved, that ſometimes it directed its Tail pretty exactly in Oppoſition to the *Sun*, as *Aug.* 30. in the Morning. But often with a notable Deviation, as is common in moſt *Comets*. Alſo its *Coma* had not always the ſame length. At firſt the Tail ſeem'd to have a length of 12°, afterwards ſometimes ſhorter, and ſometimes longer as far as 15° or 16°. But towards the End it diminiſhed continually.

Fig. 156.

A Comet. An. 1683. at Dantzick; by M. Hevelius. n. 154. p. 415.

CVIII. *An.* 1683. *Jul.* 30. at 11^h 30′. In our new Conſtellation the *Tyger* or *Lynx*, I perceived a *Comet* here at *Dantzick*; ſtretching out its Tail, which was not very long, upwards between the *Polar Star* and *Caſſiopeia*, with ſome Inclination. It made a right Line with the uppermoſt Star of the *Head* of *Auriga* and the right *Shoulder* of *Perſeus*; as alſo with the *Belly* of the greater *Bear* and the right *Shoulder* of *Auriga*; as alſo with the middle Star of the *Tail* and of the *Side* of the great *Bear*. Then taking a Tube of 10 Feet, I conſidered this Phenomenon. The Head indeed was pretty large, but the Matter was not very denſe. So that in this there appear'd no bright *Nucleus*, nor diſtinct Corpuſcles, as are generally found in moſt others. At near 12^h its Altitude was 19° 57′.

July 31. in the Evening, that is, at 12^h 30′, when its Altitude was 21° 28′ it made a right Line with the *Foot* of *Auriga* and *Capella*. The *Tail* was very dilute and thinner than Yeſterday, but ſomething longer.

Aug. 4. in the Morning, at that time it was ſo far removed from the right *Shoulder* of *Auriga*, as the ſaid *Shoulder* is diſtant from the *Head* of the *Kid*. But the left *Leg* of *Perſeus*, *Capella*, and the *Comet*, ſeem'd to be in a right Line.

Aug. 16. in the Evening, at near 11^h. the *Comet* was within four little Stars, one of which was at the upper Part of the *Comet* in Conjunction with it, at the Diſtance of only 1′, ſo near it was to the Limb. At which time I meaſured the Diameter of the *Comet* with my Micrometer, and found it 6′ 5″.

Aug. 20. in the Evening, it extended its very ſhort and thin *Coma* between *Capella* and the *Head* of the *Kid*.

On the 20 of *Aug.* in the Evening it was very near to each *Kid*, ſo that it made almoſt an Equilateral Triangle with them, the Sides of which almoſt equal'd the Diſtance of the *Kids*, which is about 47′. Beſides the *Comet* made an Equilateral Triangle with *Capella* and the Star at the bottom of the right *Foot* of *Perſeus*, the Baſe of which was the Diſtance of the ſame fixt Stars.

Aug. 24. in the Evening, it was between *Capella* and the *Pleiades*, ſo that it ſeem'd at the ſame Diſtance from each. Then *Capella*, the *Comet*, and the *Pleiades*; alſo *Almanac*, the *Head* of *Meduſa*, and the *Comet*; alſo the right *Shoulder* of *Auriga*, the *Comet*, and the Star that follows the left *Foot* of *Perſeus*; nearly made a right Line. Yet in this laſt Conſtitution, the *Comet* ſeem'd to go rather below the right Line.

Aug. 25. in the Evening; it almoſt made a right Line with *Capella* and the *Head* of the *Kid*.

Aug. 29. at 1^h 5′. in the Morning, it was accompany'd with many very minute and bright fixt Stars, that is, the *Subiescian Stars*; and was distant from the Western Point of the *Pleiades* not above 42′ 35″.

The same Day in the Evening, it was found much more advanced, contrary to the Series of the Signs; that is, almost 4° in the Space of 24 Hours.

Aug. 30. in the Evening, the *Comet* was distant not above 32′ 41″, from a certain very conspicuous little Star, and constituted a right Line with *Musca*, and that in the Base of the Triangle, and then also with the foremost in the *Foot*, and that in the *Knee* of *Perseus*.

Sept. 2. in the Morning, the *Comet* was between the *Pleiades* and the Knot of the Line, making a right Line with *Musca* and the bright Star in the *Whale's Jaw*, and with the lowest in the *Haunch* of the *Bull* and the *Jaw* a Triangle almost equicrural, the Vertex of which is the aforesaid *Jaw*. Also it made a right Line with the two in the Front of the *Whale*; also it was as far distant nearly from that towards the West, as they were from one another.

This Day I again measured very diligently with a Micrometer the Diameter of the *Comet's Head*, being 9′ 7″. *Aug.* 16. with the same Micrometer I found it only 6′ 5″, so that it had notably increased in the Space of 17 Days. It might be said the Reason is, because in the last Observation it was much nearer the *Earth*. But therefore it ought to shew its Head much clearer and brighter, especially if it were a permanent Body, as some assert, which finishing its Course in a certain Time comes again into our View. But on the contrary its Head is much more dull and rare at last, so that we might very distinctly perceive the Matter of its Head to be gradually dissolved. And this agrees much better with our Hypothesis.

Sept. 4. in the Morning the *Comet* seem'd to be in a right Line with the Star in the Western Front of the *Whale*, and the bright Star of *Aries*. Also with that in the *Mouth* and *Jaw* of the *Whale*. Besides it made almost an Equilateral Triangle with that in the *Mouth* and at the *Cheek* of the *Whale*. Lastly, it fell out as I could wish, that to the South at 3^h 40′ in the Morning nearly, with a most exact Quadrant I obtain'd its Meridian Altitude, which was 38° 15′.

D. of the Month.		Longitude.			Latitude.			Diurnal Motion.		Declination.			Right Ascen.	
		°	′		°	′		°	′	°	′		°	′
July	30	7	0	♋	29	15	N			51	30	N	100	0
	31	6	25	♋	29	0	N	0	42					
Aug.	1	5	45	♋	28	45	N	0	44					
	2	5	0	♋	28	30	N	0	46					
	3	4	10	♋	28	15	N	0	48					
	4	3	20	♋	28	0	N	0	50	51	40	N	96	0
Aug.	5	2	20	♋	27	45	N	0	52					
	6	1	20	♋	27	30	N	0	54					
	7	0	20	♋	27	15	N	0	56					
	8	29	20	♊	27	0	N	0	58					
Aug.	9	28	20	♊	26	40	N	1	0					
	10	27	20	♊	26	20	N	1	2					
	11	26	20	♊	25	55	N	1	4					
	12	25	20	♊	25	30	N	1	6					
Aug.	13	24	20	♊	25	0	N	1	8					
	14	23	20	♊	24	30	N	1	10					
	15	22	20	♊	24	0	N	1	12					
	16	21	10	♊	23	20	N	1	14	46	0	N	77	0
Aug.	17	19	20	♊	22	30	N	1	19					
	18	17	40	♊	21	30	N	1	25	44	0	N	73	30
	19	16	0	♊	20	30	N	1	35					
	20	14	20	♊	19	15	N	1	45	41	0	N	69	30
Aug.	21	12	20	♊	18	0	N	1	55					
	22	10	20	♊	16	45	N	2	5					
	23	8	20	♊	15	30	N	2	15					
	24	6	20	♊	14	15	N	2	25	25	0		60	40
Aug.	25	3	50	♊	12	45	N	2	40					
	26	1	1	♊	11	0	N	2	55					
	27	28	15	♉	9	0	N	3	10					
	28	25	15	♉	6	30	N	3	25	24	30	N	51	0
Aug.	29	22	15	♉	4	0	N	3	40	21	30	N	48	30
	30	18	55	♉	1	30	N	3	55	18	00	N	45	40
	31	16	25	♉	1	0	S	4	5					
Sept.	1	12	15	♉	1	30	S	4	20					
Sept.	2	9	55	♉	6	0	S	4	40	10	30	N	40	0
	3	6	25	♉	9	40	S	5	5					
	4	2	35	♉	11	20	S	5	30	1	30	N	34	0

From hence it may very clearly be seen, that this *Comet* moved constantly against the Order of the Signs; so that in the *Ecliptick* it went 63° 55′, but in its own *Orbit* 74° 35′, under an Angle of its *Orbit* and the *Ecliptic* of almost 39°, but of its *Orbit* with the *Equator* of 56°. Its *Latitude* at first was 29° 15′ North, and at last 11° 20′ South. So that it varied its Latitude almost 41°.

This I shall take notice of as to its Head, that at first its Diameter was much less than afterwards; but on the contrary, that at first it was far brighter than towards the End. But it exhibited no distinct and fulgent *Nuclei*, as I have seen in most, but a confused Mass of Matter, much thinner towards the End. This *Comet*, since it was seen mostly without a Tail, may truly be reckoned among Hairy Stars, or those that have Beards like Goats; for it extended upwards its very short and dilute Bristles only to *Aug.* 18. which afterwards quite disappear'd.

CIX. A new *Comet* was lately seen in the Heavens, by the sharp Eye of Ab. *Blanchini*, the Disciple of *Geminiani Montanarius*. The *Comet* was indeed but small but appear'd regular in its Orbit, of a thin Light, and as an obscure Star. But it was more luminous through the Telescope. *A Comet, An. 1684. at Rome; by S. Ciampini. n. 169. p. 920.*

June 30. New Style. *An.* 1684. the *Comet* was first seen by me in 9 Degrees of *Libra*, with some Minutes; the Latitude of the same was 8 Degrees North, with some Minutes.

Days of July.	*Longitude of the Comet.*	*Northern Latitude.*	*Days of July.*	*Longitude of the Comet.*	*Northern Latitude.*
	° ′	° ′		° ′	° ′
1	11 18 ♎	13 12	10	24 42 ♎	39 40
2	13 16 ♎	17 57	11	25 44 ♎	40 49
3	14 58 ♎	22 12	12	26 38 ♎	41 58
4	16 45 ♎	25 40	13	27 23 ♎	43 5
5	18 30 ♎	28 50	14	28 32 ♎	44 9
6	19 50 ♎	31 34	15	29 49 ♎	45 12
7	21 7 ♎	33 54	16	00 50 ♏	46 20
8	22 20 ♎	36 6	17	2 8 ♏	47 40
9	23 32 ♎	38 00			

The four last Observations require some better Calculation, for perhaps an Error of some Minutes may have crept in. The Observation of the first of *July* is very exact, for the *Comet* appear'd in the Telescope along with that Star in *Virgo* which by *Bayer* is mark'd Ω, and fell under the Girdle of the first Northern Part. The surest Observation of all is that of the sixth Day, when *Arcturus* was seen through the Tube together with the *Comet*. Also on the 14th the

the *Comet* and the Star χ in *Bootes*'s Hunting Pole, under his *Shoulder*, were ſeen at one View.

A Comet. An. 1686. *at* Leipſick; *by M.* Kirk. n. 186. *p.* 256.

CX. *Sept* 8. *ſt. v. A.* 1686. 4^h *mane*, about Day-break, M. *Kirk* found this *Comet* in the Conſtellation of *Leo*, to the Right-hand of the *Lucida in Lumbis* ♌ (as it is conceived, for the *Latin* Copy is defective in this place) and reſembling that Star in Colour and Magnitude, with a thin and ſhort Tail extended upright. Over the *Comet* in the ſame Vertical was the Star of θ ♌ of *Bayer*, or 21 *Tychonî*, diſtant therefrom, by the Micrometer, exactly a Degree; and a Line drawn from the *Lucida in Lumbis* ♌ to the *Comet*, paſſed much about half a degree to the Right-hand of the θ *Leonis*. The Diſtance of the *Comet* from *Regulus* taken by a Radius was about 17°. The next Morning *Sept.* 9. at 3^h 58′, the Diſtance thereof from θ ♌ was found by the Micrometer $2° 23'\frac{1}{2}$, and at 4^h 40′, again $2° 25'\frac{3}{4}$. To verify the Times, the Altitude of the *Lucida in Lumbis* ♌ was obſerved 11° 10′, at 4^h 8′ *manè*. A Right Line drawn by the *Comet*, and the ſaid θ *Leonis* towards β *Leonis*, or the *Lucida Colli*, left that Star a little to the Right-Hand.

This *Comet* was ſeen by a Countryman, who firſt gave Notice thereof, from the *6th* to the *12th* of *Sept.*

The Reſult of theſe Obſervations is, that the *Comet* was direct in Motion, that it mov'd about $1\frac{1}{2}$ Degree *per Diem*, and that it ſeemed rather to decreaſe in Latitude. On the *7th* of *Sept.* it was about 24′ diſtant from θ *Leonis*, but its bearing therefrom is not ſet down.

This Star, θ ♌, was then in 9° 2′ of ♍, with North Lat. $9° 41'\frac{1}{2}$. Whence at the time of the firſt Obſervation it may be concluded, that the *Comet* was in 9° 55′ of ♍, with North Lat. 9° 15′. And at the 2^d Obſervation, the Longitude of the *Comet* will be found about 11° 20′ of ♍, with much the ſame North Latitude as before.

A Comet. An. $169\frac{8}{9}$. *at* Paris; *by M.* Caſſini. *n.* 250. p. 79.

CXI. *Feb.* 19. New Style, *An.* 1699. in the *Royal Obſervatory* at *Paris*, a ſmall *Comet* began to be ſeen, which was like a nebulous Star of the third Magnitude. It reſembled that which was obſerved in *September* 1698.

Its Situation was among the unform'd Stars of the ſixth Magnitude, near the *Arctic Polar* Circle, above the *Head* of *Auriga*, almoſt at an equal Diſtance between the Weſtern *Elbow* of *Perſeus* and the *Head* of the great *Bear*. *Tycho* aſcribes them to the unform'd Stars about the leſſer *Bear*. As we continued our Obſervations, it ſeem'd by its own Motion to direct its way towards *Capella*, with a ſmall Deviation from its Circle of Declination. Its Velocity was ſuch, that in the Space of one Day it deſcribed about 7 Degrees of a great Circle, by which Motion in leſs than 4 Days it might come to the Pole, and be joyn'd to the *Polar Star*.

At 6^h after Midnight we compared the *Comet* with a Star of the 6^{th} Magnitude, which *Tycho* calls the ſecond of thoſe which are in a right Line with the Pole. In its Paſſage through the Horary Circle it preceded this Star by 15′ 53″, by which the Difference of right Aſcenſion will be given 4° 43′. But it was more to the North than this Star by 8′. whence ſuppoſing *Tycho*'s Longitude and Latitude of this Star, to this Time, the *Comet* is referr'd to 15° 51′. of *Gemini*, with North Latitude 37° 25′. This

This *Comet* moves to the oppoſite Parts of the Heavens in reſpect of thoſe to which the *Comet* of the foregoing Year tended; whereas it was almoſt at the ſame Diſtance from the *Pole*, as this *Comet* of ours when it was firſt ſeen; nor very diſtant from the ſame Place.

But the *Comet* of the Month of *September* proceeded in the ſame way as the *Comet* of the Year 1652. purſued among the Stars, as it was obſerved by us at *Bologna*. On which Occaſion we diſtinctly deſcribed that way by the ſame Stars as our *Comet* held, *An.* 1698. in Letters addreſs'd to the Moſt Serene *Fran. Eſt* Duke of *Modena*. That *Comet* paſs'd in the Month of *December* from the Southern Parts of the Heaven through the Conſtellations of the *Hare*, *Orion*, and the *Bull*, where it cut the *Ecliptic* with an Inclination of 76°, and through *Perſeus* and *Caſſiopeia*, where it ceas'd to be ſeen in the Month of *January*, *An.* 1653. This began to be ſeen in the beginning of the Month of *September*, in the ſame Part of *Caſſiopeia* where the other ceaſed to be ſeen, and thence proceeding through the *Shoulders* and the *Arms* of *Cepheus*, where it had the greateſt Latitude from the *Ecliptic* of 76° it paſs'd through the *Dragon* and the *Swan*, through the *Lion*'s *Skin* in *Hercules*, through *Ophiucus*, to the Conſtellation of *Scorpio*, which it kept in our laſt Obſervations from the 24 to the 28 of *September*. Now from theſe Obſervations we collected, that this *Comet* had its *Perigee Sept.* 7. in the Evening, with the greateſt apparent Velocity of nearly 10° in the Space of one Day.

CXII. Papers *of leſs General Uſe* Omitted. Cygnus. *n.* 21. p. 372. *n.* 65. p. 2088, 2090.

1. The Conſtellation of *Cygnus*, with the *New* Star *in Pectore* in it, by *Hevelius*; together with the Names of the Stars in that *Conſtellation* by *Tycho* and of thoſe added by *himſelf*. *Mr.* Horrox's *Lunar Syſtem.* *n.* 110. p. 219. *n.* 116. p. 368.

2. Mr. *Flamſteed* having peruſed Mr. *Street*'s Diſcourſe, and conſidered the Contrivance of his *Moon-Wiſer*, aſſures, that for the Motion of Longitude 'tis the very ſame, and for the Motion of Latitude not much better than Mr. *Horrox*'s.

But Mr. *Flamſteed* hath thought of another Contrivance, that will ſhew the *Moon*'s true Place to a Minute.

3. 1. The more Notable *Cæleſtial Appearances* calculated, by Mr. *Flamſteed*, for the *Year* 1670. *Cæleſtial Phænomena calculated.* n. 55. *p.* 1099.

2. The ſame for the *Year* 1671. n. 66. *p.* 2029.

3. The ſame for the *Year* 1672. n. 77. *p.* 2297.

4. The ſame for the *Year* 1673. n. 79. *p.* 3061. n. 86. *p.* 5040.

5. The ſame for the *Year* 1674. n. 89. *p.* 5118. n. 99. *p.* 6162.

4. 1. The *Eclipſes* of the *Satellites* of *Jupiter* viſible at *Uraniburg* the laſt four Months of the *Year* 1671. calculated by M. *Caſſini*. Satellite *Eclipſes*, n. 77. *p.* 2238. n. 151. *p.* 322.

2. The *Eclipſes* of the *Satellites* of *Jupiter* viſible at *the Obſervatory* at *Greenwich* in the three laſt Months of the *Year* 1683. calculated by Mr. *Flamſteed*.

3. The *Satellite Eclipſes* calculated by Mr. *Flamſteed* for the *Year* 1684. n. 154. *p.* 404.

4. The ſame for the *Year* 1685. n. 165. *p.* 760.

5. The ſame together with the *Parallaxes* of *Jupiter*'s *Orb* and his *Geocentrick Places*, for the *Year* 1686. *n.* 177. *p.* 1215.

6. The ſame for the *Year* 1687. n. 184. *p.* 196.

7. The *Satellites Eclipſes* calculated by Mr. *Halley*, for the *Year* 1688. *n.* 191. *p.* 435.

8. 1. An Account of the *Ephemerides* of the *Comet*, *A.* 1665. calculated by *Comets*, *n.* 1. p. 3.

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n. 2. p. 17, 18. by M. *Auzout*; and the *Principle* of his *Hypothesis* discovered by M. *Cassini*.

n. 3. p. 36. 2. An Account of the *Ephemerides* of the *Comet A.* 1665. Calculated by M. *Auzout*.

CXIII. Accounts *of* Books *and* Emendations, Omitted.

n. 102. p. 40. 1. A new Size of *Globes* about 15 Inches Diameter rectified by *R. Morden* and *Will. Berry*.

Phil. Col. n. 1. p. 44. 2. A Representation of the Heavens in two large *Hemispheres* of 30 Inches Diameter, Stereographically projected upon the Plane of the *Æquinox*; by Mr. *Fr. Lamb*.

n. 90. p. 5150. 3. Deux Machines propres à faire les *Quadrans*, avec tres grande facilité, par le P. *Ignace Gaston Pardies*, S. J. à *Paris*, 1673. in 12°.

n. 184. p. 213. 4. *Sciotericum Telescopicum*, or a new Contrivance of adapting a Telescope to an *Horizontal Dial*, for Observing the Moment of Time by Day or Night; by *Will. Molineux R. S. S. Dublin*, 1686. *in 4to.*

n. 241. p. 240. 5. The *Meridian Line* of the *Church* of St. *Petronio*, drawn and fitted for Astronomical Observations, in the *Year* 1655. revised and restored in the *Year* 1695; by *Jo. Dom. Cassini*. At *Bononia* 1695. *Fol.*

n. 66. p. 2028. n. 99. p. 6171. 6. *Joa. Hevelii* Machinæ cælestis Pars prior, *Organographiam Astronomicam* plurimis Iconibus illustratam & exornatam exhibens, *&c. Gedani* 1673 in *Fol.*

n. 109. p. 215. n. 111. p. 243. 7. *Animadversions* on the First part of the *Machina cælestis* of *Joa. Hevelius*, together with an Explication of some *Instruments* made by *R. Hook. P.* of *Geometry* in *Gresh. Coll.* and *R. S. S. London* 1674. *in 4to.* Dr. *Wallis*'s Letter to M. *Hevelius*, concerning *Divisions* by *Diagonals* there inserted, but faultily, is here *Reprinted* more correctly.

n. 175. p. 1162. Ib. p. 1164. n. 111. p. 244. n. 175. p. 1176. 8. *Joannis Hevelii* Consulis *Dantiscani* Annus Climactericus. *Gedani* 1685. *Fol.* Wherein (among other Things) M. *Hevelius* vindicates the Justness of his *Celestial Observations* against the Exceptions by some made to the *Accuracy* of them. The *Controversy* between *Him* and Dr. *Hook*, about the Use of *Telescopick* and *Plain Sights*, and Dr. *Wallis*'s Calculation, for *Dividing* the Limb of Instruments by *Diagonals*, are also here Abridged.

n. 150. p. 308. 9. Excerpta ex Literis Ill. & Clariss. Virorum ad Nob. Ampliss. & Consultiss. D. *Jo. Hevelium* Cons. *Gedanensem* perscriptis, Judicia de Rebus *Astronomicis*, ejusdemque Scriptis, exhibentia; Studio ac Operâ *Joa. Erici Olhoffii* Secretarii *Gedani* 1683. in *4to.*

n. 118. p. 440. 10. A Description of the *Helioscop*, and some other *Instruments*, made by *R. Hook*, R. S. S. *Lond.* 1675. in *4to.*

11. The Sphere of M. *Manilius* made an *English* Poem, with Annotations, and an *Astronomical Appendix*; by *Ed. Sherburn*, Esq; *Lond.* 1675. in *Fol.*

n. 101. p. 233. n. 204. p. 913. 12. *Albatenii* Observationes *Astronomicæ*, Quas ex *Arabico* in *Latinum* Transtulit *Plato Tiburtinus*. *Noribergæ* 1537; & *Bononiæ* 1645. The *Arabick Copy* of those Observations does not appear, whereby that *Translation* might be *examined*: But Mr. *Halley*, by calculating Tables from the Principles there delivered, hath here *discovered* and *corrected* above 30 considerable *Faults* in a few Pages.

n. 43. p. 868. 13. *Historia cælestis*; ex Libris & Commentariis MS. *Observationum* Vicennalium *Tychonis Brahe*, *Dani*, *Augustæ Vindelic.* Ann. 1666. in *Fol.*

n. 102. p. 27. & 29. 14. All the *Manuscripts* of the famous *Kepler*, (both *published* and *unpublished*) which are purchased, and carefully preserved by M. *Hevelius*.

15. *Je-*

15. *Jeremiæ Horroccii Angli* Opera Posthuma: una cum *Guil. Crabtræi* Observationibus Cœlestibus, nec non *Jo. Flamsteedii* de *Temporis Æquatione* Diatriba, numerisque *Lunaribus* ad Novum *Lunæ* Systema *Horroccii*. *Lond.* 1672. in 4*to*. n. 87. p. 5078.

16. *Astronomia Reformata*. Auctore *Joan. Bapt. Riccioli S. J. Stephano de Angelis*, conceiving the Arguments of this *Author*, against the *Motion* of the *Earth*, to be none of the strongest, taketh Occasion to let the world see that they are not more esteem'd in *Italy*, than in other places: *Manfredi*, in behalf of *Riccioli*, endeavours to answer the Objections of *Angeli*, and this latter replies to *Manfredi*'s Answer. The Substance of which *Controversy* is here given by Mr. *Ja. Gregory*; with some Remarks and Explications of his own upon it. n. 22. p. 394. n. 36. p. 693.

17. An Attempt to prove the *Motion* of the *Earth* from Observations, made by *R. Hook*, *F. R. S. Lond.* 1674. *in* 4*to*. The Method of this Undertaking is approved and commended by M. *Chr. Huygens*, and M. *Cassini*. n. 101. p. 12. n. 105. p. 90.

18. *Nicolai Mercatoris* Holsati, è Soc. Regia, *Institutionum Astronomicarum* Libri duo. *Lond.* 1676. in *Octavo*. n. 125. p. 611.

19. Annales Cœli & Temporum perpetui, sive Mysteria *Astronomo-Chronologica* à Seculo abscondita nunc per Dei Gratiam detecta & evidenter asserta, Libris tribus. *Kiloni*. This Book is Preparing by Dr. *Wasmuth*. n. 104. p. 74.

20. A *Catalogue* of *Fixed Stars* with their Longitudes, Latitudes, and Magnitudes, according to the *Observatiou* of *Uleg Beig*. *Oxford* 1666. n. 8. p. 145.

21. *Catalogus Stellarum Australium*, sive Supplementum Catalogi *Tychonici*; exhibens Longitudines & Latitudines *Stellarum Fixarum*, quæ prope *Polum Antarcticum* sitæ, in Horizonte *Uraniburgico*, *Tychoni* inconspicuæ fuere. Authore *Edm. Halleio*, è *Col. Reg. Oxon.* in 4*to*. n. 141. p. 1032.

22. Congietture *Physico Astronomiche* della Natura del *Universo*; da *Pietro* M. *Cavina*, in *Faenza* 1669. in 4*to*. n. 65. p. 2012.

23. Prose de *Signori Academici* di *Bologna*, in *Bologna* 1672. in 4*to*. *S. Montanari*'s Discourse concerning the admirable *Changes* and other *Novelties* observed in the *Heavens*. n. 89. p. 5125.

24. *Ismaelis Bullialdi* ad Astronomos Monita duo. Primum de *Stella nova*, quæ in *Collo* Ceti ante An. aliquot visa est. Alterum de *Nebulosa* in *Andromedæ Cinguli* parte Borea, ante Biennium iterum orta. Approv'd by M. *Hevelius*. n. 21. p. 381. n. 25. p. 460.

25. Three Letters of *Jo. Dominicus Cassinus*, concerning his *Hypothesis* of the *Sun's Motion*, and his Doctrine of *Refractions*. At *Bononia*, in 4*to*. n. 84. p. 5001.

26. Refractio *Solis Inoccidui*, in Septentrionalibus Oris circa Solstitium Æstivum, *An.* 1695. aliquot Observationibus *Astronomicis* detecta. *Holmiæ*, in 4*to*. *Translated iuto English*. *Lond.* in 8*vo*. n. 233. p. 731.

27. *Tabularum Astronomicarum* Paris prior; de Motibus *Solis* & *Lunæ* necnon de Positione *Fixarum*, ex ipsis Observationibus deductis: Authore *Ph. de la Hire*. *Paris* 1687. in 4*to*. *Some Animadversions on it are here inserted.* n. 191. p. 443.

28. 1. The *Royal Almanack* for the *Year* 1675: by *N. Stevenson*. in 12°. n. 108. p. 192.
2. ——— For the *Year* 1676. n. 120. p. 490.
3. ——— For the *Year* 1677. n. 130. p. 774.

29. Ephemeris, ad *Annum* 1686, exactissime supputata. *Lond.* in 8*vo*. n. 179. p. 35

30. The *Celestial World discovered*, or Conjectures concerning the Inhabitants, Plants, and Productions, of the *Worlds in the Planets*. Written in *Latin* by M. *Chr. Huygens*. in 8*vo*. n. 256. p. 337.

31. Rag-

n. 1. p. 2. n. 45. p. 900. n. 4. p. 69. 31. Ragguaglio de Nuove Obſervationi da *S. Gioſeppe Campani*. This Book was anſwer'd by M. *Auzout*, who gives his Opinion of *Campani*'s Glaſſes, and his new *Obſervations* of *Saturn* and *Jupiter* made with them: to this *S. Campani* publiſhes a *Reply*, and M. *Auzout* his *Animadverſions* thereon. Ib. p. 74.

n. 35. p. 688. n. 44. p. 892. 32. 1. Ephemerides *Mediceorum Siderum*, ex Hypotheſibus & Tabulis *Jo. Dom. Caſſini*, *Bononiæ* 1668. in *Fol.* The like *Tables* have been formerly publiſh'd by the Learned *J. Bapt. Hodierna* at *Rome*, about 1656.

n. 214. p. 256. 2. The Table of the *Eclipſes* of the *Firſt Satellite* of *Jupiter* by M. *Caſſini*, publiſh'd at *Paris* in the *Recueil d' Obſervations faites en pluſieurs Voyages pour perfectioner l'*Aſtronomie *& la* Geographie, being not printed with the uſual Care of the *Imprimerie Royale*, Mr. *Halley* here amends ſome of the *Errata*.

n. 14. p. 242. n. 35. p. 687. 33. *Martis*, circa Axem proprium revolubilis, Obſervationes *Bononiæ* à *Joa. Dominico Caſſino* habitæ 1666. Here M. *Caſſini* judges it evident, that the Period of this *Planet*'s *Revolution* is not performed in the ſpace of 12^h 20′, but in about 24^h 40′; and that thoſe, who affirm the former, muſt have been deceived by not well diſtinguiſhing the two *Faces*.

n. 134. p. 853. 34. *Mercurius* in *Sole* viſus; à *Jo. Hevelio*, *Ged.* 1662.

n. 6. p. 104. 35. *Prodromus Cometicus*; by *Hevelius*.

n. 17. p. 301. 36. *Joannis Hevelii* Deſcriptio *Cometæ*, *An.* Æræ *Chriſtianæ* 1665. exorti: una cum Mantiſſa *Prodromi Cometici*, obſervationes omnes Prioris *Cometæ* 1654, ex iiſque genuinum Motum accuratè deductum, cum Notis & Animadverſionibus, exhibens.

n. 40. p. 805. 37. *Jo. Hevelii* Cometagraphia. *Dantzick*, in *Fol.*

n. 35. p. 691. 38. *Staniſlai* de *Lubienietz* Theatrum *Cometicum*. *Amſtelod.* 1668. in *Fol.*

n. 53. p. 1069. 39. Del Movimento della *Cometa*, apparſa il meſe di *Decembri* 1664. da *Pietro Maria Mutoli*, in Piſa, in 4*to*.

n. 53. p. 1071. 40. *Eraſmi Bartholini* de *Cometis*, *An.* 1664 *&*. 1665. Opuſculum; ex Obſervationibus *Hafniæ*, habitis adornatum. *Hafniæ*, in 4*to*.

n. 139. p. 980. 41. *Joh. Walliſii*, De *Cometarum* Diſtantiis inveſtigandis. *Lond.* 1678.

Ib. 986. 42. Lectures and Collections made by *R. Hook*, Sec. of the *R. S. Lond.* 1678 in 4*to*.

Ph. Col. n. 4. p. 106. 43. Obſervat. of the *Comet* of 1680 and 1681. made at the *Col.* of *Clermont*; by *P. J. de Fontenay* è *S. J. Profeſſ.* of *Mathematicks*, Paris 1681.

Ib. p. 114. 44. A Treatiſe concerning the late *Comet*, publiſhed at *Turin*, 1681. by *Donato Roſſetti*, S. T. D. Canon of *Leghorn*. and *Tutor* in *Mathematicks* to the *Duke of* Savoy.

Ib. p. 116. 45. An Explication of the *Comet* which appeared at the End of 1680, and in the Beginning of 1681, upon the Obſervations of D. *Anthelme*, *Carthuſian* of *Dijon*. At *Dijon*, 1681. in one ſingle Sheet.

Ph. Col. n. 7. p. 196. 46. A ſmall Diſcourſe about *Comets* publiſhed in the *High Dutch* at *Nurenburgh* 1681; by a *Lover of Aſtronomy*.

Ib. p. 199. 47. A new Introduction, ſhewing how the *Motions* of the *Comets* may be reduced to ſome certain and Geometrical Rules, that their *Appearance* may be predicted: in *High Dutch*; by *Ja. Bernouly*, at *Bazil. An.* 1681.

n. 149. p. 272. 48. *Joannis Jacobi Zimmermanni Cometo-ſcopia*. Or, three *Aſtronomical Relations* concerning the *Comets* that have been ſeen in the *Years* 1680, 1681, 1682. *Stutgard*, 1682. in 4*to*.

CHAP.

CHAP. V.

MECHANICKS. ACOUSTICKS.

I. 1. IF an Agent as A produces an Effect as E; an Agent as 2 A, will produce the Effect 2 E, 3 A as 3 E, &c. all other things being like. And universally, *m* A will produce the Effect *m* E, whatever ratio *m* be the Exponent of. *The General Laws of Motion by Dr. Wallis. n. 43. p. 864. Nov. An. 1668.*

2. Therefore if a Force as V move the Weight P, the Force *m* V will move *m* P, other things alike: Suppose the same Space in the same Time, that is, with the same Velocity.

3. Also if in the Time T it move it the Length L; in the Time *n* T it will move it the Length *n* L.

4. Therefore if the Force V, in the Time T, move the Weight P, the Length L; the Force *m* V, in the Time *n* T; will move the Weight *m* P, the Length *n* L. And therefore as V T, (the Product of the Forces and Time,) to P L, (the Product of the Weight and Length,) so is *mn* V T, to *mn* P L.

5. Because the Degrees of Velocity are proportional to the Lengths perform'd in the same Time; or, (which comes to the same,) are reciprocally proportional to the Times spent in performing the same Length; it will be $\frac{L}{T} : C :: \frac{mL}{nT} . \frac{m}{n} C$. That is, the Degrees of Velocity are in a ratio compounded of the Direct Ratio of the Lengths, and a Reciprocal Ratio of the Times.

6. Therefore because $VT . PL :: mnVT . mnPL$; it will be $V . \frac{PL}{T} :: mV . \frac{mnPL}{nT}$. That is, $V . PC :: mV : mPC = mP \times C = P \times mC$.

7. That is, if the Force V is able to move the Weight P with the Velocity C; the Force *m* V will move either the same Weight P with the Velocity *m* C; or with the same Velocity will move the Weight *m* P; or lastly, any Weight with such Velocity, that the Product of the Weight and Velocity may be *m* P C.

8. And on this depends the Reason of the Construction of all Machines for facilitating Motions: So that in whatever ratio the Weight is increased, the Velocity must be diminish'd in the same. So that the Product of the Velocity and Weight, that is to be moved with the same Force, may always be the same. That is, $V : PC :: V : mP \times \frac{1}{m} C = PC$.

9. If the Weight P, moving with the Velocity C, by means of the Force V, ſtrikes directly againſt the Weight mP, which is at reſt, and is no ways hindred; each will be carry'd by the Velocity $\frac{1}{1+m}$C. For becauſe of the ſame Force, which is apply'd to the moving of a greater Body; the Velocity of the increaſed Body muſt be diminiſhed in the ſame ratio. That is, V : PC : : V : $\frac{1+m}{1}$P$\times\frac{1}{1+m}$C=PC. Therefore the Shock of one, (by which is meant the Product of the Weight and Velocity,) will become $\frac{1}{1+m}$ PC, and of the other $\frac{1}{1+m}m$PC.

10. If againſt the Body P, moving with the Velocity C, (by means of the Force V,) another Body follows it the ſame way with greater Velocity, and ſtrikes directly againſt it; ſuppoſe the Body mP, with the Velocity nC; (and therefore moved by the Force mnV;) both will be carry'd with the Velocity $\frac{1+mn}{1+m}$C. For it is V : PC : : mnV : mnPC : : V+mnV ($\frac{1+mn}{1}$V) : $\frac{1+mn}{1}$ PC=$\frac{1+m}{1}$P$\times\frac{1+mn}{1+m}$C. Therefore the Shock of the preceding will be $\frac{1+mn}{1+m}$ PC, and that of the ſubſequent will be $\frac{1+mn}{1+m}m$PC.

11. If Bodies moving contrary ways meet and ſtrike each other directly, ſuppoſe the Body P moves to the right Hand with the Velocity C by means of the Force V; and the Body mP moves to the left Hand with the Velocity nC, and therefore by the Force mnV; the Velocity, Shock, and Direction of each may be determined. The Body that moves to the right Hand would communicate to the other, if it were at reſt, the Velocity $\frac{1}{1+m}$C, and therefore the Impetus $\frac{1}{1+m}m$PC to the right Hand, and would retain to itſelf this ſame Velocity, and conſequently the Impetus $\frac{1}{1+m}m$PC to the right Hand, by Sect. 9. And the Body moving to the left Hand (by a like reaſoning) would communicate to the other, if at reſt, the Velocity $\frac{mn}{1+m}$C, and therefore the Impetus $\frac{mn}{1+m}$PC to the left; and would retain to itſelf this ſame Velocity, and therefore the Impetus $\frac{mn}{1+m}m$PC to the left. Now ſince the Motion is on each ſide, the Impetus of the Body that before moved to the right Hand will now be the Aggregate of $\frac{1}{1+m}$PC to the right Hand, and $\frac{mn}{1+m}$PC to the left Hand; and therefore will really be to the right Hand or left, according

ing as this or that is the biggeſt, with that Impetus which is the Difference of the two. That is, (making the Sign + to ſignifie to the right Hand, and — to the left Hand,) the Impetus will be $+\frac{1}{1+m}PC-\frac{mn}{1+m}PC=\frac{1-mn}{1+m}PC$; and the Velocity $\frac{1-mn}{1+m}C$; and therefore to the right Hand or left, according as 1 or mn ſhall be greater. And in like manner the Impetus of that Body that before moved to the left Hand will be $+\frac{1}{1+m}mPC-\frac{mn}{1+m}mPC=\frac{1-mn}{1+m}$ mPC, and its Velocity $\frac{1-mn}{1+m}C$; and therefore to the right Hand or left, according as 1 or mn is the greater.

12. But if the Bodies do not move directly the ſame way, nor directly contrary ways, but ſtrike one another obliquely; the foregoing Calculation is to be accommodated to the Degree of Obliquity. Now the Impetus of a Body ſtriking obliquely, is to the Impetus that would be produced if they ſtruck directly, other things being alike, in the ratio of Radius to the Secant of the Angle of Obliquity. Which is alſo to be underſtood, when the the Body falls not perpendicularly but obliquely on the Surface of the Body that is ſtruck, as well as when the Ways of their Motion croſs one another obliquely. This Conſideration rightly apply'd to the foregoing Calculation, will determine what will be the Velocity, Impetus, and Direction. of Bodies that impinge thus obliquely. That is, with what Impetus, with what Velocity, and towards what Parts thoſe Bodies will reflect from each other, which impinge in this manner. The Reaſon is the ſame of the Gravitation of heavy Bodies that deſcend obliquely, to the Gravitation of thoſe which deſcend perpendicularly.

13. If the Bodies ſo impinging are ſuppoſed to be not abſolutely hard, (which we have conceived hitherto,) but yielding to the Stroke, yet ſo as to be able to reſtore themſelves by their Elaſtic Force; it may hence happen that thoſe Bodies ſhall rebound from one another which otherwiſe would go together; (and indeed more or leſs, according as this Power of Reſtitution is greater or leſs;) that is, if the Impetus proceeding from the Reſtitutive Force is greater than the Progreſſive.

In Motions that are continually accelerated or retarded, that is to be eſteemed the Impetus for every Moment, which belongs to the Degree of Velocity then acquired. But when the Motion is along a Curve, that is to be reputed the Direction in every Point of the Motion, which is the Direction of the Tangent in that Place. And whenever the Motion is accelerated or retarded, and alſo along a Curve, as in the Vibrations of a Pendulum; the Impetus is to be eſtimated for every Point, both according to the Degree of Acceleration, and the Obliquity of the Tangent in that Place.

2. The Law of Nature in the Colliſion of Bodies.

The proper and moſt natural Velocities of Bodies are reciprocally proportional to thoſe Bodies. *By Sir Chriſt. Wren. Ibid. p. 867.*

 There-

Therefore the Bodies R, S, having their proper Velocities, alſo after Colliſion retain thoſe proper Velocities.

And the Bodies R, S, having improper Velocities, by Impulſe are reſtored to Equilibrium. That is, as much as R exceeds, and S is deficient from their proper Velocity before Impulſe, ſo much by the Impulſe is taken from R, and is added to S: And contrarywiſe.

Wherefore the Colliſion of Bodies having their proper Velocities is equivalent to a Balance oſcillating upon two Centers, which on each ſide are equally diſtant from the Center of Gravity. Now the Beam of the Balance is produced as there is occaſion.

Therefore there are three Caſes of equal Bodies moving improperly. But of unequal Bodies moving improperly, (whether contrary ways or the ſame way) there are in all ten Caſes, five of which ariſe by Converſion.

Fig. 157. R, S, are equal Bodies, or R is the greater Body, and S the leſſer. *a* is the Center of Gravity, or the Handle of the Balance; Z is the Sum of the Velocities of both Bodies.

R *e* S *e*	Veloc. of Body	R S	given before Impulſe.	or	S *o* R *o*	Veloc. of Body	S R	given before Impulſe.
o R *o* S	Veloc. of Body	R S	required after Impulſe.		*e* S *e* R	Veloc. of Body	S R	required after Impulſe.

The Rule. R *e*, S *e*, make *o* R, *o* S: R *o*, S *o*, make *e* S, *e* R.

[Read the Syllables (tho' disjoyn'd) R *e*, S *e*, *o* R, *o* S; or R *o*, S *o*, *e* S, *e* R, in the Line of every Caſe; and of theſe that which is written in the Scheme in the *Hebrew* manner, ſhews a Motion contrary to the Motion which the *Latin* way of writing of each Syllable denotes. A Syllable conjoyn'd ſignifies the Reſt of the Body.

Calcul.			
	R+S : S :: Z : R *a*	R *e*—2R *a*=*o* R	S *o*—2 S *a*=*e* S.
	R+S : R :: Z : S *a*	2S *a*± S *e*=*o* S	2R *a*+ R *o*=*e* R.

Nature obſerves the Rules of Specious Addition and Subtraction.

By M. Huygens. n 46. p. 927. Apr. An. 1669.

3. The Rules concerning the Motion of Bodies, after their mutual Impulſe.

1. If a hard Body ſtrikes another equal hard Body at reſt, after the Contact that will be at reſt, and the Body at reſt will acquire the ſame Velocity as was in the ſtriking Body.

2. But if the other equal Body move alſo, and in the ſame right Line, after Contact they will both continue to move, but with Velocities mutually interchanged.

3. A Body ever ſo great will be moved by a Body ever ſo little, that ſtrikes againſt it with any Velocity.

4. The General Rule for determining the Motion which hard Bodies acquire by their direct meeting is this following.

Fig. 158. Let the Bodies be A and B, of which A is moved with the Velocity AD; and let B meet it, or let it move the ſame way with the Velocity BD, or laſtly let it be at reſt, that is, in this caſe let the point D fall in B. Let the Line AB be divided in C, (which is the Center of Gravity of the Bodies A, B,) and let CE be taken equal to CD. I ſay, EA will be the Velocity of the Body A after meeting, and EB of the Body B, and each will proceed in that way as is ſhewn by the order of the Points EA, EB. Now if the Point E falls in A or B, the Bodies A or B will be reduced to reſt.

5. The Quantity of Motion of two Bodies may be increaſed or diminiſhed by Conflict: But there always remains the ſame Quantity the ſame way, taking from thence the Quantity of contrary Motion.

6. The Sum of the Products made by the Bulk of every hard Body drawn into the Square of its Velocity, is the ſame both before and after the Conflict.

7. A hard Body at reſt receives more Motion from another hard Body, whether greater or leſs, by the Interpoſition of ſome third Body which is of an intermediate Quantity, than if it had been ſtruck by it immediately. And if that interpoſed Body ſhould be a mean Proportional between the two others, it will act the moſt forcibly of all againſt the Body at reſt.

In all theſe Concluſions the Author ſuppoſes the Bodies to be of the ſame Matter, as he acknowledges himſelf; or he intends, that their Bulk may be eſtimated by their Weight.

But he adds, that he has obſerved a certain wonderful Law of Nature, which he affirms that he can demonſtrate in Spherical Bodies, and which ſeems to him to be general in all other Bodies whether hard or ſoft, or whether they ſtrike directly or obliquely; that is, that the common Center of Gravity of two, three, or any Number of Bodies, is always equally promoted the ſame way, in the ſame right Line, both before and after the Percuſſion.

Some Hiſtorical Paſſages relating to theſe Papers; by Mr. Oldenburg. *Ibid. p.* 925.

4. For ſome Months laſt paſs'd, ſeveral Members of the *Royal Society*, at their publick Meetings, had inſiſted very earneſtly, that that important Subject of the Laws of Motion ſhould at laſt undergo a ſtrict Examination, having formerly been propoſed to the Society, but not yet diſcuſs'd as it deſerved to be. It then ſeem'd proper to that Illuſtrious Society to determine, that which ever of their Members ſeem'd fitteſt for this Inquiry, into the Nature of Motion, ſhould be deſired to produce their Thoughts and Diſcoveries about it, and likewiſe to collect what had been done in this matter by other excellent Men, as *Galilæus*, *Des Cartes*, *Honoratus Faber*, *Joachimus Jungius*, *Peter Borellus*, and others. Chiefly with this View, that thus conſulting and comparing the Opinions of all, a Theory might thence be eſtabliſhed in the Philoſophical World, which might agree as much as poſſible with Obſervations and Experiments, which ſhould be often repeated with due Care and Fidelity.

This their Deſire being made known, ſeveral of the Members of the ſaid Society comply'd with it, as *Chriſtian Huygens*, *John Wallis*, and *Chriſtopher Wren*; who undertook to compleat as ſoon as might be thoſe Hypotheſes and Rules of Motion, in digeſting of which they had been employed for ſome time. Hence it was that thoſe three great Mathematicians, in the Space of a few Weeks, comunicated their Theories neatly abridged, and as it were by ſtrife, deſiring the Sentiments of the *Royal Society* upon the ſame. Firſt of all Dr. *Wallis* tranſmitted his Principles concerning the Eſtimation of Motion, by a Letter dated *Nov.* 15. 1688, which was deliver'd and read the 26 of the

ſame

ſame Month. He was ſoon ſucceeded by Sir *Chriſtopher Wren*, who on the 17th of the Month following exhibited to the Society the Laws of Nature concerning the Colliſion of Bodies. Theſe the Society ordered to be printed, having firſt obtained the Conſent of the Authors, for the more commodious Communication and ampler Diſcuſſion of this Subject.

Whilſt theſe Things were doing with us, on the 4th of *January* following, *Engliſh* Stile, the Poſt brought us Letters from Mr. *Huygens*, written on the 5th day of the ſame Month, but New Stile, containing the four firſt Rules concerning the Motion of Bodies ariſing from mutual Impulſe, together with their Demonſtrations. I had at hand a Copy of *Wren*'s Theory, which I ſent the ſame Day to Mr. *Huygens* by Way of Retaliation, the Poſt then favouring. I forbore to open Mr. *Huygens*'s Letters, ſuſpecting ſomething of this Nature to be included in them becauſe of their Bulk, and becauſe of his Promiſe, till I had an Opportunity of laying them before the moſt noble and worthy Preſident of the Royal Society, the Lord Viſcount *Brounker*. Which being done, and the Rules of each being compared by the ſaid Society, there preſently appeared a wonderful Agreement between them; which produced in us a great Deſire of committing both their Writings to the Preſs. On Mr. *Huygens*'s Side nothing was wanting but his Conſent, without which we did not think it fair to print his Diſcoveries, eſpecially as he had not given them then compleat. However we took care to regiſter his Paper in the publick Acts of the Royal Society, and to return our ſolemn Thanks to the Author on *Jan.* 11. for his agreeable Communication; afterwards adding (on *Feb.* 4.) our earneſt Wiſhes, that he would cauſe his Theory to be printed either at *Paris*, (which he might eaſily do in the Journal *des Scavans*, as it is called,) or ſuffer us to print it here at *London* in the Philoſophical Tranſactions. Which Letters being ſent to Mr. *Huygens*, we ſoon afterwards received an Anſwer from him, acknowledging the Receipt of Sir *Chriſtopher Wren*'s Paper on this Subject, but mentioning nothing about the Publication of his own Paper, either at *Paris* or *London*.

Hence I conclude it is very plain, that Mr. *Huygens* has been wanting to himſelf, in not haſtning the Publication; and by his Delay has given occaſion, that Mr. *Wren* having by his Sagacity diſcovered both the Theories, juſtly claims a right to the Glory due to this Diſcovery. Since it is beyond all doubt, that neither of the Gentlemen knew what was done by the other, before their Writings appeared together; but each by his own Ingenuity produced theſe beautiful Originals.

Indeed Mr. *Huygens*, when he was at *London* ſome Years ago, ſolved theſe Caſes of Motion which were then propoſed to him. A ſure Argument that he had even then found out the Rules, by the Evidence of which he performed this Matter. But he will not affirm, that he diſcloſed any Thing of his Theory at that Time to any of the *Engliſh*. Nay, he muſt confeſs, that though he was ſolicited by ſome of them to make this Communication, yet he could never be prevail'd on to do it till very lately.

II. Let all the Lines *a b*, *b c*, *c d*, *d e*, &c. be all equal to one another, and *b* 1, *c* 2, *d* 3, *e* 4, *f* 5, &c. increaſe equally as the Numbers 1, 3, 5, 7, 9, &c. I ſay

I say that any heavy Body falling in this Line from any Point of it, will reach the Bottom in the same Space of Time, as it would reach it if it should fall from any other Point of the same.

The Synchronism of the Vibrations made in a Cycloide; Demonstrated by a Person of Quality. n. 94. p. 6032. Fig. 159. May An. 1673.

For if you suppose $a = ab = bc = cd$, &c. and $b = b\,1$, and $x =$ any Number of either; then if xa is put for af, then xxb must represent $f\delta$, and therefore the Time of Descent will necessarily be $\frac{xxb}{xxaa}$ or $\frac{b}{aa}$. And the same obtains in all Cases. Therefore, &c.

I say moreover that this Curve is a Cycloid, which is easily demonstrated from the Construction, and from what is now shewn. For this Curve $abcdefz$ is equal to the double of the last of the right Lines, that is $2\,z\omega$, and $a\omega$ is equal to the Semicircumference of the Circle whose Diameter is $z\omega$; and in general the Triangle ♈♉♊ represents the right Line $z\omega$; and the Square ♈♊♉♋ represents the Curve $abcdefz$; and the Quadrant ♈♉♋ represents the right Line $a\omega$; and the Parts of one the Parts of the other respectively. As if ♈♎♍ represents $f\delta$, then will ♈♎♌♋ represent $a\delta$, and ♈♎♏♋ will represent af. But I have no time to pursue this farther.

Lastly I say, that a Ball suspended by a String of a due Length, and vibrating between two Cycloids, will move in a Cycloid. Wherefore such Vibrations will be Synchronous. *Q. E. D.*

III. 1. Prob. To determine the Curve-line connecting two given Points which are at different Distances from the Horizon, and not in the same Vertical Line, upon which a Body moving by its own Gravity, and beginning to move from the upper Point, shall descend to the lower Point in the shortest time.

A Problem concerning the Line of quickest Descent between two Points given; proposed by M. Jo. Bernoulli. n. 224. p. 384. Jan. An. 1697.

The Sense of the Problem is this; of the infinite number of Lines that may be drawn between those Points, from one to the other, to make choice of that, according to which if a Plate be bent having the form of a Tube or a Canal, so that a Ball being laid upon it, and suffer'd to descend freely, it may perform its Passage from one Point to the other in the shortest Time possible.

2. Yesterday I received copies of two Problems, proposed by that most acute Mathematician Mr. *John Bernoulli*, printed at *Groningen*, *Cal. Jan.* 1697. Of the first of which this is the Solution.

Solv'd; by .... *Ibid.*

From a given Point A let there be drawn an indefinite right Line APCZ parallel to the Horizon, and upon the same right Line let there be described any Cycloid AQB, meeting a right Line drawn through the other given Point B in the Point Q, as also another Cycloid ABC, whose Base and Altitude may be to the Base and Altitude of the former as AB to AQ respectively. Then this last Cycloid will pass through the Point B, and will be that Curve-line, in which a Body falling by the Force of Gravity will arrive soonest from the Point A to the Point B. Q. E. I.

Fig. 160.

3. Let AP be an Horizontal Line, P the Point from whence the heavy Body descends through the Curve-line required ADE, C and D two Points infinitely near, through which the Body will fall, CD a right Line connecting the two Points, DC and sC, DF and SG, FS and GC or sH, Moments of the Abscis of the Curve and of the Ordinate respectively. Take Dr = Ds, and tC = BC.

The Demonstration; by Mr. R. Sault. *n.* 246. p. 425. Nov. An. 1698. *Fig.* 161.

Because

Becauſe in the little naſcent Lines the Time is directly as the Way deſcribed and the Velocity inverſely, (that is, in this Caſe as the Square-root of the Altitude of the falling Body,) by Hypotheſis it will be $\frac{Ds}{\surd QD} + \frac{SC}{\surd QF} =$ the leaſt Time. And becauſe the Velocity in the Points of the ſame Altitude S and B along the Curve D s C and the right Line D B C is the ſame, the Time along D C, which evidently is the leaſt, will be as $\frac{BD}{\surd QD} + \frac{BC}{\surd QF}$. Therefore let theſe Times be equal, and then $\frac{Ds}{\surd QD} + \frac{sC}{\surd QF} = \frac{DB}{\surd QD} + \frac{BC}{\surd QF}$, that is, $\frac{DB - Ds}{\surd QD} = \frac{sC - BC}{\surd QF}$, or $\frac{Br}{\surd QD} = \frac{ts}{\surd QF}$.

But the Evaneſcent Triangles Brs, Bts, are equiangular to the Triangles DsF, HsC; therefore $\frac{Bs}{Ds} = \frac{Br}{sF}$, and $\frac{ts}{Hs} = \frac{Bs}{st}$. Let theſe two Ratios of Equality be compounded, and then $\frac{Br}{Ds \times Hs} = \frac{ts}{sF \times st}$. And *ex æquo* $\frac{\surd QD}{sF \times st} = \frac{\surd QF}{Ds \times Hs}$. Now becauſe any of the Elements may be ſuppoſed to flow equably, let us ſuppoſe Ds=sC, and the moſt ſimple Expreſſion of the Curve becomes $\frac{\surd QD}{sF} = \frac{\surd QF}{Ds}$ every where. That is, in the Point of Flexure the Curve will always be in a Ratio compounded of the Velocity directly, and of the Moment of the Ordinate reciprocally. Let $\dot{x}$, $\dot{y}$, and $\dot{z}$ be the Fluxions of the Abſciſs, the Ordinate, and of the Curve reſpectively, then $\frac{x^{\frac{1}{2}}}{\dot{y}}$ is conſtant as above. Therefore $\frac{x^{\frac{1}{2}}}{\dot{y}} = 1$; but we ſuppoſed $\dot{z}$ $(=\sqrt{\dot{x}\dot{x}+\dot{y}\dot{y}})$ to be conſtant. Therefore that this may be conſtant Unity, and may obtain its due Dimenſions, will be $\frac{x^{\frac{1}{2}}}{\dot{y}} = \frac{a^{\frac{1}{2}}}{\sqrt{\dot{x}\dot{x}+\dot{y}\dot{y}}}$; and after Reduction $\dot{y} = \frac{x^{\frac{1}{2}}\dot{x}}{\sqrt{a-x}}$, which is a known Expreſſion of the Cycloid P D E. Q. E. D.

How much the Deſcent is Quicker in the Cycloid *than in a ſtraight Line by . . . . n. 225. p. 424. Feb. An. 1697.*

IV. *Theorem.* In the *Cycloid* AV D, whoſe Baſe A D is parallel to the Horizon, and Vertex V is downwards; if from A be any how drawn the right Line A B meeting the *Cycloid* in B, from whence let there be drawn the right Line B C perpendicular to the Curve of the *Cycloid* in B, to which from A let there be drawn the Perpendicular A C. I ſay that the Time in which a Body at reſt falling from A by the force of its Gravity deſcribes the right Line A B, is to the Time in which it paſſes along the Curve AV B, as the right Line A B to the right Line A C.

Fig. 162. Through B draw B L parallel to the Axis of the *Cycloid* V E, and B K parallel to the Baſe A D, meeting the Axis in G, and a Circle deſcribed upon the Diameter

Diameter E V in F and H, and lastly meeting the *Cycloid* in K. Draw the right Line E F, which from the Nature of the *Cycloid* will be parallel to the right Line B C. Whence B M is equal to E F, and E M to B F, which because of the *Cycloid* is equal to the Arch V F. Therefore A M is equal to the Arch E H V F.

By *Prop.* 25. *Part* II. of *Huygens*'s *Horologium Oscillatorium*, the Time in which a Body falling from rest describes AV, is to the Time of the Fall along E V, as the Semicircumference is to the Diameter; and by the last Proposition of the same, the Time in which a Body describes V B, after having described AV, (which is equal to the Time in which a Body describes KV, after having described A K,) is to the Time of the Fall along AV, as the Arch V F is to the Semicircumference; and therefore to the Time of the Fall along E V, as F V to the Diameter. Wherefore the Time in which a Body describes the Curve AV B is to the Time of the Fall along E V, as the Arch E H V F to the Diameter E V. But the Time of the Fall along E V is to the Time of the Fall along L B, or E G, as E V is to E F. Therefore *ex æquo*, the Time in which a Body describes AV B is to the Time of the Fall along L B, as the Arch E H V F to the Subtense E F; that is, as the right Line A M to M B. Again, the Time of the Fall along L B is to the Time of the Fall along A B, as L B to A B. Therefore the Ratio of the Time in which the Body describes AV B to the Time in which it describes A B, is composed of the Ratio of A M to M B, and of the Ratio of L B to B A; and therefore is equal to the Ratio of $AM \times LB$ to $MB \times BA$. But $AM \times LB$ is equal to $MB \times AC$, because each is equal to the double of the Triangle A B M. Therefore the Time in which a Body falling from rest describes the Curve of the *Cycloid* AV B, is to the Time in which it describes the right Line A B, as $MB \times AC$ is to $MB \times BA$, that is as A C to A B. *Q. E. D.* And the Demonstration proceeds in the same Manner, if the Point B be between A and V.

V. 1. The upper Plate of the *Watch* is A B: The Circular *Balance-Wheel* C D, of which the *Arbor* is E F: The *Spring* turned *Spirally*, G H M, fastned to the *Arbor* of the *Balance-Wheel* in M, and to the piece that is fast to the Watch-Plate, in G, all the *Spires* or Windings of the *Spring* being free without touching any thing. N O P Q is the *Cock*, in which one of the *Pivots* of the *Balance-Wheel* turns; R S, is one of the *Indented-Wheels* of the *Watch* having a *Balancing* Motion, which the *Balance-Wheel* gives to it. And this *Wheel* R S, catches in the *Pinion* T, which holds on the *Arbor* of the *Balance*, of which by this means the Motion is entertained as much as is necessary. These *Watches* are exact for the Pocket, and when made greater, will be useful to find the *Longitudes* both by Sea and Land.

Exact Portable Watches; by M. Huygens. n. 112. p. 272. *Fig.* 163. Mar. An. 1675.

2. The Principle I thought upon some Years ago for making exact *Portable Watches*, is altogether different from that of M. *Huygens:* his depending upon a physical Observation, but mine upon a mere mechanical Reflection; which hath not been taken notice of for want of the *Art* of *Combination*, the use of which is far more general than that of *Algebra*. For, having considered

By Dr. Goth. Guil. Leibnitz. n. 113. p. 285. Apr. An. 1675.

with my ſelf, that a *Spring* being bent to the ſame Degree, will always unbend itſelf in the ſame Time, provided it find the ſame freedom of unbending it ſelf ſuddenly; I inferred from thence, that there might be imployed two ſuch one of which ſhould play, whilſt the *Firſt Mover* of the *Watch* did bend the other again.

Fig. 164. Theſe Thoughts I have executed in the following manner: Let A B be one of the *Watch-Plates*, C and M, two indented Barrels wherein the ſmall *Springs* are incloſed. The Teeth of the Barrels catch thoſe of the Pinions *d d*, which carry the Balances *e e*; and other Teeth of the ſaid Barrels are catched by thoſe of the interrupted Wheel F G. Now let us imagine, that this Wheel F G, being moved towards H F, by the force of the *Firſt Mover* of the *Watch*, and turning the Barrel C, bends the *Spring* incloſed in it, and ſtops with the Barrel as ſoon as it hath bent this *Spring*. This piece which ſerves to ſtop, is eaſy, and hath not been thought neceſſary to be marked here, to avoid embarraſſing the Figure. But whilſt one indented part of the interrupted Wheel F G, *viz.* F, turns the Barrel C, the empty part, oppoſed thereunto, which is G, anſwers to the other Barrel M, and gives Liberty to the *Spring*, it incloſeth, to unbend itſelf. Thus whilſt the *Movement* of the *Watch* bends the ſmall *Spring* of the Barrel C, in the ſame Time the ſmall *Spring* of the other Barrel M, unbends of itſelf: I ſay, in the ſame Time, except the *Spring* C, ſhall have done bending a little ſooner than the *Spring* M ſhall have unbent itſelf: So that the *Spring* C, being bent, and the Wheel F G ſtopped; both of them ſtay in this Poſture, till the *Spring* M, when it ſhall be quite unbent, doth, at the end of its Motion, touch a Piece which delivers it. And then the *Spring* C unbends of itſelf in its turn; the Teeth of the interrupted Wheel, which continues its Motion the ſame way as before, ſince 'tis delivered, not being any more able to hinder it thereform, becauſe the Barrel C, doth now meet with the empty part H, of the ſaid Wheel. But before it hath done with unbending it ſelf, the indented part L, being oppoſite to the empty part H, that turns the Barrel M, bends its *Spring* again, and having done ſo, ſtops with it; whilſt the *Spring* C, making an end of unbending itſelf, delivers them by a reciprocal good Office, and renders to the *Spring* M, the ſame Services which it had received from it, with an Expectation of receiving the like again.

Which being well conſidered, 'tis manifeſt, That the ſame *Alternative Motions* will continue always: That the Periods, taken from the very Moment of that one *Spring* begins to unbend, until the Moment it once unbends itſelf again, will always be of *equal Duration*, tho' the two ſmall *Springs* be not equally ſtrong: That the Balance of ſuch a *Watch* will be double, and may be charged more or leſs, and receive delay, by advancing or recoiling along the two Arms two equal Weights, Counter-balancing one another, that ſo the Change of the Situation may not at all prejudice the *Equality* of the *Watch*. For the reſt, we may in this kind of *Watches* ſpare the *Fuſee*, and conſequently the String or Chain. 'Tis alſo eaſy to judge, that ſuch *Watches* as theſe may be of a Size ſufficiently ſmall; that they will make no more Noiſe than ordinary *Watches*; that they will be as exact as *Pendulums*, and ceaſe not to go whilſt they are Winding up. And tho' the Motion of the Watch Wheels

may

may be altered by many Accidents, yet the *Periods* of the *ſmall Springs* will not be concerned in all or any of them, provided the Motion of the Watch Wheels have always more Strength than it needs to bend them again; which is in our Power.

The Objections that have been made againſt this Contrivance, if employed for *finding Longitudes*, are theſe; that toſſing of Ships would ſhake the *Springs* as well as other pieces; that Ruſt would ſpoil them, ſince the ſaltiſh Humidity of the Sea in remote Voyages, ſpares not the very Needles of Compaſſes tho' incloſed in Boxes; that the Changes of Seaſons and Climates will ſenſibly alter the *Springs*, eſpecially the great Heats or Rains within the Tropicks, which at length will ſomewhat untemper the Steel; as is confirmed by the Experiments of the *illuſtrious Academy of Florence*, ſhewing how eaſily that Heat and Cold do change *ſlender Springs:* Beſides that, the Air more or leſs condenſed will alſo more or leſs reſiſt the Motion of the Balance. To which may be added, that *Springs* by working are weakened: and laſtly, That there will be always ſome little *Friction*, that will make the ſeveral pieces go more or leſs eaſily, and that even in length of time they will wear out.

But I anſwer, That all theſe Defects, that proceed from the Imperfection of the Matter, may be ſurmounted by a general Remedy, without Examining them here in particular: And that is, That for executing it in great, we make uſe of maſſy Springs, as are thoſe of Croſs-Bows, we being Maſters of them, not wanting Force or Place in a Ship, to govern a great Weight that may ſerve to bend them continually again. Now theſe *maſſy Springs* that may be ſo great, and their Reſtitution ſo ſpeedy, by augmenting their Number, that all the above-named Defects will have no conſiderable Proportion to this Strength, and the Aggregate of their Repetitions will not be ſenſible till after a very long time. And 'tis eaſy to *Demonſtrate*, That by augmenting the bigneſs of the Engine, and the force of the *maſſy Springs*, we may make the Error as ſmall as we will, provided we paſs not the bounds of Conveniency, and content our ſelves with Exactneſs ſufficient for their chief End, *viz.* For finding the *Longitudes*.

A Clock Aſcendent *upon an inclined Plane; by* M. de Gennes. n. 140. *p.* 1006. *Fig.* 165. July An. 1678.

VI. The Circle, F G H, being placed upon a *Plane inclined* A B, is divided into two unequal Parts by the Line G I. To reſtore to the leſs Secture its *Æquilibrium*, there is faſtened to the Extremity of the *Radius* D F, a Weight F, which is ſufficiently heavy to recover what the leſſer Secture loſes by its Situation. That a Wheel or Clock may thus ſtand not only in *Æquilibrium*, but alſo aſcend upward, there is placed in the middle of the *Clock* a *Drum*, which incloſes the *Spring* of the *Pendulum*; upon which *Drum* is faſtened the *Radius* D F. For thus the *Spring* being mounted, enforces the *Drum* to turn, and ſo to raiſe the Weight, which it cannot raiſe without its becoming more heavy, in regard that coming to the Point E, it is farther from the Center, than when it was in F, and thus all the *Wheel* turns on that Side as the *Spring* gives way.

 VII. Altho'

A Clock Descendent on a Plane Inclined; by Mr. Maurice Wheeler. n. 161. p. 647. May An. 1684.

VII. Altho' the *Marquis* of *Worcester* is said to have contrived a *Watch* that should *Move* upon a *Declivity*, and M. *de Gennes* has given some Account of a *Clock Ascendent on a Plane inclined*; yet neither of them, nor any like them, was ever seen by me, and for ought I could ever learn, the *Reason* of their *Motions* remains to this Hour as great a Secret, as if they had never been. I shall therefore give an Account of a *Movement*, which I have design'd to measure Time after a peculiar Manner.

Fig. 166.

1. The exteriour Structure of it is a circular Body of $3\frac{1}{2}$ Inches Diameter, consisting of two Plates measured by the same *Radius*, and fixed in a parallel Position to each other by the Hoop *b*, the breadth of which is about an Inch. This Hoop and the two Plates form the Case of the *Movement*; of which, that which appears in the Front, is towards the Verge thereof inscribed with a *Horary Circle*, the Divisions whereof answer the Hours of a natural Day. The deep Shades within this Circle are intended to represent a Concave, of near half a Inch deep; and the Prominence *g*, in the middle of this Concave, is a Hemisphere of Brass or Silver, riding loosly on a Pin, which lies hid, and is the *Axis* of the *Movement*. The upper half of this Hemisphere is hollow, but the nether filled with Lead; and the small Gentleman that sits thereon, does with an erected Finger perform the Office of an Index. But this being only for Ornament, you may substitute in the room thereof any other Index, provided the *Axis* whereon it is supported, move freely in the Hole H, and the lower part thereof HL, so far preponderate to HP, as always to keep it Pendulous, with its Point to the Vertical Hour.

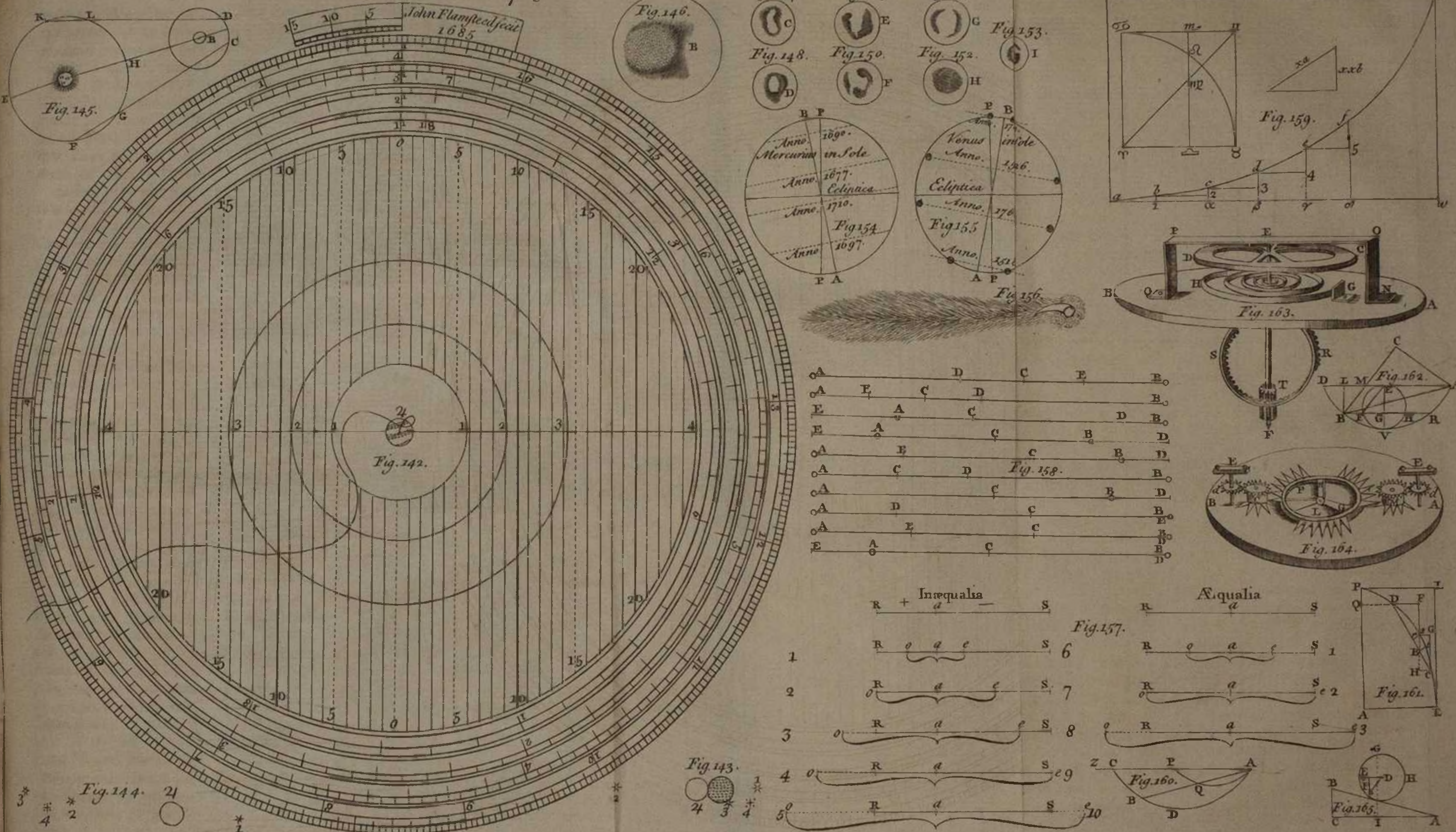
Fig. 169.

Fig. 166.

2. For the manner of its Motion, as far forth as it appears outwardly; it is thus: SE represents a Board or Shelf, of a straight and even Surface, about 6 Foot long, and so thick as not to be apt to cast with change of Weather; nor to grow camber under a small Weight; on this is the *Movement* placed, and here to perform its Course; and therefore I call it the *Stage* of the *Movement*. This *Stage* is raised at the end S, about 10 *Deg.* above the Horizon or Line of Level HE; but this Angle of its Declivity DEH, is variable. The two Plates which form the *Case* of the *Movement*, are to be extant all round without the Hoop *b*, $\frac{1}{4}$ of an Inch, and the Edges of them lightly indented; that while the *Movement* descends upon the *Stage*, it may turn only, and not slide. The *Movement* being placed as high as it may, near the point S, shall move downward towards E, with that slowness, as to finish one entire Revolution in 24 hours; and while it does so, the Divisions on the *Horary Circle* (or Dial-Plate) successively Culminating over the Point of the Index (which is always to keep the same Position) will shew the Hours of the Day and Night. And when by several repeated Revolutions, it has measured the length of its *Stage*, it is to be replaced at S, as before; which may be done in less than half the time you are Winding up a Watch; and if the *Stage* be 6 Foot long, no oftener than once in a whole Week.

3. The way of Adjusting the Motion to the exact Measure of an Hour, and Rectifying its Errors, is thus: *viz.* By the turning a Skrew inserted at S, the *Stage* may be elevated or depress'd, and accordingly the *Movement* will go Faster or Slower: Faster, if raised up, and Slower, if let down; and

Plate IV. Vol. 1. pag. 468.



and by making the *Horary Circle* moveable, and Inserting ſeveral ſmall Boſſes or Buttons, here and there upon the Verge thereof, it may with an eaſy touch of the Finger be moved to the right and left as there ſhall be Occaſion, till the juſt Time be brought to the Point of the *Suſpended Index*.

The Reaſon of this *Movement* may be thus explained: 1. Let the Circle L O D N, repreſent any Circular Body, whoſe Centers both of Gravity and Magnitude are coincident at M. Let this Circular Body be placed upon ſome Level Plane G G; and then it is evident that the Angle of its Contact with that Plane at *a*, will alſo be the Point of its Libration, and conſequently it muſt reſt there: Becauſe the Moment and Impediment are equal. Fig. 167.

2. Let D E repreſent a Deſcending Plane, making an Angle of Contact with this Circular Body at *b*: and here 'tis manifeſt it cannot reſt: becauſe the Line of Direction *r a*, which (while it inſiſted upon a Level) divided the Circular Body by the Centers of Magnitude and Gravity into parts Æquiponderate, is now removed to L D; which Line L D falling without, or beſide, the Center M, evidently deſtroys the Æquipoiſe of its Parts, and therefore muſt leave it to tumble down towards E. For here the Moment is greater than the Impediment. The Reaſon therefore of its Deſcent now being the Over-balance of the Parts L N D, to the remaining Section L D O, it muſt neceſſarily follow,

3. That if ſome Weight equal to the Exceſs of L N D, above L O D, were affix'd to the Limb of the Quadrant O *a*, as at P: then the Circular Body would reſt as quietly at *b*, as it did before at *a*. The Suppoſition cannot be denied, and the conſequence is unavoidable, becauſe $LDO + P = LND$, the Impediment is equal to the Moment.

Let then the Numbers, 1, 2, 3, 4, repreſent a Train of Wheel-work, wherein there is no material difference from what is found in a common Watch; only the numbers of the Teeth on the Wheels and Pinions are to be ſo calculated, that the Motion of the Whole Train may correſpond to the aſſigned Revolution of the Body of the *Movement* which is to be once in 24 Hours: It would be expedient alſo, That a *Spiral Spring* were applied to its *Balance*, as in later *Movements*, is uſual; but of a *Fuſee* here's no need, for the Turns of the Body of the *Movement* as it deſcends upon the *Stage*, anſwer all the Intentions of a String or Chain; and the *Contranitence* of the Weight P, to the Exceſs of LED, above LQD, ſerves inſtead of a Perpetual Spring; and the *Movement* wants only a *Perpetual Deſcent*, to make its *Motion* ſo. And whereas the great Wheel in ordinary *Movements*, is placed as near the edge of the Framing Plate *f r*, as it may be; here it muſt (with its *Axis* or Arbor M) poſſeſs the Center of the *Movement*: Becauſe this Wheel is to carry the Weight of *Power* P, by the *Vectis* M P, and that Weight P muſt always keep an Equidiſtance from the Center of the *Movement*; that while the Body thereof (*i. e.* of the *Movement*) performs its Revolutions; the ſaid Weight P, and the great Wheel (to which it is affixed) may, without any conſiderable Variation, continue in, or near the ſame Poſition, wherein they now are. Now ſuppoſe this Weight P, with its *Vectis* M P, to be taken quite out of the *Movement*, and the Body of the *Movement* to be placed on a Horizontal Plane H H, its Point of Contact in that Plane is T; where it ſhould, but cannot, reſt; Fig. 168.

becauſe

because the Weight of that part of the Train, marked with the Numbers, 2, 3, 4, removes the Center of Gravity from M; and therefore on the opposite Part of the *Movement* as about C Q, the Inside of the Hoop, which forms the Case, is to be loaded with a thin Lining of Lead, which may be a Counterpoise to that part of the Train; that so, the whole Body of the *Movement*, together with all its Furniture, within and without (excepting only P, with its *Vectis*) may, on that Horizontal Plane, or while it rides upon its own *Axis*, rest indifferently in any Point. This reducing of the *Movement* to an Equilibration of all its Parts in the Center M, must be performed *Tentando*, *i. e.* by Rasping the Lead at C Q, as much and in such Places as is needful; which, to an Artificer of Ordinary Sagacity, will not be at all difficult.

The Center of Gravity being thus reduced to M, replace the Weight P,
Fig. 170. by the Hole H, on the Arbor of the Central-Wheel M. Then let the Body of the *Movement* be placed on the Declivity DE, and supposing P+L Q D
Fig. 168. =LDE, then the Body must needs rest there: But because the Weight P, is not now fix'd to any part of the Quadrant Q D, but hangs upon the Train of Wheel-work 1, 2, 3, 4, it evidently follows, That if the Power thereof be superior to the Resistance of the Train, then the whole Body of the *Movement* must needs descend, towards E. By this you see there are two Offices assigned to the Weight or Power P. The First is, to be a Counterpoise to the excess of the Weight of L E D, above L Q D.

The Second is, that it be of force sufficient to put the Train into a Motion so adjusted, as may exactly comport with the time assigned for the Revolution of the whole Body. So that if there be any Difficulty remaining, it consists in such an exact Stating of the Weight and Power of P, that it may adequately serve both these Intentions. Now how easy this is, will be manifest from these Propositions following.

1. That whatever the intrinsick Weight of P shall be, as suppose it 4 Ounces *Troy*; yet the Power of that Weight will be augmented or diminished according to the different Degrees of its Elevation in the Quadrant TQ. Thus considering P M, as a *Vectis*, its *Hypomochlium* is M, the Point where it exerts its Power on the Train, is at V; I say then, whatever Power it has upon the Point V, in its present Elevation of 45 *Deg.* it will acquire a greater by being raised to 50, 55, *&c.* and the greatest of all in 90 *Deg.* at Q: And on the contrary, let it sink to 40°, 35°, *&c.* its Power upon the Point V, will still be diminished, insomuch that in T, it will be utterly extinguish'd. And therefore if P be of a competent Weight (*i. e.* not utterly too light) to *move* the Train at all, it will certainly move it in some Degree of Elevation or other in the Quadrant Q T.

2. If the Weight P be considered as to its Office of being a Counterpoise to the Body of the *Movement*; as I need not to prove, that it will perform this no less, while it hangs by upon the *Vectis* M P, than if it were fast riveted in the same place to the Case of the *Movement*: so, in what Point of the Quadrant soever it will move the Train, it may be also a Counterpoise to the Body of the *Movement*. For,

1. At

1. At what Point soever of the Circle L E T Q, the Line of Declivity D E, makes an Angle of Contact; on the same Point will the *Diameter* S D fall at Right Angles with D E.

2. The Line of Direction LD, will ever fall upon the Point of Contact D, making an Angle with the Diameter, as S D L.

3. The Angle S D L, will be always equal to D E H, *i. e.* As great as is the Elevation of the Line of Declivity D E, above the Horizontal E H; so great will the Angle of Distance be between the Diameter S D, and the Line of Direction L D. *Fig.* 166.

4. The greater the Angle of Declivity is, the less will be the Section L Q D; and so on the contrary, the less that Angle is, the greater the Section. And therefore,

5. The Excess of the Weight of L E D, above L Q D, must be also greater, by Raising up the *Stage* with the Skrew at S; and that Excess less by Skrewing it down.

6. The Lighter that part of the Body is, which is represented by the Section L Q D, the more Heavy ought the Counterpoise P, to be; and that either in its own Intrinsick Weight, (in Ounces and Parts of Ounces) or else in its Potential Weight, by being raised higher in the Quadrant Q T.

7. The Skrewing up the *Stage* of the *Movement* at S, will raise the Counterpoise higher in the Quadrant Q T, by *Prop.* 3. and therefore potentially heavier. And from hence appears (I take it most clearly) both the Reason of the due Adjustment of the Motion of the Train to the exact Measure of an Hour, and what Weight is to be assigned to P, that moves it; and that we are not confined to Scruples and Grains, but are allowed such a considerable Latitude, as it is not easy to err therein.

Having therefore set the *Stage* (by the help of the arched Skrew) at the Elevation of above 10 *Deg.* place the *Movement* thereon, and try what Weight, hanging at the end of the *Vectis* M P, will stir the Train; mean while holding the *Movement* with the Hand in such a Position, as the *Vectis* may make an Angle of about 30 *Deg.* with the Perpendicular M T: then let the *Movement* loose, to undulate upon the *Stage*; and when the Vibration ceases, observe to what Degree of the Quadrant the *Vectis* Points, and at the same time mind the Pulses of the Balance. If at this Observation, the Weight lies low, (as for instance, between 25 and 35 *Deg.* of the Quadrant) and the Beats of the Balance are guessed to be not much different from their due time, the Weight P, is well enough proportioned. But if it chance to be much heavier than is absolutely needful, that Excess will be moderated by Skrewing down the *Stage*; and if it be not absolutely too light, its Defect will be compensated, by Skrewing the *Stage* higher. Therefore of these two Extremes, choose the former; for the fewer Degrees that P arises in the Quadrant, beyond what is absolutely necessary, it will (for Reasons very obvious) be so much the better.

The Effects of Gravity in the Descent of Heavy Bodies, and the Motion of Projects, by Mr. Halley. *n.* 179. p. 3. Jan. An. 1686.

VIII. *Des Cartes* his Notion, I must needs confess to be to me incomprehensible, while he will have the Particles of his *Cælestial Matter*, by being reflected on the Surface of the Earth, and so ascending therefrom, to drive down

down into their Places thoſe Terreſtrial Bodies they find above them: This is, as near as I can gather, the Scope of the 20, 21, 22, and 23 *Sections* of the laſt Book of his *Principia Philoſophiæ*; yet neither he nor any of his Followers can ſhew, how a Body ſuſpended *in Libero Æthere*, ſhall be carried downwards by a continual Impulſe tending upwards, and acting upon all its Parts equally: And beſides, the Obſcurity wherewith he expreſſes himſelf, particularly, *Sect.* 23, does ſufficiently argue, according to his own Rules, the *Confuſed Idea* he had of the thing he wrote.

Others, and among them, Dr. *Voſſius*, aſſerts the Cauſe of the Deſcent of Heavy Bodies, to be the *Diurnal Rotation* of the *Earth* upon its *Axis*, without conſidering, that according to the Doctrine of Motion fortified with Demonſtration, all Bodies moved in *Circulo*, would recede from the Center of their Motion; whereby the contrary to *Gravity* would follow, and all looſe Bodies would be caſt into the Air in a Tangent to the Parallel of Latitude, without the Intervention of ſome other Principle to keep them faſt, ſuch as is that of *Gravity*. Beſides the Effect of this Principle is throughout the whole Surface of the Globe found nearly equal, and certain Experiment ſeems to argue it rather leſs near the Equinoctial, than towards the Poles; which could not be by any means, if the *Diurnal Rotation* of the *Earth* upon its *Axis* were the Cauſe of *Gravity*; for where the Motion was ſwifteſt, the Effect would be moſt conſiderable.

Others aſſign the *Preſſure* of the *Atmoſphere* to be the Cauſe of this *Tendency* towards the *Center* of the *Earth*; but unhappily they have miſtaken the Cauſe for the Effect, it being from undoubted Principles plain, that the Atmoſphere has no other Preſſure, but what it derives from its *Gravity*; and that the Weight of the upper Parts of the Air, preſſing on the lower Parts thereof, do ſo far bend the Springs of that Elaſtick Body, as to give it a Force equal to the Weight that compreſſed it, having of itſelf no Force at all: And ſuppoſing it had, it will be very hard to explain the *Modus*, how that Preſſure ſhould occaſion the *Deſcent* of a Body circumſcribed by it, and preſſed equally above and below, without ſome other Force to *Draw* or *Thruſt* it downwards. But to demonſtrate the contrary of this Opinion, an Experiment was long ſince ſhewn before the *Royal Society*; whereby it appeared, that the Atmoſphere was ſo far from being the Cauſe of *Gravity*, that the Effects thereof were much more vigorous where the Preſſure of the Atmoſphere was taken off; for a long Glaſs-Receiver, having a light Down-Feather included, being *Evacuated* of Air, the Feather, which in the Air would hardly ſink, did *in Vacuo* deſcend with nearly the ſame Velocity as if it had been a Stone.

Some think to illuſtrate this Deſcent of *Heavy* Bodies, by comparing it with the Virtue of the *Loadſtone*; but ſetting aſide the Difference there is in the manner of their Attractions, the *Loadſtone* drawing only in and about its Poles, and the Earth near equally in all Parts of its Surface, this Compariſon avails no more than to explain *Ignotum per æquè Ignotum*.

Others aſſign a certain *Sympathetical Attraction* between the Earth and its Parts, whereby they have, as it were, a Deſire to be united, to be the Cauſe we enquire after: but this is ſo far from explaining the *Modus*, that it it is little more, than to tell us in other terms, that *Heavy* Bodies descend, because they descend.

But tho' the *Efficient Cause* of *Gravity* be so obscure, yet the *Final Cause* thereof is clear enough; for it is by this single Principle that the Earth and all the Celestial Bodies are kept from Dissolution: the least of their Particles not being suffered to recede far from their Surfaces, without being immediately brought down again by Virtue of this *Natural Tendency*, which, for their Preservation, the Infinite Wisdom of their *Creator* has ordained to be towards each of their Centers; nor can the Globes of the *Sun* and *Planets* otherwise be destroyed, but by taking from them this power of keeping their Parts united.

The Properties of Gravity. Ibid. p. 6.

The Affections or *Properties* of *Gravity*, and its manner of acting upon Bodies Falling, have been in a great measure discovered, and most of them made out by Mathematical Demonstration in this our Century, by the accurate Diligence of *Galilæus*, *Torricellius*, *Huygenius*, and others; and now lately, by our worthy Countryman Mr. *If. Newton*. Which *Properties* I shall here enumerate.

1. The first *Property* is, That by this *Principle* of *Gravitation*, all Bodies do *descend* towards a *Point*, which either is, or else is very near to the *Center* of *Magnitude* of the *Earth* and *Sea*, about which the Sea forms itself exactly into a Spherical Surface, and the Prominences of the Land, considering the Bulk of the whole, differ but insensibly therefrom.

2. That this *Point*, or *Center* of *Gravitation*, is *fix'd* within the Earth, or at least has been so, ever since we have any Authentick History: For a Consequence of its Change, tho' never so little, would be the over-flowing of the Low-lands on that side of the Globe towards which it approached, and the leaving new Islands bare on the opposite side, from which it receded; but for this Two Thousand Years it appears, that the Low Islands of the *Mediterranean* Sea (near to which the Antientest Writers lived) have continued much at the same Height above the Water, as they now are found; and no Inundations or Recesses of the Sea arguing any such Change, are recorded in History, excepting the *Universal Deluge*, which can no better way be accounted for, than by supposing this *Center* of *Gravitation* removed for a time, towards the Middle of the then Inhabited Parts of the World; and a Change of its place, but the two thousandth Part of the Radius of this Globe, were sufficient to bury the Tops of the Highest Hills under Water.

3. That in all Parts of the Surface of the Earth, or rather in all Points equidistant from its Center, the *Force* of *Gravity* is nearly *Equal*; so that the length of the Pendulum vibrating Seconds of Time, is found in all Parts of the World to be very near the same. 'Tis true, at St. *Helena*, in the Latitude of 16 *Deg.* South, I found that the Pendulum of my Clock, which vibrated Seconds, needed to be made shorter than it had been in *England*, by a very sensible Space (but which at that time I neglected to observe accurately) before it would keep Time; and since, the like Observations have been made by the *French Observers* near the Equinoctial: yet I dare not affirm, that in mine it proceeded from any other Cause, than the great Height of my Place of

Obſervation above the Surface of the Sea, whereby the *Gravity* being diminiſhed, the Length of the Pendulum, vibrating Seconds, is proportionably ſhortned.

4. That *Gravity* does *equally affect* all *Bodies*, without regard either to their Matter, Bulk, or Figure; ſo that the Impediment of the Medium being removed, the moſt compact and moſt looſe, the greateſt and ſmalleſt Bodies, would deſcend the ſame Spaces in equal Times; the Truth whereof will appear from the Experiment I before cited. In theſe two laſt Particulars, is ſhewn, the great Difference between *Gravity* and *Magnetiſm*, the one affecting only Iron, and that towards its Poles, the other all Bodies alike in every part. As a *Corollary*, from hence it will follow, That there is no ſuch thing as *Poſitive Levity*, thoſe things that appear *Light*, being only comparatively ſo; and whereas ſeveral things riſe and ſwim in Fluids: 'tis becauſe, Bulk for Bulk, they are not ſo heavy as thoſe Fluids; nor is there any Reaſon why *Cork*, for inſtance, ſhould be ſaid to be *Light* becauſe it *ſwims* on *Water*, any more than *Iron*, becauſe it *ſwims* on *Mercury*.

5. That this *Power Increaſes* as you *Deſcend* to, and *Decreaſes* as you *Aſcend* from the *Center*, and that in the Proportion of the *Squares* of the *Diſtances* therefrom *Reciprocally*, ſo as at a double Diſtance to have but a quarter of the Force: This Property is the Principle on which Mr. *Newton* has made out all the Phænomena of the Celeſtial Motions, ſo eaſily and naturally, that its Truth is paſt diſpute. Beſides that, it is highly rational, that the *Attractive* or *Gravitating Power* ſhould exert itſelf more vigorouſly in a ſmall Sphere, and weaker in a greater, in proportion as it is contracted or expanded; and if ſo, ſeeing that the Surfaces of Spheres are as the Squares of their *Radii*, this Power at ſeveral Diſtances will be as the Squares of thoſe Diſtances *Reciprocally*; and then its whole Action upon each Spherical Surface, be it great or ſmall, will be always equal. And this is evidently the Rule of *Gravitation* towards the Centers of the *Sun*, *Jupiter*, *Saturn*, and the *Earth*, and thence reaſonably inferred, to be the general Principle obſerved by Nature in all the reſt of the *Celeſtial Bodies*.

Theſe are the principal Affections of *Gravity*, from which the Rules of the *Fall* of *Bodies*, and the *Motion* of *Projects*, are Mathematically deducible. Mr. *Iſ. Newton* hath ſhewed how to define the Spaces of the Deſcent of a Body, let fall from any given Height, down to the Center, ſuppoſing the *Gravitation* to increaſe, as in the Fifth Property; but conſidering the Smallneſs of Height, to which any *Project* can be made to aſcend, and over how little an Arch of the Globe it can be caſt by any of our Engines, we may well enough ſuppoſe the *Gravity Equal* throughout, and the *Deſcents* of *Projects* in *Parallel Lines*, which, in Truth, are towards the *Center*; the Difference being ſo ſmall as by no means to be diſcovered in Practice.

Propoſitions concerning the Deſcent of Heavy Bodies, and the Motion of Projects.
Ibid. p. 9.

Prop. I.] *The Velocities of falling Bodies, are Proportionate to the Times from the Beginning of their Falls.*

This follows, for that the Action of *Gravity* being continual, in every ſpace of Time, the falling Body receives a new Impulſe, equal to what it had before,

fore, in the ſame Space of Time, received from the ſame *Power:* For inſtance, in the Firſt Second of Time, the falling Body has acquired a *Velocity*, which in That Time would carry it to a certain Diſtance, ſuppoſe 32 Foot, and were there no new Force, would deſcend at that Rate with an Equable Motion; but in the next Second of Time, the ſame *Power* of *Gravity* continually Acting thereon, ſuperadds a New *Velocity* equal to the former; ſo that at the end of two Seconds, the *Velocity* is double to what it was at the end of the firſt; and after the ſame manner may it be proved to be Triple, at the end of the Third Second, and ſo on. Wherefore the *Velocities of Falling Bodies, are proportionate to the Times of their Falls.* Q. E. D.

Prop. II.] *The Spaces deſcribed by the Fall of a Body, are as the Squares of the Times from the Beginning of the Fall.*

Demonſtration.] Let A B repreſent the Time of the Fall of a Body, B C, perpendicular to A B, the Velocity acquired at the End of the Fall, and draw the Line A C; then divide the Line A B, repreſenting the Time, into as many equal Parts as you pleaſe, as *b*, *b*, *b*, *b*, *&c.* and through theſe Points draw the Lines, *bc*, *bc*, *bc*, *bc*, *&c.* parallel to B C; 'tis manifeſt that the ſeveral Lines, *bc*, repreſent the ſeveral *Velocities* of the Falling Body, in ſuch parts of the Time, as A *b*, is of A B, by the *Former Propoſition.* It is evident likewiſe that the Area, A B C, is the Sum of all the Lines *bc*, being taken according to the *Method* of *Indiviſibles* infinitely many; ſo that the Area A B C, repreſents the Sum of all the *Velocities* between none and BC, ſuppoſed infinitely many; which Sum is the Space deſcended in the Time repreſented by A B. And by the ſame Reaſon, the Areas A *bc*, will repreſent the Spaces deſcended in the Times A *b*; ſo then the Spaces deſcended in the Times A B, A *b*, are as the Areas of the Triangle, A B C, A *b c*, which by the 20*th* of the 16*th* of *Euclid* are as the *Squares* of their *Homologous Sides* A B, A *b*, that is to ſay, of the *Times:* Wherefore the *Deſcents* of *Falling Bodies*, are as the *Squares* of the *Times* of their *Fall.* Q. E. D. Fig. 171.

Prop. III.] *The Velocity which a falling Body acquires in any ſpace of Time, is double to that, wherewith it would have moved the Space deſcended by an Equable Motion, in the ſame Time.*

Demonſtration.] Draw the Line E C parallel to A B, and A E parallel to B C, and compleat the *Parallelogram* ABCE; it is evident that the *Area* thereof may repreſent the Space, a Body moved *Equably* with the *Velocity* BC, would deſcribe in the Time AB; and the *Triangle* A B C, repreſents the Space deſcribed by the Fall of a Body, in the ſame Time AB, by the *Second Propoſition.* Now the *Triangle* A B C, is Half of the *Parallelogram* A B C E; and conſequently the Space deſcribed by the *Fall*, is Half what would have been deſcribed by an *Equable Motion* with the *Velocity* B C, in the ſame Time: wherefore the *Velocity* BC, at the End of the Fall, is *Double to that Velocity*, which in the Time A B, would have deſcribed the Space fallen, repreſented by the *Triangle* ABC, with an *Equable Motion.* Q. E. D.

Prov. IV.] *All Bodies on or near the Surface of the Earth, in their Fall, descend so, as at the End of the First Second of Time, they have described* 16 *Feet, one Inch*, London *Measure, and acquired the Velocity of* 32 *Feet, two Inches, in a Second.*

This is made out from the 25*th Prop. Par.* 2. *Porol. Oscill. Hugen.* wherein he demonstrates the Time of the least Vibrations of a Pendulum, to be to the Time of the Fall of a Body, from the Height of Half the Length of the Pendulum, as the Circumference of a Circle to its Diameter: whence, as a *Corollary*, it follows, That as the Square of the Diameter to the Square of the Circumference, so half the Length of the Pendulum vibrating Seconds, to the Space described by the Fall of a Body in a second of Time: and the Length of the Pendulum vibrating Seconds, being found 39,125, or $\frac{1}{8}$ Inches, the Descent in a Second will be found, by the aforesaid Analogy, 16 Foot and one Inch: and by the *Third Proposition*, the Velocity will be double thereto; and near to this it hath been found by several Experiments, which by reason of the Swiftness of the Fall, cannot so exactly determine its Quantity.

From these Four Propositions, all Questions concerning the *Perpendicular Fall* of Bodies are easily solved, and either *Time*, *Height*, or *Velocity* being assigned, one may readily find the other two. From them likewise is the *Doctrine of Projects* deducible, assuming the two following *Axioms: viz.*

1. *That a Body set a moving, will move on continually in a Right Line with an Equable Motion, unless some other Force or Impediment intervene, whereby it is accelerated, or retarded, or deflected.*

2. *That a Body being agitated by two Motions at a Time, does by their Compounded Forces pass thro' the same Points, as it would do, were the two Motions divided and acted successively.* As for instance,

Fig. 172. Suppose a Body moved in the Line G F, from G to R, and there stopping, by another Impulse suppose it moved in a Space of Time equal to the former from R towards K to V; I say the Body shall pass thro' the point V, tho' these two several Forces acted both in the same Time.

Prop. V.] *The Motion of all Projects is in the Curve of a Parabola.*

Demonstration.] Let the Line G R F be the Line in which the *Project* is directed, and in which by the *first Axiom* it would move equal Spaces in equal Times, were it not deflected downwards by the Force of *Gravity*. Let G B be the Horizontal Line, and G C a perpendicular thereto. Then the Line GRF, being divided into equal Parts, answering to equal Spaces of Time, let the Descents of the Project be laid down in Lines Parallel to G C, proportioned as the *Squares* of the *Lines*, GS, GR, GL, GF, or as the *Squares* of the *Times*, from S to T, from R to V, from L to X, and from F to B; and draw the Lines T H, V D, X Y, B C, Parallel to G F: I say, the Points T, V, X, B, are Points in the *Curve* described by the *Project*, and that that *Curve* is a *Parabola*. By the *second Axiom* they are Points in the *Curve*; and the Parts of the Descent G H, G D, G Y, G C,=to S T, R V, L X, F B, being as the Squares of the Times (by the *second Prop.*) that is, as the *Squares* of the *Ordinates* H T, D V, Y X, C B, equal to GS, GR, GL, GF, the Spaces measured

measured in those Times; and there being no other *Curve* but the *Parabola*, whose Parts of the *Diameter* are as the *Squares* of the *Ordinates*, it follows, that the *Curve* described by a *Project* can be no other than a *Parabola:* And saying, as RV, the Descent in Time, to GR, or VD, the direct Motion in the same Time; so is VD, to a third Proportional; that Third will be the Line called by all Writers of *Conicks*, the *Parameter* of the *Parabola* to the *Diameter* GC; which is always the same in *Projects* cast with the same *Velocity:* and the *Velocity* being defined by the number of Feet, moved in a Second of Time, the *Parameter* will be found by dividing the Square of the *Velocity* by 16 Feet 1 Inch, the Fall of a Body in the same Time.

Lemma. [*The Sine of the Double of any Arch, is equal to twice the Sine of that Arch into its Co-Sine, divided by Radius; and the versed Sine of the Double of any Arch, is equal to the Square of the Sine thereof divided by Radius.*

Let the Arch BC be double the Arch BF, and A the Center; draw the Radii AB, AF, AC, and the Chord BDC, and let Fall BE, perpendicular to AC; and the Angle EBC, will be equal to the Angle BAD, and the Triangle BCE, will be like the Triangle ABD; wherefore it will be, as AB to AD; so BC, or twice BD, to BE; that is, as *Radius* to *Co-Sine*, so *twice Sine* to *Sine of the double Arch*; and as AB to BD, so twice BD or BC, to EC; that is, as *Radius* to *Sine*, so *twice that Sine* to the *versed Sine of the double Arch:* which two *Analogies* resolved into Equations, are the *Propositions* contained in the *Lemma* to be proved. *Fig. 173.*

Prop. VI.] *The Horizontal distance of Projections made with the same Velocity, at several Elevations of the Line of Direction, are as the Sines of the doubled Angles of Elevation.*

Let GB, the horizontal Distance be $=z$, the Sine of the Angle of Elevation, FGB, be $=s$, its Co-sine $=c$, Radius $=r$, and the Parameter $=p$. It will be, as c to s, so z to $\frac{sz}{c}=$ FB $=$ GC, and by reason of the *Parabola* $\frac{psz}{c}$ $=$ to the Square of CB, or GF. Now as c to r, so is z to $\frac{zr}{c}=$ GF, and its Square $\frac{zzrr}{cc}$ will be therefore $=$ to $\frac{psz}{c}$: which *Equation* reduced, will be $\frac{psc}{rr}=z$. But by the former *Lemma* $\frac{2sc}{r}$ is equal to the Sine of the double Angle, whereof s is the Sine: Wherefore 'twill be, as *Radius* to *Sine* of double the Angle FGB, so is half the *Parameter*, to the *horizontal Range* or *Distance* sought: and at the several *Elevations*, the *Ranges are as the Sines of the double Angles of Elevation.* Q. E. D. *Fig. 172.*

Coroll.] Hence it follows, That *half the Parameter is the greatest Random, and that that happens at the Elevation of* 45°, *the Sine of whose double is Radius.*

 Likewise,

Likewise, That *the Ranges equally distant above and below 45° are equal, as are the Sines of all doubled Arches, to the Sines of their doubled Complements.*

Prop. VII.] *The Altitudes of Projections made with the same Velocity, at several Elevations, are as the versed Sines of the doubled Angles of Elevation.*

As c is to s, so is $\frac{psc}{rr}=$ G B, to $\frac{pss}{rr}=$ B F, and V K = R V = $\frac{1}{4}$ B F, the *Altitude* of the *Projection* $=\frac{pss}{4rr}$. Now by the foreging *Lemma* $\frac{2ss}{r}=$ to the *versed Sine* of the *double Angle*; and therefore it will be, as *Radius* to *versed Sine* of double the Angle F G B, so $\frac{1}{4}$ of *Parameter* to the *height* of the *Projection* V K; and so those *Heights* at several *Elevations* are as the said *versed Sines.* Q. E. D.

Coroll.] From hence it is plain, That *the greatest Altitude of the Perpendicular Projection is a 4th of the Parameter, or half the greatest horizontal Range:* The *versed Sine of* 180 Degrees being $=2r$.

Prop. VIII.] *The Lines* G F, *or Times of the Flight of a Project cast with the same degree of Velocity at different Elevations, are as the Sines of the Elevations.*

As c is to r, so is $\frac{psc}{rr}=$ G B (by the 6 *Prop.*) to $\frac{ps}{r}=$ G F, that is, as *Radius* to the *Sine* of *Elevation*, so the *Parameter* to the Line G F; so the *Lines* G F are as the *Sines* of *Elevation*, and the *Times* are proportional to those *Lines:* wherefore the *Times* are as the *Sines of Elevations*; therefore the *Proposition* is manifest.

Prop. IX. Prob. 1.] *A Projection being made, as you please, having the Distance and Altitude, or Descent of an Object, thro' which the Project passes, together with the Angle of Elevation of the Line of Direction; to find the Parameter and Velocity*; that is, (having the Angle F G B, G M,) and M X.

Solution.] As Radius to Secant of F G B, so G M the distance given, to G L;
Fig. 172. and as Radius to Tangent of F G B, so G M to L M. Then L M—M X in *Heights*, or + M X in *Descents:* or else M X—M L, if the *Direction* be *below* the *Horizontal-Line*, is the *Fall* in the *Time* that the direct Impulse given in G, would have carried the Project from G to L=L X=G Y; then by reason of the *Parabola*; as L X, or G Y, is to G L or Y X; so is G L, to the *Parameter* sought. To find the *Velocity* of the *Impulse*, by *Prop.* 2 and 4. find the *Time* in Seconds that a Body would *fall* the Space L X, and by that dividing the Line G L, the Quote will be the *Velocity*, or *Space moved* in a Second sought, which is always a mean *Proportional* between the *Parameter* and 16 Feet, 1 Inch.

Prop. X. Prob. 2.] *Having the Parameter, Horizontal Distance, and Height or Descent of an Object; to find the Elevations of the Lines of Direction necessary* *to hit the given Object*; that is, having GM, MX, and the greateſt Random equal to half the Parameter; to find the Angles FGB.

Let the Tangent of the Angle ſought be $=t$, the horizontal Diſtance GM$=b$, the Altitude of the Object MX$=h$, the Parameter$=p$, and Radius $=r$; and it will be, as r to t, ſo b to $\frac{tb}{r}=$ ML; and $\frac{tb}{r}\mp h$ $\left\{\begin{array}{l}\text{in Aſcents}\\ \text{in Deſcents}\end{array}\right\}=$ LX, and $\frac{ptb}{r}\mp ph=$ GL$q.$ $=$ XY$q.$ *ratione Parabolæ*; *but* $bb+\frac{ttbb}{rr}=$ GL$q.$ (47. 1. *Euclid.*) Wherefore $\frac{ptb}{r}\mp ph=bb+\frac{ttbb}{rr}$; which Equation tranſpoſed, is $\frac{ttbb}{rr}=\frac{ptb}{r}\mp ph-bb$; divided by bb, is $\frac{tt}{rr}=\frac{pt}{br}\mp\frac{ph}{bb}-1$. This Equation ſhews the Queſtion to have two Anſwers, and the Roots thereof are $\frac{t}{r}=\frac{p}{2b}\mp\sqrt{\frac{pp\mp4ph}{4bb}-1}$; from which I derive the following Rule. Divide half the *Parameter* by the *Horizontal Diſtance*, and keep the Quote, *viz.* $\frac{p}{2b}$; then ſay, as Square of the *Diſtance* given to half the *Parameter*, ſo half Parameter $\left\{\begin{array}{c}-\\ +\end{array}\right\}$ double $\left\{\begin{array}{l}\text{Height}\\ \text{Deſcent}\end{array}\right\}$ to the Square of a *Secant* $=\frac{pp+4ph}{4bb}$, the *Tangent* anſwering to that *Secant* will be $\sqrt{\frac{pp\mp4ph}{4bb}-1}$, or rr: ſo then the Sum and Difference of the afore-found *Quote* and this *Tangent*, will be the Root of the *Equation*, and the *Tangents* of the *Elevations* ſought.

Note here, That in *Deſcents*, if the *Tangent* exceed the *Quote*, as it does when ph is more than bb, the *Direction* of the lower *Elevation* will be below the *Horizon*; and if $ph=bb$, it muſt be *directed Horizontal*, and the *Tangent* of the upper *Elevation* will be $\frac{pr}{b}$: *Note* likewiſe, That if $4bb+4ph$ in *Aſcents*, or $4bb-4ph$ in *Deſcents*, be equal to pp, there is but one *Elevation* that can *hit* the *Object*, and its *Tangent* is $\frac{pr}{2b}$; and if $4bb\mp4ph$ in *Aſcents*, or $4bb-4ph$ in *Deſcents*, do exceed pp, the *Object* is without the *Reach* of a *Project* caſt with that *Velocity*, and ſo the thing impoſſible.

From this *Equation* $4bb\mp4ph=pp$, are determined the utmoſt *Limits* of the *Reach* of any *Project*, and the Figure aſſigned, wherein are all the Heights upon each horizontal Diſtance, beyond which it cannot paſs; for by Reduction of that *Equation*, h will be found $=\frac{1}{4}p-\frac{bb}{p}$ in *Heights*, and $\frac{bb}{p}-\frac{1}{4}p$ in *Deſ-*

 cents;

cents; from whence it follows, that all the Points b are in the *Curve* of the *Parabola*, whose *Focus* is the Point from whence the *Project* is cast, and whose *Latus Rectum*, or *Parameter ad Axem* is $= p$. Likewise from the same *Equation* may the least *Parameter* or *Velocity* be found capable to reach the Object proposed; for $bb = \frac{1}{4}pp \mp pb$ being reduced, $\frac{1}{2}p$ will be $= \sqrt{bb \mp bh}$ $\left\{ \begin{array}{l} +h \text{ in } \textit{Ascents} \\ -h \text{ in } \textit{Descents} \end{array} \right\}$ which is the *Horizontal* Range at 45°, that would just reach the *Object*, and the *Elevation* requisite will be easily had; for dividing the so found *Semi-Parameter* by the *Horizontal Distance* given, b, the Quote into *Radius* will be the *Tangent* of the *Elevation* sought.

But if a *Geometrical Construction* of this *Problem* be required, I think I have one, that is as easy as any can be expected, which I deduce from the foregoing *Analytical Solution*, *viz.* $\frac{t}{r} = \frac{p}{2b} \pm \sqrt{\frac{\frac{1}{4}pp \pm ph - bb}{bb}}$; and it is this: Having made the right Angle LDA, make DA, DF$=p$, or greatest *Range*, DG$=b$, the *Horizontal Distance*, and DB, DC$=h$, the
Fig. 174. Perpendicular *Height* of the Object; and draw GB, and make DE $=$ thereto. Then with the Radius AC, and Center E, sweep an Arch, which if the thing be possible, will intersect the Line AD, in H; and the Line DH, being laid both ways from F, will give the Points K, and L, to which draw the Lines GL, GK; I say, the Angles LGD, KGD, are the *Elevations* required for *hitting* the *Object*, B. But *Note*, that if B be below the Horizon, its *Descent* DC$=$DB, must be laid upon A, so as to have AC$=$AD$+$DC. *Note* likewise, that if in *Descents*, DH be greater than FD, and so K fall below D, the Angle KGD shall be the *Depression* below the Horizon.

n. 216. p. 69. When I gave the preceding Solution of this Problem, *viz. To hit an Object*
Mar. An. 1695. *above or below the Horizontal Line, with the greatest Certainty and least Force*, I was not aware, that the *Elevation* there sought did constantly *bisect* the Angle between the Perpendicular and the Object, as is demonstrated from the Difference and Sum of the *Tangent* and *Secant* of any Arch, being always equal to the *Tangent* and *Co-Tangent* of the half Complement thereof to a Quadrant. But having discover'd this, I think nothing can be more compendious, or bid fairer to compleat the *Art* of *Gunnery*, it being as easy to shoot with a *Mortar* at any Object on demand, as if it were on the Level; neither is there need of any Computation, but only simply laying the Gun, to pass in the middle Line between the Zenith and the Object, and giving it its due *Charge*. Nor is there any great need of Instruments for this Purpose: For, if the *Muzzle* of the *Mortar* be turned truly Square to the Bore of the Piece, as it usually is, or ought to be, a Piece of Looking-Glass Plate applied Parallel to the *Muzzle*, will, by its Reflection, give the true Position of the Piece; the *Bombardier* having no more to do, but to look perpendicularly down on the Looking-Glass, along a small Thread with a Plummet, and to raise or depress the *Elevation* of the *Piece*, till the Object appear reflected on the same Point of the *Speculum* on which the Plummet falls; for the Angle of *In-*

Incidence and *Reflection* being *Equal*, in this caſe a Line at Right Angles to the *Speculum*, as is the *Axis* of the *Chaſe* of the Piece, will *biſect* the Angle between the Perpendicular and the Object, according as our *Propoſition* requires.

Prop. XI. Prob. 3.] *A Shot being made on an inclined Plane, having the Horizontal Diſtance of the Object it ſtrikes, with the Elevation of the Piece, and the Angle at the Gun between the Object and the Perpendicular; to find the greateſt Horizontal Range of that Piece, laden with the ſame Charge; that is, half the* Latus Rectum *of all the* Parabolæ *made with the ſame* Impetus.

Take half the Diſtance of the Object from the *Nadir*, and take the Difference of the given Elevation from that Half; the *verſed Sine* of twice that Difference ſubtract from the *verſed Sine* of the Diſtance of the Object from the *Zenith*: then ſhall the Difference of thoſe *verſed Sines* be to the Sine of the Diſtance of the Object from the *Zenith*, as the Horizontal Diſtance of the Object ſtruck to the greateſt Horizontal Range at 45°.

Prop. XII. Prob. 4.] *Having the greateſt Horizontal Range of a Gun, the Horizontal Diſtance and Angle of Inclination of an Object to the Perpendicular; to find the two Elevations neceſſary to ſtrike that Object.*

Halve the Diſtance of the Object from the *Nadir*; this Half is always equal to the Half Sum of the two *Elevations* we ſeek. Then ſay, as the greateſt Horizontal Range, is to the Horizontal Diſtance of the Object; ſo is the Sine of the Angle of Inclination, or Diſtance of the Object from the Perpendicular, to a 4*th* Proportional; which 4*th* being ſubtracted from the *verſed Sine* of the Diſtance of the Object from the *Zenith*, leaves the *verſed Sine* of the Difference of the *Elevations* ſought; which *Elevations* are therefore had, by adding and ſubtracting that half Difference to, and from, the aforeſaid half Sum.

Prop. XIII.] *To determine the Force or Velocity of a Project, in every Point of the Curve it deſcribes.* n. 172. p. 18.

To do this, we need no other *Præcognita*, but only the *Third Propoſition*, viz. *That the Velocity of falling Bodies is double to that, which in the ſame Time would have deſcribed the Space fallen by an equable Motion*: For the *Velocity* of a *Project* is compounded of the conſtant equal *Velocity* of the impreſſed Motion, and the *Velocity* of the Fall, under a given Angle, *viz.* the Complement of the *Elevation*: For inſtance, In the Time wherein a *Project* would move from G to L, it deſcends from L to X, and by the *third Propoſition* has acquired a *Velocity*, which in that Time would have carried it by an equable Motion from L to Z, or twice the Deſcent L X; and drawing the Line G Z, I ſay the *Velocity* in the Point X, compounded of the *Velocities* G L, and L Z, under the Angle G L Z, is to the *Velocity* impreſſed in the Point G, as G Z is to G L; this follows from our *ſecond Axiom*; and by the 20*th* and 21*ſt Prop. Lib.* 1. *Conic. Midorgii*, X O, Parallel and Equal to G Z, ſhall touch the *Parabola* in the Point X. So that the *Velocities* in the ſeveral Points, are as the Lengths of the *Tangents* to the *Parabola* in thoſe Fig. 172.

 Points,

Points, intercepted between any two Diameters: and theſe again are as the *Secants* of the Angles, which thoſe *Tangents* continued make with the Horizontal Line G B. From what is here laid down, may the comparative Force of a Shot in any two Points of the Curve, be either Geometrically or Arithmetically diſcovered.

Coroll.] From hence it follows, that the force of a Shot is always leaſt at V, or the *Vertex* of the *Parabola*; and that at equal Diſtances therefrom, as at T and X, G and B, its force is always equal; and that the leaſt Force in V, is to that in G and B, as *Radius* to the *Secant* of the Angle of *Elevation*, FGB.

The *Tenth Propoſition* contains a *Problem*, untouched by *Torricellius*, which is of the greateſt uſe in Gunnery, and for the ſake of which this Diſcourſe was principally intended. It was firſt ſolved by Mr. *Anderſon*, in his Book of the *Genuine Uſe and Effects of the Gun*, printed in the Year 1674, but his *Solution* required ſo much Calculation, that it put me upon Search, whether it might not be done more eaſily; and thereupon in the Year 1678, I found out the *Rule* I now publiſh, and from it the Geometrical Conſtruction: Since which time, there has a large Treatiſe of this Subject, Intitled, *L' Art de Jetter les Bombes*, been publiſhed in *France* by M. *Blondel*, wherein he gives the *Solutions* of this *Problem*, by Meſſieurs *Bout*, *Romer*, and *de la Heir*: but none of them are the ſame with mine, or in my Opinion more eaſy.

n. 216 ... 68. It was formerly the Opinion of thoſe concerned in *Artillery*, that there was a certain requiſite of *Powder* for each *Gun*, and that in *Mortars* where their Diſtance was to be varied, it muſt be done by giving a greater or leſſer *Elevation* to the *Piece*. But now our later Experience has taught us, That the ſame thing may be more certainly and readily performed, by *increaſing and diminiſhing the quantity of Powder*, whether regard be had to the Execution to be done, or to the Charge of doing it. For when *Bombs* are diſcharged with great *Elevations* of the *Mortar*, they fall too Perpendicular, and bury themſelves too deep in the ground, to do all that Damage they might, if they came more oblique, and broke upon or near the Surface of the Earth; which is a thing acknowledged by the *beſieged* in all *Towns*, who unpave their Streets to let the *Bombs* bury themſelves, and thereby ſtifle the force of their Splinters. A *Second Convenience* is, that at the Extreme *Elevation*, the *Gunner* is not obliged to be ſo curious in the Direction of his *Piece*, but it will ſuffice to be within a Degree or two of the Truth; whereas in the other method of *Shooting*, he ought to be very curious. But a *Third* and no leſs conſiderable *Advantage* is, in the ſaving of the *King*'s *Powder*, which in ſo great and ſo numerous *Diſcharges*, as we have lately ſeen, muſt needs amount to a conſiderable Value. And for *Sea Mortars* it is ſcarce practicable otherwiſe to uſe them, where the *Agitation* of the Sea continually changes the *Direction* of the *Mortar*, and would render the Shot very uncertain, were it not that they are placed about 45° *Elevation*, where ſeveral degrees above or under makes very little difference in the *Effect*.

Ibid. p. 70. It only remains, by good and valid *Experiments*, to be aſſured of the *Force* of *Gun-Powder*; how to make and conſerve it equal; and to know the Effect thereof in each *Piece*; that is, how far differing *Charges* will caſt the ſame Shot out of it; which may moſt conveniently be Engraven on the outſide thereof, as a ſtanding Direction to all *Gunners*, who ſhall from thence forward have occaſion to uſe that *Piece*: and were this Matter well aſcertained, it might be worth the while to make all *Mortars* of the like *Diameter*, as near as may be alike in *Length* of *Chaſe*, *Weight*, *Chamber*, and all other *Circumſtances*.

Now the foregoing Rules would be rigidly true, were it not for the *Oppoſition* of the *Medium*, whereby not only the direct impreſſed Motion is continually retarded, but likewiſe the Increaſe of the Velocity of the Fall, ſo that the Spaces deſcribed thereby, are not exactly as the *Squares* of the *Times*: But what this *Oppoſition* of the *Air* is, againſt ſeveral *Velocities*, *Bulks*, and *Weights*, is not ſo eaſy to determine. 'Tis certain, That the *Weight* of the *Air* to that of *Water*, is nearly as 1 to 800; whence the *Weight* thereof, to that of any *Project* is given: 'tis very likely, that to the ſame *Velocity* and *Magnitude*, but of different Matter, the *Oppoſition* ſhould be *reciprocally* as the *Weights* of the *Shot*; as likewiſe that to *Shot* of the ſame *Velocity* and *Matter*, but of different *Sizes*, it ſhould be as the *Diameters reciprocally*: Whence generally the *Oppoſition* to *Shot* with the ſame *Velocity*, but of differing Diameters, and Materials, ſhould be as their *Specifick Gravities* into their *Diameters reciprocally*: but whether the Oppoſition to differing *Velocities* of the ſame *Shot*, be as the *Squares* of thoſe *Velocities*, or as the *Velocities* themſelves, or otherwiſe, is yet a harder Queſtion. However it be, 'tis certain, That in large Shot of Metal, whoſe Weight many thouſand times ſurpaſſes that of the *Air*, and whoſe Force is very great in proportion to the Surface wherewith they preſs thereon, this *Oppoſition* is ſcarce *diſcernible*: For by ſeveral *Experiments*, made with all Care and Circumſpection, with a *Mortar-Piece*, extraordinary well fixed to the Earth on purpoſe, which carried a ſolid Braſs Shot of $4\frac{1}{2}$ Inches Diameter, and of about 14 Pound Weight, the Ranges above and below 45°, were found nearly equal; if there were any Difference, the under Ranges went rather the fartheſt, but thoſe Differences were uſually leſs than the Errors committed in ordinary Practice, by the unequal Goodneſs and Dryneſs of the ſame ſort of *Powder*, by the Unfitneſs of the Shot to the *Bore*, and by the Looſeneſs of the *Carriage*. In a ſmaller Braſs Shot of about an Inch and half Diameter, caſt by a *Croſs-Bow*, which ranged it at moſt about 400 Foot, the Force being much more equal than the *Mortar Piece*, this Difference was found more curiouſly, and conſtantly, and moſt evidently, the under *Ranges* out-went the upper. From which Trials I conclude, that altho' in ſmall and light Shot the *Oppoſition* of the *Air*, ought and muſt be accounted for; yet in ſhooting of great and weighty *Bombs*, there need be very little or no Allowance made: and ſo theſe Rules may be put in practice to all Intents and Purpoſes, as if this *Impediment* were abſolutely removed.

179. p. 79.

The Meaſure of the Air's Reſiſtance to Bodies, moved in it; by Dr. Wallis. n. 186. p. 269. Jan. An. 1687.

IX. 1. In order to compute the *Reſiſtance* of the *Air* to all *Projects*, I firſt premiſe this *Lemma* (as the moſt rational that doth occur, for my firſt footing)

ing) *That* (ſuppoſing other things equal) *The Reſiſtance is a proportional to the Celerity*. For in a double *Celerity*, there is to be removed (in the ſame time) twice as much *Air* (which is a double *Impediment* ;) in a treble, thrice as much ; and ſo in other Proportions.

2. Suppoſe we then the Force impreſſed (and conſequently the *Celerity*, if there were no *Reſiſtance*) as 1 ; the *Reſiſtance* as r (which muſt be leſs than the Force, or elſe the Force would not prevail over the Impediment, to create a *Motion*.) And therefore the effective Force at a firſt *Moment*, is to be reputed as $1-r$: That is, ſo much as the Force impreſſed, is more than the *Impediment* or *Reſiſtance*.

3. Be it as $1-r$ to 1, ſo 1 to m (which m is therefore greater than 1.)

4. And therefore the effective Force (and conſequently the *Celerity*) as to a firſt *Moment*, is to be $\frac{1}{m}$ of what it would be, had there been no *Reſiſtance*.

5. This $\frac{1}{m}$ is alſo the remaining Force after ſuch firſt Moment ; and this remaining Force is (for the ſame Reaſon) to be proportionably abated as to a ſecond Moment : That is, we are to take $\frac{1}{m}$ thereof, that is, $\frac{1}{mm}$ of the impreſſed Force. And for a third Moment (at equal Diſtance of Time) $\frac{1}{mmm}$; for a fourth $\frac{1}{m^4}$; and ſo onward infinitely.

6. Becauſe the Length diſpatched (in equal Times) is proportional to the *Celerities* ; the Lines of Motion (anſwering to thoſe equal Times) are to be as $\frac{1}{m}$, $\frac{1}{m^2}$, $\frac{1}{m^3}$, $\frac{1}{m^4}$, *&c.* of what they would have been in the ſame Times, had there been no *Reſiſtance*.

7. This therefore is a *Geometrical Progreſſion* ; and (becauſe of m greater than 1) continually decreaſing.

8. This decreaſing Progreſſion infinitely continued (determining in the ſame Point of Reſt, where the Motion is ſuppoſed to expire) is yet of a finite Magnitude, and equal to $\frac{1}{m-1}$ of what it would have been in ſo much time, if there had been no *Reſiſtance*: As is demonſtrated in my *Algebra*, *Chap.* 95. *Prop.* 8. For (as I have elſewhere demonſtrated) the *Sum* or Aggregate of a *Geometrical Progreſſion* is $\frac{VR-A}{R-1}$, (ſuppoſing V the greateſt Term, A the leaſt, and R the common Multiplier ;) That is, $\frac{VR}{R-1}-\frac{A}{R-1}$. Now in the preſent Caſe (ſuppoſing the Progreſſion infinitely continued) the leaſt Term A, becomes infinitely ſmall, or $=0$: And conſequently $\frac{A}{R-1}$ doth alſo vaniſh, and thereby the Aggregate be-

comes $= \frac{VR}{R-1}$. That is (as will appear by Dividing V R by R—1;) $V+\frac{V}{R}+\frac{V}{RR}+\frac{V}{R^3}+$, &c. $= \frac{VR}{R-1}$; (ſuppoſing the Progreſſion to begin at V = 1.) That is, (dividing all by R, that ſo the Progreſſion may begin at $\frac{V}{R}=\frac{1}{m}$:) $\frac{V}{R-1}=\frac{V}{R}+\frac{V}{RR}+\frac{V}{R^3}+$ &c. That is, in our preſent Caſe (becauſe of V=1, and R=m) $\frac{1}{m}+\frac{1}{mm}+\frac{1}{m^3}$, &c. $=\frac{1}{m-1}$. That is (putting $n=m-1$) $\frac{1}{n}$, of what it would have been, if there had been no *Reſiſtance*.

9. This infinite Progreſſion is fitly expreſſed by an *Ordinate* in the *Exterior Hyperbola*, parallel to one of the *Aſymptotes*; and the ſeveral Members of that, by the ſeveral Members of this, cut in *Continual Proportion:* As is there demonſtrated at *Prop.* 15. For let S H be an *Hyperbola* between the *Aſymptotes*, A B, A F: And let the *Ordinate* D H (in the *Exterior Hyperbola*, parallel to A F) repreſent the impreſſed Force undiminiſhed; or the Line to be deſcribed in ſuch Time, by a *Celerity* anſwerable to ſuch Undiminiſh'd Force. And let B S (a like *Ordinate*) be $\frac{1}{m}$ thereof; which therefore, being leſs than D H (as being equal to a part of it) will be further than it from A F. In A B (which I put =1,) let B d be ſuch a part thereof, as is B S of D H. Now becauſe (as is well known) all the inſcribed Parallelograms, in the *Exterior Hyperbola*, A S, A H, &c. are equal; and therefore their Sides reciprocal: Therefore as A $d = 1 - \frac{1}{m}$, (ſuppoſing B d, to be taken from B toward A,) to A B=1, (or as $m-1$ to m:) ſo is B S$=\frac{1}{m}$D H to $d\,h$, which is therefore equal to $\frac{1}{m-1}$ of D H; that is (as will appear by Dividing 1 by $m-1$,) to $\frac{1}{m}+\frac{1}{mm}+\frac{1}{m^3}$ &c. of D H. *Fig.* 175.

Or if B d be taken beyond B: then as A $d = 1+\frac{1}{m}$, to A B = 1, or as $m+1$ to m, ſo is $\frac{1}{m}$ D H, to $d\,b$, which is therefore equal to $\frac{1}{m+1}$ D H; that is, (as will appear by like dividing of 1 by $m+1$) $=\frac{1}{m}-\frac{1}{mm}+\frac{1}{m^3}-$, &c. of D H.

10. Let ſuch *Ordinate* $d\,b$, or (equal to it in the *Aſymptote*) A F, be ſo divided in L, M, N, &c. (by Perpendiculars cutting the *Hyperbola* in l, m, n, &c.) *Fig.* 176.

n, &c.) as that F L, L M, M N, be as $\frac{1}{m}$, $\frac{1}{m\,m}$, $\frac{1}{m^3}$, &c. That is, so continually decreasing, as that each Antecedent be to its Consequent, as 1 to $\frac{1}{m}$, or as *m* to 1.

11. This is done by taking A F, A L, A N, &c. in such Proportion. For, of continual Proportionals the Differences are also continually Proportional, and in the same Proportion. For let A, B, C, D, &c. be such Proportionals, and their Differences, *a*, *b*, *c*, &c. That is, A—B=*a*, B—C=*b*, C—D=*c*, &c.

Then because, A, B, C, D, &c. are in continual Proportion;
That is, A : B :: B : C :: C : D ::, &c.
And dividing, A—B : B :: B—C : C :: C—D : D ::, &c.
That is, *a* : B :: *b* : C :: *d* : D ::, &c.
And Alternately; *a.b.c.* &c. :: B. C. D. &c. :: A. B. C. &c.
That is, In Continual Proportion, as A to B, or as *m* to 1.

12. This being done; the *Hyperbolick Spaces* F *l*, L *m*, M *n*, &c. are equal, as is demonstrated by *Gregory San-Vincent*; and as such is commonly admitted.

13. So that F *l*, L *m*, M *n*, &c. may fitly represent equal Times in which are dispatched unequal Lengths, represented by F L, L M, M N, &c.

14. And because they are in Number infinite, (tho' equal to a finite Magnitude) the Duration is infinite; and consequently the impressed Force, and Motion thence arising, never to be wholly extinguished (without some further Impediment) but perpetually approaching to A, in the Nature of *Asymptotes*.

15. The Spaces F *l*, F *m*, F *n*, &c. are therefore as *Logarithms* (in *Arithmetical Progression* increasing) answering to the Lines, A F, A L, A M, &c. or to F L, L M, M N, &c. in *Geometrical Progression* decreasing.

16. Because F L, L M, M N, &c. are as $\frac{1}{m}$, $\frac{1}{m\,m}$, $\frac{1}{m^3}$, &c. infinitely terminated at A; therefore (by *Prop.* 8.) their Aggregate FA, or *d h*, is to DH, (so much Length as would have been dispatched in the same Time, by such impressed Force undiminished) as 1 to $m-1=n$.

17. If therefore we take, as 1 to *n*, so A F to D H; this will represent the Length to be dispatched, in the same Time, by such undiminished Force.

18. And if such D H be supposed to be divided into equal Parts innumerable (and therefore infinitely small;) these answer to those (as many) Parts unequal in FA, or *h d*.

19. But, what is the Proportion of *r* to 1, or (which depends on it) of $1-r$ to 1, or 1 to *m*; remains to be enquired by *Experiment*.

20. If the Progression be not infinitely continued; but end (suppose) at N, and its least Term be A = M N: Then out of $\frac{V}{R-1} + \frac{1}{m} + \frac{1}{mm} = \frac{1}{m^3}$, + &c. is to be subducted $\frac{A}{R-1}$, (as at *Prop.* 8.) that is, (as by Division will appear) $\frac{A}{R} + \frac{A}{R^2} + \frac{A}{R^3} +$ &c. That is, (in our present Case) $\frac{a}{m} + \frac{a}{mm} + \frac{a}{m^3} +$ &c. And so the Aggregate will be $\frac{1-a}{m} + \frac{1-a}{mm} + \frac{1-a}{m^3} +$ &c. $= \frac{1-a}{n}$.

And thus as to the Line of *Projection*, in which (secluding the *Resisting*) the Motion is reputed uniform; dispatching equal Lengths at equal Times. Consider we next the Line of Descent.

21. In the Descent of Heavy Bodies, it is supposed, that to each Moment of Time, there is superadded a new Impulse of *Gravity* to what was before: And each of these, secluding the Consideration of the *Air's Resistance*, to proceed equally (from their several beginnings) thro' the succeeding Moments. As, (in the Erect Lines) 1 1 1 1 &c. 1 1 1 &c. 1 1 &c. 1. &c. and so continually, as in the Line of *Projection*.

1
1 1
1 1 1
1 1 1 1
&c. &c. &c. &c.

22. Hence ariseth (in the Transverse Lines) for the first Moment 1, for the second 1 + 1, for the third 1 + 1 + 1, and so forth, in *Arithmetical Progression.* As are the *Ordinates* in a Triangle at equal Distance.

23. And such are the continual Increments of the Diameter, or of the *Ordinates* in the *Exterior Parabola*, answering to the interior *Ordinates*, or Segments of the *Tangent*, equally increasing; as is known, and commonly admitted.

24. If we take in the Consideration of the *Air's Resistance*; we are then for each of these equal Progressions, to substitute a decreasing *Progression Geometrical*; in like manner (and for the same Reasons) as in the Line of Projection.

25. Hence ariseth for the first Moment $\frac{1}{m}$; for the second $\frac{1}{m} + \frac{1}{m^2}$; for the third $\frac{1}{m} + \frac{1}{m^2} + \frac{1}{m^3}$; &c. And such is therefore the Descent of a heavy Body falling by its own Weight. The several impulses of Gravity being supposed equal.

$\frac{1}{m}$
$\frac{1}{m^2}\ \frac{1}{m}$
$\frac{1}{m^3}\ \frac{1}{m^2}\ \frac{1}{m}$
$\frac{1}{m^4}\ \frac{1}{m^3}\ \frac{1}{m^2}\ \frac{1}{m}$
&c. &c. &c. &c.
Fig. 176.

26. That is, as FL, FM, FN, &c. in the Line of Descent, answering to FL, LM, MN, &c. in the Line of Projection.

27. But tho' the Progressions for the Line of Projection, are like to each of those many in the Line of Descent: It is not to be thence inferred, that therefore $\frac{1}{m}$ in the one is equal to $\frac{1}{m}$ in the other: But in the Line

of

of Projection (suppose) $\frac{1}{m}f$, (such a part of the Force impressed, and a Celerity answerable :) in the Line of Descent $\frac{1}{m}g$, (such a part of the impulse of Gravity.)

28. Those for the Line of Descent (of the same Body) are all equal each to other: Because g, (the new Impulse of Gravity) in each Moment is supposed to be the same.

29. But what is the Proportion of f to g (that is, of the Force impressed, to the Impulse of Gravity, in each Body) remains to be enquired by *Experiment*.

30. This Proportion being found as to one known Force; the same is thence known as to any other Force (whose Proportion to this is given) in the same *uniform Medium*.

31. And this being known as to one *Medium*, the same is thence known as to any other *Medium*, the Proportion of whose Resistance to that of this is known.

32. If a heavy Body be projected downward in a Perpendicular Line, it descends therefore at the rate $\frac{1}{m}$, $\frac{1}{m^2}$, $\frac{1}{m^3}$, &c. of f (the impressed Force) increase by $\frac{1}{m}$, $\frac{1}{m}+\frac{1}{m^2}$, $\frac{1}{m}+\frac{1}{m^2}+\frac{1}{m^3}$, &c. of g, the Impulse of Gravity: (by *Prop.* 5. and *P.* 25.) because both Forces are here united.

33. If in a perpendicular Projection upwards; it ascends in the Rate of the former, abated by that of the latter. Because here the Impulse of Gravity is contrary to the Force impressed.

34. When therefore this latter (continually increasing) becomes equal to that former (continually decreasing) it then ceaseth to ascend; and doth thenceforth descend at the rate wherein the latter continually exceeds the former.

35. In an Horizon or oblique Projection: If to a Tangent, whose Increments are as F L, L M, M N, &c. that is, as $\frac{1}{m}f$, &c. be fitted Ordinates (at a given Angle) whose Increments are as F L, F M, F N, &c. that is, as $\frac{1}{m}g$, &c. The Curve answering to the Compound of these Motions, is that wherein the Project is to move.

36. This Curve (being hitherto without a Name) may be called *Linea Projectorum*, the Line of Projects, or things projected; which resembles a *Parabola deformed.*

37. The *Celerity* and *Tendency*, as to each Point of the Line, is determined by a Tangent at that Point.

38. And that against which it makes the greatest Stroak or Percussion, is that which (at that point) is at right Angles to that Tangent.

39. If the Projection (at *pag.* 25.) be not infinitely continued, but terminate

minate (ſuppoſe) at N, ſo that the laſt Term in the firſt Column or Series erect, be a; and conſequently in the ſecond, ma; in the third mma, &c. (each Series having one Term fewer than that before it:) then (for the ſame Reaſons as at *P.* 20.) the *Aggregates* of the ſeveral Columns (or erect Series) will be $\frac{1-a}{n}$, $\frac{1-ma}{n}$, $\frac{1-mma}{n}$, and ſo forth, till (the Multiple of a becoming $=1$,) the Progreſſion expire.

40. Now all the Abatements here, a, ma, mma, &c. are the ſame with the Terms of the firſt Column taken backward. For a is the laſt, ma the next before it; and ſo of the reſt.

41. And the Aggregate of all the Numerators is ſo many times 1, as is the Number of Terms (ſuppoſe t) wanting the firſt Column; that is, $t-\frac{1-a}{n}$ or $\frac{nt-1+a}{n}$; and this again divided by the common Denominator n, becomes $\frac{nt-1+a}{nn}$. And therefore, $\frac{nt-1+a}{nn}g$, is the Line of Deſcent by its own Gravity.

42. If therefore this be added to a projecting Force downward in a Perpendicular, or ſubducted from ſuch projecting Force upward; that is, to or from $\frac{1-a}{n}f$: The Deſcent in the firſt Caſe, will be $\frac{1-a}{n}f+\frac{nt-1+a}{nn}g$; and the Aſcent in the other Caſe $\frac{1-a}{n}f-\frac{nt-1+a}{nn}g$. And in this latter Caſe, when the Ablative part becomes equal to the Poſitive part, the Aſcent is at the higheſt: And thenceforth (the Ablative part exceeding the Poſitive) it will deſcend.

43. In an Horizontal or Oblique Projection, having taken $\frac{1-a}{n}f$, in the Line of Projection, and thence (at the Angle given) $\frac{nt-1+a}{nn}g$, in the Line of Deſcent; the Point in the Curve anſwering to theſe, is the place of the Project anſwering to that Moment.

44. I am aware of ſome Objections to be made, whether to ſome Points of the Proceſs, or to ſome of the Suppoſitions. But I ſaw not well how to wave it, without making the Computation much more perplexed. And in a manner ſo nice, and which muſt depend upon phyſical Obſervations, 'twill be hard to attain ſuch Accuracy, as not to ſtand in need of ſuch Allowances.

45. Somewhat might have been further added, to direct the *Experiments* ſuggeſted at *P.* 19. and 29. But that may be done at leiſure, after Deliberation had, which way to attempt the *Experiment*.

46 The like is to be ſaid of the different Reſiſtance which different Bodies may meet with in the ſame *Medium*, according to their different Gravities, (extenſively or intenſively conſidered) and their different Figures and

Positions in Motion: Whereof hitherto we have taken no account; but supposed them, as to all these, to be alike and equal.

Fig. 176, 177. 47. The Computation (in *P.* 39, 40, 41) may, if that be also desired, be thus represented by Lines and Spaces. The Ablatives, *a*, *m a*, *m m a*, *&c.* (being the same with the first Column taken backward) are fitly represented by the Segments of N F, (beginning at N) and therefore by Parallelograms on these Bases, assuming the common height of F *b*, or N Q; the Aggregate of which is N *b*, or F Q, and so many times 1, by so many equal Spaces, on the same Bases, between the same Parallels terminated at the *Hyperbola*; the Aggregate of which is *b* F N Q *n*. From whence if we subduct the Aggregate of Ablatives F Q; the remaining Trilinear *b* Q *n*, represents the Descent.

48. If to this of Gravity, be joined a Projecting Force; which is to the impulse of Gravity, as *b* K to *b* F, (be it greater, less or equal) taken in the same Line; the same Parallels determine proportional Parallelograms, whose Aggregate is K Q.

49. And therefore, if this be a Perpendicular Projecting downwards; then *b* K *k m* (the Sum of this with the former) represents the Descent.

50. If it be a Perpendicular upwards; then the difference of these two represents the Motion, which, so long as K Q is the greater, is Ascendant; but Descendant, when *b* Q *n* becomes greater; and it is then at the highest when they be equal.

51. If the Projection be not in the same Perpendicular, (but Horizontal or Oblique) then K Q represents the Tangent of the Curve; and *b* Q *n*, the Ordinates to that Tangent, at the given Angle.

52. But the Computation before given, I take to be of better use than this Representation in Figure. Because in such methametical Enquiries, I chuse to separate (as much as may be) what purely concerns Proportions; and consider it abstractly from Lines or other matter wherewith it is incumbred.

As to the Question proposed; Whether the resistance of the Medium does not always take off such a proportional part of the Force, moving thro' it, as is the Specifick Gravity of the Medium to that of the Body moved in it (for if so, it will save us the trouble of Observation;) I think this can by no means be admitted; for there be many other things of Consideration herein, beside the intensive Gravity (or, as some call it, the Specifick Gravity) of the Medium.

A viscous *Medium* shall more resist than one more fluid, tho' of like intensive Gravity.

And a sharp Arrow shall bore its way more easily thro' the Medium than a blunt-headed Bolt, tho' of equal Weight and like intensive Gravity.

And the same Pyramid with the Point, than with the Base forward.

And many other like Varieties intended in my *P.* 46.

And this I think may be admitted, namely, that different Mediums, equally liquid, (and other Circumstances alike) do in such Proportion resist, as is their intensive Gravity. Because there is, in such proportion, a heavier Object to be removed, by the ſame Force; which is one of the things to which *P.* 31. refers.

And again: the heavier Project once in motion, (being equally ſwift, and all other Circumſtances alike) moves thro' the ſame Medium in ſuch Proportion more ſtrongly, as is its intenſive Gravity: for now the Force is in ſuch Proportion greater, for the Removal of the ſame Reſiſtance. And this is part of what my *P.* 30 inſinuates.

But where there is a Complication of theſe Conſiderations one with another, and with many other Circumſtances, whereof each is ſeverally to be conſidered; there muſt be reſpect had to all of them.

Experiments, to Determine the Point Blank Diſtance; the Charge of Powder; and beſt Size of Guns. Propoſed; by Sir Rob. Moray. *n.* 26. p. 473. June An. 1667.

X. 1. *To know how far a Gun ſhoots Point Blank, (as they call it) that is, ſo near the Level of the Cylinder of the Piece, that the Difference is either not diſcernible, or non conſiderable.*

On a fit *Platform*, place and point the *Gun* at a Mark as large as the Bullet, ſome 50, 60, or more Yards diſtant, ſo as the underſide of the Mark may be in the ſame Level or Line with the underſide of the *Cylinder* of the *Piece.* Then between the *Gun* and the Mark at convenient Diſtances, place pieces of Canvas, Sheets of Paper paſted together, or the like, upon ſtakes fix'd in the ground; ſo as the underſide, being level with the Horizon, may juſt touch the viſual Line, that paſſeth from the Eye to the upperſide of the Mark, when the Eye is placed in the Line that paſſeth from it to the upperſide of the *Cylinder* of the *Gun*; the Canvas being ſo broad and long, that if the *Bullet* paſs thro' it 2 or 3 Foot higher than the Level of the Mark, or of either hand, the Hole it makes may make it known, how much it flieth higher than the Level of that Place. If the *Bullet* falls lower than the Mark, and touch not the Canvas, the *Gun* may be the next time raiſed a little, and ſo on, till the *Bullet* hit the Mark, or as high as it. If it fall as high as the Mark and cut the Canvas, the Mark and Canvas may be brought nearer the *Gun:* But if it fall as high as the Mark and do not cut the Canvas, the Mark may be removed to greater and greater Diſtances.

If this way of *Experiment* be made for further Diſtances and Raiſings of the *Piece*, as high as conveniently may be above the Level, and the Diſtances meaſured, and then all *Randoms* above theſe likewiſe tried and meaſured; the Diſtance of an Object, to be ſhot at, being known, and other neceſſary Cautions, beneath to be mentioned, carefully obſerved; good *Gunners* may with great Confidence undertake to hit the Mark, be the Diſtance what it will, ſo it exceed not the reach of the *Gun.*

2. *To know what Quantity of Powder is the juſt Charge of any Piece, ſo as it maketh the fartheſt Shot, and Fires totally.*

1. Raiſe the *Gun* to a mean *Random*, as of 20°, or 25°, and ſhoot with the ordinary *Charge* of *Powder*, in ſome convenient Ground where the Fall of the Bullet may be eaſily ſeen, and having made a Shot, meaſure the Diſtances with a Chain between the Hole made by the Bullet and the *Muzzle* of the *Gun.*

2. Then inſtead of a full *Charge* of *Powder* uſed in the firſt Shot, take $\frac{1}{16}$ part leſs, or ſome ſuch Proportion, for the next Trial, doing all things elſe as before.

3. For a third, fourth, or more Trials, diminiſh ſtill the *Quantity* of *Powder*, by $\frac{1}{16}$ at a Time, till the Shot be conſiderably ſhorter than at firſt.

4. Then take $\frac{1}{16}$ more than the firſt *Charge*, and do all things elſe as before, and ſo continue more Trials, increaſing ſtill the *Quantity* of *Powder* in the ſame Proportion every new Trial, till you find the Increaſe of the *Charge* does not make the *Piece* ſhoot further: Only over-charge not ſo far as to endanger the *Gun*.

5. Three or more Shot are to be made with every different *Charge*, and at every ſeveral Trial, that the Certainty may the better appear.

6. The firſt Shot being meaſured and marked, the reſt may all be meaſured from it, or from one another, to ſave Labour.

7. The *Gun* is to be pointed, placed and ordered, every time in one and the ſame Place and Poſition, aiming ſtill at the ſame Mark, or Pointing ſtill at the ſame Line or Azimuth; that ſo all the Shot may fall in the ſame Line as near as is poſſible.

8. The *Powder* (which ought to be all of the ſame *Goodneſs*) muſt be exactly weighed every time the *Piece* is *Charged*, leſt it having been weighed long before, the Weight may be altered; tho' *Experiment* may be made with *Cartridges* and without.

9. The *Powder* and *Bullet* is to be *Rammed* home, equally at every Shot; tho', the looſer the *Powder* lie, it *fire* the better.

10. When the right *Charge* of a *Piece* is found, that makes the *fartheſt Shot* in the ordinary and plain way of *Charging*, M. *de Sons*'s Contrivance of a *Wedge* may be tried, to make it ſhoot farther; which is a piece of Board, ſo long, as being thruſt home to the *Breech* of the *Piece* at one End, the other may reach farther out than the Outſide of the Bullet being *Rammed* up to its place; broad about an Inch, and thin ſo far as the *Wadd* before the *Bullet* reaches on the outſide; there it is to have a Shoulder, from which forward to the end, it is to be cut a-ſlope, like a *Wedge*, being of ſuch thickneſs, as that at the Place, where the Center of the *Bullet* is to be, it may make it ſtick ſo faſt, that the *Powder* finding more Reſiſtance, may at length drive it out with the greater Violence.

11. Another of this Nature is a *Wooden Tampion*, like a piece of a Cylinder, big enough to fill the hollow *Cylinder* of the *Gun*, the length ſomewhat more than the Diameter of it, and hollow'd towards the *Bullet*, ſo as to fit it; and either flat or (which is better) hollow likewiſe towards the *Powder*, and ſerving inſtead of *Wadd*. Theſe, and ſuch others, will probably render the Effect of the *Powder* greater, than otherwiſe it would be: But care muſt be had that they do not endanger the *Piece*.

12. The *Strength* of the *Powder* muſt be examin'd by a *Powder Trier*, that raiſeth a Weight, ſuch an one as hath been contrived by Mr. *Hook*.

13. The ſame *Bullet* is to be made uſe of, if it can be had, till the Figure
of

of it be marred; otherwiſe another as near of the ſame Size, Shape and Weight, as is poſſible.

14. Obſerve the *Strength* and *Poſition* of the *Wind*, and at what *Azimuth* the Mark ſtands from the *Gun* at every time of Shooting: And take preciſe notice what effect it hath upon the *Bullet* in carrying it further, in hindring, or turning it aſide.

15. Note the Figure, Dimenſions, and Weight of the *Gun*, *Carriage*, and *Wheels*; and record every thing exactly in a Book, as alſo every Accident and Obſervation.

16. After all other *Experiments* are made, every *Piece* may be tried with the right *Charge* of *Powder*, laying every time more and more Weight upon the *Carriage*; and at laſt fixing the *Gun*, ſo as it may not *Recoil* at all, obſerving every Time how far the *Bullet* goes, and how much leſs *Powder* than the full *Charge* will ſerve to ſhoot the *Bullet*, when the *Piece* is Fixt, as far as the whole *Charge* does, when it *Recoils* freely.

17. The *Right Charge* found, the beſt *Random* is to be ſought, by trying all Randoms, by Degrees at a time.

3. *To know what Gun ſhoots fartheſt.*

1. A *Gun* to be prepared of *Culverine-Bore* (as being held the beſt for ſhooting far) but much longer (double the ordinary Length may do well) and without any *Ring* about the *Muzzle*, is to be placed as in the former *Experiments*, and charged with the Ordinary Charge of a *Culverine*, or rather with that Quantity, which by the former *Experiments* ſhall be found the beſt; and being ſhot, the Fall of the *Bullet* is to be mark'd, and Diſtance meaſured.

2. Then try leſs, and more *Powder* in her, as before.

3. Then cut off two Inches of the *Muzzle* with a Saw, and place the Pieces ſo cut off in the *Carriage*, or their *Weight* of Lead in a convenient Figure, that the *Recoil* may ſtill be the ſame: and try as before, doing every thing in the ſame manner: and ſo cut off ſtill for new Trials, till the Shot begin to fall ſhorter than before.

4. The ſame may be done with *Guns* of different *Bores*.

2. *Mar.* 18. 1651. At 200 Yards diſtance from the Platform for great Ordinance at *Woolwich*, there were raiſed three *Butts*, one behind another: The ſpace between the firſt and ſecond *Butt* was 14 Yards; between the ſecond and the third, eight. The thickneſs of each *Butt* was 19 Inches, whereof 13 was of Beams of Maſſey oak faſtened into the Ground, and ſet ſo cloſe that they touched each other: On each ſide were Planks of Oak, 3 Inches a-piece in Thickneſs, and theſe were joined cloſe, and faſtened on both Sides with Iron bolts, and ſtrong Pins of Wood; and on the Back, at the Ends, and on the Middle there were 3 Braces of Elm, a Foot in Breadth, and 5 Inches in Thickneſs.

Experiments for Trying the Force of Great Guns; by Mr. Greaves. n. 173. p. 1090.

The firſt *Experiment* was with an *Iron Demy-Cannon*, having a *Cylinder Bore* of 3500 *lib. Weight*, the *Bullet* 32 *lib.* of Iron, the *Powder* 10 *lib.* which pierced thro' the two firſt *Butts*, and ſtuck in the third, ſo as the Ball was almoſt quite within, but the Timber not ſhivered (ſmall) nor ſcarce

ſpilt.

ſpilt. The *Butts* being touched by me, felt not warm; the like *Execution* was done when it was charged with 9 *lib.* as alſo when with 8 *lib.* of *Powder*.

The ſecond *Experiment* was with an *Iron Demy-Cannon*, having a *Taper Bore* and being 3600 *lib.* in *Weight*, and 4 Inches longer than the former; the *Iron Bullet* 32 *lib.* and the *Powder* 7. *lib.* which in three Trials ſeemed to have the ſame Force with the firſt. One of the *Shots* piercing thro' the ſecond *Butt*, and lighting near the Edge of the middle * *Butt* of Elm, tore it, but by the yielding of it, the *Bullet* glanced aſide off the third *Butt*, and entered into the Earth.

* *Brace.*

The third *Experiment* was with a whole *Culverine* in *Braſs*, of 5300 *lib.* in *Weight*, 11 Foot 1 Inch in *Length*, with a *Taper Bore*: the *Iron Bullet* was 18 *lib.* in *Weight*; the *Powder* in the firſt Trial 10 *lib.* in the ſecond 9 *lib.* in the third 8 *lib.* which laſt Proportion did the beſt *Execution*, and paſſed thro' the two *firſt Butts*, entering gently into the Third, which the former two did touch, but not enter.

The fourth *Experiment* was with a whole *Culverine* in *Braſs*, made at *Amſterdam* for the *French*, with this Mark, 3580, being 10 Foot long, and not very thick in the *Breech*; the firſt Shot with 9 *lib.* of *Powder*, 18 *lib.* of *Bullet-Iron*, paſt thro' the three *Butt*, and entered one Foot into the Ground; it paſſed by the Joints of the Timber, two Planks having been beat down before. The ſecond Shot with 8. *lib. Powder*, paſſed thro' two *Butts* and *grazed* between them. The third with 8 *lib.* paſt two *Butts*, and 7 Inches into the third; but the firſt *Butt* was much battered before, where it entered.

The fourth Shot paſſed, with 8 *lib.* of *Powder*, two *Butts*, and in both *Butts* thro' the midſt of a Maſſey ſtrong Beam (below) that had not been battered.

The Fifth *Experiment* was with an *Iron Demy-Culverine*, having 9 *lib. Bullet* in *Iron*, and 4 *lib. Powder*; this paſt one *Butt* (which was torn before) and entered the ſecond.

This ½ *Culverine* was ſhot 8 Times, as faſt as they could *charge* it with *Powder*, and the *Iron-Bullet*, and yet was but ſcarce lukewarm at the *Breech*, a little more in the midſt, moſt at the *Muzzle*, and this laſt ſcarce ſo hot as my Hand; and yet the *Gunners* in charging her, wet not at all the *Scoop*, or *Spunge*.

The Sixth *Experiment* was with a *Braſs Demy-Culverine*, the *Breech* of her was 13 Inches $\frac{5}{8}$, the Mouth $9\frac{5}{8}$. The firſt *Shot*, with 4 *lib.* of *Powder*, 9 *lib. Iron-Bullet*, paſt two *Butts*: The Second *Shot* with 3 *lib.* of *Powder*, paſt almoſt two *Butts*: This proved to be the beſt Shot, becauſe the Timbers were the ſtrongeſt.

Shooting by the Rarefaction of the Air, by Dr. Papin. *n.* 179. *p.* 21.

XI. Whereas ordinary *Wind Guns* do their Effect by the *Compreſſion* of the *Air*: *Ottho Ghericke* hath found a New ſort that ſhoots by *Rarefaction*; and he hath publiſhed that Device at large in his Book about *Pneumatick Experiments*. I have contrived another which I take to be better.

A A is a *Pipe*, very equal from one end to the other. *Fig.* 178. Jan. An. 1686.

B B a small *Pipe* solder'd in a Hole near the End of the *Pipe* A A, and apply'd to the *Plate* of the *Pneumatick Engine*.

CCCC some kind of *Stool*, to bear up the hinder part of the *Pipe* A A.

D a *Piece* of *Lead* fitted to the Bore of the *Pipe* A A.

The *Pipe* A A is to be shut at both Ends by *Valves* outwardly applied, and so the said *Pipe* A A, tho' never so big, may be *exhausted* of *Air* by means of the *Pneumatick Engine:* Which done, the *Valve* towards D must be suddenly opened, so that the whole Pressure of the Atmosphere acting upon the *Lead* D, may drive it along the *Pipe* A A, with such a Swiftness, that it will be able to carry it to a great Distance: and because such a *Valve* shutting a great Hole, would prove very difficult to be opened, when the *Pipe* A A, is of a great Bore, the Aperture towards D, may be left much smaller than the *Pipe*; the Swiftness of the Air being so great, that even thro' a pretty small Aperture, it pressed the *Lead* D, as freely almost, as if the whole Bore was quite open.

Having prepared a *Barrel* carrying a *Lead* of two Ounces, the *Experiment* was shewn before the *Royal Society* and the Effect was found very considerable, the Force being little less than that of the *Wind-Gun* by *Compression*; the same *Experiment* being afterwards repeated with a longer *Barrel*, 'twas found that the length in this way of *Shooting* was very little, if any Advantage.

XII. My way of computing the *Velocity* of the *Air* (which I think is better than the Trial made by the *Royal Academy* at *Paris*) is grounded upon this *hydrostatical Principle*, That *Liquors have a Strength to ascend as high as their Source is*; and altho' the *Resistance* of the *Medium* does always hinder *Jets d'Eau* in the open Air from reaching quite so high, nevertheless the *Liquor* at its first spouting out, hath the necessary *swiftness* to come to that *height*. *The Velocity wherewith the Air rushes into an exhausted Receiver; by* Dr. Papin. n. 184. p. 193. Oct. An. 1686.

Prop. I.] From this Principle may be easily deduced this Proposition, That, *of two different Liquors driven by the same Pressure, that which is in Specie lighter must ascend higher than that which is heavier, and their heights will be reciprocally in the same Reason as their specifick Gravities are.*

Prop. II.] From the foregoing *Proposition* another may easily be deduced, *viz.* That, *of differing Liquors bearing the same Pressure, those that are lighter in Specie must acquire a greater Swiftness, and their differing Velocities are to one another * as the Roots of the specifick Gravities of the said Liquors.* * *Reciprocally.*

For we have seen, *P.* I. That the *Heights* to be attained are * in the same Reason as the *specifick Gravities:* Now *Gallilæus*, *Hugenius*, and others, have demonstrated, That the *Velocities* of Bodies are to one another, as the *square Roots* of the *Heights* to which they may ascend: And so in this occasion they are also * as the *Roots* of the *specifick Gravities*. * *Reciprocally.* * *Reciprocally.*

If therefore we would know what is the *Velocity* of the *Air* being driven by any degree of *Pressure* whatsoever, we ought but to find what would be the

the *Velocity* of *Water* under the ſame *Preſſure:* And then take the *ſquare Roots* of the *ſpecifick Gravities* of theſe two Liquors; becauſe as much as the *ſquare Root* of the *ſpecifick Gravity* of *Water* doth exceed the *ſquare Root* of the *ſpecifick Gravity* of the *Air*; ſo much in proportion will the *Velocity* of *Air* exceed the *Velocity* of *Water.* For Example; when I would compute what ſhould be the *Swiftneſs* of a *Bullet* ſhot by my *Pneumatick Engine*, I ſhould at firſt compute what was the *Velocity* of the Air itſelf that drove the Bullet: I did therefore take notice, That in this Occaſion the *Air* bears a Preſſure much about the ſame as that of *Water* when its *Spring* is 32 Foot high. Now ſuch *Water* would ſpout out with a ſufficient *Velocity* to aſcend 32 Foot Perpendicular, and therefore according to the Rules and Obſervations of *Galliæus*, *Hugenius*, and others, ſuch Water hath the *Velocity* of 45 Foot in a Second. It remains therefore but to know the Proportion of the *Gravity* of the *Air* to that of *Water:* And we have found it not to be always the ſame; becauſe the Height, the Heat, and the Moiſture, of the *Atmoſphere*, are variable: Nevertheleſs, we may ſay in general, That the Reaſon between the *ſpecifick Gravities* of *Water* and *Air* is much about 840 to 1. Taking then their *Square Roots* as I have ſaid above, which *Roots* are 29 and 1, we may conclude that the *Velocity* of *Air* muſt exceed that of *Water*, by 29 Times: And ſo multiplying 45, the *Velocity* of *Water*, by 29, we ſhall find, that the *Velocity* of *Air* driven by the whole Preſſure of the *Atmoſphere*, is about 1305 Foot in a Second.

Wind produced by the Fall of Water; by Dr. Wal. Pope. n. 2. p. 25. *Fig.* 179. Apr. An. 1665.

XIII. In the *Braſs-Works* at *Tivoli*, the *Waters blow* the *Fire*, not by moving the *Bellows*, but by affording the *Wind*. Thus: A, is the *River*. B, the *Fall* of it. C, the *Tube* into which it falls. LG, a *Pipe*. G, the *Orifice* of the *Pipe*, or *Noſe* of the *Bellows*. GK, the *Hearth*. E, a Hole in the Pipe. F, a Stopper to that Hole. D, a place under ground, by which the Water runs away. Stopping the Hole E, there is a perpetual ſtrong *Wind*, iſſuing forth at G; and G being ſtopt, the *Wind* comes out ſo vehemently at E, that it will, I believe, make a *Ball* play, like that at *Freſcati*..

The beſt Form of Horizontal Sails for a Mill; by Dr. Rob. Hook, *Phil. Coll.* n. 3. *p.* 61. Dec. Ann. 1681.

XIV. Whatever Men may imagine concerning *Horizontal Sails*, I doubt there never will be found a better, and more advantageous way, for receiving the Strength of the *Wind*, or *Motion* of the *Air*, than *Perpendicular Vanes* made of a true Form, ſo as every part thereof may draw alike. But becauſe I find divers have of late attempted *Horizontal Vanes* for *Mills*, I ſhall explain a way of making *Horizontal Vanes* capable of performing the *moſt* that is *poſſible* with *Vanes* of equal *Extenſion.*

The *Invention* is founded upon the ſame Principle with that of the *Sailing* of *Ships*, and other *Veſſels* upon the Sea; namely, upon diſpoſing and ordering of the Vane or Sail ſo, as *to ſtand in the beſt Poſture 'tis poſſible to move the Arms of the Mill, or the Body of the Ship, in that way it is to be moved, by the Force of the Wind blowing thus or thus againſt them.*

The *Firſt Principle* then common to both, is, that the *Vane* or *Sail* be as near as 'tis poſſible, a perfect *plane* and *ſmooth Superficies*, without any Belly-ing, Bunting, or Curvity in the Superficies thereof, upon which the Motion or Force of the *Wind* is impreſſed.

Secondly, That the *Air* may have as many *Paſſages* between the parts of the *Vane* or *Sail*, as may be, that the moved *Air* may come to it as *freely* as may be, without being intercepted by a *ſtagnant Air* before it, to impede or divert its Force.

Thirdly, That the *Plane* of the *Vane* or *Sail* be put in the *middle Inclination*, between the *way* of the *Wind* and the *way* of the *Arm*, or that of the Body of the Ship.

The *Contrivance* itſelf is *This*.

Let A B ſignify the *Stream* or *Current* of the *Air* or *Wind*, moving from A to B, and let C repreſent the Center of the *Axis* or *Spindle*, ſtanding perpendicular to the *Horizon*, upon which, at the Top, is fixed at right Angles, the piece D H, making the two *Arms* C D, and C H, upon the Ends of which the *Vanes* M N, are moved on *Spindles*; ſo as that the Plane of the *Vane* doth always paſs thro' the Point D: I ſay, theſe *Vanes* ſo ordered, ſhall be always placed in the moſt *advantageous Poſture* for moving the *Arms* round upon the ſaid *Spindle*, whoſe Center is C, in the Order of D E F G H I K L D. *Fig.* 180.

Firſt, For the *Vanes* placed at D and H, I ſay, They are ſet in the moſt *advantageous Poſture* poſſible, in thoſe two Points: For *Firſt*, the *Vane* M N at D being to move directly againſt the *Wind*, the moſt *advantageous Poſture* is to turn its edge directly againſt the *Wind*, and thereby to give the leaſt *Reſiſtance* poſſible, that being the only Point in which the *Vane*, ſuppoſed only a Superficies, draws not. And *Secondly*, For the *Vane* M N placed at H, it ſtandeth the moſt *advantageouſly*, becauſe its Motion being directly from or before the *Wind*, it ſtandeth full *Croſs*, or oppoſed to the Motion thereof.

Secondly, The *Vanes* at E, F, G, and I, K, L, ſtand the moſt *advantageouſly* becauſe they *divide* the *Angle*, between the *way* of the *Wind*, and that of the *Arms* in thoſe Points into two equal Parts, and conſequently the Wind impreſſeth the greateſt Force in the moſt direct Way: For it is eaſy to be demonſtrated, That the Force impreſſed on the *Vane* by the *Wind*, is perpendicular to the Surface, and conſequently that the Obliquity of the Force to the way of the *Arms*, increaſed by the *Vanes* ſtanding more full againſt the *Wind*, will have a leſs Proportion of Power to promote the Motion thereof, than in the Poſture here ſet. And ſuppoſing the *Vanes* ſet ſharper to the *Wind*, the Diminution of the Force impreſſed by the *Wind* on its Surface, will be greater than the Augmentation of its Power by being moved more directly to the *way* of the *Arms*. This is eaſy enough to be *geometrically demonſtrated*.

The *Vane* may be ſo ordered, as always to ſtand in this *Poſture* by a great many ways: I ſhall only inſtance in one, not the beſt for *Practice*, but the moſt eaſy to be underſtood and *demonſtrated*.

Let the *Vane* be equally expanded on each ſide of its *Axis*, by which the Preſſure on the Extremes of it are always counterpoiſed; then faſten upon the lower end thereof a Wheel, which may be in a Diameter about $\frac{1}{2}$ of the length *Fig.* 181.

length of the *Arms* from Hole to Hole; then fix a Wheel upon the Frame in which the *Spindle* of the *Arms* do move, that ſhall be of half the Diameter with the former, and to contain half the number of Teeth. Then by a third ſmall Wheel, fixed under the *Arms*, of a convenient bigneſs, communicate the Motion of the one to the other; for by this means each *Vane* being ſo provided, they will, being once ſet right, always continue to be moved and diſpoſed in the true Poſture deſired.

This *Contrivance* will not only be uſeful for all manner of common *Wind-Mills*, but alſo for *Water-Mills* in *Rivers*, where there can no *Dam* be made; as may alſo the *perpendicular Vanes* of other *Mills*, neither of which has been ſo much as *hinted*, by any Perſon whatſoever, that I have hitherto heard of.

An Account of Flying; by Dr. Hook. *Phil. Coll.* n. 1. p. 14.

XV. 1. The Art of *Flying* hath been in all Ages *attempted* by many, particularly in the Times of our famous Friar *Roger Bacon*, who lived about 500 years ſince. He was believed a *Magician* or *Conjurer*, and to have performed what was related of him, by the help of *diabolical Magick*; but from the peruſal of ſeveral of his excellent *Works* yet extant, I eſteem him no ſuch Perſon. I rather find him to have been a good *Mathematician*, a knowing *Mechanick*, a rare *Chemiſt*, and a moſt accompliſhed *experimental Philoſopher*, which was a Miracle for that dark Age. This Man affirms the Art of *Flying* poſſible, and that he himſelf knew how to make an *Engine*, in which a Man ſitting, might be able to carry himſelf thro' the *Air* like a *Bird:* And affirms, that there was then another Perſon who had actually tried it with good Succeſs. We have not wanted later Inſtances in *England*, of ſeveral ingenious Men, who have employed their Wits and Time about this *Deſign*. Particularly, I have been credibly informed, that one Mr. *Gaſcoigne* did about 40 Years ſince try it with good Effect; tho' he ſince dying, the Thing alſo died with him. And even now, there are not wanting ſome in *England*, who affirm themſelves able to do it, and that they have proved as much by *Experiment*. We have little or no account of the ways they have taken to effect their Deſigns; but we may conclude them defective in ſomewhat or other, ſince we do not find them brought into *common Uſe*.

The Art of Flying; by S. Beſnier. *Ibid.* p. 15. *Fig.* 182.

2. The Sieur *Beſnier*, a Smith of *Sable* in the County of *Maine*, hath invented an *Engine* for *Flying*. It conſiſts of two *Poles* or *Rods*, which have at each end of them an oblong *Chaſſie* of Taffety; which *Chaſſie* folds from above downwards, as the frame of a folding *Window Chaſſie*. He ſits theſe *Poles* upon his Shoulders, ſo that two of the *Chaſſies* may be before him, and the other two behind him. The *order* of *moving* them is thus: When the *right Hand* ſtrikes down the *right Wing* before, A, the *left Leg* by means of the String E, pulls downwards the *left Wing* behind, B; then immediately after, the *left Hand* moves or ſtrikes downwards the *left Wing* before, C; and at the ſame time the *right Foot*, by the String F, moves or pulls down the *right Wing* behind, D; and ſo ſucceſſively, or alternately, the diagonally oppoſite *Wings* always moving downwards, or ſtriking the Air together.

3. 1. *P.*

3. 1. *P. Francifco Lana* in his *Prodromo*, finding by an *Experiment*, That the *Weight* of the *Air* is $\frac{1}{640}$ part of the *Weight* of a like quantity of *Water*, he concludes certainly, That if we could make a Veffel of Glafs or other Matter that might *weigh* lefs than the *Air* that is in it, and fhould draw out all its *Air*, this Veffel would be *lighter* in *Specie* than *Air* itfelf, and therefore would fwim in it and afcend on high. This he *fuppofes* may be done, by making a round Veffel of thin *Plate Brafs* (*weighing* 3 Ounces in a *fquare Foot*) of the *Diameter* of 14 Foot. For the *Surface* of the Veffel will be 616 *fquare Feet*, and the *Brafs* will *weigh* no more than 1848 Ounces; whereas the *Content* will be $1437\frac{1}{2}$ *Cub. Feet*, and that Quantity of *Air* will weigh $2155\frac{2}{3}$ Ounces: So that that Air being evacuated, the Veffel will be $307\frac{2}{3}$ Ounces *lighter* than *Air*, and therefore will not only afcend into the Air, but alfo carry up with it a weight of $307\frac{2}{3}$ Ounces. And thus by encreafing the Bulk of the Veffel, without encreafing the Thicknefs of the Plates of Brafs, he fuppofes a kind of Ship may be made, to fwim in the Air, and to carry two or three Men in it.

A Flying Chariot; by Fr. Lana. *Ibid. p.* 18.

2. The fallacy of the *Author*'s Reafoning lies in this; he fuppofes *Copper* of 3 Ounces in a Foot Square to be of fufficient Thicknefs to refift the Preffure of the *Air* in a *Globe* of 14 Foot *Diameter*, nay of any *Dimenfion*. But in this we can no wife affent to him: For the Preffure from without inwards, tho' it be always the fame upon equal Surfaces, yet upon unequal Surfaces the Cafe is quite otherwife, for there the Preffure will be found not the fame, but to encreafe always in the fame Proportion with the Surface, and thence confequently the *Thicknefs* of his *Copper*, or any Metal or Material, which he fhall make ufe of, muft increafe in the fame Proportion, with the *Diameter* of the *Sphere*, and confequently the *Weight* of his *Copper* muft always *increafe* in the fame Proportion at leaft to the *Solidity* of his *Sphere*; fo that by his augmenting the *Quantity* of his *Sphere*, he has no manner of Advantage of making it proportionably *lighter* than the *Air*, and proportionably *ftrong*, but the contrary: For it is manifeft, That a bigger *Sphere* fo made of any Matter we yet know, has lefs Power of refifting the fame *Preffure* of the *Air* than a lefs, becaufe of finite Refiftance of Matter to Preffure, there being fome degree of *Preffure* that will *crufh* every Body.

Shewn Impracticable by Dr. Hook. *Ibid. p.* 27.

XVI. This *Engine* is compofed of four principal *Parts*; the *Serpent* A A two Foot-Steps or *Treddles* B B, one *Clapper* C, and two *Arms* D D, D D.

An Engine to make Linnen Cloth; by M. de Gennes. n. 140. p. 1007. *Fig.* 183. July, An. 1678.

The *Serpent* or Iron Bar A A, has two Elbows, E E, whereto the Ends of the Ropes are fix'd that raife and put down the *Foot-fteps* B B; F F are two fourths of a Circle, that fucceffively reft upon two Arches or Bows of Iron, G G, which are above the *Clappler* C, to raife it. H H are two Teeth of Iron, added to the *Serpent*, making an Angle of 25 *Deg.* with F F, and K K; which ferve to put down a *Bafcule*, or *Sweep*, which is in the Arm that carries the *Shuttle*. The Foot-fteps or *Treddles* differ in nothing from thofe that are ufually made ufe of, only the Cords that hold them pendent from the Ground are fix'd in the Elbows of the *Serpent*, which in turning raifes raiſes and puts them down by the help of two little Pullies, upon which the Ropes turn.

The *Clapper* is ſupported between two Pillars, with a Rope double twiſted, which occaſions it to make a kind of Spring, and cauſes it naturally to give forwards to beat the Cloth.

L M, is one of the *Arms* which paſſes freely into the Canal or Pipe N N, ſupported by four Pillars of Wood O O O O. The Motion of it proceeds from the following Parts. P Q, is a *Baſcule*, which tho' unequally divided by its Supporter R, is yet *in Æquilibrio*, the end P R being made to weigh exactly as much as R Q.

At the Extremity of this *Baſcule* is ty'd a Cord which paſſes thro' the Pulley S, and terminates at the Extremity of the *Arm*, where it is faſtened, to a little Bow M. At the other Extremity of the ſame *Arm*, that is to ſay, towards L, is alſo faſtened underneath, a Cord, which paſſes thro' the Pulſey T, and which carries the Weight V.

At the ſame end of the *Arm* is added a little *Niche* Z, about the bigneſs of half the *Shuttle* : Then over a little Bar X Y, which paſſes a-thwart the *Arm*, there are two other little pieces of Wood, having at the end of them two Teeth which enter into the *Niche* Z, thro' two Holes which are there, of the one ſide and t'other.

To the Ends of theſe little pieces of Wood, there is a little *Bow* of Whalebone or Steel, which keeps the two Ends aſunder, and forces the Teeth, which are at the other end, to enter into the *Niche* before the ſaid pieces can themſelves. At the Points 1 1, are two Ropes that paſs thro' the Pullies 2 2, faſtened to the Pillars, *o* 3 *o* 4, and have each of them a little weight at the end big enough to keep it from paſſing thro' a little Bowl which is under each Pulley.

This *Arm* thus diſpoſed, goes and comes in the Hole N N, in the following manner. One Tooth of the *Serpent*, already deſcribed, ſtrikes upon the Extremity of the *Baſcule* P Q, and ſo cauſes the End Q to riſe up, which drawing the Cord faſtened to the Point Q, makes the *Arm* L M, to advance forward. But when afterwards the Tooth of the *Serpent* is come forth again, then the Weight V, tied to the other End of the ſame *Arm*, by a Cord that paſſes thro' the Pulley T, forces the ſaid *Arm* by its own Weight to return again.

When the *Arm* L M is in its ordinary place, the two little pieces of Wood into which enters the Bar XY, encloſe the *Shuttle* by means of the Whalebone-*Spring*. But when the ſaid *Arm* approaches the other oppoſite Arm, then the Cords tied to the Points 1 1, being a little too ſhort, and the Weight which is at the end of them not being able to paſs thro', the *Spring* gives way a little, and ſo the *Shuttle* is no longer encloſed by the *Arm* which carries it, but is wholly received and graſp'd by the other ; which likewiſe in its turn, delivers it back again in the ſame manner.

The Motion of the whole *Machine* is made at the rate as you move the Handle of the *Serpent*, for then the *Arms* cauſe the Threads to open, and immediately one of the *Arms* begins to ſlide in towards the oppoſite Arm,

to

to which it carries the *Shuttle*, and retires immediately: At the fame time, one of the Quarters of a Circle, which held the *Clapper* elevated, forfakes it, and leaves it for to flap, and then the oppofite Quarter of a Circle elevating itfelf, the other Elbow changes the Threads, and the other *Arm* retires; and fo fucceffively.

The *Advantages* of this *Engine* are thefe. 1. One *Mill* will fet 10 or 12 of thefe *Looms* at Work. 2. You may make the *Cloth* of what *Breadth* you pleafe. 3. There will be fewer Knots in the *Cloth*, fince the Threads will not break fo faft as in other *Looms*, becaufe the *Shuttle* that breaks the greateft Part, can never touch them. In fhort, The *Work* will be carried on *Quicker*, and at lefs *Charge*, in regard that inftead of feveral Work-Folks which are required in making very large *Cloths*, One Boy will ferve to tie the Threads at the feveral *Looms* as faft as they break, and to order the *Quills* about the *Shuttle*.

XVII. I ordered a *Model* of a part of a *Waggon* to be made confifting of four *Wheels*, two *Axes*, and a *Board* nailed upon the *Axes*. The *Leffer Wheels* were $4\frac{1}{3}$ Inches high, and the *Bigger-Wheels* $5\frac{1}{3}$ Inches high, *viz*, $\frac{1}{12}$ of the ordinary Height of the *Wheels* of a *Waggon*: The Weight of the *Model* was almoft $1\frac{1}{2}$ *lib.* I had alfo two other *Wheels* made $5\frac{2}{3}$ Inches high to be put on inftead of the Leffer. The Middle of the two *Axes* were $6\frac{1}{4}$ Inches afunder. All the *Wheels* turned very eafily upon the *Axes*.

Advantages of High Wheels experimented; by a Member of the Oxford Society. n. 167. p. 856. Jan. An. 1685.

A piece of Lead $50\frac{3}{4}$ *lib. Averdupoife*, was laid upon the *Model*, fo forward, that the *Leffer Wheels* feemed to bear above $\frac{2}{5}$ parts of the *Weight*. Then the *Model* was drawn with a String laid over a Pulley, the Top whereof was $\frac{1}{4}$ of an Inch higher than the Top of the *Hinder Axis*, and the Middle of this Pully was $7\frac{1}{2}$ Inches from the Middle of the *Fore Axis*.

The *Leffer Wheels* being put on, and the String being tied to the Top of their *Axis*,

1. *Three* Pound drew the *Model* on the fmooth Level Table.

2. *Twenty* Pound drew the *Leffer Wheels* over a Squared Rod $\frac{1}{4}$ of an Inch thick.

3. *Thirty* Pound drew *them* over a round Rod a little more than $\frac{1}{2}$ an Inch thick.

4. *Thirty One* Pound drew *them* over a Square Rod half an Inch thick.

5. *Twelve* Pound drew the *Hinder Wheels* over the bigger Square Rod.

The String being laid under the *Axis*, *viz.* $\frac{5}{8}$ of an Inch lower than before,

6. *Twenty nine* Pound drew the *Leffer Wheels* over the Bigger Square Rod.

Then the two *Bigger Wheels* being put on inftead of the *Leffer*, and the String lying over the *Axis*,

7. *Three* Pound drew the *Model* on the Table.

8. *Twenty five* Pound drew the *Fore Wheels* over the Round Rod.

9. *Twenty five* Pound drew them over the Bigger Square Rod.

10. The String lying under the *Axis*, 16 Pound drew them over the leaft Rod.

11. *Twenty three* Pound drew them over the round Rod.

12. *Twenty*

12. *Twenty three* Pound drew them over the Bigger Square Rod.

13. *Thirteen* Pound drew the *Hinder Wheels* over the Bigger Square Rod.

In all theſe *Experiments*, the *Lead* was laid exactly upon the ſame part of the *Board*, but yet when the *Leſſer Wheels* were taken off, the *Lead* did not lean ſo much forward, ſo that the *Hinder Wheels* were ſomewhat more preſſed than they were before.

By comparing the ſecond, third, and fourth *Experiments*, with the tenth, eleventh and twelfth, it appears how much more eaſily a *Waggon*, *&c.* might be drawn in rough Ways, if the *Fore Wheels* were as high as the *Hinder Wheels*, and if the *Thills* were fixed under the *Axis*. Such a *Waggon* as this, would likewiſe be drawn more eaſily, where the *Wheels* cut in *Clay* or *Sand*, or any Soft Ground. And moreover, *High Wheels* would not cut ſo deep as *Low Wheels*.

Low Wheels indeed are better for Turning in a narrow Compaſs than high Ones: But it ſeems probable that *Waggons* with four *High-Wheels*, might be ſo contrived, that there ſhould be no great Inconvenience in that reſpect; at leaſt, ſuch *Waggons* as ſeldom have occaſion to turn ſhort, as *Carriers Waggons*, and ſuch like.

The Difference which you may obſerve in the eighth and eleventh *Experiments*, is agreeable to what is ſaid by *S. Stevinus*, and *Dr. Wallis*, *viz.* That if a *Coach*, *&c.* muſt be drawn over rough, uneven Places, it is beſt to fix the *Traces* to the Coach lower than the Height of the Horſes Shoulders.

14. A Table $2\frac{1}{2}$ Foot long, was ſet with one End $8\frac{1}{2}$ Inches higher than the other End, and the *Model* being loaded as before, leſs Weight by 6 Ounces drew it up the Table, when the four *Bigger Wheels* were on, than when two *Bigger* and two *Leſs* were on. Becauſe, in the firſt Caſe there was almoſt the ſame *Direction* of the Motion of the *Model* and of the String that drew it; but not in the ſecond Caſe, when the *Fore Axis* was ſo much lower than the Top of the Pulley.

A new ſort of Caleſh deſcribed by Sir R. B. n. 172. *p.* 1028. June. An. 1685.

XVIII. This *Caleſh* goes on two Wheels; carries one Perſon, is light enough; tho' it hangs not on *Braces*, yet it is eaſier than the Common Coach: A common Coach will overturn, if one Wheel go on a Superficies a Foot and a half higher than that of the other, but this will admit of the Difference of $3\frac{1}{3}$ Foot in Height of the Superficies, without danger of Over-turning: We choſe all the irregular Banks, and ſides of Ditches to run over; and I have this Day ſeen it at five ſeveral Times turn over and over, and the Horſe not at all diſordered. If the Horſe ſhould be in the leaſt unruly, with the help of one Pin, you diſingage him from the *Caleſh* without any Inconvenience. I myſelf have been once overturned, and knew it not till I looked up, and ſaw the Wheel flat over my Head; and if a Man went with his Eyes ſhut, he ſhould imagine himſelf in the moſt ſmooth way. tho' at the ſame time there be three Foot Difference in the Height of the Ground of each Wheel.

The Contrivance of a perpetual Motion; by M. .... explained; by Dr. Papin. n. 177. p. 1240. Fig. 184. Dec. An. 1685.

XIX. Let DEF, be a pair of *Bellows* 40 Inches long, that may be opened by removing the Part F, from E: Let them be exactly ſhut every where, but at the Aperture E; and Let a Pipe EG, 20 or 22 Inches long, be ſodered to the ſaid Aperture E, having its other end in a Veſſel G, full of *Mercury*, and placed near the middle of the *Bellows*.

A, is an *Axis* for the *Bellows* to turn upon.

B, a Counterpoiſe faſtened to the lower end of the *Bellows*.

C, a Weight with a Claſp to keep the *Bellows* upright.

Now if we ſuppoſe the *Bellows* opened only to $\frac{1}{3}$, or $\frac{1}{4}$, ſtanding upright, and full of *Mercury*; it is plain that the ſaid *Mercury* being 40 Inches high, muſt fall, as in the *Torricellian Experiment*, to the Height of about 27 Inches, and conſequently the *Bellows* muſt open before F, and leave a *Vacuity* there. This *Vacuity* muſt be filled with *Mercury* aſcending from G thro' the Pipe GE, the ſaid Pipe being but 22 Inches long: By this means the *Bellows* muſt be opened more and more till the *Mercury* continuing to aſcend, makes the upper part of the *Bellows* ſo heavy, that the lower part muſt get looſe from the Claſp C, and the *Bellows* ſhould turn quite upſide down; but the Veſſel G, being ſet in a convenient place, keeps them Horizontal, and the part F, engageth there in another Claſp C; then the *Mercury* by its Weight runs out from the *Bellows* into the Veſſel C, thro' the Pipe EG, and the *Bellows* muſt ſhut cloſer and cloſer until the part EF comes to be ſo light, that the Counterpoiſe B is able to make the part F, get looſe from the Claſp C; then the *Bellows* comes to be upright again as before; the *Mercury* left in them falls again to the height of 27 Inches, and conſequently all the other Effects will follow, as we have already ſeen, and the *Motion* will *continue for ever*. *Fig. 185.*

And ſhewn inſufficient by him. Ib. p. 241. n. 182. p. 138.

Upon this, it is to be obſerv'd, That the *Bellows* can never be opened by the internal Preſſure, unleſs the ſaid Preſſure be ſtronger than the external. Now in the Caſe before us, it is plain, That altho' the lowermoſt part of the *Bellows* be preſſed outward by 40 Inches of *Mercury*, yet the upper part having no *Mercury* above it, bears none at all; the parts that lie in the middle near the Axis of the *Bellows* bear but 20 Inches, and ſo all the reſt muſt bear more or leſs, according as they lie higher or lower: It is evident therefore, That there are as many parts that bear leſs than 20 Inches, as there are that bear more, and the Increaſe of Preſſure following an *Arithmetical Progreſſion*, it is undeniable, that all theſe Preſſures added together, will do more than one uniform Preſſure, that would be equal to 20 Inches every where. It is alſo plain, that the Weight of the *Atmoſphere*, cannot come at the inward part of the *Bellows*, but thro' the Pipe GE, which containing 22 Perpendicular Inches of *Mercury*, doth counterpoiſe ſo much of the Weight of the *Atmoſphere*; ſo that this being ſuppoſed to be 27 Inches of *Mercury*, it cannot preſs the inward part of the *Bellows*, but with a weight equivalent to 5 Perpendicular Inches of *Mercury*. So that we find, the Inward Preſſure both of the *Mercury* and the *Atmoſphere*, is equivalent but to 25 Inches of *Mercury* in all: whereas the Preſſure of the *Atmoſphere* upon the Outſide is every where equal to 27 Inches; from whence it appears, That

That the Preſſure without is ſtronger than the Preſſure within. From this we may conclude, that the *Bellows* ſtanding upright will rather ſhut than open.

n. 185. p. 267. I ſhall ſay nothing to the Alterations this *Author* may make in his *Engine*, reſolving to leave it to others to ſhew him, that upon that Principle all he can do ſignifies nothing. And I doubt not, but if he pleaſes to conſult *M. Perault*, *De la Hire*, or any other at *Paris*, he will find them of the ſame Opinion with Mr. *Boyle*, and Mr. *Hook*, and others *here*.

The Speaking Trumpet improved; by Mr. J. Conyers. *n.* 141 *p.* 1027. Sept. An. 1678. *Fig.* 186.

XX. This *reflecting Trumpet* conſiſts of two Parts. The utmoſt *B b*, is a large Concave Pyramid, about a Yard long, (or may be of any manageable Length) open at the Baſe B, and cloſed not with a Flat, but a Concave Head, at the Cone B. Within this is faſtened a bended Tube *A a*. This *Trumpet* did at a Meeting of the *Royal Society* at *Arundel Houſe*, diſtinctly deliver ſome Words, croſs the Garden and the River *Thames*, and that againſt the Wind which was then ſtrong; and the words were written down by one, that was ſent over for that purpoſe: Whereby it appeared, That a *reflecting Trumpet*, after this, or ſome other like manner of Wood, Tin, Pewter, Stone, or Earth, or which may be beſt, of Bell-Metal, will carry the Voice as far, if not farther, than the long one invented by Sir *Samuel Moreland*. Beſides that, it ſeems to take off from the aſtoniſhing Noiſe near at hand, which happens in uſe of the ſaid long *Trumpet*: By Sir *Sam. Moreland*'s *Trumpet* angularly arched in the middle, the delivery of Sound to any diſtant Place was much ſhortened; and by another with three large angular Arches, reaching almoſt from one end to the other, the Sound was almoſt wholly obſtructed.

Fig. 187.
Fig. 188.

The Swiftneſs of Sounds and their Reflections or Ecchoes; by Mr. Walker. n. 247. p. 433. Dec. An. 1698.

XXI. I provided a *Pendulum*, of ſmall virginal Wire, with a Piſtol Bullet at the end of it, which had two Vibrations in one Second of Time. I took this *Pendulum*, and ſtanding over-againſt a high Wall, I clapt two pieces of ſmall Boards together, and obſerved how long it was ere the *Eccho* returned; and I removed my Station till I found the place whither the *Eccho* returned in about half a Second. But that I might diſtinguiſh the Time more nicely, I clapt every Second of Time, 10 or 15 times together; ſo that by this means, I could the better diſcover whether the diſtances betwixt the Claps and the *Ecchoes*, and the following Claps, were equal. And tho' it be very difficult to be exact, yet I could come within ſome few Yards of the place I ſought for, thus I obſerved the two Places, where I could but juſt diſcover that I was too near, and where I was too far off; and from the midway betwixt them I meaſured to the Wall, which Meaſure doubled, was the Space that the *Sound moved* in half a Second.

Here follow the Numbers of *Engliſh* Feet which a *Sound moved* in one Second of Time at ſeveral Trials.

Trials

Trials.	*Feet.*	*Trials.*	*Feet.*	*Trials.*	*Feet.*
1	1256	5	1292	9	1278
2	1507	6	1378	10	1290
3	1526	7	1292	11	1200
4	1150	8	1185		

Merſennus mentions an *Experiment* wherein he found the *Motion* of the *Sound* to be 1474 *Feet* in a *Second.* The *Academy del Cimento* cauſed 6 Harquebuſſes, and 6 Chambers to be fired one after another at the Diſtance of 5739 *Engliſh Feet*, and from the Flaſh to the Arrival of the Report each was 5″: And repeating the *Experiment* at the Midway, the *Motion* was exactly in half the time; and Mr. *Boyle* obſerved, that the *Motion* of *Sound* paſſes above 400 *Yards* in a *Second.*

When the *Firſt Trial* was made, there was ſome Wind ſtirring, tho' not much; the 2d, 3d and 6th were made in a Calm Morning. In the 8th, the *Eccho* was returned from a Wall at 395 *Yards* Diſtance in two *Seconds*; and in the 9th and 10th, at 213 and 215 *Yards* Diſtance, in one *Second.* The 4th was made at one end of St. *John's Cloiſter*, in *Oxford*, which is 104 *Feet* 7 *Inches* long, where the *Sound* was *Reflected* 11 *times* in two *Seconds*: And the 5th, on the North ſide of *New College Cloiſter* (which is 160 *Feet* 8 *Inches* long) where there are about 7¼ *Ecchoes* in two *Seconds.*

By ſome of thoſe *Experiments* that I tried, I am inclined to think, That the *Sound* moved quicker when it was Calm, than in a Wind, even when the *Sound* moved half way with the Wind; and that it moves ſwifter at firſt, than afterwards.

There is ſeldom any *Eccho*, where there is not ſome Wall, Wood, Bank, or ſuch like, directly oppoſite, that may Reflect the *Sound* to the Perſon that makes it; but in St. *John's Grove*, if you ſtand near the Gate leading from the *College* to the *Grove*, and Clap, the *Eccho* will return to you from the *Ball Court*, tho' a Line drawn from you to the *Ball Court* be not perpendicular to the Wall there, but as much oblique as the Line A B is to the Line B C; where A repreſents the Gate, B C the *Ball Court* Wall, and B D another Wall. Or, if you ſtand at E, the Corner of the *Grove* next to *Trinity*, and clap, the *Eccho* will return to you from the *Ball Court*. *Fig.* 182.

In the ſame *Grove* I ſtood about 20 Yards from the ſame Gate, and the Gate being ſhut, Clapt, and at other times Stamped, and the *Eccho* returned from the Gate as loud, if not louder than the Clap or Stamp.

An *Eccho* reflected from a Gate or Door, has uſually a baſer and duller Sound than that which is returned from a Wall, this being much briſker.

As I have been walking towards a Wall, I have clapped my Hands together ſeveral times, and I could diſtinguiſh the *Eccho* from the Clap, till I came within 7 or 8 Yards of the Wall.

In the *Cloiſters*, where, as we ſaid before, the *Eccho* was repeated ſeveral times, the firſt Repetition ſeemed to be ſlower than the ſecond or third; but

but of all the Repetitions, besides the first, the subsequent seemed slower than the precedent.

I have observed the Tossing of a *Sound* forward and back again, in very many Places where there are Parallel Walls; and where the Distance of the Walls is less, there the *Ecchoes* follow one another quicker.

Wheresoever a *Sound* was thus tossed betwixt two Walls, if I stood about the Middle, I could hear the *Sound* twice as quick, that is twice as often repeated in one Second, as if I stood near one Wall: The *Sound* being reflected to me from both ends, when I stood in the middle.

Fig 190. In *Trinity Ball Court*, when I stood and clapt at B, three or four Yards from the End of the Wall C, or at A, which is opposite to B, the *Sound* was tossed betwixt the opposite Walls, but not half so long as when I stood betwixt the Walls. In Places where there are Parallel Walls, not above six or eight Yards asunder, as in *Trinity Ball Court*, and at the Entrance into St. *John*'s *Grove*, *&c.* I have heard the Ecchoes of a Clap following one another distinctly enough: But there the *Ecchoes* of a *musical Note*, which was longer than a Clap, were so confused, that they seemed one continued *long Sound*: which makes me think, that the *Eccho* in some Vaults, is nothing else but the *Sound* tossed betwixt the side Walls, and betwixt the top and bottom. This also makes me conjecture, That the Reason why *stringed Musical Instruments* give a greater and longer *Sound* to the *Strings* than if the *Strings* were fixt to a single Board, may be this; because the *Sound* is tossed from side to side in the Belly of the *Instrument*.

The Doctrine of Sounds; by Narcissus, *Bishop of* Ferns *and* Leighlin. n. 156. *p.* 472. *Nov.* An. 1683.

XXII. I cannot better explain the *Usefulness* of this *Theory* of *Sounds*, than by making a Comparison 'twixt the Faculty of *Seeing* and *Hearing*, as to their Improvements. In order to which, I observe, That *Vision* is threefold, *direct*, *refracted*, *and reflex'd*; answerable whereunto we have *Opticks*, *Dioptricks*, and *Catoptricks*.

In like manner *Hearing* may be divided into *direct*, *refracted*, and *reflex'd*; whereto answer three Parts of our Doctrine of *Acousticks*, which are yet nameless, unless we call them *Acousticks*, *Diacousticks*, and *Catacousticks* (or in another Sense, but to as good Purpose) *Phonicks*, *Diaphonicks*, and *Cataphonicks*.

Direct Vision has been improved two ways.

1. *Ex parte Objecti*, by the Arts of *producing*, *conserving*, and *imitating*, and duly *applying*, Light and Colours.

2. *Ex parte Organi vel Medii*, by making use of *Tubes without Glasses*, or, a Man's closed Hand to look thro'. So likewise *direct Hearing*, partly has, and partly may further receive great and notable Improvements, both *ex parte Objecti*, and *ex parte Organi vel Medii*.

1. As to the Object of *Hearing*, which is *Sound*, Improvement has been and may be made, both as to the *begetting*, and as to the *conveying* and *propagating* (which is a kind of *Conserving*) of *Sounds*.

1. As to the *begetting* of *Sounds*. The Art of imitating any *Sound* whether by speaking (that is pronouncing) any kind of Language (which really

is

is an *Art*; and the *Art* of Speaking perhaps one of the greateſt) or by Whiſtling, or by Singing (which are allowed *Arts*) or by Hollowing, or Luring, (which the *Huntſman* and *Falconer* would have to be an *Art* alſo) or by imitating with the Mouth (or otherwiſe) the Voice of any Animal; as of Quails, Cats, and the like; or by repreſenting any Sound begotten by the Colliſion of ſolid Bodies, or after any other manner; theſe are all *Improvements* of *direct Hearing*, and may be *improved*.

Moreover, the Skill to make all ſorts of *muſical Inſtruments*, both antient and modern, whether *Wind Inſtruments* or *ſtring'd*, or of any other ſort, whereof there are very many (as *Drums*, *Bells*, the *Syſtrum* of the *Egyptians*, or the like) that *Beget* (and not only *Propagate*) *Sounds*: The Skill of making theſe, I ſay, is an *Art*, that has much improv'd *direct Hearing*; and an *harmonious Sound* exceeds a ſingle and rude one, that is an *immuſical Tune*: which *Art* is yet capable of farther Improvement And I hope, That by the Rules which may happily be laid down, concerning the *Nature*, *Propagation*, and *Proportion* or *adapting* of *Sounds*, a way may be found out, both to improve *Muſical Inſtruments* already in uſe, and to invent new ones, that ſhall be more ſweet and luſcious than any yet known. Beſides, that by the ſame means *Inſtruments* may be made, that ſhall imitate any *Sound* in Nature, that is not *Articulate*; be it of Bird, Beaſt, or what thing elſe ſoever.

2. The *Conveying* and *Propagating* (which is a kind of *Conſerving*) of *Sounds*, is much helped by duly *Placing* the *ſonorous Body*, and alſo by the *Medium*.

For if the *Medium* be *Thin* and *Quieſcent*, and the *ſounding Body placed* conveniently, the *Sound* will be eaſily and regularly *propagated* and mightily *conſerved*.

1. The *Medium* muſt be *Thin* and *Quieſcent*: Hence in a ſtill Evening, or the Dead of the Night (when the Wind ceaſes) a *Sound* is better ſent out, and to a greater Diſtance, than otherwiſe.

2. The *ſonorous Body* muſt be *placed* conveniently, *viz.* Near a *ſmooth Wall*, either *Plain* or *Arched* (*Cycloidically* or *Elliptically*, rather than otherwiſe; tho' a *Circular* or any *Arch* will do; but not ſo well.) Hence in a *Church*, the nearer the *Preacher* ſtands to the *Wall* (and certainly it's much the beſt way to place *Pulpits* near the *Wall*) the better is he heard, eſpecially by thoſe who ſtand near the *Wall*; alſo, tho' at a greater Diſtance from the *Pulpit*, thoſe at the remoteſt End of the Church, by laying their Ears ſomewhat cloſe to the Wall, may hear him eaſier than thoſe in the middle.

Hence alſo do ariſe *Whiſpering Places*. For the Voice being applied to one end of an *Arch*, eaſily rowls to the other. And indeed were the Motion and *Propagation* of *Sounds* but rightly underſtood, 'twould be no hard matter to contrive *Whiſpering Places* of infinite variety and uſe. And perhaps there could be no better or more pleaſant hearing a *Concert* of *Muſic*, than at ſuch a Place as this; where the *Sounds* rowling long together before they come to the Ear, muſt needs conſolidate and imbody in one; which becomes a true *Compoſition* of *Sounds*, and is the very Life and Soul of *Concert*.

2. If the *ſonorous Body* be placed near *Water*, the *Sound* will eaſily be *convey'd*, yet mollify'd; as Experience teacheth us from a *Ring* of *Bells* near

 a River,

a River, and a great Gun ſhot off at Sea; which differ much in the ſtrength, and yet Softneſs and Continuance, or *Propagation* of their *Sounds*, from the ſame at Land; where the *Sound* is more harſh and more periſhing, or much ſooner decays.

3. In a *Plain* a Voice may be heard at a far greater Diſtance, than in uneven Ground. The Reaſon of all which laſt nam'd *Phænomena* is the ſame; becauſe the *ſonorous Air* meeting with little or no Reſiſtance upon a *Plane* (much leſs upon an *Arch'd*) ſmooth *Superficies*, eaſily rowls along it, without being let or hindred in its Motion, and conſequently without having its Parts disfigured, and put into another kind of Revolution, than what they had at the firſt *begetting* of the *Sound*, which is the true Cauſe of its *Preſervation* or *Progreſſion*; and fails much when the *Air* paſſes over an uneven Surface, according to the degrees of its Inequality; and ſomewhat alſo, when it paſſes over the *Plain Superficies* of a Body that is hard and reſiſting.

Wherefore the *ſmooth* Top of the *Water* (by reaſon of its yielding to the *Arched Air*, and gently riſing again with a kind of *Reſurge*, like to *Elaſticity* tho' it be not ſo; by which *Reſurge* it quickens and haſtens the *Motion* of the Air rowling over it, and by its yielding preſerves it in its *Arched Cycloidical* or *Elliptical* Figure) the *ſmooth* Top of the *Water*, I ſay, for theſe Reaſons, and by theſe Means, *conveys* a *Sound* more entire, and to a greater Diſtance, than the *Plane Surface* of a piece of Ground, a Wall, or any other ſolid Body whatever, can do.

2. The *Organ*, which is the *Ear*, is helpt much by *placing* it near a Wall, (eſpecially at one end of an *Arch*, the *Sound* being *begotten* at the other) or near the Surface of Water, or of the Earth; along which the *Sounds* are moſt eaſily and naturally conveyed; as was before declared. And 'tis Incredible, how far a *Sound* made upon Earth (by the Trampling of a Troop of Horſes, for Example) may be heard in a ſtill Night, if a Man lays his Ear cloſe to the Ground in a large *Plain*.

Otacouſticks here come in for helping the *Ear*; which may be ſo contrived (by a right underſtanding the *Progreſſion* of *Sounds*, which is the principal Thing to be known for the due regulating all ſuch kinds of Inſtruments) as that the *Sound* might enter the *Ear* without any *Refraction*.

2. *Refracted Viſion* (which is always made *ex parte Medii*) ariſes from the different *Denſity*, *Figure*, and *Magnitude* of the *Medium*; which is ſomewhat altered by the divers *Incidence* of the *Viſive Rays*, and ſo it is in *Refracted Hearing*, all theſe Cauſes concur to its *Production*; and ſome others to be hereafter conſidered.

Now as any Object (a Man for Example) ſeen thro' a Thickened Air, by *Refraction* appears greater than really he is: So likewiſe a *Sound* heard thro' the ſame Thickened part of the *Atmoſphere*, will be conſiderably vary'd from what it would ſeem to be, if heard thro' a Thinner *Medium*. And this I call a *Refracted Sound*.

Improvements of *Refracted Viſion* have been made, by Grinding or Blowing Glaſſes into a certain Figure, and placing them at due Diſtances; whereby the

the Object may be (as 'twas) enabled to ſend forth its Rays more Vigorouſly, and the *Viſive Faculty* impowered the better to receive them. Thus,

1. A fine Glaſs Bubble, filled with clear Water, and placed before a burning Candle or Lamp, does help it to dart forth its Rays to a prodigious Length and Brightneſs.

2. The *Viſive Faculty* is much helped,

1. By *Spectacles* and other Glaſſes, which are made to help the *Purblind* and *Weak Eyes*, to ſee at any competent Diſtance.

2. By *Perſpective Glaſſes* and *Teleſcopes*, which help the *Eye* to *ſee* Objects at a very great *Diſtance*, which otherwiſe would not be diſcernible.

3. By *Microſcopes* or *Magnifying Glaſſes*, which help the *Eye* to ſee *near Objects*, that by reaſon of their Smallneſs were *Inviſible* before.

4. By *Polyſcopes* or *Multiplying Glaſſes*, whereby one thing is repreſented to the *Eye* as many, whether in the ſame or different Shapes.

After the ſame manner, Inſtruments may be contrived for aſſiſting both the *Sonorous Body* to ſend forth its *Sound* more ſtrongly, and the *Acouſtick Faculty* to receive and diſcern it more eaſily and diſtinctly. And thus;

1. An Inſtrument may be invented, that applied to the *Mouth*, (or any *Sonorous Body*) ſhall ſend forth the *Voice* diſtinctly as to a prodigious *Diſtance* and *Loudneſs*. For if the *Stentoro-phonicon* (which is but a Rude and Inartificial Inſtrument) does ſuch great Feats; what might be done with One compoſed according to the Rules of Art, whoſe *Make* ſhould comply with the Laws of *Sonorous Motion*, which that does not?

2. There are ſome *Inſtruments*, and more ſuch may be Invented to help the *Ear*: As,

1. *Otacouſticks* (and better may be made) to help *Weak Ears* to hear at a reaſonable Diſtance alſo. Which would be as great a help to the *Infirmity* of *Old Age*, as the other Invention of *Spectacles* is, and perhaps greater; foraſmuch as the *Hearing* what's ſpoken is of more daily Uſe and Concern to ſuch Men, than to be able to *read* Books, or to view Pictures.

2. A ſort of *Otacouſticks* may be ſo contrived, as that they ſhall receive in *Sounds* made at a very great *Diſtance*, which otherwiſe would have been *Inaudible*: and theſe *Otacouſticks*, in ſome Reſpects, would be of greater uſe than *Perſpectives*.

1. In Time of *War*, for diſcovering the *Enemy* at a good *Diſtance*, when he marches or lies incamp'd behind a Mountain or Wood, or any ſuch Place of Shelter, which hinder the Sight from reaching very far.

2. At *Sea*, when in dark *Hazy Weather* the Air is too thick, or in *Stormy* Tempeſtuous *Weather*, the Waves riſe too high, for the *Perſpective* to be made uſe of.

3. In Dark Nights, when *Perſpectives* become almoſt inſignificant, and yet at ſuch times, generally, Soldiers take their March, when they would ſurprize their Enemies.

4. *Microphones*, or *Micracouſticks*, that is, *Magnifying Ear Inſtruments*, which may be contrived after that manner, that they ſhall render the moſt *Minute Sound* in Nature diſtinctly Audible by *Magnifying* it to an unconceiveable

ceiveable Loudneſs: By the help whereof we may hear the diffe rent Cries and Tones of the ſmalleſt *Animals*.

5. A *Polyphone*, or *Polycouſtick*, ſo ordered that One *Sound* may be heard, either of the Same, or a different *Note:* Inſomuch that who uſes this *Inſtrument*, he ſhall at the Sound of a *Single Viol* ſeem to hear a whole *Concert*, and all True *Harmony*. By which means this *Inſtrument* has much the Advantage of the *Polyſcope*.

I have called it *Refracted Hearing*, becauſe made thro' a *Medium*, *viz.* Thick Air, or an Inſtrument, thro' which the *Sound* paſſing is broken or *Refracted*,

3. *Reflected Viſion* (which is always made *ex parte Objecti*) hath been improv'd by the Invention of *Looking Glaſſes* and *Poliſh'd Metals*, whether *Plane*, *Concave* or *Convex*, of ſeveral Figures, and placed at Determinate Diſtances.

In like manner *reflex'd Audition* (which is only made *ex parte Corporis Oppoſiti*) may be improved by contriving ſeveral ſorts of *Artificial Ecchoes*. For (ſpeaking in general) any *Sound* falling *directly* or *obliquely* upon any denſe Body of a ſmooth (whether Plane or Arch'd) Superficies, is beat back again and reflected, or does *Eccho* more or leſs.

I ſay, (1.) *Falling directly* or *obliquely*; becauſe, if the *Sound* be ſent out and Propagated Parallel to the Surface of the denſe Body, there will be no *Reflection* of *Sound*, no *Eccho*.

I ſay, (2.) Upon a Body of a *ſmooth Superficies*; becauſe if the Surface of the *Corpus Obſtans* be uneven, the Air by *Reverberation* will be put out of its regular Motion, and the *Sound* thereby broken and extinguiſh'd: So that, tho' in this caſe alſo the Air be beaten back again, yet *Sound* is not reflected, nor is there any *Eccho*.

I ſay, (3.) It does *Eccho more or leſs*, to ſhew, that when all things are, as is before deſcrib'd, there is ſtill an *ecchoing*, tho' it be not always heard, either becauſe the *direct Sound* is too weak to be beaten quite back again to him that made it; or that it does return home to him, but ſo weak, that without the help of a good *Otacouſtick* it cannot be diſcerned; or that he ſtands in a wrong Place to receive the *reflected Sound*, which paſſes over his Head, under his Feet, or to one ſide of him; which therefore may be heard by a Man ſtanding in that place, where the *reflected Sound* will come, provided no interpos'd Body does intercept it; but not by him that firſt made it.

Theſe *Ecchoes* (like *Reflected Viſion*) may be ſeveral ways *produced*, as;

1. A *Plane Corpus Obſtans reflects the Sound* back in its due *Tone* and *Loudneſs*; if allowance be made for the porportionable Decreaſe of the *Sound* according to its Diſtance.

2. A, *Convex Corpus Obſtans repels* the *Sound* (inſenſibly) ſmaller: but ſomewhat quicker (tho' weaker) than otherwiſe it would be.

3. A *Concave Corpus Obſtans ecccboes* back the *Sound* (inſenſibly) bigger, ſlower, (tho' ſtronger) and alſo *inverted*; but never according to the order of Words. Nor do I think it poſſible for the Art of Man to contrive

a ſingle *Eccho*, that ſhall invert the *Sound* and repeat backwards; becauſe then the Words laſt ſpoken, that is, which do laſt occur to the *Corpus Obſtans*, muſt firſt be *repell'd*; which cannot be. For where in the mean time ſhould the firſt Words hang and be conceal'd or lie dormant? Or how, after ſuch a Pauſe be reviv'd and animated again into Motion? Yet in complicated or compound *Eccboes*, where many receive from one another, I know not whether ſomething that way may not be done.

From this determinate *Concavity* or *Archedneſs* of theſe *reflecting Bodies*, it comes to paſs, that ſome of them from a certain Diſtance or Poſiture, will *eccho* back but one determinate *Note*, and from no other Place will they *reverberate* any; becauſe of the undue Poſition of the *ſounding Body*. Such an one (as I remember) is the *Vault* in *Merton College* in *Oxford*.

4. *The Echoing Body*, being removed farther off, *reflects* more of the *Sound*, than when nearer. And this is the Reaſon, why ſome *Eccboes* repeat but one Syllable, ſome one Word, and ſome many.

5. *Echoing Bodies* may be ſo contriv'd and plac'd, as that *reflecting* the *Sound* from one to the other, either *directly* and *mutually*, or *obliquely* and by Succeſſion, out of one *Sound* ſhall many *Echoes* be begotten; which in the firſt Caſe will be all together and ſomewhat involv'd or ſwallow'd up of each other; and thereby confuſed (as a Face in Looking-Glaſſes obverted;) in the other they will be diſtinct, ſeparate and ſucceeding one another, as moſt *multiple Echoes* do.

Moreover, a *multiple Echo* may be made, by ſo placing the *Echoing Bodies*, at unequal Diſtances, that they *reflect* all one way, and not one on the other; by which means a *manifold ſucceſſive Sound* will be heard (not without Aſtoniſhment; one Clap of the Hands like many; one *Ha* like a Laughter; one ſingle Word like many of the ſame *Tone* and *Accent*; and ſo one *Viol* like many of the ſame kind, imitating each other.

Furthermore, *Echoing Bodies* may be ſo ordered, that from any one *Sound* given, they ſhall produce many *Echoes* different both as to their *Tone* and *Intention*. By this means a *muſical Room* may be ſo contrived, that not only one *Inſtrument*, played on in it, ſhall ſeem many of the ſame Sort and Size; but even a *Concert* of (ſomewhat) different ones; only by placing certain *Echoing Bodies* ſo, as that any *Note* (played) ſhall be returned by them in *3ds*, *5ths*, and *8ths*, which is not poſſible to be done otherwiſe than was mentioned before in *refracted Audition*.

I have been thus large, that I might give you a little Proſpect into the Excellency and Uſefulneſs of *Acouſticks*, and that thereby I might excite others to bend their Thoughts, towards the making of *Experiments* for the compleating this (yet very Imperfect tho' Noble) *Science*; a *Specimen* whereof I will give in theſe three *Problems*.

Prob. I.] *To make the leaſt Sound (by the help of Inſtruments) as loud as the greateſt; a Whiſper to become as loud as the Shot of a Cannon.*

Prob.

By the help of this *Problem* the moſt minute Sounds in Nature may be clearly and diſtinctly heard.

Prob. II. *To propagate any (the leaſt) Sound to the greateſt Diſtance.*

By the help thereof any *Sound* may be conveyed to any and therefore heard at any *Diſtance*, (I muſt add, within a certain, tho' very large Sphere.)

Moreover by this means a *Weather-Cock* may be ſo contrived, as that with an ordinary blaſt of Wind it ſhall cry (or whiſtle) loud enough to be heard many Leagues. Which haply may be found of ſome Uſe, not only for Pilots in mighty tempeſtuous Weather, when *Light Houſes* are rendered almoſt uſeleſs: But alſo for the meaſuring the Strength of Winds, if allowance be made for their different Moiſture. For I conceive, That the more dry any Wind is, the louder it will whiſtle, *cæteris paribus*: I ſay, *cæteris paribus*, becauſe, beſides the ſtrength and dryneſs of *Winds* or Breath, there are a great many other things (hereafter to be conſider'd) that concur to the increaſe of *magnifying of Sounds*, begotten by them in an Inſtrument expoſed to their Violence, or blown into.

Prob. III.] *That a Sound may be convey'd from one extreme to the other (or from one diſtant Place to another) ſo as not to be heard in the middle.*

By the Help of this *Problem* a Man may talk to his Friend at a very conſiderable Diſtance, ſo that thoſe in the middle Space ſhall hear nothing of what paſſed betwixt them.

Fig. 197. I ſhall here add a *Semiplane* of an *Acouſtick* or *Phonical Sphere*, as an Attempt to explicate the great Principle in this *Science*, which is the *Progreſſion* of *Sounds*.

You are to conceive this (rude) *Semiplane* as *Parallel* to the *Horizon*; for, if it be *Perpendicular* thereunto, I ſuppoſe the upper *Extremity* will be no longer Circular, but *Hyperbolical*, and the lower part of it ſuited to a greater Circle of the Earth. So that the whole *Phonical Sphere* (if I may ſo call it) will be a ſolid *Hyperbola*, ſtanding upon a *Concave Spherical Baſe*. I ſpeak this concerning *Sounds* made (as uſually they are) nigh the *Earth*, and whoſe *ſonorous Medium* has a free Paſſage every way. For if they are generated high in the Air, or directed one way, the Caſe will be different; which is partly deſigned in the Inequality of the Draught.

XXIII A Paper, *of leſs General Uſe*, omitted, *viz.*

Carriages. n. 161. p. 666. EXperiments *to be made, relating to* Carriages; *propoſed by Sir* William Petty.

XXIV. Ac-

XXIV. Accounts *of Books* and Additions, omitted.

1. DE *vi Percussionis*, Joh. Alphons. Borelli. Bononiæ, 1667. *in 4to.* n. 32. p. 626.

2. *De Motionibus à* Gravitate *dependentibus Liber*, Jo. Alphonsi Borelli, *in Academia* Pisana *Mathefeos* Professoris, Regio Julio 1670. *in 4to.* n. 73. p. 2210.

3. *Dialogi Physici, quorum Primus de Lumine; Secundus & Tertius de* Vi Percussionis & Motu; *Quartus de Humoris Elevatione per Canaliculum; Quintus & Sextus de Variis Selectis. Auth.* Honor. Fabry, *S. Jesu.* Lugduni Galliarum. 1669. *in 8vo.* n. 67. p. 2057.

4. Mechanica, *sive de* Motu, *Tractatus Geometricus: Auth.* Joa. Wallis, *S. S. Th. D.* Londini, 1670. 1671. *in 4to. The* Author *here makes some* Additions *to* Prop. I. Chap. XV. p. 753. *concerning the* Center of *Gravity of the* Hyperbola. n. 54. p. 1086. n. 61. p. 2005. n. 76. p. 2286. n. 87. p. 5074.

5. *Exercitationes Mechanicæ*, Alexandri Marchetti. Pisis, 1669. *in 4to.* n. 61. p. 2008.

6. *De Resistentia Solidorum*, Alexandri Marchetti *in* Pisana *Academia Phil. Prof.* Florentiæ. 1665. *in 4to.* n. 82. p. 4050.

7. *Hypothesis, Physica nova, sive Theoria* Motus Concreti, *una cum Theoria* Motus Abstracti. *Auth.* Gothfredo Gulielmo Lebnitio. *J. V. D.* Lond. 1621 *in* 12°. *Of this Book Dr.* Wallis *here gives his Opinion.* n. 73. p. 2215. n. 74. p. 2227.

8. *La Statique ou la Science de* Forces Mouvantes, *par le P.* Ignace Gaston Pardies, *S. J. à* Paris 1673. *in* 12°. *The first Part being of* Local Motion. *Printed at* Paris, 1670. *was* Englished *and printed at* London, *the same Year, in* 12°. n. 94. p. 6042. n. 65. p. 2010.

9. Christiani Hugenii Zulichemii *Horologium Oscillatorium.* Parisiis, 1673. *in* Fol. n. 95. p. 6068.

10. *A Discourse made before the* Royal Society *concerning the Use of* Duplicate Proportion *in sundry important Particulars; together with a* New Hypothesis *of* Elastique *or* springy Bodies: *By Sir* William Petty. n. 109. p. 209.

11. *Traité de la* Percussion *ou* Choq *des* Corps, *&c. par M.* Marriotte, *de l'* Academie Royal *des* Sciences. *A* Paris, 1673. *in* 12°. n. 134. p. 859.

12. *Philosophiæ Naturalis Principia Mathematica. Authore* If. Newton *Lond. in 4to.* n. 186. p. 291. n. 226. p. 445.

13. *Traité de* Mouvement *des* Eaux *& des autres* Corps Fluids. *Par feu M.* Mariotte, *à* Paris, 1686. *in 8vo.* n. 181. p. 191.

14. *Mechanick Exercises; or,* The Doctrine of *Handy Works. By Mr.* Jos. Moxon. Lond. 1677. *in 4to.* n. 138. p. 967. n. 139. p. 967.

15. *The* Speaking-Trumpet, *as it hath been contrived, and Published, by Sir* Samuel Moreland; *together with its* Uses *both at* Sea *and* Land. Lond. 1671. n. 79. p. 3056.

CHAP. VI.

Hydrostaticks. Hydraulicks.

To weigh Water, or other Fluids; by . . . . . . A new Areometer; *by M.* Homberg. n. 262. p. 530. *Fig.* 192.

I. 1. TAKE a *Viol* with a very *narrow Body*, and, when it is almost full, the Water is to be dropt into it, drop by drop, till it can hold no more. Then *weigh* it exactly, and deduct the *weight* of the *empty Viol.*

2. A is a Glass Bottle like a little *Matracium*, of which the *Neck* BC is so small that a drop of Water therein takes up the Space of 5 or 6 Lines; near that *Neck* is a little Capillar Tube D, about 6 Lines long, and Parallel to the *Neck* BC; the opening B is a little dilated, in the Fashion of a Tunnel, for pouring more easily the Liquors into the Bottle, and the little Tuble D is for giving a way to the Air contained in that Vessel to go out, when the Liquor is poured in at B; the Point C is a little Mark at the same height as the end of the little Tube D.

When we fill the Vessel, we pour the Liquors into it, by the opening B, until it goes out by the little Tube D, and, if the Height of the Liquor is even to the Mark C, 'tis well; if it is lower, we must fill more to that Point; if it is higher, we must strike softly upon the opening B, till the Overplus of the Liquor be even to the Point C in the *Neck* of the Bottle. By that means we have always exactly the same *Volume* of *Liquor*, and we can know how the same *Volume* of the several Liquors weighs more one than another precisely. But we must consider the variation of the Weather, when we compare the weight of a Liquor which we weigh in Summer time, with the weight of another which we have weighed in the Winter; for the same Liquor, being more rarified in the hot time, and condensed in the cold, the same Volume of it will be more weighty in cold Weather than in warm.

A New *Essay Instrument; by M.* Boyle. n. 24. p. 447. n. 115. p. 329. *Fig.* 193.

II. 1. Many Years ago I made use of a little *Glass Instrument*, consisting of a *Bubble*, and furnished with a *long* and *slender Stem* to compare the *specifick Gravities* of *different Liquors* by its more or less *sinking* in *them*: And I have since employed it to discover the *specifick Gravities* of *several Solids*, *appended*, by its being more or less depressed by them in the *same Liquor*. For 'tis clearly deducible from the Grounds of *Hydrostaticks*, that any solid Body heavier than Water, loses in the Water as much of the weight it had in Air, as Water of equal Bulk to the immersed Solid would weigh in the Air; and consequently, since Gold is by far the most ponderous of Metals, a piece of Gold, and one of equal Weight of Copper, Brass, or any other Metal, being proposed, the Gold must be less in bulk, than the Copper or Brass. And by this means, if both of them be weighed in the Water, the Gold must

muſt loſe in that Liquor leſs of its former Weight than the Braſs or Copper; becauſe the baſer Metal, as well as the Gold, grows lighter by the weight of a bulk of Water equal to it: and the baſer Metal being the more voluminous, the correſpondent Water muſt weigh more than that which is correſpondent to the Gold. Whence I concluded, that the *floating Inſtrument* abovementioned would be made to ſink deeper by an Ounce, for Inſtance of Gold, hanging at it under Water, than by an Ounce of Braſs, or any other Metal, which, by reaſon of its greater Bulk than Gold, loſing more of its weight by the Immerſion, muſt needs retain leſs, and ſo have leſs power to depreſs the Inſtrument 'twas faſtened to. Which Concluſion will alſo hold (tho' the Diſparity be not ſo great and conſpicuous) in reference to other Metals, as Lead and Tin, that differ in *ſpecifick Gravity*.

This *Inſtrument* may be of *Glaſs*, *Copper*, *Silver*, or almoſt any other ſolid *Body*, that is, or may be made, fit to *float* on the Water, with a *Guinea*, *&c. hanging* at it, and of a Texture cloſe enough to keep out the Water. It conſiſts of three Parts: The *Ball* or Globulous part; the Stem or Pipe; and that which *holds* the *Coin*.

The *Ball* or Round part of BCDE (if of *Metal*) conſiſts of two thin concave Plates, exactly ſoldered together in the middle; and at the diſtanteſt parts from the Commiſſure, there ought to be left two oppoſite *Holes*, one in each Plate, for the two other parts of the Inſtrument. This middle part, tho' for Brevity's ſake we name it the *Ball*, ſhould not be exactly round; but of any Shape that ſhall be found fit to make the *Inſtrument* keep to its erect poſture ſteadily in the Water. It muſt contain as much Air as may ſerve to keep the whole *Inſtrument*, when loaded, from ſinking beneath the top of the *Stem*. *Fig.* 194.

The *Stem* AB is to be ſoldered on to the *Ball* at the uppermoſt of the two mention'd *Holes*. It may be either hollow or ſolid: But it ought to be made very ſlender, that the different Depreſſions of the Inſtrument in the Water may be the more notable. And, for the ſame reaſon, it ought not to be too ſhort, eſpecially if it be to be applied to other Uſes than the Examining of *Guineas*.

At the undermoſt of the two *Holes* in the *Ball*, is inſerted and ſolder'd the undermoſt part of the *Inſtrument*, which I call the *Screws*, or the *Stirrup*. The *Screw* F is a very ſhort Piece of Braſs with a broad Slit in it, capable of receiving the Edge of the *Guinea*, which with one turn or two of a ſmall and ſlight lateral Screw may be kept faſt in it, and readily, the Operation being ended, taken out again. The *Stirrup* G is made of a piece of Wire, that, a little beneath the bottom of the *Ball*, is bent round, ſo as to ſtand Horizontally, that the *Guinea* may be laid on it. *Fig.* 195.

It would be convenient, that the *undermoſt Stem* and the *Screw* be made by itſelf, that it may be at pleaſure thruſt upon the Stem, and taken off again. For, by this means, if the *Ball* of the *Inſtrument* be made large enough, you may have room to put on for Ballaſt, as occaſion ſhall require, one, two, or three flat and round pieces of Copper, Lead, *&c.* with each of them a Hole *Fig.* 196.

 in

in the middle fitted to the Size of the Stem, ſo that they may be put on as near the lower part of the *Ball* as you think fit, and then the *Screw* may be thruſt on after them, not only to take Hold of the *Coin* or metalline Mixture to be *examin'd*, but to ſupport the thin Plates.

To adjuſt this *Inſtrument* for the uſe of examining *Guineas*, which are by far the moſt uſual *Gold Coins* that paſs in *England*, you muſt by the help of the *Stirrup* or *Screw*, hang, at the bottom of it, a piece of that *Coin* which you know to be genuine, and having carefully ſtopt the Orifice of the Stem (if it be a Pipe) that no Water may get in at it, immerſe the *Inſtrument* leiſurely and perpendicularly into a Veſſel full of clean Water, 'till it be depreſt almoſt to the top of the Stem, and then letting it alone; if, being ſettled, it continue in the ſame Station and Poſture, your work is done, if it emerge, you muſt add a little weight to it, either by putting into the *Stem*, if it be Hollow, ſome Duſt Shot, Filings of Lead, or ſome other minute and heavy Body, or elſe by putting on the ſhort Stem abovementioned, that comes out beneath the *Ball*, a flat, round and perforated piece of Lead, of Weight ſufficient to enable the *Guinea* to depreſs the Weight as low as 'tis deſired: But, if it ſink quite under Water, you muſt lighten it either with a File, or by ſcraping or grating off a little of the Ballaſt Plate abovementioned; or, if you have put any Weight into the Cavity to poize it, by taking out ſome of that, till you have made it light enough: This being done, a Mark, H, is to be made juſt at the place where the Surface of the Water touches the *Stem*, and then taking out your *Inſtruments*, ſubſtitute, in the place of your *Guinea*, a little round Plate of Braſs, of the ſame Weight, or a Grain or two heavier, in the Air; and, putting the *Inſtrument* into the Water, as before, ſuffer it to Settle, and make another Mark I, at the Interſection of the *Stem* and the Horizontal Surface of the Water.

Fig. 193.
Fig. 194.

There may (tho' 'tis like there very ſeldom will) happen a Caſe wherein, tho' the Principle our *Inſtrument* is framed on, will hold good, yet the Practical Application may be Unſecure. For, if a *Falſifier* of *Money* have the Skill, by *Waſhing* or otherwiſe, to take off much of the Quantity or Subſtance of the *Guinea*, without altering or impairing either the *Figure* or *Stamp*, the piece of Coin will not be able to depreſs our Inſtrument to the uſual Mark, and may thereby make it to be judged *Counterfeit*, when 'tis indeed but too *Light*. But it preſently ſhews, that the propoſed *Guineas*, if it be not *Counterfeit*, is otherwiſe *Abuſed*; and, tho' it does not clearly determine, whether that likewiſe proceed from the want of *Specifick Gravity* in the Metal, or from the Coins having been *waſhed*, or otherwiſe fraudulently *leſſened*; yet it probably reſolves the doubt, becauſe, if the want of Weight appear by the *Inſtrument* to be very great, as it uſually does, where the piece has been robbed of ſome of its Subſtance, 'tis a ſtrong Preſumption, that 'tis rather *Waſhed*, &c. than *Counterfeited*. However, it will be ſure to prompt him that uſes it, to employ the Balance, which will preſently aſſiſt him to reſolve his Doubt. For if the Suſpected Coin have in the Air its due Weight, 'twill argue that the great Lightneſs of it in the Water proceeds

ceeds from its not being of the requisite Finenefs; and, if it want much of its due Weight in the Air, 'tis very probable, that 'tis *Washed*, *&c.* rather of another Metal than *Gold*.

Any other kind of *Gold Coin*, that is near about the Weight of a *Guinea*, may be examined by our *Instrument* after the Manner above deliver'd. If the *Coin* be heavier than a *Guinea*, as is a twenty Shilling piece of *Broad Gold*, the *Ballast*, whether internal or external, of the *Instrument*, muft be taken off, that fo heavy a *Coin* may not quite fink it. But, if it be lighter than a *Guinea*, one may add as much *Gold* (of the fame *Alloy*) beaten into thin Plates, as with the Coin propofed, will make up in the Air the Weight of a *Guinea*. For then this Aggregate, being examined, as if it were a *Guinea*, will difcover in the Water, whether the *Coin* be *Right* or *Counterfeit*.

This *Instrument* may be alfo made to ferve to examine fome forts of *white Money*, lefs heavy than Half-Crowns. And, becaufe it may be ufeful to know in General, what Coins may, and what may not, be examined by this or that particular *Instrument* propofed, I fhall here add a general way that is not difficult for finding this out; namely, firft by Weighing the piece of *Gold* or *Silver* in the Air, and afterwards in the Water, and Subtracting the latter from the former, to obtain the Difference of the two Weights: And next by Weighing alfo in the Air and in the Water a piece of Copper or Brafs, if this be the likelieft to be employed in *Counterfeiting* the *Coin*, and obferving likewife the Difference between thofe Weights. For, if the leffer of thefe Differences being fubtracted from the greater, the Remains will fhew, how much the true piece of Coin will out-weigh the other in the Water, and confequently, if fo many Grains, as this refidue amounts to, being added to the Weight of the lighter Metal, do make a fufficiently manifeft Depreffion of it below the Mark it would ftay at without that Addition, one may probably conclude, that the Difference between a True and Counterfeit piece of Coin propofed, will be difcoverable by the *Instrument*. But it may be Expedient, for thofe that have frequent Occafions to examine various forts of Coin, to have a feveral *Instrument* adjufted for each of them, to fave themfelves fome Pains and Trouble.

With this *Instrument*, Pure *Tin* may be certainly diftinguifh'd from fuch as is adulterated. For, as *Gold*, being the Heavieft of Metals, cannot be allay'd by any other that will not deprefs our *Instrument* lefs than *Gold* can do; fo *Tin*, being the lighteft of Metals, cannot be mixed with any other that will not fink it lower than unmix'd *Tin*, (ftill fuppofing the Weight to be the fame in the Air.)

After the fame manner may *Pewter* be compared and examin'd. For, having once obferv'd how much the *Instrument* is depreft by a piece of two, three, or four Drams, or even an Ounce Weight of *Pewter*, which is known to be good, and to contain fuch a proportion of Lead in reference to the Tin, if you load the *Instrument* with an equally Heavy piece of any other Mafs of *Pewter* propounded, if the *Instrument* fink deeper, 'twill be a fign that the former Proportion of Lead may be very probably argued to exceed in the

mixture;

mixture; I say probably, because perhaps 'tis possible to embase *Pewter* by mixing not only Lead, but other Mineral Substances, whose *Specifick Gravity* is not well known: But yet I say very probably, because the Addition of too much Lead is the most Gainful way of Adulterating *Pewter*.

This *Instrument* may also assist us, to make such an Estimate as will not much deceive us of the *Fineness* of *Gold* and its differing *Allays* with Silver, or some other determinate Metal.

In order to this, the *Instrument* may be fitted to sink to the tip of the Pipe with some determinate Weight of the *Finest Gold*, as of 24 *Carats*, as they call that which is most *Pure* and *Fine*. But 'twill be convenient, that this Metal, in the Air, be just an Ounce, or half an Ounce, or some such Determinate Weight, that is commodiously Divisible into many aliquot Parts. Then you may make a Mixture, that contains a known Proportion of the Metal wherewith you *allay* the *Gold*; as if it hold 19 or 15 parts of *Gold*, and one of *Silver*; and, letting the *Instrument* settle in the Water, mark the place where the Surface of the Water cuts the *Stem* or *Pipe*. And then putting in another Mixture, wherein the Silver has a new and greater Proportion to the *Gold*; as if the former be an 18*th* or 14*th* part of the latter, you may observe, how much less than before this depresses the *Instrument*, and so you may proceed with as many Mixtures or Degrees of *Allays* as you think fit, or can be distinguish'd conveniently on the *Stem*; being always careful, that, whatever be the Proportion of the two Ingredients, the Weight of the Mass in the Air be just the same with that of the *Pure Gold*, which we may have lately supposed to be an Ounce, or half an Ounce.

By the same Method may be examined the differing *Allays* of *Pure Silver*, upon the Admixture of such and such determinate Proportions of Copper, or any other Metal lighter in Specie than *Silver*; and by the same way, with a slight Variation, 'twill not be difficult to estimate, how much divers *Coins*, whether of *Silver* or *Gold*, are more or less embas'd by the known ignobler Metal that is mix'd in the piece proposed. These Estimates (which may be made without much Trouble) will come nearer the Truth, not only than the Estimates wont to be made by the *Touch-Stone*, but perhaps too, than some of those that divers make with Trouble, Inconvenience, and Charge.

It may be also employ'd to examine other *Mixtures* besides *Allay'd Coins*, and that if the *Instrument* be adjusted to an Ounce, for instance, of Pure Copper, it may help Men to make an Estimate of the *Allay* of *Tin*, or the Quantity of it that is oftentimes added to *Copper*, to make different Sorts of *Bell-Metal*, and of those *Metalline Specula*, whether Plane or Concave, that are call'd Steel Glasses, as also of Solders consisting of certain Proportions of Silver aud Brass, or Copper; in all which, and divers others, the Discovery of the Proportion of the Ingredients may, on some Occasions, be useful to Tradesmen, as well as desirable to *Virtuosi*. And tho' I have observed, that by Mixture, Tin and Copper acquire a *Specific Gravity* somewhat differing from what their Ingredients promise; yet, since the *Instru-*

ment is to be fitted for ſuch Eſtimates, not by Calculation, but by Trials, the Eſtimates may be made near enough to the Truth.

2. Long ſince I took Notice, how light and Silver-like the Pewter was, which deſcended to us; but, as ſoon as, to follow the Faſhion, we changed it, the Weight and the very Colour was altered; and is in every Change more and more embaſed. And, if our *Silver-Smiths* hold on their degrading Mixtures, I ſhall queſtion, whether our Silver-Plate may not ſhortly come down to approach our Fore-Fathers Pewter: I mean, in the Country, where 'tis never or ſeldom tried.

Farther conſidered; by . . . . n. 116. p. 553.

III. A Glaſs Bubble, of about the Bigneſs of a Pullet's Egg, was purpoſely blown at the Flame of a Lamp, with a ſomewhat long Stem turn'd up at the End, that it might the more conveniently be broken off. This Bubble being very well heated to rarify the Air, and thereby drive out a good part of it, nimbly ſealed at the End, and, by the help of the Figure of the Stem, was by a convenient Weight of Lead depreſſed under Water, the Lead and Glaſs being tied by a String to one Scale of a good Balance, in whoſe other there was put ſo much Weight, as ſufficed to Counterpoiſe the Bubble, as it hung freely in the midſt of the Water. Then with a long Iron Forceps I carefully broke off the Seal'd End of the Bubble under Water, ſo as no Bubble of Air appear'd to emerge or eſcape thro' the Water, but the Liquor by the Weight of the Atmoſphere ſprung into the unrepleniſh'd part of the Glaſs-Bubble, and filled the whole Cavity about half full; and preſently, as I foretold, the Bubble ſubſided, and made the Scale it was faſtened to, preponderate ſo much, that there needed 4 Drachms and 38 Grains to reduce the Balance to an *Æquilibrium.* Then, taking out the Bubble with the Water in't, we did, by the help of the Flame of a Candle, warily applied, drive out the Water (which otherwiſe is not eaſily excluded at a very narrow Stem) into a Glaſs counterpoiſed before; and we found it, as we expected, to weigh about 4 Drachms and 30 Grains, beſides ſome little that remained in the Egg, and ſome ſmall matter that may have been rarify'd into Vapours, which added to the Piece of Glaſs that was broken off under Water and loſt there, might very well amount to 7 or 8 Grains. By which it appears not only, that Water hath ſome Weight in Water, but then it weighs very near or altogether as much in Water, as the ſelf ſame Portion of a Liquor would weigh in the Air. We repeated the Experiment with another ſeal'd Bubble as big as a great Hen-Egg, with like Succeſs.

The Weight of Water in Water; by Mr. Boyle. n. 50. p. 6001.

IV. *Apr.* 7. 1680. Being off of *Pantalara* near *Sicily* in a Calm, I let down a *Bottle* 70 *Fathom*, ſtop'd with an excellent good tender Cork, well fitted, and the Cork came up in the Bottle ¾ full of Salt Water. The Bottle was again fitted with an excellent good Cork, but of a Woodineſs or Hardneſs as ſome Corks are; with the which, being let down in like manner, the Cork continued in its Place; but as it were bruiſed, and the Bottle, as before, about ¾ full of Salt Water: Whereupon I took a good Ox Bladder, and

The Preſſure of Water in great Depths; by a Perſon of Honour. n. 193. p. 504.

and bound it four fold over the Mouth of the Bottle without any Cork at all, only I put a piece of Leather to keep the Glaſs from cutting the Bladder; and ſo ordered, it was let down as before, but taken up without any Water, or the leaſt Moiſture in it.

May 18. 1680. Being in a ſtark Calm ſome Leagues diſtant from the Coaſt of *South Spain*, off the great Hills of *Granada*, we took a Bottle and clapt a Leather on the mouth of it, tying over that a ſingle part of the Bladder, the which we let down 75 *Fathom*, but it came up again entire: We then made a Hole in the Leather, about the bigneſs of a large Pea, and let the ſame down again 75 *Fathom*, but it came up perforated in the vacant place where the Leather had the Hole in it, and almoſt full of Water: we then bound over another part of the Bladder ſingle, and let it down but 30 *Fathom*, but it came up whole and entire; whereupon immediately we let it down 50 *Fathom*, but it came up broke and full of Water. Then, we again fitted the Bottle with the ſaid perforated piece of Leather and a double Bladder, and let it down 50 *Fathom*; but it again came up entire: So again, immediately we let it down 75 *Fathom*, but then it came up broken and full of Water.

June 24. 1680. Being in $39\frac{1}{4}$ Degrees of *Latitude*, and by the Ship's Account 150 Leagues Weſtward of *Portugal*, I cauſed a *Florence Flask* to be well ſtopped with a Bladder over the Mouth of it, and lower'd it down 30 *Fathom*, but it was taken up broken. Whereupon, imagining that the roughneſs of the Lead's halling ſo tender a Body ſo violently thro' the Water might be the breaking thereof, I cauſed another *Flaſk* in like manner to be fitted, and cloſe by it I tied likewiſe another *Flaſk*, ſo as to be borne with the Mouth downwards, as were the other, but which was not ſtopp'd; and theſe I cauſed to be taken up when they had been but 10 *Fathom* under Water, and found them both entire; but the open *Flaſk* almoſt full of Water; the which being emptied, were both let down again and taken up at 20 *Fathom*, when the open *Flask* was entire, tho' full of Water, but the other broken to pieces.

By Dr. Oliver. n. 204. p. 908. *Jun.* 8. 1693. In the *Bay* of *Biſcay*, when we had 100 *Fathom* of Water, we took a Quart-Glaſs-Bottle ſtopt with a large Cork: And, faſtening it to our Plumbing-Rope with the Lead at the end, we ſunk it to the *Bottom* of the Sea, which as ſoon as we perceived, we drew it up again, and found the Cork quite preſſed thro' the Neck of the Bottle into its Cavity, and the Bottle full of Salt Sea-Water. We repeated our *Experiment* with another Bottle and Cork in the ſame manner as before; but, the Cork being not ſound, the Sea-Water ſoaked thro' it, and the Bottle was half full of Water; ſo the Cork remained in the Mouth of the Bottle, not preſſed down at all. We repeated our *Experiment* a third time in 90 *Fathom* of Water, with a very ſound Cork, and much larger than the Mouth of the Bottle. We beat it in with a Hammer as far as it would go, leaving about an Inch of the Cork above the Mouth of the Bottle. The Cork at this *Trial* was preſſed down only into the Neck, and became level with the Mouth of the Bottle: But I really believe, had we we had 10 or 20 *Fathoms* of Water more, it would have ſucceeded as at our firſt *Trial.*

V. 1. The following Bodies were poured gently into a Veſſel of well ſeaſoned Oak, whoſe Concave was an exact *Cubick Foot.* Thoſe in the Twelve firſt Experiments were weighed in *Scales* turning with two Ounces, but the laſt Seven were weighed in *Scales* turning with one Ounce. The *Pounds* and *Ounces* here mentioned are *Averdupois.*

The Weight of divers Bodies try'd by the Direction of the Phil. Society *at* Oxford. n. 169. p. 926. Mar. An. 1685.

		℔	℥
1	A Foot of *Wheat* (worth 6*s.* a Buſhel.)	47	8
2	*Wheat* of the beſt ſort (worth 6*s.* 4*d.* a Buſhel.)	48	4
3	The ſame ſort of *Wheat* meaſured the ſecond time.	48	2
	Both ſorts were *Red Lammas Wheat* of the laſt Year.		
4	*White Oats* of the laſt Year.	29	8
	The beſt ſort of *Oats* were 2*d.* in a Buſhel better than theſe.		
5	*Blue Peaſe* (of the laſt Year,) and much Worm-eaten.	49	12
6	*White Peaſe* of the laſt Year but one.	50	8
7	*Barley* of the laſt Year: (the beſt ſort ſells for 1*s.* 6*d.* in a Quarter more than this.)	41	2
8	*Malt* of the laſt Year's *Barley*, made two Months before.	30	4
9	Field *Beans* of the laſt Year but one.	50	8
10	*Wheaten Meal* (unſifted.)	31	0
11	*Rye Meal* unſifted.	28	4
12	*Pump Water.*	62	8
13	*Bay Salt.*	54	1
14	White *Sea Salt.*	43	12
15	*Sand.*	85	4
16	*Newcaſtle Coal.*	67	12
17	*Pit Coal* from *Wedneſbury* 63, but this is very uncertain in the filling the Interſtices between the greater pieces	63	0
18	*Gravel.*	109	5
19	Wood-*Aſhes.*	58	5

Pump-

2. *The* Specifick Gravities *of several Bodies, by the Direction of the* Phil. Society *at* Oxford. *Ib. p.* 927.

Pump-Water.	1000
Fir Dry.	546
Elm Dry.	600
Cedar Dry.	613
Walnut-Tree Dry.	631
Crab-Tree meanly Dry.	765
Ash meanly Dry, and of the Out-side lax part of the Tree.	734
Ash more Dry, about the Heart.	845
Maple Dry.	755
Yew of a Knot or Root 16 Years Old.	760
Beech meanly Dry.	854
Oak very Dry almost Worm-eaten.	753
Oak of the Out-side sappy Part, Fell'd a Year since.	870
Oak Dry; but of a very sound close Texture.	929
The *same* tried another Time.	932
Logwood.	913
Claret.	993
Moil Cyder, not Clear.	1017
Sea-Water settled Clear.	1028
College Plain *Ale* the same.	1028
Urine.	1030
Milk.	1031
Box the same.	1031
Redwood the same.	1031
Sack.	1033
Beer Vinegar.	1034
Pitch.	1150
Pit-Coal of Staffordshire.	1240
Speckled Wood of *Virginia.*	1313
Lignum Vitæ	1327
Stone Bottle.	1777
Ivory.	1826
Alabaster.	1872
Brick.	1979
Heddington Stone, the Soft Lax kind.	2029
Burford Stone, an old Dry Piece.	2049
Paving Stone, a hard Sort from about *Blaidon.*	2460
Flint.	2542
Glass of a Quart Bottle.	2666
Black Italian Marble.	2704
White Italian Marble tried twice.	2707
White Italian Marble of another sort, of a visibly Closer Texture.	2718
Block *Tin.*	7321
Copper.	8843
Lead.	11345
Quick Silver.	14019
Quick Silver.	13593

The

The laſt *Experiment* was tried with another quantity of *Quick-Silver*, which had been uſed in Water in the preceding *Experiment*: However, I rather truſt the laſt, for that I found a ſmall miſtake (tho' here in the Calculation allow'd for) in the Weight of the *Glaſs* containing the *Quick-Silver*, in the Trial before.

The *Solids* here mentioned, were examined *Hydroſtatically* by weighing them in Air and Water; but the *Fluids*, by weighing an equal Portion of each in a Glaſs holding about a Quart. The Numbers ſhew the Proportion of *Gravity* of equal Portions of theſe Bodies: But if of theſe Bodies we take Portions equally heavy, their *Magnitudes* will be *reciprocally* proportional to their correſpondent Numbers: *e. g.* a Cubic Foot of *Water* is to a Cubic Foot of *Alabaſter* in *Gravity* as 1000 to 1872; but a Pound Weight of *Water*, is to a pound Weight of *Alabaſter* in *Magnitude*, as, 1872 to 1000. So that, knowing by the former Table, the Weight of a Cubic Foot of *Water*, and by this Proportion in *Gravity* betwixt *Water* and *Alabaſter*, we may by the Rule of Three find the *Weight* of a Cubic Foot of *Alabaſter*, and ſo of any other of theſe Bodies; or we may know their *Magnitude* by knowing their *Gravity*. So that, an irregular piece or quantity of theſe Bodies being offered, 'tis but *weighing* them, and we may know their juſt *Magnitude* without farther trouble.

Pump-Water.	1000
Cork.	237
Saſſafras Wood.	482
Juniper Wood (Dry.)	556
Plum-Tree (Dry.)	663
Maſtic.	849
Santalum Citrinum.	809
Santalum Album.	1041
Santalum Rubrum.	1128
Ebony.	1177
Lignum Rhodium.	1125
Lignum Aſphaltum.	1179
Aloes.	1177
Succinum Pellucidum.	1065
Succinum Pingue.	1087
Jet.	1238
The Top part of a *Rhinoceros Horn.*	1242
The Top part of an *Ox Horn.*	1840
The (Blade) *Bone* of an *Ox.*	1656
An *Human Calculus.*	1240
Another *Calculus Humanus.*	1433
Another *Calculus.*	1664
Brimſtone, ſuch as commonly ſold.	1811
Borax.	1720

By Mr. J. C. n. 199. p. 904. Apr. An. 1693.

 A

A Spotted *Factitious Marble.*	1822
A *Galley-Pot.*	1928
Oyſter Shell.	2092
Murex Shell.	2590
Lapis Manati.	2270
Selenitis.	2322
Wood *Petrified* in *Lough Neagh.*	2341
Onyx Stone.	2510
Turcois-Stone.	2508
Engliſh Aget.	2512
Grammatias Lapis.	2515
A *Cornelian.*	2568
Corallachates.	2605
Talo.	2657
Coral.	2689
Hyacinth (Spurious.)	2631
Jaſper (Spurious.)	2666
A Pellucid *Pebble.*	2641
Rock Cryſtal.	2659
Cryſtallum Diſdiaclaſticum.	2704
A Red *Paſte.*	2842
Lapis Nephriticus.	2894
Lapis Amiantus from *Wales.*	2913
Lapis Lazuli.	3054
An *Hone.*	3288
Sardachates.	3598
A *Granat.*	3978
A Golden *Marcaſite.*	4589
A Blue *Slate* with ſhining Particles.	3500
A *Mineral Stone*, yielding 1 part in 160 *Metal.*	2650
The *Metal* thence extracted.	8500
The (reputed) *Silver Ore* of *Wales.*	7464
The *Metal* thence extracted.	11087
Biſmuth.	9859
Spelter.	7065
Spelter Solder.	8362
Iron of a Key.	7643
Steel.	7852
Caſt *Braſs.*	8100
Wrought *Braſs.*	8280
Hammer'd *Braſs.*	8349
A Falſe *Guinea.*	9075
A True *Guinea.*	18888
Sterling Silver.	10535
A *Braſs* Half Crown.	9468

Electrum

Electrum a *Britiſh Coin.* — — — —	12071
A *Gold Coin* of *Barbary.* — — — —	17548
A *Gold Medal* from *Morocco.* — — — —	18420
A *Mentz Gold Ducat.* — — — — —	18261
A *Gold Coin* of *Alexander*'s. — — —	18893
A *Gold Medal* of Queen *Mary*'s. — — — —	19100
A *Gold Medal* of Queen *Elizabeth*'s. — — —	19125
A *Medal* eſteem'd to be near *Fine Gold.* — — —	19636

VI. M. *Homberg* has given us the following Table of the *Various Weights* of ſome more uſual Liquors in the *Coldeſt* Time, and in the *Hotteſt.* *The Different Weight of ſeveral Liquors in* Winter *and* Summer; *by M.* Homberg, n. 262. p. 530. *Vid. Sup.* §. I. 2. Mar. An. 1700.

	In Summer.			In Winter.		
	℥	ʒ	gr.	℥	ʒ	gr.
The *Aerometer* full of *Mercury.*	11	0	6	11	0	32
Oil of *Tartar.*	1	3	8	1	3	31
Spirit of *Urine.*	1	0	32	1	0	43
Oil of *Vitriol.*	1	3	58	1	4	4
Spirit of *Nitre.*	1	1	40	1	1	70
Spirit of *Salt.*	1	0	39	1	0	47
Aqua Fortis.	1	1	38	1	1	55
Vinegar.	0	7	55	0	7	60
Spirit of Wine.	0	6	47	0	6	61
River Water.	0	7	53	0	7	57
Diſtilled Water.	0	7	50	0	7	54
This Empty *Aerometer* Weighs				0	1	28

VII. 1. Having poured a ſtrongly *Alcalizat Menſtruum* (I uſed that made of *Fix'd Nitre*, diſſolved by the moiſture of a Cellar) into a Pipe of Glaſs, ſealed at one end, and not full a quarter of an Inch in Bore; that the Cavity, which in a greater depth would ſeem leſs deep, might be the more conſpicuous: We gently poured on it ſome highly Dephlegm'd *Spirit* of *Wine*, which we knew would not mix with it, but ſwim above it, and preſently, as we had gueſs'd, we found the *Figure* of the *Surface* of the lower Liquor changed, and the Cavity quite deſtroyed; the *Surface* that ſeemed, as it were, common to the two *Contiguous Liquors*, appearing flat or Horizontal. And ſuch a level Superficies we had, by putting theſe two *Liquors* together in a much wider Glaſs. *Experiments about the Superficial Figures of* Fluids, *eſpecially Liquors Contiguous to other Liquors, and their Reflective Powers; by Mr.* Boyle. n. 131. p. 775. Jan. An. 1676.

2. We found alſo, that by Employing *Oyl* of *Turpentine*, inſtead of the *Spirit* of *Wine*, the Liquor did almoſt totally loſe its Cavity.

3. But

3. But, if, inftead of *deliquated Tartar*, we put common Water into the Pipe, we found this Liquor to retain its Concave Surface, tho' we put to it fome *Oil* of *Turpentine*, and left it to reft upon the Water a good while.

4. Having provided fome pure *Oil* of the *Gum* of *Guaiacum*, and poured a little of it into a flender Pipe, we found the upper Superficies of it to be Concave almoft, if not altogether, like that which Water would have had in the fame Pipe. But when I put a little Water upon this *Oil*, it prefently changed the Figure of its Surface, which became vifibly, tho' not very much, Protuberant or Convex.

5. Having put fome *Oil* of *Tartar* into the flender Pipe, and put fome Drops of the *Oil* of *Guaiacum* to it, we found, that this Liquor did not manifeftly alter the Concave Figure of the Surface of the Liquor *Alcali*, as the *Oil* of *Turpentine* had done: And having for Curiofity's fake warily poured a little Water upon the *Oil* of *Guaiacum*, I found, as I had reafon to fufpect, that the upper Superficies of it changed prefently from a Concave Figure to a Convex; fo that this *Oil* in the midft of the other two Liquors appeared like a little red Cylinder; which, inftead of having Circular Bafes, was protuberant at both ends; but more at that which touched the *Oil* of *Tartar*.

6. I put fome *Effential Oil*, (as *Chymifts* call it) of *Cloves* into a new flender Pipe, and, having obferved it to be fomewhat Concave at the Top, where it was Contiguous to the Air, we caufed [a little common *Water* (perhaps a quarter of a Spoonful or lefs) to be put to it, and found, as we expected, the Surface of this *Oil* alfo to be tumid. And in regard this Liquor, as well as the forementioned *Oil* of *Guaiacum*, tho' it were fo heavy as to fink into Water, would not do fo in *Deliquated Salt* of *Tartar*, we did, into another flender Pipe, put firft fome of this laft named Liquor, then fome of the *Aromatic Oil*, and laftly, a little common Water; by which means we found, that the little *Cylinder* of *Oil*, did, like that of the *Oil* of *Guaiacum*, appear Convex at both ends, but was unlike it in one Circ umftance, that the *Oil* of *Cloves* appear'd more Convex at the upper end where 'twas Contiguous to the *Water*, than at the lower, that leaned upon the Surface of the *Oil* of *Tartar*.

7. Having taken a little flender Glafs, that was much longer, but of the like Bore with the former, we put into it a fmall quantity of *Quick-Silver*; and having taken notice how the upper Superficies fwelled in the middle above the Level of the Parts where it touched the Glafs, we poured fome Water upon it, and found a manifeft and confiderable Depreffion of the Surface, tho' the Protuberance were not quite fuppreffed.

8. This *Phænomenon*, having been for greater fecurity feveral times repeated, fometimes it feem'd, that when the *Aqueous Cylinder* was much longer, the Depreffion of the *Mercurial* Surface was fomewhat greater. But this did not fo conftantly happen; but we often obferved, that tho' a very little Water fufficed by its Contact to make, in the Judgment of the Eye, a manifeft Abatement of the Protuberance of the *Quik-Silver*; yet it had not the fame effect on that ponderous Fluid, that it had, when being increafed almoft as high as the length of the Pipe would permit, a greater Weight of it was

incumbent on the *Mercury*, for then I manifeftly perceived, and fhewed to others, that the Surface of the *Quick-Silver* being depreffed almoft to a Level in thofe Parts of it that were near the infide of the Glafs, there was about the Middle of the Surface an Elevation of *Mercurial Matter*, that appeared to be rather more than a half Globe, and was, to the Height of its full Semidiameter, raifed above the reft of the *Mercurial* Surface, and in that State it continued, as long as I thought fit to let it do fo. And, left this Trial fhould impofe upon me, I caufed it to be more than once repeated; and, the better to confirm it, I afterwards caufed the incumbent *Water* to be little by little fuck'd up, and found as I expected, that, when the incumbent *Water* began to be too much fhorten'd, the little Teat or Segment of a Sphere, lately mentioned, began to be fomewhat flatned, and fubfided more and more as the *Water* was further taken off.

9. Having conveyed into one of our *Pneumatical Receivers*, a Couple of fuch flender Pipes as have been already defcribed, one of them furnifh'd with *Common Water*, and the other with *Quick-Silver*, we caufed the Common Air to be diligently pump'd out, without obferving any fenfible Change in the Concave Figure of the *Water:* but as for the *Quick-Silver*, I knew not what to conclude about it. For, having repeated the Trial twice or thrice, the *Murcury* fometimes feem'd manifeftly to fwell, to be more protuberant upon the Exhauftion of the *Receiver*, than when it was put in, efpecially when its Figure was attentively viewed, and the external Air, that was pump'd out but flowly, was fuffer'd to re-enter with all convenient Celerity. But that which yet kept me doubtful was, that I obferved, That, upon the diligent withdrawing of the Air's Preffure upon the *Quick-Silver*, there difclofed themfelves fome little Bubbles, which, I fear'd, we had not been able to free it altogether from, and which might be fufpected to have fome Intereft in the *Phænomenon*. We alfo conveyed into our *Receiver*, a clear *Chymical Oil* that was heavier than *Water*, and, whilft it was contiguous to it, had not a Concave but a Convex Surface, and, having placed the Pipe furnifh'd with both Liquors in the *Pneumatical Receiver*, we pump'd out the Air, without finding that the *Oil* fenfibly altered its protuberant Surface, as neither did the Water lofe the concave Figure of its Surface.

10. I took *Fix'd Nitre*, (or which is Analogous to it, *Salt* of *Tartar*) refolved *per Deliquium* into a Tranfparent Liquor, and having filled a clear Viol half full with this, I poured on it a convenient Quantity of *Vinous Spirit* exactly *Rectified*, that there might be no Phlegm to occafion an Union betwixt the two Liquors, which ought, as ours did, to retain Diftinct Surfaces, and fpeedily regain them, though the Glafs were well fhaken. Then, having found by a Trial formerly mentioned, that Common *Oil* of *Turpentine*, if employ'd in a competent Quantity, will not totally (and much lefs will readily) diffolve in *Spirit* of *Wine*, and alfo having obferved (what may feem fomewhat ftrange) that if this *Spirit* of *Wine* be exquifitely dephlegm'd, the *Oil*, tho' a *Chymical* One, will not fwim on it, but fink in it; I warily let fall fome drops of *Oil* into the *Spirit*, and had the pleafure to fee, as I expected, that they fell towards the bottom of the Glafs, till their Defcent was ftop'd by the

the Horizontal (for it was not Concave) Surface of the *Alcalizat Liquor* of *fix'd Nitre*. And, becauſe my deſign was chiefly to obſerve the ſuperficial Figure of a Fluid encompaſſed by other Fluids without touching any ſolid Body, I ſhall here take notice of the chief *Phænomena* that were produced of that kind, without ſtaying to enquire into the Cauſes or the Conſequences of them.

1. If the *Oily Drops* were but ſmall, they ſeem'd to the Eye exactly enough *Spherical*. For, the *Oil* differing but very little in the *ſpecifick Gravity* from the *Spirit* of *Wine*, the Drops did but juſt touch the Surface of the ſubjacent *Alcali*; and, the ſame Drops being but ſmall, their own Weight was not great enough viſibly to depreſs them, and hinder that Roundneſs which the Preſſure of the Ambient *Spirit*, or their own Viſcoſity, endeavoured to give them.

2. If an Aggregate of Drops were conſiderably bigger than thoſe newly mentioned, as if it had about a third part of an Inch in Diameter, it would then manifeſtly lean upon the *Alcalizat Liquor* as upon a Floor, and appear ſomewhat *Elliptical*, (for ſome little part of the bottom was a Plane;) the Weight of the upper parts depreſſing the Drops, and making the horizontal Diameter ſomewhat longer than the Tranſverſe.

3. If a yet greater Portion of *Oil* were let fall upon the heavy Liquor, it would for a pretty while appear in the Form of a ſomewhat imperfect *Hemiſphere*, or ſome other large Section of a *Sphere*, the lower part being cut off; (as if a Globe were divided by a Plane) by the horizontal Surface of the *deliquated Salt*.

4. But if the Quantity of *Oil* were not too great, 'twas pretty to obſerve, that, tho' at firſt putting in, it did perhaps ſpread itſelf over the ſubjacent Liquor, and lie as it were flat upon it; yet, by little and little, (for 'twas but ſlowly) it would by the Action of the ambient, concurring with it's own Tenacity, be raiſed above the Surface of the Fluid *Nitre*, and be reduced to the Figure, either of half a Globe, or of a greater Segment of a Globe, or even of an Imperfect Ellipſis, according to the Bulk or Weight of the *Oil*.

5. Tho' theſe Globules, or Portions of *Oil*, did oftentimes readily mingle, when they touched one another, yet divers times alſo we obſerved, that, having warily approach'd them, we were able to make them touch without mingling, inſomuch that we have with pleaſure made them ſo far bear againſt one another's Surfaces, as manifeſtly to preſs them inwards, tho' being parted they would preſently reſume their former Figure. But, in caſe any of theſe *Oily Portions* came by a more preſſing Contact to be united, they would then alter the Figures they had whilſt ſeparate, and take another ſuitable to the Bulk of the Aggregate.

9. When a large Portion of *Oil* reſted upon the *Saline Liquors*, if then the ambient *Spirit* was moderately and warily agitated, 'twas not unpleaſant to obſerve the various Figurations, which the Convex and Protuberant part of the mutilated Globe would be put into by the Shakes, without any viſible Solution of Continuity, or conſiderable Motion of the whole Body, which would very quickly recover its former Figure. Tho', if the Agitation were too ſtrong,

ſtrong, ſome Portions would be quite broken off, and preſently turned into little Globes.

11. I tried to produce another *Phænomenon*, that would not have been unpleaſant, by putting together in a ſomewhat large Veſſel, with other Liquors, two *Oils*, (whereof one, if I miſtake not, was from *Turpentine*) which firſt by reaſon of the Oleaginous Nature wherein they agreed, might exactly mingle and make a compounded Liquor; and then, by reaſon of their being one heavier, and the other lighter in *Specie* than Water, might by this Liquor be again ſeparated, and include betwixt them the Liquor that had divided them. But I found that the *Oils* being once united would not be eaſily parted, but according to the Prevalency of the lighter or heavier Ingredient, in the mixture, the compounded *Oil*, would almoſt totally either emerge to the top of the Water, or lie beneath at the bottom of it; I ſay, almoſt totally, becauſe ſome Part of the *Oil*, which was not perhaps all uniformly mix'd, did not keep in a Body with the reſt; but either was ſeparated from the Maſs in the form of Globules, or elſe ſticking to the Side of the Glaſs, had the other Part of its Superficies, which was contiguous to the Water, very *Variouſly Figured*, according as the bulk and degree of Gravity of the adhereing *Oil*, and other Circumſtances happen'd to determine.

Theſe are ſome of the *Phænomena* I obſerved in *Oil* of *Turpentine*, when 'twas invironed only with Fluids; but, if it were permitted to be contiguous to the Inſide of the Glaſs, and ſo to faſten part of its Surface to a Solid, the greater part of the Surface, which remained expoſed to one or both of the contiguous Liquors, would partly by their Action, and partly by the Gravity of the *Oil* itſelf, be put into *Figures* ſo *Various* and ſometimes ſo *Extravagant*, that 'twas much more pleaſant to behold them, than it would be eaſy to deſcribe them.

12. *Confining Fluids* may have diſtinct Surfaces, without having, at leaſt in many Poſitions, *Refractions* differing enough, or *Reflections* ſtrong enough, to make the Plane that diſterminates them, obvious to the Eye. Thus, when the *Oil* of *Tartar*, or *Nitrous Alcali*, that I employed, happen'd to be very clear and colourleſs, I have more than once made highly *Rectified Spirit* of *Wine* float upon it ſo, that in moſt Poſitions the Vial ſeem'd to have in it but one uniform Liquor; the Plane that divided the two Fluids being unapt to be diſcerned, but in a Poſition, wherein the Rays of Light paſſing thence to the Eye, fell very obliquely on it; and indeed, when there was no little Duſt or other Feculency, ſwimming upon the Surface of the *Oil* of *Tartar*, I had ſometimes much ado to convince ordinary Spectators, that the Vial, in two diſtinct Regions of it, contained two *Unſociable Liquors*.

13. We took a *Deliquated Alcali*, made of *Nitre* and *Tartar*, and deeply tinged with *Cochineal*; and, that the Liquors might not only be heterogeneous, but as differing in *Gravity* and *Denſity*, as we could make them, we poured on it a peculiar kind of *Oil* lighter than *Spirit* of *Wine*, and holding the Plane where the two Liquors were contiguous in a convenient Poſition, in reſpect of the Light and the Eye, I obſerved it to make a ſtrangely vivid

Reflection of the incident Beams of Light: So that this Physical Surface which was flat, look'd almost, for 'twas not so *Specular*, like that of *Quick-silver*; and, when I kept it till Night, and considered it by the Light of a Candle, the bright Figure of the Flame was strongly reflected almost as from a close specular Body; which tempted me to suspect, that there might be something else than the bare Smoothness of the Surface of the *Alcalizat Liquor* to produce so brisk a *Reflection*; and the rather, because I did not observe, that the Remains of the same Tinged *Alcali*, which I kept in another Glass, nor a Portion of the same *Oil*, which I had also by me in a separate Vial, did either of them afford so vivid a *Reflection* from its Surface; tho' I did the less wonder at this, because of the great Disposition to *reflect Light*, which I had formerly the Curiosity to observe in the forementioned *Oil*, when I joined it with other Liquors. I shall add, that looking on this Liquor, as a Body which tho' it have all the necessary Qualities of an *Oil*, does in regard of its Origin, and some Properties I have found in it, differ from common *Chymical Oils*; I was invited the more to observe its *Phænomena* in reference to *Reflection*; and I found among other Things (not pertinent to this Place,) *First* that the confining Plane often mentioned between the tinged *Alcali*, and this *Liquor*, did not appear Red itself, nor communicate that Colour to the Image of the Flame of a Candle reflected from it. *Secondly*, That when I warily shook the Vial, which contained the two Liquors, the uppermost would be reduced into a seeming Froth, consisting of a great number of imperfectly Globular Bodies, which after a while would make a kind of a rude Physical Plane; which, tho' neither very Horizontal, nor sensibly Smooth, would at its upper Superficies, send back the *incident Light* with more Briskness than one would expect; and, when the seeming Froth consisted of smaller Particles, these, when they were of a certain Size, and conveniently placed, in reference to the Flame of a Candle and the Eye, would (as more than one Trial informed me) reflect the *incident Light* so many ways, and so visibly, that they seemed, for Multitude and Splendor, like little sparkling Corpuscles of polished *Silver*; or almost like those Glistering ones, that appear when a clean Plate of *Copper* is first immersed into a much allayed Solution of good *Silver*, made in *Aqua Fortis*. *Thirdly*, That tho' pure *Spirit* of *Wine* be so thin a Liquor, and our *Oil* is nevertheless so light as to swim upon it, yet I found the confining Surface very strongly *Reflexive*.

I have also found, that some other *Essential Oils* (as *Chymists* call those that are distilled with Water in *Limbecks*) and particularly an Unsophisticated *Oil* of *Lemons*, did with our *tinged Alcali* afford most of the same Phænomena; but not so brisk a *Reflection*: I say most, chiefly because with *Spirit* of *Wine*, their *Subtile Oils*, as I formerly noted, will readily be confounded, tho' our Anomalous *Oil* be unsociable with it.

N. 1[illegible], p. 799. 14. In Cold Weather we took *Essential Oil* of *Anniseeds*, whose Property it is to Coagulate in such Weather, and, having in a gentle Warmth brought it to be fluid, we poured it into a slender Vial more than half filled with *Common*

mon Water ; that had been alſo a little warmed, that the *Oil* might not be too haſtily reduced to its former State. This Oil being lighter than ſo much *Water*, and being poured on in a convenient Quantity, had its upper Surface ſomewhat Concave, as that of the Water was ; but the lower Surface, ſurrounded by the Water was very Convex, appearing almoſt (for it was not perfectly) of the Figure of a great Portion of a *Sphere*. This being done, the Vial was ſtopt, and ſuffered to reſt for ſome time in a cold Place, by which means the *Water* continuing Fluid as before, the *Oil* of *Anniſeeds* was, as I expected, found *coagulated*, in a Form approaching to that it had whilſt in a Fluid State ; I ſay approaching, becauſe it was not eaſy to diſcern the exact Figure in the Vial I was fain to make uſe of: And I ſuſpected that the *Oil* grown conſiſtent was become leſs Convex than before. But 'twas worth obſerving, how great a Difference there was between the dull *Reflection* it made when it was *coagulated*, and the fine *Reflection* it had made whilſt 'twas a Liquor. The latter of which *Reflections* brought into my mind, how vivid the *reflective Power* of ſome *Fluids* is in compariſon of that of the Generality of *Solid Bodies*.

15. Having obſerved, That *Quick-Silver*, and rectified *Oleum Petræ*, are, the Former of them the Heavieſt, and the Latter the Lighteſt, of all the Viſible Fluids that are yet known to me ; I put ſome (Diſtill'd) *Quick-Silver* into a ſmall Vial, and held it in ſuch a poſture, that the *incident Light* was ſtrongly remitted to my Eye: I then ſlowly put to it ſome *Petroleum*, that being well rectified was very clear, and obſerved, that, as this Liquor covered the *Quick-Silver*, there was at the imaginary Plane, where they both confined, a briſker *Reflection* than the *Quick-Silver* alone had given before. On this occaſion it will not be amiſs to take notice, that either the Surface of the Air itſelf, as thin and yielding a Fluid as it is, or the Surface of a Solid contiguous to included Air, or ſome interpoſed *Subtile Matter*, may *reflect the incident* Beams of *Light* more ſtrongly than moſt Men would expect. To this purpoſe, I remember, that a curious Perſon having one Day brought me a couple of Rarities, which he told me were two pieces of a Solid, but tranſparent Body, that he had caſually found ; in one of which there was a *Pearl*, Large, Round, and Orient, and in the other a leſs perfect One: One of them was opened, and that which had appeared a *Pearl* was found to be but a Cavity, that contained no groſſer Subſtance than Air. And I have by me a well ſhap'd piece of Glaſs of a good Thickneſs, with an Aereal Bubble in the middle, which by ſome Qualities, particularly its Pearl-like Shape and vivid *Reflection*, does not ill reſemble a fair, tho' not *orient Pearl*. But in ſuchlike Obſervations, the Poſition of the Eye, and that wherein the Body receives the Beams of Light, may be very conſiderable. For I have by me a Small Stone (with which I have puzzled a ſkillful Jeweller to determine what kind of *Gem* it was) that being laid flat upon ones Hand, or a piece of Paper, and look'd on directly downwards, looks almoſt like a piece of common Glaſs, and is tranſparent: But, if the Eye be ſo placed, that the *incident* Beams of *Light*, by whoſe *Reflection* it's ſeen, fall with a conveni-

ent Degree of Obliquity upon the Stone, it makes an exceeding pretty ſhew, ſometimes appearing like a fine *Opal*, and ſometimes not very unlike an *orient Pearl*.

16. We made a competent Quantity of a *Reſinous* or *Gummous Subſtance*, that looked like high-coloured *Amber*, but was eaſy to melt. This we put into a deep round Glaſs with a wide Mouth, and held it by the Fire-ſide in a moderate Warmth, till it was brought into a fluid State; then we tranſferr'd it into one of our *Pneumatical Receivers*, where we preſumed, that this temporary Liquor, would, as well as Liquors that are conſtantly ſuch, diſcloſe *Aereal Bubbles*, when the Preſſure of the Air was withdrawn from it; and accordingly having cauſed the Air to be pumped out by degrees, we found, that ſtore of Bubbles appeared at the Top of the Liquor, and made there a copious Froth, many of them being by reaſon of the Viſcoſity of the Fluid, very large, and divers of them, becauſe of the Nature and Texture of it, and the thinneſs of the Films, being adorned with the Colours of the Rainbow, whoſe Vividneſs made them pleaſant to behold, and ſuggeſted to us ſome *optical Conſiderations*. But, notwithſtanding this Froth, I cauſed the Pumping to be continued, that thoſe Bubbles that had moſt of common Air in them, and which therefore are wont to riſe firſt, might get to the Top, and the ſubſequent Bubbles might meet with more Reſiſtance from the Liquor ſtill tending to grow cold, and ſo might be the more expanded, and yet kept from emerging by the Concretion of the reſinous Subſtance; and anſwerable to this we found, That, when this Subſtance had reſumed its conſiſtent Form, there were intercepted, between the upper and the lower Surfaces of it, ſome Bubbles that were not ſmall, which yet had a conſiderable *Reflection*, notwithſtanding the ſmall Quantity of the groſſer Particles of the Air, that may be ſuppoſed to be contained in Bubbles ſo very much expanded.

17. 'Tis taken for granted, That the *falling Drops* of *Rain* are *Spherical*, yet their Deſcent is ſo ſwift, that I fear 'tis rather *ſuppoſed* than *obſerved* that their Figure is *Spherical*; which will be the more queſtionable, if it be true, which is vulgarly thought, That *Hail* is but *Rain Frozen* in its Paſſage thro' the *Air*. For 'tis evident, That the Grains of *Hail* are frequently of other Figures than truly Orbicular. But the *Surface* of *Water* may have differing Figures, according as 'tis totally encompaſs'd with heterogeneous Fluids, or as 'tis only in ſome places contiguous to one or more of them. In the former caſe we found it not ſo eaſy to make an Obſervation, becauſe, we know not of any two Liquors (ſetting *Mercury* aſide) that will not mingle either with one another, or with Water. We therefore cautiouſly convey'd into ſome *chymical Oil* of *Cloves* ſome Portions of *common Water* of differing Bigneſſes, taking care, as far as we could, that they might not touch one another; by which means the *Oil* being tranſparent, and yet ſomewhat coloured, 'twas eaſy to obſerve, that the ſmaller Portions of *Water* were ſo near totally invironed with the *Oil*, that they were reduced into almoſt perfect *Globes*; Thoſe Portions, that were ſomewhat bigger (as about twice the Bigneſs of a

Pea,)

Pea,) would be of a Figure ſomewhat approaching to that of an *Ellipſis* (for 'twas not the ſame) and thoſe Portions that were yet ſomewhat larger, tho' they ſeemed to be ſunk almoſt totally beneath the *Oil*; yet, they held to it by a ſmall Portion of themſelves, whoſe Surface was eaſily enough diſtinguiſhable from that of the *Oil*. Theſe larger Portions of *immerſed Water* being almoſt wholly inviron'd with the other Liquor, were by it reduced into a round Figure, which was ordinarily ſomewhat *Elliptical*, but more depreſſed in the Middle than that Figure requires.

18. Having into a ſlender Pipe, of that ſort that has been deſcribed before, put a little *Oil* of *Cloves*, and upon this ſome *Oil* of *Turpentine*, that ſo the *Water* might both above and beneath be touched by Heterogeneous Liquors, I obſerved not the *Oil* of *Cloves* to be very manifeſtly Tumid at the top, nor the lower Surface of the *Oil* of *Turpentine* (for the upper was Concave) to be very Convex: For ſomewhat Convex it was downwards. And, from this 'twill be eaſy to conclude the Figure of the Cylindrical Portion of *Water* intercepted between theſe two Oils.

19. I took *Oil* of *Anniſeeds*, thaw'd by a gentle Warmth, and *Common Water*, and, having put them together in a convenient ſhap'd Glaſs, they were ſuffered to ſtand in a Cold Place, till the *Oil* was coagulated; which done, it was parted from the *Water*, and by the Roughneſs of its Superficies manifeſted, as I expected, that, when its Parts were no longer agitated and kept eaſily Diſplaceable, by the ſubtile permeating Matter, or whatever other Agent or cauſe it were to which it owed its *Fluidity*, when the contiguous *Water* grew unable to inflect, or otherwiſe place them after the manner requiſite to conſtitute a *ſmooth Surface*. And, what happen'd to that part of the *Oil*'s Surface that was touched by the *Water*, happen'd alſo to that which was contiguous to the *Air*, ſave that the *Aſperity* of the laſt named Surface was differing from the other, which, whether it were an accidental or conſtant *Phænomenon*, farther Trial muſt determine. But I have often obſerv'd, that the upper Surface of *Oil* of *Anniſeeds*, when this Liquor comes to be Coagulated by the cold Air, was far enough from being *Smooth*, being variouſly aſperated by many flaky Particles, ſome of which lay with their broad, and others with their edged Parts upwards.

20. An Inequality and Ruggedneſs of Superficies I have alſo obſerved in *Water*, when, having covered it with *chymical Oil* of *Juniper*, and expoſed it in very cold Weather, tho' the *Oil* continued *Fluid*, yet the *Water* being Frozen had no longer a *ſmooth Superficies*, as whilſt in its Liquid State 'twas contiguous to the *Oil*. And the like Inequality, and rather a greater, we obſerved in the Surface of *Water* Frozen, which had *chymical Oil* of *Turpentine* ſwimming over it, yet a no leſs, if not a much greater, Roughneſs may be oftentimes obſerved in the Surfaces of divers Liquors that abound with *Water*, when, thoſe Liquors being Frozen, their Surfaces have an immediate Contact with the Air. I ſhall here add, that, having purpoſely cauſed a Strong and Blood red Decoction of the Soot of Wood to be expoſed in a large Glaſs in a very cold Night, I was more pleaſed than ſurprized, to find in

in the Morning a Cake of Ice, that was curiouſly Figured, being full of large flakes ſhap'd almoſt like the broad Blades of Daggers, but neatly fringed at the Edges. But that which I chiefly mention theſe Figures for, is, that they ſeem to be as it were Imboſt, being both to the Eye and the Touch raiſed above the horizontal Plane or Level of the other Ice.

21. I have ſometimes obſerved the like *Phænomenon* in one and the ſame Liquor, and particularly not long ſince in froſty Weather, on a Vial where I had long kept *Oil* of *Vitriol*, I perceived that the Cold had reduced far the greateſt part of the *Menſtruum* into a conſiſtent Maſs, whoſe upper Surface was very rugged and oddly figur'd, tho' it lay covered all over with a pretty deal of high-coloured Liquor, that was not frozen or coagulated, nor ſeem'd to be diſpoſed to be ſo, at leaſt, in that degree of Cold.

22. This may be alſo obſerv'd in the beſt ſort of what the *Chymiſts* call *Regulus Martis Stellatus*, where the Figure of a Star, or a Figure ſomewhat like that of the Decoction of the Soot lately mentioned, will frequently appear imboſt upon the upper Superficies of the *Regulus*; and ſuch a raiſed Figure I have ſeen on a Maſs of *Regulus* made of *Antimony* without *Mars*. But if to thoſe two Bodies *Copper* be alſo ſkilfully added, the Superficies will be oftentimes adorned with new Figures according to the Circumſtances; tho' the moſt uſual I took notice of was that of a Net, that ſeemed to cover the Surface of the compounded *Regulus*. But this is not ſo conſtant, but that I have by me a Maſs of the Conical Figure conſiſting of two very contiguous, but eaſily ſeparable Parts, whereof the lowermoſt, which abounds more in *Metal*, hath its upper Surface covered with round Protuberances, in ſhape and bigneſs not unlike to ſmall Peaſe cut in two: and theſe are ſo really imboſt and elevated above the reſt of the Superficies, that the other part of the Cone which is of a more ſcorious Nature, as in its lower Surface, which exactly fits the upper of the *Regulus*, Cavities, for number, Shape and Bigneſs, anſwering to the Protuberances lately mentioned; which argues that the *Regulus* cooled firſt with that Inequality of Surface we have deſcribed, and that the Lighter and more Recrementitious Subſtance, continuing longer Fluid, had thereby opportunity to accommodate itſelf to the Superficial Figure of the *Regulus*, on which it firſt leaned, and afterwards coagulated.

Why Bodies Diſſolved Swim in Menſtrua Specifically Lighter than themſelves; by Mr. Will. Molyneux. n. 181. p. 88. *May* An. 1686.

VIII. 1. My Brother, Mr. *Tho. Molyneux* (in the *Nouvelles de la Republique des Lettres*) has given this Reaſon for the *Phænomenon*, *viz.* That the *Internal Motion* of the Parts of the Liquor does keep up the Particles of the *diſſolved Solid*, for they, being ſo very minute, are moveable by the leaſt Force imaginable, and the Action of the *Particles* of the *Menſtruum* is ſufficient to drive the Atomes of the *Diſſolved Solid Body* from place to place; and conſequently, notwithſtanding their *Gravity*, they do not *Sink in the Liquor Lighter than themſelves.*

But I conceive another Account may be given of this *Appearance*, and that the Prime Law of *Hydroſtaticks* is a little deficient. 'Tis true indeed, if we conſider only the *Specifick Gravity* of a Liquor, and the *Specifick Gravity* of a

Solid

Solid Particle Floating therein, the forementioned Rule is exact; but in *Sinking* there is requisite a *Separation* of the Parts of the Liquor by the *Sinking* Body; and there being a Natural Inclination in the Parts of all Liquors to *Union*, arising from an Agreement or *Congruity* of their Parts, there is a Resistance therein to any thing that separates this Conjunction: Now, unless a Body have Weight enough to overcome this *Congruity* or Union of Parts, such a Body will float in a Liquor *Specifically Lighter* than itself. But that a Heavy Body, as *Mercury* or *Iron*, may have its Parts reduced to that Minutness that their *Gravity* or Tendency downwards, is not strong enough to Separate the *Cohesion* or Union of Parts of a Liquor, will be manifest, if we consider, that the Resistance made by the Medium to a *Falling* Body, is according to the Superficies of the Body: But, as the Body decreases in Bulk, its Superficies does not proportionably Decrease: thus a Sphere of an Inch Diameter has not *eight* times less Superficies than a Sphere of two Inches Diameter, tho' it have *eight* times less bulk, and consequently passing thro' a Medium, as suppose Air or Water, the Sphere of an Inch Diameter, is proportionably to its Bulk, more *Resisted*, than a Sphere of two Inches Diameter in proportion to its Bulk; and hence it will come to pass, that at last a Body may be reduced to that Minuteness, that its Gravity *Pressing* downwards (which is according to its Bulk) may be less than the *Resistance* of the Medium, which operates on the Surface of the Body; seeing, as I said before, the Surfaces of Bodies do not decrease so fast as their Bulks, these decreasing in a Triplicate, but those in a Duplicate *Ratio* of the Bodies Diameters.

But because I have said that the forementioned Law of *Hydrostaticks* is a little defective, I desire to explain myself a little further in that point. In Weights *Falling* thro' the Air, were Gravity only considered, the Proportions of their Descents would be exactly as *Galileo* has demonstrated; but it is allowed by all, that the *Resistance* of the Air, not being considered in those Demonstrations, they are not *Mathematically* True in Practice, but that really there is something of that Proportion hindred by the Air's *Resistance*. Now, what is this less than to say, that the *Resistance* of the Air takes off some of the Operation of Gravity, or is able to withstand or oppose part of its Action? And if so, what shall we say, were an *Iron* Sphere let thro' a Medium of Water? Surely the Proportions of its Descents, would be much more disturbed herein, as Water is much more solid and difficult to be separated or passed thro' than Air, and consequently we must needs grant, that more of the Operation of Gravity is taken off or resisted by this Opposition of the Water, than that of the Air. And if so, surely there may be a certain Degree of Gravity, that may be quite taken off by the *Resistance* of the Water. Were a Pistol Bullet let fall thro' the Air, it would descend imperceptibly nigh the Proportions that *Galileo* has assigned; but were a single Grain of Sand so let fall, it would be much hindered in its Course, and half of this Grain would be more obstructed; what shall we then say of the Ten thousandth part, or of a part of the Ten thousand Millionth of this, and again of the infinite Subdivisions of that, till at last we come to a Part that would be

wholly

wholly *resisted*, or kept up; such I conceive the minute Particles of a Body *dissolved* in a *Menstruum*.

On this Account, 'tis, I say, that the forementioned Principle of *Hydrostaticks* is a little defective; for it considers not the natural *Congruity* of the Parts of a Liquor, whereby they desire as 'twere to unite and keep together, just as we see two drops of Water on a dry Board being brought together, do jump and coalesce, and therefore Liquors have an innate Power of *resisting* a certain degree of Force that would *separate* them; such as I suppose the degree of Gravity, in the most minute Particles of a Body *dissolved* in a *Menstruum*.

The forementioned Rule holds true to the most nice Sense in great Bodies; but in those that are by many Millions of Divisions smaller, it seems to fail.

I would not however be thought wholly to reject my *Brother's Solution* of this *Problem*: for certainly that *Motion* (whatsoever it is) in a *Menstruum*, which is able to dissolve such a solid Body as Iron, that is, which is able to disturb the close and strong Cohesion of the Parts of Iron, may very well be supposed sufficient to disturb or keep up these Parts from resting in the Bottom of the Vessel, wherein the *Solution* was made; and certainly no better Account can possibly be given of such *Solutions*, than by supposing such an *internal Motion* in the parts of the *Menstruum* insinuating themselves into the solid Body, and loosening its parts. But I leave to others to consider what kind of *Motion* and peculiar Conformation of Parts is requisite both in the *Menstruum* and in the dissolved Body, that a Solution may result from their Commixture.

Considered; by Mr. Tho. Molyneux. *ibid. p.* 93.

2. Tho' Liquors consist of Parts united, and tho' this Union be easily destroyed, yet of necessity it requires some degree of Force for effecting it; yet this *Property* ought not to be rely'd on as the sole Cause of this *Appearance*: For, in this *Solution* of the *Problem*, We first suppose the minute Particles of a heavy Body rais'd, and then give the Reason of their not sinking; whereas 'tis not to be questioned, but that that Force which raised them, is the same that keeps them from falling to the bottom.

An Undertaking for Raising of Water; by Sir Sam. Moreland. n. 102. p. 25. Apr. An. 1674.

IX. Sir *Sam. Moreland* undertakes to demonstrate, (contrary to the common and received Opinion thro' *England* and all *Europe*)

1. That he will force Water 60 Foot high with treble the Weight that shall raise it 20 Foot; and so proportionably, *in infinitum*.

2. That by how much wider the Barrel is, in which the Forcer works, than the Pipe thro' which the Water is forced up, by so much is the *Engine* pressed with unnecessary Weight.

A Siphon *performing the same things with the* Sipho Wurtemburgicus; *by Mr.* J. Davis. n. 167. p. 846. Nov. An. 1684.

X. 1: The last Summer, *Ann.* 1684. a Treatise fell into my Hands, called *Sipho Wurtemburgicus*, or an inverted Siphon with Legs of the same Height, running and running back again, such as had never been heard of before. The Author speaks wonderful Things of this Machine, but begs

the

the Reader's Pardon for his moſt Serene Patron, who had a mind to preſerve the Structure of it for himſelf. When I read this I conſider'd how a Siphon might be conſtructed, which ſhould perform the ſame Things as are ſaid of that of *Wurtemburg*. Having therefore in my Hands a certain Siphon of Glaſs, I erected it as nearly perpendicular as I could over two Veſſels; and when it was fixt in this Situation, I poured Water into one of the Veſſels, till the Orifice of the Siphon was a little immerſed in it, and preſently as I expected the Water run out into the other Veſſel. Then that Veſſel being emptied into which I had firſt poured Water, I poured it into the other, and immediately that Water run back into the firſt Veſſel. Tho' I would not venture to compare the Artifice of this Siphon of mine with that of *Wurtemburg*, yet as to its uſe I doubt not but that it will not much yield to that, eſpecially if a certain Inſtrument be added to it, which I have contrived in a particular manner.

2. In the Treatiſe concerning the *Wurtemburg* Siphon, which was lately printed at *Stutgard* by Dr. *Salomon Reiſelius*, certain wonderful Things, and before unheard of, are told of that new Siphon, of which the Characteriſtic Properties are declared in theſe Words. *By Dr.* Papin. *Ibid. p.* 847.

1. *The Orifices of the two Legs of his* Siphon *being placed Horizontally, are dipped into the Brims of the Veſſels; whereas in thoſe of the antient Invention the longer Leg always deſcends below the Brim or the Equilibrium.*

2. *The Orifices being either partly or half fill'd with Water, yet the Water flows out when drawn over a Mountain. Whereas in other* Siphons *the whole Orifice muſt be fill'd with Water, or immerged in the Water.*

3. *Tho' the Machine be at reſt and in continual Dryneſs, yet it will produce its Effect, Water being apply'd again.*

4. *Either of the Eyes or Orifices being open, and the other after ſome Hours or a Day being ſtop'd by a Cone or Stopper, yet the Water will run out; whereas in others both the Eyes muſt be open at the ſame time.*

5. *The Orifices being placed in an Horizontal Line, and the Legs being equal in height, yet the Liquor will run out; but in the Machines of* Porta *and others, the Legs muſt be unequal, and the Perpendicular greater.*

6. *The Water being poured into either Veſſel will aſcend and run into either; whereas in thoſe of the Antients it runs out at one Leg only, and that the longer, but never runs back.*

Theſe are the Words of the Author; but by what Method, or what Artifice theſe notable Effects may be produced, he adds not a Syllable. Therefore the Royal Society order'd me to prepare a Machine, which might exhibit the ſame Phenomena as theſe deſcribed in the Book. I have perform-ed the thing after three different manners; but to avoid being tedious, I judge the following, as the eaſieſt and ſimpleſt, will be ſufficient.

A A are two Veſſels of Metal, in which are inſerted the two Extremities of the *Siphon*. BCDEDCB is a *Siphon*, the Eyes of which BB are to be diſpoſed in the ſame Horizontal Line. F is a little Tube ſoder'd into a Hole *Fig.* 197.

Hole in the upper part of the *Siphon*, and carefully closed, after the *Siphon* is totally replenished with Water. Now it is plain, that the Water contain'd in the Parts C D will hinder the Influx of the external Air, so that it cannot penetrate to the upper Part of the *Siphon* E. So that the *Siphon* being always full of Water, (provided it does not exceed a due height,) will most surely produce its Effect, as soon as the Water in the Vessels A A shall fill any part of either of the Orifices B. And when both the Orifices being partly fill'd with Water, in either Vessel A the Surface of the Water shall come to the same Horizontal Line, if you pour never so little Water to either Vessel, part of it will presently be carry'd through the *Siphon* into the other Vessel; and in the same Method the other *Phænomena* may be exhibited as are described in the *Book*.

By D. Salomon Reiselius himself. n. 178. p. 1272. Dec. An. 1685.

3. That I may not any longer be wanting in satisfying the Desire of the most serene Society, I confess that the *Siphon* of the most excellent Dr. *Papin* to be the very *Wurtemburgic Siphon*, also made with a Recurvation of the Feet. Nor is there any other Mystery at the Top, as the Inventer has described, than that it must be fill'd by a Funnel, without which Impletion it cannot run away. This shall soon be confirm'd farther by the Press, because it would be long and tedious to set down every thing here.

Now that I may shew at present that I have already perform'd something about the Effects of the *Siphon*, here is something that has occur'd to me in making my Experiments. For I have learn'd how the Water may run out either at the top, or at the sides, which many have promis'd to do, but hardly any one has perform'd.

A New way of Raising Water Enigmatically proposed; by Dr. Papin. n. 173. p. 1093. July An. 1685. *Fig.* 198.

XI. 1. A A is a great Glass made like a Tumbler, but much bigger, and laid upon the Chimney Board, B B.

C C is the *Engine* like a small *Rock*, that doth constantly *spout* out Water by the two Holes D D: This *Rock* is kept at a distance from the bottom of the *Glass* A A; so that it may plainly be seen that it cannot receive any Water by subterraneal Tubes.

E E is a factitious *Coral*, reaching from the Center of the *Rock* C C, to the Center of the *Crown* F F.

F F is a *Crown* bearing upon the Aperture of the *Glass* A A, and holding the *Rock* C C, suspended at a considerable distance from the Bottom.

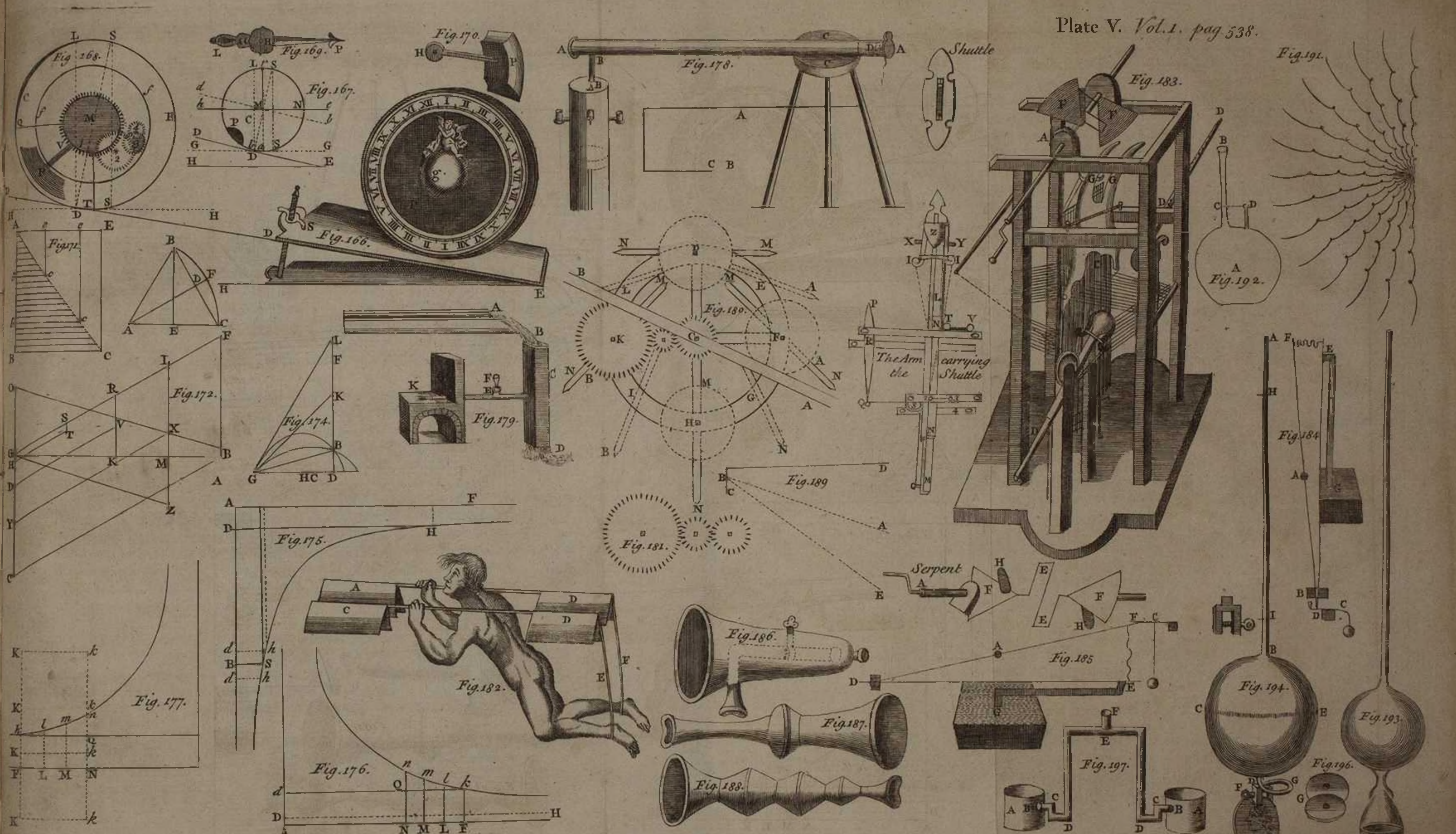
G G a Glass open at both ends, apply'd to the *Rock* C C, to keep the Water upon it from falling down.

The Water in this Engine runs constantly. H H, Two *Shells* to receive the Water from the *Jetto's*.

Solv'd; by Dr. Nath. Vincent. n. 177. p. 1238. Dec. An. 1686.

2. Within the *Rock* C C, there may be a Vessel placed, which shall be made like the Body of a Pair of *Bellows*, or those *Puffs* heretofore used by *Barbers*, which being filled with Water, a Piece of *Clockwork* put under it may produce the *Jetto's*; the Water being received into the *Shell* H H, and running thence into the hollow of the *Coral* E E, may be thereby conveyed into the follicular Cavity in the same quantity it is ejected from the two

Plate V. Vol. 1. pag. 538.



two emerging Tubes; and it will circulate according to the going of the *Clock-work*.

3. I conceive that the Air is forced into the *outer Glaſs* at the bottom thereof. *By Mr. R. A. ib. p. 1259.*

That it then paſſes up between the two *Glaſſes*.

That the *outer Glaſs* or Caſe being cloſe luted at the Head or *Crown* to which the *inner Glaſs* is hung by the *Coral*, the Air is forced into the Mouth of the *inner Glaſs*.

That the Air ſo forced, preſſing on the Surface of the Water that covers the *Rock*, forces the Water to riſe thro' thoſe two extream Parts that are not at all clogg'd, or covered with Water.

4. A B D E, ſignifies a Cylindrical Veſſel, cloſed on every ſide, and divided into two Rooms by the Floor E F. *In a Letter Subſcribed, W. Tenon.*

G L M H, is another Cylindrical Veſſel within that upper Room, cemented with its Mouth downwards to the Floor, and full of Water up to the Surface I K; the upper Part thereof G I K H, being full of Air. *n. 178. p. 1254. Fig. 199. Dec. An. 1685.*

Q O, R P, Two Pipes, open above and below, and let thro' the Upper Room into this Veſſel, and reaching almoſt down to the Floor E F.

V W, a Pipe open above and below, and let into the Upper Room. Theſe Pipes muſt be cloſe joined round about them to the Floors C D and G H.

X Y, Two little Hemiſpherical Bladders prepared with Oil or ſome Oily Subſtance (as Butter and Turpentine) againſt Water, and Cemented with their Mouths upward to the Floor E F, underneath.

a, β Two Valves Opening out of the Upper Room in the Bladders.

γ, δ Two other Valves Opening out of the Bladders into the inner Veſſel above.

N Z, a *Pendulum* playing upon the Centre N, and having two Battledoor Arms *a*, *b*, to ſqueeze alternately the Bladders which reſt upon them.

Let the Upper Room be filled with Water at the Pipe V W, and if the *Pendulum* be made to play by *Clock-work*, the Bladders will perpetually pump it thence into the Inner veſſel, and the compreſt Air G I K H in the Upper part of that Veſſel preſſing upon the Surface of the Water I K, will force it thence into the Pipes O Q, P R, out of which ſpouting with a perpetual even Stream into the Spoons S T, it will run down by the Pipe W V, into the Upper Room again: the *Pendulum* will play moſt eaſily when the upper Room is filled to the Top of the Pipe W V. Inſtead of the Bladders may be other Contrivances, as of Suckers or little Organ Bellows, playing alternately with two Leaves about an Axis in the middle.

5. A A, is the *Great Tumbler*, that muſt have ſome little Hole in the bottom, as I. *By Dr. Papin. Ibid. p. 1274. Fig. 200.*

I L L, a ſlender Pipe hidden by the Chimney Board B B, whereby the *Tumbler* A A, hath Communication with the *Pump* or *Bellows* M M.

MM, ſome kind of *Pump* or *Bellows* well ſhut, and having no other Aperture, but thro' the Pipe I L L. Theſe are put in ſome ſecret Place where a Body may play the ſame and not be ſeen.

N N, a ſlender Pipe, that makes a Communication between the *Glaſs* AA, and the *Crown* F F; this Pipe reacheth near to the cover of the *Crown*, that the Water contained in it may not run down by that Aperture.

E E, the *Factitious Coral*, hollow within, ſhut at the Bottom and open at the Top.

D D, D D, Two crooked Pipes ſoldered to the Sides of the *Coral* E E, ſo that the Water running down the *Coral* may ſpout out at the Holes D D.

O O, a Pipe hidden in the *Coral* E E, paſſing thro' the bottom of the ſame, where it muſt be well ſolder'd, and reaching near to the Bottom of the *Rock* C C.

P P, a Pipe to convey the Water from the *Glaſs* G G, into the *Rock* CC; this Pipe is well ſolder'd to the *Cover* of the ſaid *Rock*.

Q, a *Valve* working by a Spring at the Bottom of the Pipe P P, to keep the Water, that gets in that Way, from returning back.

R, another *Valve* at the top of the Pipe O O, that the Water getting up that way, may not fall thro' the ſame.

Now it is plain, that the *Rock* C C, being filled partly with Water, partly with Air; if we open the *Bellows* M M, the Air from the *Crown* F F, muſt run thro' the Pipe N N, into the *Tumbler* A A; and thence thro' the Pipe I L L, into M M, to fill the Vacuity made therein: The Air in the *Crown* F F, being thus rarified, gives liberty to the Air in the *Rock* C C, to rarify too, by driving the Water thro' the Pipe O O. The Water being got up into the *Crown* F F, runs down the *Coral* E E, and thro' the crooked Pipes D D, D D, ſpouts out at their upper Apertures, and from the *Shells* H H falls upon the *Rock* C C: If we come afterwards to ſhut the *Bellows* M M, the Air got into their Vacuity muſt run back into the *Tumbler* A A, and preſs upon the Water at the Top of the *Rock* C C: But the Air in the ſaid *Rock* having been rarified, its Spring is not ſufficient to reſiſt this Preſſure, and ſo the Water is forced into the ſaid *Rock* thro' the Pipe P P: And thus opening and ſhutting the *Bellows* M M, the Water muſt conſtantly circulate by the Ways aforeſaid.

The Uſe of this Contrivance.

As for the Uſes this way for *raiſing Water* may be applied to, this I do conceive: The Glaſſes, being merely to conceal the Secret, muſt be left out; and their may be made ſeveral Receptacles above one another to receive the raiſed Water, ſo as doth the *Crown* F F: And there ſhould be as many *Bellows* to communicate every one with one *Receptacle*: Theſe *Bellows* ſhould be moved by an Axis, ſo that, when the *Firſt* is open, the *Second* ſhould be ſhut; the *Third* open, the *Fourth* ſhut; and ſo forth, alternatively; which may be eaſily done: By this Means, the firſt or loweſt *Receptacle* would give the neceſſary Supply of Water to the *Second*, the *Second* to the *Third*, and the *Third* to the *Fourth*, &c. till the Water would be *raiſed* to the intended Height. Such Receptacles might eaſily be ſet at 12 or 15 Foot above one another,

another, and ſo but few of them might *raiſe Water* to a conſiderable Height, as well as ordinary *Pumps* do: But this new Way would have this advantage, that in the ordinary Pumps the Strength to be applied lieth near the Water to be raiſed, but by this Contrivance the Stream of a River may be applied to draw Water out of a Mine far diſtant from it. By the ſame way the Stream of the *Thames* might keep conſtant *Water-Works* in *Windſor-Caſtle*, as eaſy almoſt as in the loweſt Fields: The River *Seine* might do the ſame at St. *Germain*, and perhaps at *Verſailles* too, notwithſtanding the great Diſtance. For it is to be obſerved, That the Pipes of Communication between the *Bellows* and the *Engine*, being merely for the conveying of the Air, which moves very ſwiftly, they may be ſlender enough, and ſo contain but a ſmall Quantity of Air to be rarified; and beſides, they will not be ſubject to burſt or leak, ſince the Preſſure they bear, being all external to the Pipe, will rather ſtrengthen than break the ſame. From whence it follows, That the ſaid Pipes need not be ſtrong, but may be made at very ſmall Charges. It is alſo to be obſerved, That thoſe *Bellows* which are open, have the Air in them very much rarified, ſo that the outward Air lieth heavy upon (to ſhut) them; by which means the Motion of the *Engine* muſt be help'd in Lifting up the oppoſite *Bellows*, that are to be opened: And this Obſervation may anſwer the greateſt difficulty that might be objected againſt this Contrivance. So that I don't queſtion but this way for *raiſing Water*, may on ſeveral Occaſions be of great Advantage.

A B, A B, Are ſeveral *Receptacles* ſet above one another, which muſt be well ſhut and ſoldered every where. *Further Explained.* *Fig.* 201.

CDD, CDD, Are two ſlender Pipes, whereby the Firſt and Third *Receptacles* have a Communication with the *Pump* H H.

EFF, EFF, Two other ſlender Pipes, whereby the Second and Fourth *Receptacles* have a Communication with the *Pump* I I.

HH, I I, Two *Pumps* whoſe Plugs are ſo moved by the Axis L L, that when one goeth down the other goeth up.

MM, a Wheel faſtened to the Axis L L, that it may be moved by the Stream of a River.

NO, PQ, NO, PQ, Are big Pipes for the Water to go up, from a lower into a higher *Receptacle*.

O, Q, O, Q, Are *Valves* fitted to the Top of the aforeſaid Pipes, that the Water may not go down thro' the ſame.

Now it is plain, that, when the *Plug* in the *Pump* HH, is going up, the Air comes in thro' the Pipes C D D, and ſo it is rarified in the firſt and third *Receptacles* marked A, A: And by that Means the Water may be driven up into the ſaid *Receptacles* thro' the Pipes N O, becauſe at the ſame time the *Plug* in the *Pump* II, going down, cauſeth the Air to return to its ordinary Preſſure in the ſecond and fourth *Receptacles*, that it may be able to drive up the Water thro' the ſaid Pipes N O, and the loweſt Pipe draws the Water that lies open to the Air. By the ſame reaſon when the *Plug* in the *Pump* I I, goeth up, the Air muſt come in thro' the Pipes

EFF: And so it is rarified in the second and fourth *Receptacles* marked B, B, and by that means the Water may be driven up into the said *Receptacles* thro' the Pipes P Q, P Q, because at the same time the *Plug* in the *Pump* AH, going down, causes the Air to return to its ordinary Pressure in the first and third *Receptacles*, so that it is able to drive up the Water thro' the said Pipes P Q.

Several Objections made by M. Nuis, *Answered; by Dr.* Papin. n. 186. *p.* 263. Jan. An. 1687.

6. 1. To keep the *Receptacles* from being fill'd too much, the Water may be let out by inserting into each a crooked Pipe, reaching a pretty way downwards, and having its lower Aperture shut up with a Valve, whereby the Water may run out when the *Receptacle* shall be fill'd to a certain Height: And I may add, to prevent new difficulties, that, lest the *Pumps* should be fill'd too much, a Valve may be made that shall open as soon as the Air in the *Pump* should be more comprest than the outward Air: So the Air getting in thro' any Pores would be constantly let out.

2. I have not positively promised a good Success, but for *Windsor* and St. *Germain*; but when I spoke of *Versailles* I used the Word *perhaps*, thereby shewing that, before any one should go about such a great Undertaking, he should reflect upon it more than I would then do, not having occasion for such Work. But I now make the following computation.

Let the Distance of *Versailles*, as *M. Nuis* supposeth, be 12000 Foot, and the Capacity of each *Receptacle* be about one half of a cubic Foot: I might make the Wheel with the Axis to make their *Revolution* in one Minute of Time, and so order all things that the Air under the ascending *Plugs* might come to be rarified to such a Degree, that by its Elasticity it might not counterpoise more than 7 Foot of Water; but at the same time the Air in the *Receptacles* A,A,B,B, would, even in its greatest Dilatation, be able to counterpoise 17 Foot; So it is plain, that the Air will be driven from the *Receptacles* into the *Pumps* by a Strength equivalent to 10 Foot of Water: Now if we compute the * *Velocity* of Air driven by such a Pressure we shall find that the said *Velocity* will be about 740 Foot in a Second: So that in half a Minute, during which the Plug goeth up, this Air might pass above 22000 Foot, altho' it were not rarified at all; but being rarified, as we do suppose it to be, it might go a great deal further.

* *Vid. Sup. Cap.* v. §. XII.

I must now take notice, that according to the Honourable Mr. *Boyle*'s Experiments, the *Rarefaction* of the *Air* is much lesser than *M. Nuis* takes it to be: for the Water contained in the Pipe N O, is so far from causing the Air to fill up a Space four times bigger, that it will not extend itself to a Space once bigger than before: Considering therefore the *Velocity* of the Air, and the small Dilatation it doth suffer, if any one will take the Trouble to compute, he will find, that if the *Pumps* have in Diameter the Diagonal of a square Foot, and the same Height; and if the small Tubes of Communication be made of $\frac{1}{9}$ part of an Inch in Diameter; so that being 12000 Foot long, they may contain about one cubic Foot of Air; that would be more than sufficient to make the necessary *Rarefaction* in the *Receptacles*.

But

But for the good Succefs of the *Engine* it is not enough to make the Air pafs from the *Receptacles* into the *Pumps*, it muft alfo return from the *Pumps* into the *Receptacles*: Now for this Intent it would be neceffary to fet the *Receptacles* but five Foot above one another; fo to drive the Water up the Pipe N O, it would be enough that the Air in the *Receptacle* B, fhould prefs with a Strength equivalent to 23 Foot of Water: for it is plain, that five Foot in the Pipe N O, together with a Preffure equivalent to 17 Foot, which I have fuppofed to be in the upper *Receptacle* A, will make but 22 Foot in all; and therefore 23 Foot depreffing in the *Receptacle* B, muft prevail, and caufe the Water to afcend: Now the Preffure in the *Receptacle* being but 23 Foot, and the Air in the *Pump* returning to its ordinary Preffure, which is about 33 Foot, it is plain that the Air going back to the *Receptacle*, will be driven by a Strength equivalent to 10 Foot, as well as it had been in coming from the *Receptacle* towards the *Pump*: and fo the Bignefs affigned for the *Communication Pipes* will alfo prove more than fufficient to this Effect.

From what I have been faying, it is plain, That in great diftances there fhould be made as many *Pumps* as *Receptacles*, as hath been already propounded: And for to raife Water but 16 Foot High, there fhould be required 13 or 14 *Receptacles*, and as many *Pumps* of the Bignefs aforefaid. Some People may take this for a great Difficulty: But I anfwer, That in this *Engine*, this is not fo much as it feems at firft; becaufe, the Preffure being all from without, there is no need of any great Strength to refift it; and fo the Metal for the *Pump* will coft but little: There may alfo be found Occafions where to make fo good ufe of them, that fuch an *Engine* as I have defcribed would in a Year's time fave Labour enough to pay for many *Pumps*, fince it might every hour raife about 1800 Pounds of Water to the Height of 60 Foot. Mean while I don't pretend to have given here the beft Proportion for the Bignefs of every part of the *Engine*; but it may be, by altering the Capacity of the *Pumps*, of the *Pipes*, or of the *Receptacles*, a much more confiderable Effect might be produced.

3. The Water doth not at any time afcend higher than from a lower *Receptacle* into the next upper *Receptacle*; which Height is but 12 Foot: So that it is plain enough, that the Preffure of the Air may be fufficient to drive it up. It is indifferent, whether it be by Rarefaction or otherwife that the Water comes into the *Receptacle* A; it is enough that the Water is there, and that the Air preffes upon it with fuch a Strength as will prevail againft all that oppofeth it.

4. Tho' the Ufe of the Pipes be merely for conveying of Air, they may neverthelefs be eafily fill'd with Water, when need requires; and fo the Defects in them may as well be found out as in Pipes that are ufed for the conveying of Water.

An Engine for Raifing Water, by the Help of Fire; by Mr. Tho. Savery. n. 253. p. 228. *Fig.* 202. June An. 1699.

XII. A, The *Furnace*.
B, The *Boyler*.
CC, Two *Cocks*, which convey the *Steam* by turns to the *Veffels* D D.

DD, The *Vessels* which receive the Water from the Bottom, in order to Discharge it again at the Top.

EEEE, *Valves*.

FF, *Cocks* which keep up the Water, while the *Valves* on Occasion are cleans'd.

G, The *Force* Pipe.

H, The *Sucking* Pipe.

I, The *Water*.

An Hydraulique Engine; by .... n. 128. p. 679. Fig. 203. An 1675.

XIII. This *Engine* is a *Chest* of Copper A, pierced with many Holes above BB, and holds within it the Body of a *Pump* EFM, whose *Sucker* DE, is raised and abased by two *Levers* C, O; these *Levers* having each of them two Arms, and each Arm being fitted to be laid hold on by both Hands of a Man. Each *Lever* is pierced in the middle by a Mortise *aa*, in which an Iron Nail, which passes thro' the Handle of the *Sucker*, turns when the *Sucker* is raised or lower'd. Near the Body of the *Pump* there is a *Copper Pot* IHL, joined to it by the Tube G, and having another Tube KNL, which in N may be turned every way.

To make this *Engine* play, Water is pour'd upon the *Chest* to enter in at the Holes that are in the Cover thereof. This Water is drawn into the Body of the *Pump*, at the Hole F, at the time when the *Sucker* is raised: and when the same is let down, the *Valve* of the same Hole F shuts, and forces the Water to pass thro' the Hole M into the Tube G, of which the *Valve* H being lifted up, the Water enters into the *Pot*, and filling the bottom, it enters thro' the Hole K, into the Tube KNL, in such a manner, that when the Water is higher than the Tube KNL, and the Hole of the Tube G is shut by the *Valve* H, the Air enclosed in the *Pot* hath no Issue; and it comes to pass, that, when you continue to make the Water enter into the *Pot* by the Tube G, which is much thicker than the Aperture of the end L, at which it must issue, it must needs be, that the surplus of the Water that enters into the *Pot*, and exceeds that which at the same time issues thro' the small end of the *Jet*, compresses the Air to find Place in the *Pot*: which makes that, whilst the *Sucker* is raised again, to make new Water to enter into the Body of the *Pump*, the Air which has been *Compress'd* in the *Pot* drives the surplus of the Water by the Force of its *Spring*, mean time that a new *Compression* of the *Sucker* makes new Water to enter, and causes also a new *Compression* of Air. And thus the Course of the Water, which issues by the *Jet*, is always entertained in the same State.

A cheap Pump: by Mr. Conyers. n. 136. p. 888. June An. 1677.

XIV. A A, the Body of a square Taper *Pump*, made of Oak, Elm, or Deal Planks; with a *Valve* at bottom, *aa*.

BB, the *Bucket*, in the midst of which there is a *Valve* *b*, not visible in the Figure, being concealed by the Sides of the Leather, *bb*.

CCC, the Iron to raise the *Bucket*.

D D, the Wood at the bottom of the *Bucket* containing the *Valve.*

E E, the *Handle* for raiſing the *Bucket*, to be managed by fewer Hands than ordinary *Pumps* are; which may be altered ſo as to employ a Horſe, or Mill, or other ſuch-like way, more advantageous than that of this Handle managed by the ſtrength of Men. *Fig.* 204.

F F, a Square *Taper-Box* with Holes in the Sides, and open at the Bottom; into the narrower part of which is encloſed the narrower End of the Body of the *Pump.*

G G, an additional *Bucket* of a larger Dimenſion, to be placed on the Iron Work of the *Pump* about H, when it ſhall be needful to lengthen the Taper of your *Pump*, and thereby to raiſe the Water more forcibly to a greater height.

I I, the *Spout* of the *Pump*, to caſt out the Water, of the ſame Breadth with the ſide of the *Pump.*

K K, the Iron or Wooden Work ſet off, or bent back (if need be) and placed at the back of this *Pump*, for the eaſier and more capacious Motion of the *Pump-Handle*, in which it moves.

This *Pump* was by me contrived in 1673. when the *New Canal* of *Fleet-River* in *London* was enlarged: It was found to raiſe at leaſt twice as much Water proportionably as thoſe of the ſame, or rather bigger Bore, that were firſt made uſe of and caſt by. It was $8\frac{1}{2}$ Foot long, and 1 Foot 8 Inches broad at the top, and about 8 Inches broad at the bottom, where it is inſerted in the *Box*; and did caſt out 8 Gallons at a *Stroke*, and 21 *Strokes* being made in one *Minute*, there were delivered about 169 Gallons in a *Minute*'s time; whence it is eaſy to compute, what Quantity is thrown out in an *Hour*. This kind of *Pump* may by the ſame Contrivance be made of a Tree bored thro' with a Taper Bore: and a Baſket may be uſed at the bottom of the *Pump* inſtead of the *Box-Colender.*

XV. Papers *of leſs General Uſe*, (*Extracted from a Book of* Jo. Alph. Borellius *de Motu Animalium*) omitted.

1. A *Way how a Man may* Swim *under* Water, *and* breathe *by the Help of a Bag about his Head.* *Phil. Coll.* n. 2. p. 35.

2. *Another way of* Breathing *under* Water *by the help of a Leathern Pipe kept open by wreathed Wires, and extended by the* Swimmer's *Head to the top of the Water.*

3. *A way to make a* Submarine Veſſel *accommodated with ways to* Row *it, and to make it* Riſe *and* Sink *in the Water.*

XVI. Accounts *of Books* omitted.

n. 8. p. 145. n. 10. p. 173. n. 226. p. 481. 1. HYdrostatical Paradoxes, *made out by New* Experiments, *(for the most part Physical and Easy) by the Honourable* R. Boyle, *Esq*;

2. Recueil de diverses Pieces touchant quelques *Nouvelles Machines*, &c. Par *D. Papin, M. D. A Cassel.* 1695. *in* 8*vo*.

CHAP. VII.

Geography. Navigation.

A new place for the first Meridian propos'd; by a Professor of Math. *at* Seville. n. 118. p. 425. Oct. An. 1675.

I. THE *Longitude* of a Place upon the *Earth* is an Arch of the Equinoctial intercepted between two Meridians; or the Space of Time which is number'd by the Equinoctial between two Places. Wherefore it would be proper to fix the beginning of *Longitude* in the Equinoctial it self. Moreover since the Equinoctial Circle divides the *Globe* into Northern and Southern Hemispheres, if the fixing of this *Primary Meridian* is appointed in it, there will be an Equality and Conformity between the Northern and Southern Parts. Besides it is necessary for exactness, that the Place of this first *Meridian* should be small, that the Reckoning of the *Longitude* may be express'd the more exactly; not as some have done, who have assumed all the *Fortunate Islands* for their Beginning of Longitude, and did not regard the Distance of two Degrees which they have from one another; which surely is very absurd. Again care must be taken that the *Primary Meridian* may not be confounded with the Land and Draughts of Places which are described on the *Globes* or *Maps*, which must be if it passes thro' the midst of Countries. And if it divides the principal Parts of the Earth, as *America*, *Africa*, *Europe*, passing over the Seas, it will be so much the fitter and more convenient in the Representation of the *Terraqueous Globe*. Now considering all these Reasons here mentioned, I have found, that as Nature, which provides nothing in vain, has placed a certain Island under the very *Equinoctial Circle*, near *Brasile*, formerly called *Abroxos*, which Island is distant from the *Pike* of *Teneriff* 9 Degrees to the West, and from *Uraniburg* 42 Degrees to the West, in which all the Circumstances are found proper to constitute the first *Meridian*, as by me have been enumerated.

II. 1. Those

II. 1. Those that intend to make use of *Pendulum Watches* at *Sea*, must have two of them at least; that, if one of them should by mishap or neglect come to stop, or (being by length of time become foul) need to be made clean, there may likely always remain one in Motion.

M. Huygen's *Instructions for finding the* Longitude *with Pendulum Watches; Enlarged by . . . .* n. 47. p. 937. May An. 1669.

2. The *Watches* on *Shipboard* are to be hung in a Close Place, where they may be freest from Moisture or Dust, and out of Danger of being disordered by knocking or touching.

3. Before the *Watches* be brought on *Shipboard*, 'tis convenient they be adjusted to a *Middle* or *Mean Day*; the Use of them being then most easy.

To adjust the Watches.

4. Here take notice, That the Sun passeth the 12 Signs, or makes one entire Revolution in the *Ecliptick* in 365 Days, $5^h\ 48'$. or thereabout; and that those Days, reckoned from Noon to Noon, are of different Lengths; as is known to all that are versed in *Astronomy*. Now between the Longest and the Shortest of those Days, a Day may be taken of such a Length, as 365 such Days, $5^h\ 49'$. &c. make up, or are equal to that Revolution; and this is called the *Equal* or *Mean Day*, according to which the *Watches* are to be Set; and therefore the Hour or Minute shewed by the *Watches*, tho' they be perfectly just and equal, must needs differ almost continually from those that are shew'd by the *Sun*, or are reckoned according to its Motion. But this Difference is Regular, and is otherwise called the *Æquation*; which is accounted from the first of *February* in the following *Table*.

	Januar.		*Februar*		*March.*		*April.*		*May.*		*June.*	
	′	″	′	″	′	″	′	″	′	″	′	″
1	6	10	0	0	4	46	14	23	19	25	16	24
2	5	47	0	2	5	03	14	39	19	28	16	13
3	5	24	0	4	5	21	14	55	19	29	16	01
4	5	02	0	8	5	39	15	10	19	29	15	49
5	4	41	0	12	5	57	15	25	19	29	15	37
6	4	21	0	16	6	15	15	39	19	28	15	24
7	4	02	0	21	6	33	15	53	19	26	15	11
8	3	44	0	26	6	51	16	07	19	24	14	58
9	2	27	0	32	7	09	16	21	19	21	14	45
10	3	11	0	40	7	27	16	34	19	18	14	32
11	2	55	0	48	7	45	16	47	19	15	14	19
12	2	39	0	57	8	03	16	59	19	11	14	06
13	2	23	1	06	8	22	17	11	19	07	13	53
14	2	07	1	16	8	41	17	22	19	02	13	40
15	1	52	1	26	9	01	17	33	18	57	13	27
16	1	38	1	37	9	21	17	43	18	51	13	15
17	1	25	1	49	9	41	17	53	18	45	13	03
18	1	13	2	02	10	01	18	03	18	39	12	52
19	1	02	2	15	10	21	18	13	18	33	12	41
20	0	51	2	28	10	40	18	23	18	26	12	30
21	0	41	2	42	10	59	18	32	18	18	12	19
22	0	32	2	56	11	18	18	39	18	10	12	08
23	0	24	3	11	11	37	18	46	18	01	11	58
24	0	18	3	26	11	56	18	53	17	51	11	48
25	0	13	3	41	12	15	18	59	17	41	11	38
26	0	9	3	56	12	34	19	04	17	30	11	28
27	0	6	4	12	12	53	19	09	17	19	11	18
28	0	3	4	29	13	12	19	14	17	08	11	09
29	0	1			13	31	19	18	16	57	11	00
30	0	0			13	49	19	22	16	46	10	52
31	0	0			14	06			16	35		

	July.		*August.*		*Septem.*		*October.*		*Novem.*		*Decemb.*	
	′	″	′	″	′	″	′	″	′	″	′	″
1	10	45	11	07	19	41	29	16	31	13	21	14
2	10	38	11	16	20	01	29	30	31	03	20	44
3	10	31	11	25	20	22	29	43	30	53	20	14
4	10	25	11	36	20	43	29	56	30	43	19	44
5	10	19	11	48	21	04	30	09	30	32	19	14
6	10	13	12	01	21	25	30	22	30	20	18	44
7	10	07	12	14	21	47	30	34	30	08	18	14
8	10	02	12	28	22	09	30	45	29	55	17	44
9	9	58	12	42	22	31	30	55	29	40	17	14
10	9	54	12	57	23	52	31	04	29	23	16	44
11	9	51	13	12	23	13	31	12	29	06	16	14
12	9	49	13	27	23	33	31	19	28	48	15	44
13	9	47	13	43	23	53	31	26	28	30	15	14
14	9	46	13	59	24	13	31	32	28	11	14	43
15	9	46	14	10	24	33	31	38	27	51	14	12
16	9	46	14	33	24	53	31	43	27	30	13	41
17	9	47	14	50	25	13	31	47	27	08	13	10
18	9	49	15	08	25	33	31	50	26	45	12	40
19	9	52	15	26	25	52	31	53	26	22	12	10
20	9	56	15	45	26	11	31	55	25	58	11	40
21	10	00	16	04	26	30	31	55	25	34	11	10
22	10	04	16	23	26	49	31	55	25	10	10	40
23	10	08	16	42	27	08	31	55	24	45	10	10
24	10	13	17	01	27	26	31	54	24	20	9	41
25	10	18	17	21	27	43	31	52	23	55	9	13
26	10	23	17	41	28	00	31	50	23	30	8	45
27	10	28	18	01	28	16	31	47	23	04	8	17
28	10	34	18	21	28	32	31	43	22	38	7	50
29	10	41	18	41	28	47	31	37	22	11	7	23
30	10	49	19	01	29	02	31	30	21	43	6	58
31	10	58	19	21			31	22			6	34

By the help of the foregoing *Table* you will always know what a Clock it is by the *Sun* preciſely, and conſequently, whether the *Watches* have been ſet to the right Meaſure of the *Mean Day*, or no; uſing the *Table* as follows.

When you firſt Set your *Watch* by the *Sun*, you are to ſubduct from the Time obſerved by the *Sun*, the *Æquation* adjoined to that Day of the Month in the *Table*, and to Set the *Watches* to the remaining Hours, Minutes and Seconds; that is, the *Watches* are to be Set ſo much Slower than the Time of the *Sun*, as (in the *Table*) is the *Æquation* of that Day; ſo that the *Æquation* of the Day added to the Time of the Clock, is the true Time by the *Sun*. And when after ſome Days, you deſire to know by the *Watch* the Time by the *Sun*, you are to add to the Time ſhewed by the *Watch*, the *Æquation* of that Day; and the Aggregate ſhall be the Time by the *Sun*, if the *Watch* hath been perfectly well adjuſted after the Meaſure of the *Mean Days*; for the Doing of which, this will be a convenient Way.

Draw a *Meridian Line* upon a Floor, and then hang two Plummets, each by a ſmall Thread or Wire, directly over the ſaid *Meridian*, at the diſtance of ſome two Foot or more one from the other, as the Smallneſs of the Thread will admit. When the Middle of the *Sun* (the Eye being placed ſo, as to bring both the Threads into one Line) appears to be in the ſame Line exactly (for the better and more ſecure diſcerning whereof, you muſt be furniſh'd with a Glaſs of a dark Colour, or ſomewhat Black'd with the Smoak of a Candle) you are then immediately to Set the *Watch*, not preciſely to the Hour of 12, but by ſo much leſs as is the *Æquation* of that Day; *e.g.* If it were the 12*th* of *March*, the *Æquation* of that Day being by the Table, $8' \ 3''$ theſe are to be ſubducted from 12 Hours, and the Remainder will be $11^h, 51' \ 57''$, to which Hours, Minutes, and Seconds, you are to Set the *Index* of the *Watch* reſpectively: Then after ſome Days you are to obſerve again in the ſame manner, and likewiſe to note the Hour, Minute, and Second of the *Watch*; to which you are to add the *Æquation* of theſe Days, taken out of the Table; and if the Aggregate do juſt make 12 Hours, the *Watch* is adjuſted to the Right Meaſure; but if it differ, you are to divide the Minutes and Seconds of that Difference by the Number of the Days between both the Obſervations, to get the daily Difference.

Let us ſuppoſe this ſecond Obſervation to have been made the 20th of *March*, *viz.* Eight Days after the firſt, and finding that the Middle of the *Sun*, being ſeen in the Meridian in the ſame Line with the two Threads, as before,

The *Watch* points, — —— —— —— ——	11	51′	07″.
The *Æquation* of the 20*th* of *March*, by the *Table*, is —	00	10	40
Which being added to the Time ſhow'd by the *Watch*, gives	12	01	47

If this had been juſt 12 Hours, the *Watch* would have been well adjuſted, but being $1' \ 47''$, more than 12, it hath gone ſo much too faſt in eight Days. And theſe $1' \ 47''$, that is $107''$, being divided by 8, there come $13\frac{3}{8}$ Seconds for the Difference of every 24 Hours; which Difference being known, if you want time, or have no mind to take the Pains to adjuft the *Watch* to its right meafure, (this being not neceffary, fince you may bring it thus on *Ship-Board*) note only the daily Difference, and regulate yourfelf accordingly. But if you will adjuft it better, you muft remove the lefs Weight of the *Pendulum* a little downwards, which will make it go flower, and then you muft begin a-new to obferve by the *Sun*, as before. If it had gone too flow, you muft have removed the mentioned Weight fomewhat upwards. And this is of that Importance in the finding out of *Longitudes*, that if it be not obferved, you may fometimes in the fpace of Three Months mifreckon 7 Degrees, and more yet (without any fault in the *Watches*;) which under the *Tropicks* will amount to above 400 *Englifh Miles*.

The *Watch* may alfo be adjufted *on Board*, when a Ship Rides at Anchor, thus: In the Morning, when the *Sun* is juft half above the *Horizon*, Note what Hour, Minute and Second, the *Watch* points at, if it be going; if not, fet it a going, and put the *Indexes* at what Hour, Minute, and Second you pleafe. Let them go till *Sun-Set*, and when the Body of the *Sun* is juft half under the *Horizon*, fee what Hour, Minute, and Second the *Indexes* of the *Watch* point at, and note them too; and reckon, how many Hours, *&c.* are paffed by the *Watch* between the one and the other. Then take the half of that Number, and add it to the Hours, *&c.* of the Morning Obfervation, and you fhall have the Hours, *&c.* which the *Watch* did fhow, when the *Sun* was in the *South*; whereunto add the *Æquation* in the Table belonging to that Day, and note the Sum. Then, fome Days being pafs'd, (the more the better) you are to do juft the fame: And if the Hour of this laft Day be the fame that was noted before, your *Watch* is well adjufted; but if it be more or lefs, the Difference divided by the Number elapfed between the two Obfervations, will give the daily Difference. And if you will, you may let it reft there, or otherwife, removing the leffer Weight of the *Pendulum*, you may adjuft it better. You may alfo, inftead of the *Sun's Rifing* and *Setting*, take two equal Altitudes of the *Sun*, before, and after Noon, and having noted the time given by the *Watches* at the time of both the Obfervations, proceed with it in the fame Manner, as was juft now directed for obferving the *Sun* in the *Horizon*. In either of which ways there may be fome Error, caufed by the *Sun's Refraction*; which is inconfiderable, and therefore needs not to be taken notice of.

5. Give to each of the *Watches* a Name, or a Mark, as A, B, C; and before you fet *Sail*, fet them to the Time obferved by the *Sun* in the Place where you are, and whence you are departing, allowing for the *Æquation* of the Day whereon you make your Obfervation; which Day you are to note, if the *Watches* be not well adjufted; otherwife it is not neceffary. *To find by them the* Longitude *at Sea.*

Then afterwards being at *Sea*, and defiring to know the *Longitude* of the Place where you are; that is, How many degrees the Meridian of that Place is more Eafterly or Wefterly, than the Meridian of that Place where you did fet the *Watches*; you muft obferve by the *Sun* or *Stars*, what Time of the Day it is, as precifely as is poffible, and Note at the fame time, to what Hour,

Minutes,

Minutes and Seconds the *Watches* do point (which Time, if the *Watches* be not ſet to the Right Meaſure, is by the known Daily Difference to be adjuſted,) adding thereunto the *Æquation* of the preſent Day, which gives you the time of the Day, ſhewed by the Sun, at the Place where the *Watches* were ſet: And, if this time of the Day be the ſame with that obſerved where you are, then you are under the ſame Meridian with the Place where the *Watches* were Set by the *Sun*; but, if the time of the Day, obſerved where you are, be greater than that ſhewed by the *Watches*, you may be aſſured, that you are come under a more Eaſterly Meridian; and if leſs you are come under a more Weſterly. And counting for every hour of difference of Time, 15 Degrees of *Longitude*, and for every Minute, 15 Minutes, or $\frac{1}{4}$ of a Degree, you ſhall then know, how many Degrees, Minutes, &c. the ſaid *Miridians* do differ from one another. *E. g.* Suppoſe the *Watches* A, B, C, were Set at the Place, whence you parted, on the 20*th* of *February*, to the time of Day obſerved by the *Sun*, abating the *Æquation* of the 20*th* of *February*, (*viz.* 2′ 28″.) and ſuppoſe that the *Watch* A, be ſet to its Right Meaſure, but that B goes every Day 7″ too Slow, and C every Day 12″ too faſt: Some Days after, ſuppoſe the 5*th* of *May*, deſiring to know the *Longitude* of the Place where you are at *Sea*.

You obſerve the Time of the Day there to be	05ʰ	18′	10″
And you find the *Watch* A to point at	02	06	00
But the *Watch* B to point at	1	57	22
Going too ſlow by 7″ every Day, which makes in 74 Days (*viz.* From the 20*th* of *Feb.* to the 5*th* of *May*,)	00	08	38
Which being added to its own Time, gives the ſame with that of the *Watch* A, *viz.*	02	06	00
You find alſo the *Watch* C to point at	02	20	48
Going 12″ too faſt every Day, which makes in 74 Days	00	14	48
Which being Subducted from its own Time, gives again	02	06	00
The Time of the Day therefore by the *Watches* being	02	06	00
Add thereunto the *Æquation* of the 5*th* of *May*	00	19	29
And ſo you have for the Time of Day at the Place where the *Watches* were ſet	02	25	29
But the Time obſerved being	05	18	10
Exceeds this by	02	52	41
Wherefore the *Meridian* of the Place, where you are *May* 5. is more Eaſterly, than the place where the *Watches* were ſet by	02	52	40
Which being reduced to Degrees, reckoning 15 Degrees for an Hour, comes to	43°	10′	15″

'Tis true, that from the ſame Reckoning it may be concluded, that you are 180 Degrees more Eaſterly; which happens, becauſe the *Hour Index* goes round in the ſpace of 12 Hours in the *Watches*; but the Difference is ſo great, that one cannot be deceived in it; elſe the *Watch* might be ſo made, that the *Index* ſhall go round about once in 24 Hours.

To find the Time of the Day at Sea.

6. Since that for finding the *Longitude*, the Time of the Day, at the Place where you are, muſt be known, (as hath been ſaid above you muſt have a

care to obſerve that Time as preciſely as poſſible. For every Minute of Time, that you miſreckon, makes ¼ Degree in *Longitude*, which amounts, near the *Æquator*, to above 15 *Engliſh Miles*, but leſs elſewhere. Wherefore to find the Time of the Day with Certainty, the beſt way is to obſerve the *Sun's Altitude* when it is in the Eaſt or Weſt, (the nearer the better :) for, being there, its *Altitude* changes in a ſhort time more ſenſibly than before or after ; and thus from the Height of the Pole and the Declination of the Sun, the Hour may be calculated.

7. At the *Riſing* and Setting of the *Sun*, when it is half above the *Horizon*, mark the time of the Day, which the *Watches* then ſhew ; and tho' ye have in the mean time ſailed on, it is not conſiderable. Then reckon by the *Watches* what Time is elapſed between them, and add the half thereof to the Time of the *Riſing*, and you ſhall have the Time by the *Watches*, when the *Sun* was at *South*, to which is to be added the *Æquation* of the preſent Day by the Table. And, if this together makes 12 Hours, then was the Ship at Noon under the ſame *Meridian* where the *Watches* were ſet with the *Sun*. But, if the Sum be more than 12, then was ſhe at Noon under a more *Weſterly Meridian* ; and, if leſs, then under a more *Eaſterly* ; and that by as many Times 15 *Deg*. as that Sum exceeds or comes ſhort Hours of 12 ; as the Calculation thereof hath been already deliver'd. *An Eaſier Way.*

Suppoſe, *e.g.* that the *Watches* A and B, as before, were ſet with the *Sun* at the Place whence you parted, the 20 of *February* ; and the *Indexes* ſet to the Hour, Min. and Sec. ſhewed by the *Sun*, abating the *Æquation* of that Day, *viz.* 2′ 20″ ; the *Watch* A being reduced to the right Meaſure, and B going too ſlow by 7″ a day. Afterwards on the 22*d* of *May* deſiring to know the *Longitude* of the Place to which you are come, you obſerve in the

Morning the *Sun* half above the Horizon, when the *Watch* points at	2^h	30′	10″
And in the *Evening*, the *Sun* being half under the *Horizon*, when the ſame *Watch* points at ——— ——— ———	3	8	40
To find the Time elapſed between them, ſubducting the Time of the *Riſing* ——— ——— ——— ——— ———	2	30	10
From ——— ——— ——— ——— ——— ——— ———	12	0	0
There Remains ———	9	29	50
Adding thereunto the Time of the *Setting* ——— ———	3	8	40
You have for the Time elapſed between the Obſervations —	12	38	30
Whereof the Half ——— ——— ——— ——— ———	6	19	15
Being added to the Time of *Riſing* ——— ——— ———	2	30	10
You have the Time by the *Watch* A, when ☉ was in the *South*	8	49	25
And after the ſame manner you are to ſeek the Time by the *Watch* B, when the *Sun* was in the *South* ; which let be —	8	38	48
But this *Watch* going 7″ a day too ſlow, it is retarded in 19 days (from the 20*th* of *Febr.* to the 22*d* of *May*) ——— ———	0	10	37
Which therefore added to the ſaid Time gives ——— —	8	49	25
That is the ſame Time given by the *Watch* A. Now adding to this Time of the *Watches*, the *Æquation* of the 22*d* of *May*	0	18	10
You have ——— ——— — — — — — —	9	7	35

Which

Which is the ſame time of the Day with that of the Place, where the *Watches* were ſet when the *Sun* was in the ſame *Meridian* with the Ship, or where the Ship was at Noon.

	h	′	″
The Difference is ——— ——— ——— ——— ———	2	52	25

	°	′	″
Wherefore this laſt *Meridian* is by ſo much more *Eaſterly* than the firſt, which being reduced to Degrees, as (as hath been formerly directed) make ——— ——— ——— ——— ———	43	6	15

'Tis manifeſt, that by this way you find preciſely enough the *Longitude* of the Place, where you were at Noon, or the Time of the Sun's being in the South: Which altho' it differs from the *Longitude* of the Place, where you are when you obſerve the *Setting* of the *Sun*; yet you may eſtimate near enough, how much you have advanc'd or chang'd the *Longitude* in theſe few Hours, by the *Log-Line*, or other ordinary Practices of *Reckoning* the Ship's Way; or (which is the ſurer Way) by the Degrees paſſed in 24 Hours by a former day's Obſervation.

You may alſo, inſtead of obſerving the *Sun's Riſing* and *Setting*, obſerve the *Setting* firſt, and then the next Morning the *Riſing*; marking at both Times the Time ſhew'd by the *Watches*; and find thence, after the ſame manner as before, the *Longitude* of the Place where the Ship was at Midnight.

Finally, You may alſo, inſtead of the *Riſing* and *Setting* of the *Sun*, obſerve before and after Noon two *Equal Altitudes* of the *Sun*, noting the Time ſhown by the *Watches*, and reckoning in the ſame manner, as hath been ſaid of the *Riſing* and *Setting:* Yet it is to be conſider'd, that the *Altitudes* of the *Sun* are beſt taken, when it is about Eaſt and Weſt, as hath been already intimated. But note, that in Sailing North and South you make not the Obſervations at the Sun's *Riſing* and *Setting*, but at its being due *Eaſt* and *Weſt*.

8. But you may put the Rule here preſcribed in Practice, by taking two *Equal Altitudes* of ſome known *Star*, that riſeth high above the Horizon. For you ſhall thence, according to the mentioned Rule, know at what time by the *Watches* the Star hath been in the South; and ſo the *Right Aſcenſion* of that Star being known, as alſo the *Right Aſcenſion* of the *Sun*, you may thence eaſily calculate, what Time it then was: Which, being compared with the Time of the *Watches*, as before, ſhall give the *Longitude* of the Place where you were, when you had the Star in the Meridian.

9. If the *Watches* that have gone exactly for a while, ſhould come to differ from one another (as in length of time it may well happen, that the one or other fail a Minute, more or leſs;) in that caſe it will be beſt to reckon by that which goes faſteſt; unleſs you perceive an apparent Cauſe, why it goes too faſt (as it may happen when the *Cheeks* retain not their proper Figures) ſeeing it is not ſo eaſy for theſe *Pendulum-Watches* to move faſter than at firſt, as it is to go ſlower. For the *Wire*, on which the *Pendulum* hangs, may perhaps by the violent Agitation of the Ship, come to ſtretch a little, but it cannot grow ſhorter; and the little weight of the *Pendulum* may perhaps ſlip downwards, but cannot get up higher.

If it ſhould be ſaid, that upon any Foulneſs the *Watch* will go faſter by reaſon of the ſhorter Vibrations of the *Pendulum*, it is to be conſidered, that this is only True when the *Watches* have no *Cheeks*, but when they have them 'tis not ſo. n. 48. p. 976.

10. When you get Sight of any known Country, Iſland or Coaſt, be ſure to note the *Longitude* thereof as exactly as you can by the help of the Rules here preſcribed. Firſt, thereby to correct the *Sea-Maps*, after the *Longitude* of a Place ſhall have been found at divers times to be the ſame, ſo that you doubt no more of it. For all *Maps* are very defective as to the Situation of Places in reſpect of Eaſt and Weſt, chiefly where Seas are interpoſed. Secondly, to be able always to know in the Proſecution of your Journey, how far you have ſailed from any Place to the Eaſt or Weſt. And, if by any notable Miſchance or Careleſſneſs all the *Watches* ſhould come to ſtand ſtill, yet you may at any Place, whereof the *Longitude* is certainly known, ſet them a going again, and adjuſt them there by the *Sun*, and ſo reckon the *Longitudes* from that ſame Meridian. For you are to know, that you are not at all obliged to put one certain Meridian of any known Place as a Beginning of the *Longitude Reckoning*; this happening only in *Maps*, or Tables of *Longitude*: As when you take for that purpoſe the *Meridian* of the *Pico* in *Teneriffe*, or that of the Iſlands of *Corvo* and *Flores* (the moſt Weſterly of the *Azores*) or any others. Yet it were very fit, that all *Geographers* agreed and pitched upon one and the ſame *Firſt Meridian*, that ſo all Places might be known by the ſame Degrees as well of *Longitude* as *Latitude*; tho' in Voyaging, it is ſufficient to obſerve only the Difference of *Longitudes*, beginning to reckon from the *Meridian* of any Place, you pleaſe, as if it were the *Firſt*. n. 47. p. 951.

11. If it happen that being at *Sea* all the *Watches* ſtop, you muſt as ſpeedily as is poſſible, ſet them a moving again, that you may know how much you advance from that Place towards the *Eaſt* or *Weſt*.

The Journal for the Watches.

12. The *Watches* being diſtinguiſhed by Marks, as A, B, or the like, every Day about Noon, or when moſt conveniently you can, obſerve the Time of the Day by the *Sun*, or by the *Stars* at Night, and ſubduct thence the Minutes and Seconds, that are adjoined to that Day in the Table, and write the Remainder down in the Paper, wherein 9 Columns or more are marked, placing them in the ſecond Column, having placed the Day of the Month in the Firſt; and at the ſame time write down the Hours, Minutes, and Seconds, of each *Watch* in a diſtinct Column, all oppoſite one to another. Then in another Column write down the Difference between the Time taken by Obſervation, and that given by the *Watches*, or one of them. Then one Column for the *Latitude*: One for the *Longitude* by the Ordinary Way of *Reckoning*: Another for the *Longitude* taken from the Difference between the Time found by Obſervation, and that given by the *Watches*: And at laſt a large Column to note the Accidents, that befal the *Watches*, &c.

The Succeſs of Pendulum Watches; by Maj. Holmes. n. 1. p. 13.

2. Major *Holmes* having left the Coaſt of *Guinea*, and being come to the Iſle of St. *Thomas* under the *Line*, he adjuſted his *Watches* there, and put to Sea, and Sailed Weſtward, 7 or 800 Leagues, without Changing his Courſe; after

after which finding the Wind favourable, he steered towards the Coast of *Africk* N. N. E. But, having sailed upon that Line, a matter of two or three hundred Leagues, the Masters of the other Ships under his Conduct, apprehending that they should want Water, before they could reach that Coast, did propose to him to steer their Course to *Barbadoes*, to supply themselves with Water there. Whereupon the said *Major*, having called the Masters and Pilots together, and caus'd them to produce their Journals and Calculations, it was found that those Pilots did differ in their Reckoning from that of the *Major*, one of them about 80 Leagues, another about a hundred, and the third more; but the *Major*, judging by his *Pendulum Watches* that they were only some thirty Leagues distant from the Isle of *Fuego*, which is one of the Isles of *Cape Verde*, and that they might reach it next day, and having a great Confidence in the said *Watches*, resolved to steer their Course thither; and having given Order so to do, they got the very next Day about Noon a Sight of the said Isle of *Fuego*, finding themselves to sail directly upon it, and so arrived that Afternoon as he had said.

M. Huygens being informed of this Success, wrote to *Paris* to this effect; Ib. *p.* 14. I did not magine that the *Watches* of this First Structure would succeed so well, and I had reserved my main Hopes for the New ones. But, seeing that those have already served so successfully, and that the other are yet more just and exact, I have the more Reason to believe, that the Invention of *Longitudes* will come to its Perfection. In the mean time I shall tell you, that the States did receive my Proposition, when I desired of them a Patent for these New *Watches*, and the Recompence set apart for the Invention, in Case of Success; and that without any difficulty they have granted my Request, commanding me to bring one of these *Watches* into their Assembly, to explicate unto them the Invention, and the Application thereof to the *Longitudes*; which I have done to their Contentment.

Longitudes from the Moon's Places; by Math. Professor *at* Seville. n. 118. *p.* 427.

III. 1. I have at last found a Method of knowing the *Moon's Place*, by the Assistance only of a small Instrument, within a Scruple or two; and which is wonderful, neither the *Refractions* nor the *Parallaxes* are any hindrance to my Observations, because the Contrivance of the Method delivers me from these Niceties. I can use this Method either at Sea or Land; therefore the *Tables* of the *Moon* being corrected, a Method will no longer be wanting of taking the so much desired *Longitudes* of Places, wherever they are, either at Sea or Land,

Considered by Mr. Flamsteed. Ib. *p.* 431.

2. What the Professor of *Seville* writes about his little Instrument, with his leave I must own exceeds my belief. In our *Horizon* the *Moon* cannot be clear'd either of *Refraction* or *Parallax*, unless she could ascend as high as the *Zenith*. For the *Refractions* are extended so far, and make her Place troublesome to find. Once only in a Day the *Parallax* of *Longitude* ceases in the Nonagesim Degree of the *Ecliptic*, but not so that of *Latitude*. Nor can I comprehend how an Instrument shall be constructed, which together with the *Parallax* takes notice of the *Refraction*, the Increase of which is owing to very different Principles.

IV. The

IV. The Obſervation of *Lunar Occultations* is of Singular Uſe to determine the *Longitude* of Places, eſpecially thoſe that are far remote. Longitudes *from Lunar Occultations; by* M. Halley. n. 181. p. 87.

V. The *Revolution* of *Jupiter* upon his *Axis* being the ſwifteſt, and the moſt Regular Motion that is hitherto known in the Heavens, a Traveller alone, even without having any Correſpondence with other Obſervers, may make uſe of it to find the *Longitudes* of the moſt remote Places of the Earth. Longitudes *by the Revolution of* Jupiter *upon his Axis; by M.* Caſſini. n. 82. p. 4042.

VI. 1. I am ſomething in doubt whether the *Eclipſes* of *Jupiter*'s Satellites are equally convenient for inveſtigating the Difference of *Meridians*, as the Occultations of the fixt Stars by the *Moon*. Particularly becauſe of the too ſlow Motion of the *Jovials*, tho' the Obſervations were perform'd with a more accurate Tube. Longitudes *by the Satellite Eclipſes; by M.* Hevelius. n. 78. p. 3030.

2. The *Eclipſes* of the *Satellites* of *Jupiter*, which happen almoſt every Day, afford a fair Way for eſtabliſhing the *Longitudes* over all the Earth. For, beſides that theſe *Eclipſes* are very frequent, the *Emerſion* and *Immerſion* of theſe *Satellites*, eſpecially in the Shadow of *Jupiter*, is ſo Momentary and ſo Senſible, that they may be obſerved with the greateſt Exactneſs, being altogether exempt from thoſe Eſſential Inconveniences that accompany the *Eclipſes* of the *Sun* and *Moon*, which alſo are rare, and whoſe *Beginning* and *End* are always doubtful by reaſon of a certain *Ambiguous Light*. *By M.* Borelli. n. 128 p. 691.

The *Longitude* of Places at *Sea*, *Capes*, *Promontories*, and divers *Iſlands*, being once exactly known by this Means, would doubtleſs be of Great Help, and conſiderable Uſe to *Navigation*.

3. The *Eclipſes* of *Jupiter*'s *Satellites* have been eſteemed, and certainly are a much better Expedient for the Diſcovery of the *Longitude* than any yet known, by reaſon that they happen frequently, and are eaſily obſervable with a Teleſcope of 12 Foot, or for need with one of eight. *By Mr.* Flamſteed *n.* 151. *p.* 322.

The *Longitude* might be alſo attained by Obſervations of the *Moon*, if we had Tables that would anſwer her Motions exactly; but after 2000 Years Experience (for we have ſome Obſervations of *Eclipſes* much Ancienter) we find the beſt Tables extant erring ſometimes 12 Minutes or more in her apparent Place, which would cauſe a Fault of half an Hour, or $7\frac{1}{2}$ *deg.* in the *Longitude* deduced, by comparing her *Place* in the Heavens with that given by the *Tables*. I undervalue not this Method, for I have made it my Buſineſs, and have ſucceeded in it, to get a large Stock of good *Lunar Obſervations* in order to the Correction of Her *Theory*, and as a Ground-Work for better *Tables*; but, if we ſhould happily attain what we ſeek, yet the Calculation will be ſo perplexed and tedious, that it will be found much more inconvenient and difficult than that I propoſe by obſerving the *Eclipſes* of *Jupiter*'s *Satellites*, which however at preſent I muſt prefer. For I am perſuaded, that the *Eclipſes* of the *Firſt* will ſcarcely be found above 4 Minutes of Time different from my Calculations, and I hope it will ſcarce ever be found to err ſo much. But, if the ſame *Eclipſe* may be obſerved in two diſtant n. 154. p. 404. n. 165. p. 760.

diſtant Places at the ſame time, or compared with an Obſervation of the ſame *Satellite*, made within a Week elſewhere, the *Difference* of *Meridians* will be had ſomething better, than by comparing two Obſervations of the ſame *Phaſis* of a *Lunar Eclipſe*, made in diſtant Places. For, whereas it is ſomewhat difficult by reaſon of the *Penumbra* to determine the True Time of the Application of either of the *Moon*'s Limbs to the Shadow, the *Satellite Eclipſes*, eſpecially thoſe of the *Firſt*, are almoſt Momentary.

And, whereas there can rarely happen 4 *Eclipſes* of the *Moon* Viſible, the ſame Year, thoſe of the *Satellites* happen ſo frequently that there are more of them Viſible in one Year than we count Days in it, tho' the Planet *Jupiter* lie hid under the *Sun*'s Rays every Year a whole Month together.

I know our *Navigators* will object againſt this Method, that it is difficult to practiſe at *Sea*, becauſe long Teleſcopes are required, which the Motion of the *Ship* will not permit them to manage aboard. But, if it be not practicable at *Sea*, they cannot deny but that it is at *Land*; and that the *True Longitude* of remote Coaſts from us is the firſt thing deſired for the Correction of their Charts; (Let them attempt theſe firſt, and I doubt not but the Succeſs will encourage them ſo much, that they will readily find means to put it in Practice at *Sea*;) That the *French* have uſed this Method ſucceſsfully both in *Denmark* and in their *Own Country*; That a Teleſcope of 14 Foot Long at moſt, or for need one of 8 Foot, with broad *Eye Glaſſes*, will be ſufficient for this purpoſe; That the Difficulty cannot be known till it be tried, and that Uſe renders may things eaſy, which our firſt Thoughts conceived impracticable.

If it be required to know whether any of thoſe *Eclipſes* which are inviſible with us, be viſible in any other given Place, convert the *Difference* of *Meridians* betwixt it and *London* into Time; and, if the Place lie to the *Eaſt* of *London*, add it to, if to the *Weſt*, ſubtract it from, the Time of the Appearance at *London*; the Sum or Difference accordingly ſhall be the true Time of the *Eclipſe* under that *Meridian*, at which, if *Jupiter* be above the Horizon, and the *Sun* beneath it, the *Eclipſe* is there viſible, otherwiſe not.

Or, By the help of the *Ephemerides* of the *Planet*'s Places, and a *Terreſtrial Globe*, the ſpace on it, in which any of theſe *Eclipſes* will be viſible, may be found thus:

Firſt ſeek the true Places of the *Sun* and *Jupiter* with his *Latitude* in the *Ephemerides*, whereby you may find their Declinations and Right Aſcenſions, either by the Vulgar Tables, or the Globe itſelf, exactly enough for this Method.

Bring *London* on the *Globe* to the *Meridian*, and, detaining it there, note what *Deg*. of the *Æquator* is cut by it. From this ſubtract the Time of the *Eclipſe* after Noon converted into *deg*. and *min*. the Remainder ſhews you the *Longitude* of that *Meridian* on the Earth, where it is then Noon when the *Satellite* is *Eclipſed*, which I therefore call the *Meridional Longitude* of the *Eclipſe*. Bring this *Meridional Longitude* under the *Meridian*, and elevate the nearer Pole to the *Sun* as much as is his *Declination*: keep the

2 *Globe*

Globe in this Poſition, and, if *Jupiter* be in *Conſequence* of the *Sun*, draw a Line of the *Globe* along the *Eaſtern Horizon*, it paſſes over all thoſe Places where the *Sun* is Setting at that Time; but, if *Jupiter* be in *Antecedence* of the *Sun*, draw the ſaid Line on the *Globe* by the *Weſtern* Edge of the *Horizon*, it paſſes over all thoſe Places where the *Sun* is then Riſing. *Jupiter* being in *Conſequence* of the *Sun*, add the Difference of *His* and the *Sun*'s Right Aſcenſions to the *Meridional Longitude* afore-mentioned, bring the *Deg.* of the *Æquator* anſwering their Sum under the Meridian, raiſe the *Pole* next *Jupiter* equal to his Declination, and, detaining the *Globe* in this Poſition, draw a Line again to the *Eaſtern Horizon*; the Space intercepted betwixt this and the Line of the *Sun*'s Setting, before deſcribed on the *Globe*, comprehends all thoſe Places on the Earth where this *Eclipſe* is ſeen from *Sun* Setting till *Jupiter* is Set. But, if *Jupiter* were in *Antecedence* of the *Sun*, Subtract the difference of his and the *Sun*'s Right Aſcenſions from the *Meridional Longitude*, ſet the *Degree* of the Æquator anſwering the Remainder under the *Meridian*, and elevate the *Pole* next *Jupiter* equal to his Declination: Keeping the *Globe* in this Poſition, draw a Line by the *Weſtern* Edge of the *Horizon*, the Space included betwixt this and the Line of the *Sun*'s Riſing contains all thoſe places on the Earth, where this *Eclipſe* is viſible betwixt *Jupiter*'s *Riſing* and *Sun-riſe*.

When any *Eclipſe* of theſe is obſerved, the Difference betwixt the noted Time and that given by the *Tables* ſhall be the *Difference* of *Meridians* betwixt the Place of the Obſervation and *London*.

As the *Sun* removes from the Conjunction of *Jupiter*, the *Ingreſſes* of the *Satellites* into his Shadow become obſervable. When he is about 30° from it, the *Emerſions* of the *Fourth*, and at 60°, of the *Third*, began to be ſeen betwixt the Shadow and Body, continuing ſo till the *Sun* be arrived within 60° of the Oppoſition of *Jupiter*, when the *Emerſions* of the *Third* fall behind his Body, but the *Emerſions* of the *Fourth* continue viſible till he be leſs than 30° diſtant from ☍: at which Time they alſo are hid behind him, all the Appearances being made really to the Right Hand, or in Antecedence of *Jupiter*, tho' with inverting Teleſcopes, they appear on the contrary, to the Left.

After the Oppoſition of the *Sun* and *Jupiter* we begin to ſee the *Immerſions* of the *Satellites* from the Shadow now on the Left Hand, or in Conſequence of *Jupiter*, but thro' inverting Glaſſes on the Right; when the *Sun* is near 30 *deg.* from the Oppoſition, the *Ingreſſes* of the *Fourth*, when 60 *deg.* from it, of the *Third*, begin to be obſervable betwixt the Body and Shadow, continuing ſo till the *Sun* arrive at the ſame or rather within ſomething a wider Diſtance for the Conjunction of *Jupiter*.

4. By this very eaſy Method, to be perform'd with a very little Apparatus of Inſtruments, the Foundations for reſtoring *Geography* may be laid. The Precepts leading to this cannot be unknown to *Aſtronomers*. It may be proper to take notice of one thing, that with a Tube of 7 or 8 feet, which is eaſily portable, the Moments of theſe *Eclipſes* may be obſerved diſtinctly enough, particularly in the exterior *Satellites*; eſpecially if the Aperture of the Object-glaſs be $2\frac{1}{2}$ or 3 Inches. For thus the greateſt Quantity of the

By Mr. Halley, n. 121. *p.* 435.

the refracted Rays will come to the Eye, whence these very little Stars near *Jupiter* may be seen, which otherwise would be extinguish'd by his greater Light. And tho' they may be tinged with Colours, and the Limb of *Jupiter* may seem not very bright, yet since we are only concern'd about the Moment of losing or of recovering the Light, it is sufficient that they strike our Eyes surely, increased in their Light as much as may be.

n. 214. p. 237. The *Eclipses* of the *First Satellite* of *Jupiter* are found by the *Royal Academy* at *Paris*, in Ascertaining the Geographical Site of the Principal Ports of *France*, almost *Instantaneous*, and with good Telescopes discernible almost to the very Opposition of *Jupiter* to the *Sun*. So that, could the *Satellites* be observed with Telescopes manageable on *Shipboard*, a *Ship* at *Sea* might be enabled to find the Meridian she was in to a very great exactness, beyond what we can yet hope to do by the *Moon*, tho' they seem to afford us the only Means practicable for the *Seamen*. However, before *Sailors* can make use of the Art of *Finding* the *Longitude*, it will be requisite that the *Coast of the whole Ocean* be first laid down truly; for which Work this Method by the *Satellites* is most apposite.

Long. and Lat. of Derby; *by Mr.* Flamsteed. n. 55. p. 1103, 1106. n. 111. p. 237.

VII. The *Longitude* of *Derby* from *London* W. is 5 or 6 *min.* the *Latitude* 52° 57′ or 59′.

Lat. of Ecton. n. 76. p. 2272.

VIII. The *Latitude* of *Ecton* in the County of *Northampton* is 52° 15′.

Long. and Lat. of Townley. n. 127. p. 664.

The observed *Latitude* of *Townley* in the County of *Lancaster*, (as Mr. *Townley* writes,) is 53° 44′. and its *Longitude* from the Meridian of *London*, is about 9 Minutes to the West.

Lat. of Tredagh. n. 164. p. 749.

X. The *Latitude* of *Tredagh* in *Ireland* is 53° 40′.

Long. of Oxford *and* Dantzick; *by Mr.* Halley. n. 129. p. 724. *vid sup. Cap.* IV. § LXII.

XI. Having carefully considered the *Parallaxes* of the *Moon* in the Observations of the Occultations of *Mars*, *Aug.* 21. 1676 at *Dantzick* and *Greenwich*, I find from the *Immersion* the *difference* of *Meridians* between *Greenwich* and *Oxford* 4′ 57″, between *Greenwich* and *Dantzick* 1^h 14′ 50″: By the *Emersion* the *first* of these *Differences* is found 4′ 59″, the latter 1^h 14′ 41″; which near Agreement shews the exactness of the Observations.

Long. of Paris; *by M.* Cassini. n. 117. p. 390. *vid. sup. Cap.* V. XLV.

XII. 1. I took much Pleasure in comparing the Observations of Mr. *Flamsteed*, about the Eclipse of the *Moon*, *July* 7. New Style 1675. with ours made in the *Royal Observatory*. For from them I see confirm'd within a few Seconds, by ten new Comparisons, the *Difference* of *Meridians* which formerly I had defined to be 11 Minutes, by a Comparison of our Observations.

Obser-

Mr. *Flamsteed*'s Observations.	h. ′ ″	Our Observations.	h. ′ ″	Diff. of Meridians.	′ ″
Pentadactylus cover'd	1 55 15	The same, or *Seleucus*	2 06 15		11 00
Porphyrites cover'd	2 02 20	The same, or *Aristarchus*	2 12 40		10 20
The first Limb of *Sinai*	2 05 30	Of the same, or of *Tycho*	2 16 30		11 00
		or *	16 25		10 55
The first Limb of *Ætna*	2 06 00	Of the same, or of *Copernicus*	2 16 30		10 30
		or *	16 40		10 40
The first Limb of *Besbicus*	2 23 05	Of the same, or of *Manilius*	2 34 15	Less than	11 05
Horminius cover'd	2 26 03	To the same, or *Dionysius*	2 36 15	Greater than	10 12
It touch'd the first Limb of *Corocondometes*	2 39 30	Of the same, or *Palus Somni*	2 50 20		10 50
It touch'd *Palus Mæotis*	2 45 00	The same, or *Mare Caspium*	2 55 20		10 20
		or *	55 40		10 40
Mæotis wholly cover'd	2 50 40	The same	3 01 10		10 30
Immersion	2 56 55	*Immersion*	3 07 45		10 50
		or *	3 07 40		10 45

This Mark * denotes the particular Determination of Mr. *Cassini*; in the rest he agreed with Mr. *Pichard* and Mr. *Romer*.

2. The Middle of the Lunar *Eclipse*, *Jan.* 1. New Style, 1675. is derived from a Comparison of the Beginning and End — h. ′ ″ 3 20 00 n. 123. p. 562.

Of two equal *Phases* — — — — 3 20 15 vid. sup. Cap. IV. §. XLVI.

From the Observations of Mr. *Flamsteed* the Middle of the *Eclipse* may be derived in a like manner. For he at 2^h 29′ 30″, observed the Distance of the Cuspids to be 17′ 16″. and at 3^h 52′ 45″, the *Eclipse* decreasing he observed the same Distance 18′ 57″, that is, 1′ 41″. greater. Therefore the Middle of the *Eclipse* is nearer the latter Observation than the former. The middle Time between both the Observations was 3^h 11″ 7″. Therefore the Middle of the *Eclipse* is derived from hence something later. Whence the Difference of *Meridians* would come out less than 9′; which but little agrees with more certain Observations of the preceding Summer *Eclipse*, from whence I determin'd it to be $10\frac{3}{4}$ Minutes. Our first Observation compared with the former of Mr. *Flamsteed*, which was a little later, exhibits the Difference of *Meridians* greater than 8′ 35″. Our latter, being later than Mr. *Flamsteed*'s latter Observation, would exhibit the Difference of *Meridians* less than 9′ 40″.

	h. ′ ″
The End as estimated by Mr. *Flamsteed*	4 07 15
And by us	4 15 25
Would infer the Difference of *Meridians* to be	0 08 10
The Beginning observed by Dr. *Halley* at *London*	2 16 00
With that observed by us	2 24 35
Would make the Difference of *Meridians* to be	0 08 35

Therefore from this *Eclipse* the Difference of *Meridians* would come out less by about two Minutes than by the *Eclipse* of the foregoing Summer, which yet I prefer far before this. Not only in respect of the greater Facility of determining the Times of the Appulses and Emersions in in that total *Eclipſe* than in this partial one; but alſo becauſe of the clearneſs of the Air, which we enjoy'd alike in that *Eclipſe*, whereas in this the Air was very ſerene at *Paris*, but at *London* it was cloudy. Therefore I judge we ought to ſtand by the former, till we can determine the Matter with greater exactneſs, by Obſervations of the *Immerſions* and *Emerſions* of the *Satellites* of *Jupiter*, which I think better fitted for this purpoſe.

Ib. p. 564.

By Mr. Flamſteed. *Ib. p.* 565.

3. I can hardly truſt to the Difference of *Meridians*, derived from the *Eclipſe* of the *Moon* of *June* 27. 1675. obſerved at *London* and at *Paris*; for tho' I believe the Times of the *Phaſes* obſerved by you to be very accurately determin'd; yet as I was obliged to make uſe of a Quadrant of only 20 Inches, becauſe a larger was not at hand, which had only plain Sights, in in order to correct my Clock; therefore I can hardly be ſure of the Moment of any Phaſis, nearer than to a Minute. I was better furniſhed when I obſerved the laſt *Eclipſe* of *Dec.* 22. 1675. but as the Air was ſomething cloudy, and becauſe of the oblique Incidence of the *Moon* into the *Earth*'s Shadow, its Appulſe to the Spots was very ſlow, and thence this *Eclipſe* was not ſo proper for this purpoſe. Therefore the Difference of our *Meridians* ſtill remains uncertain within two Minutes, which I do not queſtion but we ſhall ſometime determine to our Wiſhes.

Long. of Straſburg *and* Paris; *by* M. Bullialdus. n. 125. p. 610. *vid. ſup. Cap.* IV. § XLVI.

XIII. The Meridian of *Paris* is diſtant from that of *Straſburg* 22′ 48″. from the End of the *Lunar Eclipſe* of *Jan.* 1. New Style 1676. And by this *Eclipſe* the *Meridian* of *Paris* is diſtant from that of *London* 6′ 38″ Eaſterly. From the Obſervation of the *Eclipſe* of *July* 7. 1675. this appear'd to be 10′, as alſo from the *Eclipſe* of *Jan.* 11. of the ſame Year.

Long. of Avignon; *by Mr.* Halley. *Ph. Col.* n. 5. p. 126.

XIV. *Avignon* is 19′ 40″, or 4° 50′ to the Eaſtward of *London*.

Long. & Lat. of ſeveral Places in France n. 163. p. 718, 719, 720. *vid. ſup. Cap.* IV. § XXXIV.

XV. *M. Caſſini* having compared together the Obſervations of the *Solar Eclipſes* of *July* 12. *ſt. n.* 1684. and made ſuch Reductions as the Parallax requires, lays down the *Longitudes* from *Paris* to

Aix in *Provence* 14′ E. The *Lat.* by M. *Gautier* is 43° 30′.
Avignon 8 ½′ E.
Lyons 8′, or 13′ E.
Roſes 4′ E. The *Lat.* by M. *Chaſſeles* 42° 10′.
Honfleur 7′ W.
Pau 11′. W. The *Lat.* by *P. Richaud* 43° 30′.

Long. of Lisbon n. 146. p. 151. *vid. ſup. Cap.* IV. §. XLIX.

XVI. Mr. *Jacobs* an *Engliſh* Merchant reſiding at *Lisbon*, informed Mr. *Flamſteed*, that he obſerved the *Beginning* of the *Lunar Eclipſe*, Feb. 11. 168½ there at 8h 31′, *p. m.* which gives the *Difference* of the *Meridians* betwixt the *Obſervatory* at *Greenwich* and *Lisbon*, 41½ *Minutes* of Time, or 10° 22′, conſiderably different from our *Maps* and *Sea-Charts*.

Lat. of Madrid; *by* E. *of* Sandwich. n. 22. p. 390.

XVII. The *Earl* of *Sandwich* eſteem'd, by the *Sun*'s Altitude in the *Solſtice*, and by other Meridian Altitudes, the *Latitude* of *Madrid* to be 40° 10′; which differs conſiderably from that aſſigned by others, the General Chart of *Europe* giving to it 41° 30′, the General Map of *Spain* 40° 27′, and a large Provincial Map of *Caſtile* 40° 38′.

XVIII. 1. By many concurring Obſervations of the *Moon* and other Planets, I have found that the Diſtance of the City of *Seville* from *Uraniburg*, as to *Longitude*, is 90′ or 1½ Hour, the Diſagreement being within 2′. *Long. of* Seville *and* Uraniburg; *by* .... Math. Profeſſor *at* Seville. n. 118. p. 427. *by Mr.* Flamſteed. *ib.* p. 431. *vid ſup. Cap.* IV. §. XLIV.

2. The Profeſſor ſhould conſider again, how he can make the Diſtance of the *Meridians* of *Seville* and *Uraniburg* to be 90′. For the Obſervations of the Eclipſe of the *Moon*, *Jan.* $\frac{1}{11}$, 1675. make the Middle at *London* 7ʰ 11′½ after Noon; to which the Obſervations at *Paris* agree. The Obſervations of the ſaid *Profeſſor* fix the Middle at *Seville* at 6ʰ 47′. Therefore the Difference of our *Meridians* is 24′½. But between us and *Uraniburg* there are only 52′. Therefore the Difference of *Meridians* between *Seville* and *Uraniburg* can be only 1ʰ 16′½. I am afraid theſe Obſervations of the *Profeſſor* are made only with his naked Eye; for the Times of *Incidence* and *Emerſion* make 1ʰ 5′. whereas our Obſervations, thoſe of *Paris*, and of *Hevelius*, do not make thoſe Times greater than 1ʰ 1′½. or perhaps ſomething leſs.

XIX. The *Difference* of *Meridians* of *Copenhagen* and *Paris* by the Obſervations of *Jupiter*'s *Satellites*, is found by Mr. *Picard* to be 0ʰ 41′ 40″. *Long. of* Copenhagen; *by M.* Picard. n. 146. p. 145.

XX. *An.* 1680. *Oct.* 23. *St. v. S. Joſ. Ponthia*, and *Marco Antonio Cellio*, with a Teleſcope of 25 Palms, obſerved the Total *Immerſion* of the *firſt Satellite* in *Jupiter*'s Shadow at *Rome*, at 10ʰ 7′ 53″; *p. m.* which in our *Obſervatory* here I noted at 9ʰ 15′ 41″, whoſe Difference is the *Difference* of our *Meridians* = 52′ 12″, or 13° 03′. Again, *Jan.* 28. 1685. S. *Francis Blanchini* obſerved the Total *Immerſion* of the *Firſt* at *Rome*, at 11ʰ 19′¾ which I ſaw not here, but my Numbers give at 10ʰ 27′¼: Therefore the *Difference* of *Meridians* is 52′½, and *Rome* lies ſo much more *Eaſterly* than the *Obſervatory* at *Greenwich*; agreeing with the former Obſervation. *Long. of* Rome *and* Uraniburg; *by Mr.* Flamſteed. n. 177. p. 1215.

The Noble *Tycho* judged therefore not much amiſs, when he placed *Uraniburg* and *Rome* under the ſame *Meridian*; for by ſeveral Obſervations of *Satellite Eclipſes* it is evident, that the *Difference* of *Meridians* betwixt *Uraniburg* and our *Obſervatory* is 51′ 10″ of Time; ſo that *Rome* lies only one *Minute* of Time, or ¼ of a *Deg.* to the *Eaſt* of *Uraniburg*.

XXI. 1. *Dantzick* is by many and undoubted Obſervations proved to be 1ʰ 15′ 30″, more *Eaſterly* than *London*. *Long. of* Dantzick; *by Mr.* Halley. *Ph. Col.* n. 5. p. 124.

2. *An.* 1683. On the very Day of the *Summer Solſtice*, *Jun.* 21. New Style, at *Dantzick*, the *Sun*'s Altitude at Noon was 59° 7′. by a certain ſmall Braſs Quadrant, but very exact. But on the Day of the *Autumnal Equinox* the *Sun*'s Altitude at Noon was found 35° 27′. *Lat. of* Dantzick; *by M.* Hevelius. n. 151. p. 330. n. 154. p. 424.

XXII. The *Longitude* of *Nuremburg* has been formerly ſtated 11° from *London*, and ſince found to be ſo by Obſervations of the *Eclipſe* of the *Sun* *July* 2. 1684. which made it 44′½ of Time. *Long. of* Nuremburg; *by* ..... n. 182. p. 147.

Long. of Mosco, Leipsick and Aleppo; by . . . . n. 192. p. 453. vid. sup. Cap. V. §. LIII.

XXIII. The *Duration* of the *Lunar Eclipse*, *Apr.* 5. 1688. is made by M. *Timmerman* from 7^h 38′ about 10^h 45′, which agrees within 8 or 10 *Minutes* with our *Tables*, that never err ſenſibly in the Continuance of *Eclipſes*; and ſo much ought to be allowed to an Obſerver not ſufficiently inſtructed to diſtinguiſh the *Penumbra* from the true Shadow, tho' a ſmall *Teleſcope* were uſed in this Obſervation. Let us conclude then, that the *End* was at 10^h 40′ at *Moſco*. We do not find that this *Eclipſe* was obſerved at *London*: However this defect is in good part ſupplied by an Obſervation thereof made at *Leipſick*, by M. *Gotfrid Kirck*, and publiſhed in his *Ephemerides* for the Year 1689; where the *End* is determined at 8^h 54′ *p. m.* Hence *Moſco* will be 1^h 46′ to the *Eaſtward* of *Leipſick*; and the *Difference* of *Meridians* between *London* and *Leipſick* being already determin'd 49′, it will follow that *Moſco* is 2^h 35′ to the *Eaſt* of *London*, or 38° 45′ of Longitude, which from other Accounts we find to be very near that of the City of *Aleppo* in *Syria*.

n. 181. p. 86. it is 52 Min.

Lat. of ſeveral Places in Ruſſia. *ib. p. 454.*

By the ſame hand we have procured the Latitudes of the following Places, obſerved, as 'tis ſaid, with a large Quadrant.

Moſco	55°	34′
Tereſlaw	57	44
Wologda	59	19
Woſtak	61	15
Arch-Angel	64	30

Latitudes of ſome remarkable Places; *by Mr.* Francis Vernon. n. 124. p. 582. Jan. An. 1675.

XXIV. I have been as curious as I could in taking the Latitudes of ſome remarkable Places: As I find them, I ſhall give them you.

Athens	38°	05′	*Patras*	38°	40′
Corinth	38	14	*Delphos*	38	50
Sparta	37	10	*Thebes*	38	22
Corone	37	02	*Negropont* or *Chalcis*	38	31

Latitudes of Conſtantinople *and* Rhodes; *directed to* A. B. Uſher; *by Mr.* Greaves. n. 178. p. 1295. Dec. An. 1685.

XXV. Upon Intimation of your *Grace*'s Deſires, and upon Importunity of ſome Learned Men, having finiſhed a Table, as a Key to your *Grace*'s exquiſite Diſquiſition, touching *Aſia properly ſo called*; I thought myſelf obliged to give both you and them a Reaſon, why, in the ſituation of *Byzantium* and the Iſland *Rhodes*, (which two eminent Places I have made the παραπέγματα and Bounds of the *Chart*) I diſſent from the Traditions of the *Antients*, and from the Tables of our *late* and beſt *Geographers*; and conſequently, Diſſenting in theſe, have been neceſſitated to alter the Latitudes (if not Longitudes) of moſt of the remarkable Cities of this diſcourſe. And firſt for *Byzantium*, the received Latitude of it by *Appianus*, *Mercator*, *Ortelius*, *Maginus*, and ſome others, is 43° 5′. And this alſo we find in the *Baſil Edition* of *Ptolemy*'s *Geography*, procured by *Eraſmus* out of a Greek MS. of *Pettichius*. The ſame likewiſe is confirmed by another choice MS. in Greek, of the moſt Learned and Judicious Mr. *Selden*, to whom for this Favour and

and ſeveral others I ſtand obliged. And as much is expreſſed in the late *Edition* of *Ptolemy* by *Bertius*, compared and corrected by *Sylburgius*, with a Manuſcript out of the *Palatine Library*. Wherefore it cannot be doubted, having ſuch a croud of Witneſſes, but that *Ptolemy* aſſigned to *Byzantium*, as our beſt *Modern Geographers* have done, the Latitude of 43° 5′. And this will farther appear, not only out of his *Geography*, where it is often expreſſed, but alſo out of his Μεγάλη Σύταξις, or *Almageſt* as the *Arabians* term it, where deſcribing the Parallel paſſing διὰ Βυζαντίȣ, he aſſigns to it 43° 5′. What was the Opinion, concerning *Byzantium*, of *Strabo* preceding *Ptolemy*, or of *Hipparchus* preceding *Strabo*, or of *Eratoſthenes* antienter, and it may be accurater than all of them, (for *Strabo Lib.* 2. calls him τελευ]αῖον πραγματευσάμενον περὶ τῆς γεωγραφίας) tho' *Tully* (*Lib. Ep. ad Att.*) makes *Hipparchus* often reprehend *Erateſthenes*, as *Ptolemy* after him doth *Marinus*, their Writings not being now extant, (unleſs thoſe of *Strabo*) cannot be determined by us. But as for *Strabo*, in our Inquiry we can expect little Satisfaction; for his Deſcription of Places having more of the *Hiſtorian* and *Philoſopher*, (both which he hath performed with ſingular Gravity and Judgment) than the Exactneſs of a *Mathematician*, who ſtrictly reſpects the Poſition of Places, without Inquiſition after their Nature, Qualities and Inhabitants, (tho' the beſt *Geography* would be a Mixture of them all, as *Abulfeda*, an *Arabian Prince* in his *Rectification of Countries* above 300 Years ſince hath done;) I ſay, for theſe Reaſons we can expect little Satisfaction from *Strabo*, and leſs may we hope for from *Dionyſius Afer*, *Arrianus*, *Stephanus Byzantius*, and others. Wherefore, next having recourſe to the *Arabians*, who in *Geography* deſerved the ſecond Place after the *Græcians*, I find in *Naſſir Eddin* the Latitude of *Byzantium*, which he terms *Buzantiya*, *Conſtantiniya*, to be 45°, and in *Uleg. Beg*'s *Aſtronomical Tables* the ſame to be expreſſed. *Abulfeda* chiefly follows four Principal Authors as his Guides, in the compiling of his *Geographical Tables*; thoſe are, *Alfaras*, *Albiruny*, *Hon Saiid Almagraby*, laſtly *Ptolemy*, whoſe *Geography* he terms a *Deſcription of the Quadrant*, (or the fourth Part of the Earth) inhabited; and all theſe, according to his Aſſertion, place *Byzantium* in 45° of Latitude. And here it may juſtly be wondred, how this Difference ſhould ariſe between the *Greek Copies* of *Ptolemy*, and thoſe tranſlated into *Arabick* by the Command of *Almamon*, the Learned *Caliph* of *Babylon*; for *Abulfeda* expreſly relates, that *Ptolemy* was firſt interpreted in his Time, that is, in the Computation of *Almecinus* in *Erpenius*'s *Edition*, and of *Emir Cond* a *Perſian Hiſtoriographer*, more than 800 Years ſince: Concerning which *Abulfeda* writes thus, *This Book* (diſcourſing of *Ptolemy*'s *Geography*) *was tranſlated out of the Grecian Language into the Arabick for Almamon:* And in this I find (by three fair MSS of *Abulfeda*) *Byzantium* to be conſtantly placed in 45°. and as conſtantly in the *Greek Copies* in 43° 5′. But in the πρόχειροι Κάνονες of *Chryſococca*, out of the *Perſian Tables*, made about the Year 1346. in *Scaliger*'s Calculation, it is placed in 45°. To reconcile the Difference between the *Greeks* and *Arabians* may ſeem impoſſible, for the common Refuge of flying to the Corruption of Numbers by Tranſcribers, and laying the Fault on them which ſometimes is the Author's, will

2 not

not help us in this particular; ſeeing the *Greek Copies* agree among themſelves, and the *Arabick Copies* amongſt themſelves. The beſt way to end the Diſpute will be, to give Credit concerning the *Latitude* of *Byzantium*, neither to the *Greeks* nor *Arabians*. And that I have reaſon for this Aſſertion, appears by ſeveral Obſervations of mine at *Conſtantinople*, with a Braſs Sextant of above 4 Foot *Radius*. Where taking, in the *Summer Solſtice*, the Meridian Altitude of the *Sun* without uſing any προσταφαίρεσις for the Parallax and Refraction, (which at that time was not neceſſary,) I found the *Latitude* to be 41° 6′. And in this *Latitude* in the *Chart* I have placed *Byzantium*, and not in that either of the *Greeks* or *Arabians*. From which Obſervation, being of ſingular Uſe in the *Rectification* of *Geography*, it will follow by way of Corollary, that all *Maps* for the *North Eaſt* of *Europe*, and of *Aſia*, adjoining upon the *Boſphorus Thracius*, the *Pontus Euxinus*, and much farther, are to be corrected; and conſequently the Situation of moſt Cities in *Aſia, properly ſo called*, are to be brought more Southerly than thoſe of *Ptolemy* by almoſt two entire Degrees, and than thoſe of the *Arabians* by almoſt four.

Concerning *Rhodes*, it may be preſumed, that, having been the Mother and Nurſe of ſo many eminent Mathematicians, and having long flouriſhed in Navigation, by the Direction of theſe, and by the Vicinity of the *Phœnicians*, they could not be ignorant of the preciſe *Latitude* of their Country, and that from them *Ptolemy* might receive a true Information. Tho' it cannot be denied, but that *Ptolemy* in Places remoter from *Alexandria* hath much erred. I ſhall only inſtance in our own Country, where he ſituates Λονδίνιον, that is *London*, in 54° of *Latitude*; and the τὸ μέσον or the middle of the *Iſle* of *Wight*, (which in the printed Copies is falſely termed Οὐίκλεσις, but in the MSS. rightly Οὐηκτὶς,) in 52° and 20′ of *Latitude*. Whereas *London* is certainly known to have for the Altitude of the *Pole*, or *Latitude* of the Place, only 51° and 32′, and the Middle of the *Iſle* of *Wight* not to exceed 50° and ſome Minutes.

But in my Judgment *Ptolemy* is very excuſable in theſe and the like Errors, of ſeveral other Places far diſtant from *Alexandria*; ſeeing he muſt for their Poſition neceſſarily have depended either upon Relations of *Travellers*, or Obſervations of *Mariners*, or upon the *Longitude* of the *Day* meaſured in thoſe times by *Clepſydræ*: All which how uncertain they are, and ſubject unto Error, if ſome Celeſtial Obſervations be not joined with them, and thoſe exactly taken with large Inſtruments, (in which kind the *Antients* had not many, and *Our Times* (excepting *Tycho Brahe*, and ſome of the *Arabians*) but a few,) I ſay no Man, that hath converſed with *Modern Travellers* and *Navigators*, can be ignorant. Wherefore to excuſe thoſe Errors of his (or rather of others fathered by him) with a greater Abſurdity, by aſſerting the *Poles* of the *World* ſince his time to have Changed their ſite, and conſequently all Countries their *Latitudes*, as *Mariana* the Maſter of *Copernicus*, and others after him have imagined: Or elſe to charge *Ptolemy*, being ſo excellent an Artiſt, with Ignorance, and that even of his own Country, as *Cluverius* hath done, (from which my Obſervations at *Alexandria*, and

Memphis, may vindicate him,) the former were too great a Stupidity, and the latter too great a Preſumption. But to return to *Rhodes*, an Iſland (in *Euſtathius*'s *Comment* upon *Dionyſius*'s Περιήγησις) of 920 Furlongs Circuit, where according to *Ptolemy* the Parallel paſſing διὰ Ῥόδȣ hath 36° of *Latitude*, and ſo hath *Lindos*, and Ἰηλυσσός, the Chief Cities of the Iſland; the ſame is confirmed by the MS. but where the printed Copy and *Euſtathius* read Ἰηλυσσός, which *Mercator* renders *Talyſſus*, the MS. renders Ἰλισός. *Abulfeda* in ſome Copies ſituates the Iſland *Rhodes*, (for he mentions no Cities there) in the *Latitude* of 37 *Deg.* and 40 *Min.* and the *Geography* of *Said Ibn Aly Algiorgany*, commended by *Gilbertus Gaulmyn*, in 37°, if it be not by a Tranſpoſition in the MS. of the Numerical Letters in *Arabick*, 37 for 36. which by reaſon of their Similitude are often confounded in *Arabick* MSS. By my Obſervations under the Walls of the City *Rhodes*, with a fair Braſs *Aſtrolable* of *Gemma Friſius*, containing 14 Inches in the Diameter, I found the *Latitude* to be 37° and 50′. A larger Inſtrument I durſt not adventure to carry on Shore in a Place of ſo much Jealouſy. And this *Latitude* in the Chart I have aſſigned to the City *Rhodes*, (from the Iſland ſo denominated, upon which on the North Eaſt ſide it ſtands ſituated) better agreeing with the *Arabians* than with *Ptolemy*, whom I know not how to excuſe.

XXVI. In the Second Book of the *Voyage de Siam des Peres Jeſuites*, are related two Obſervations of the *Satellites* of *Jupiter*, capable, if well made, to aſcertain the *Longitude of the Cape of good Hope*. The firſt was there made *June* 2d. *ſt. n.* 1685, when at 11h 29′ 20″, the *Firſt* or innermoſt *Satellite* touched the *Weſtern Edge* of *Jupiter*, and at 11h 30′ 50″ it appeared no more: This Obſervation is ſaid to be made with an excellent Teleſcope of 12 Foot. The other was on *June* the 4*th* following *ſt. n.* when the *Emerſion* of the ſame *Satellite* was obſerved at 9h 37′ 40″, from which latter is concluded, that the *Longitude* of the *Cape* is 18° to the *Eaſt* of *Paris*; for that the ſaid *Emerſion*, according to the *Calculus* of *Caſſini*, in the *Meridian* of *Paris*, ought to have happen'd at 8h 26′. This ſame *Emerſion* is computed by Mr. *Flamſteed*, at 8h 19′, at *London*, that is 3 *min.* later than by *S. Caſſini*; and conſidering that neither is verified by Obſervation in *Europe*, the *Longitude* hence deduced is doubtful at leaſt 3 *min.* if this had been the only Obſervation. But the former being conſidered will yet ſhew that there is a much greater Doubt ſtill remaining: For from certain Aſtronomical Principles the *Parallax* of the *Orb*, or Difference between the Place of *Jupiter* ſeen from the *Sun* and *Earth* was, at the Time of the firſt Obſervation, 9° 9′; which Arch that *Satellite* moves in 1^h 6′. and the utmoſt Duration of an *Eclipſe* hereof in this Poſition of *Jupiter* being ſcarce 2^h 20′ (as appears by the accurate Obſervations of M. *Caſſini* and M. *Flamſteed*) it will follow, that from the *Immerſion* behind *Jupiter*'s Weſtern Edge to the *Emerſion* out of the Shadow, there could not be full 3^h 26′. Wherefore the *Emerſion* out of the Shadow, on *June* 2*d*, ought according to the time of *Immerſion*, to be at 14^h 56′, at the lateſt at the *Cape*; which by

Long. and Lat. Cape of good Hope. n. 185. p. 253.

Mr.

Mr. *Flamsteed*'s *Calculus* was at *London* 13^h 51′. or according to *S. Cassini* at 13^h 58′ at *Paris*. Hence the Longitude of the Cape will be found but 14 *deg.* and a half at most to the *East* of *Paris*; so that these 2 Observations will differ in the Result about a quarter of an Hour, which is a little too much. However there are some Reasons that seem to argue for this latter Longitude rather than the former; for it is much easier to observe what becomes of a luminous Object that appears, than to wait upon the first Appearance of a Star eclipsed: And it is probable that the *Satellite* might in the latter Time be several Minutes *emerged* out of the Shadow, when they might first perceive it; but they could not but see the Application to the Body of *Jupiter* in the Former, if we may suppose their Telescopes so good as they are said to be. And that the *Cape of good Hope* is not more than an Hour to the *East* of *Paris*, is proved by the constant Consent of our *Navigators*, who find by their Reckonings that the Island of St. *Helena* is about 22 or 23 *deg.* of *Longitude* to the *Westward* of the *Cape*: (and that Sailing both backwards and forwards, 'tis the same, which takes away the Objection of Currents) Now by accurate Observations made at St. *Helena*, and compared with others made in *Europe* at the same time, the *Longitude* of that Isle is certainly about 8 $\frac{1}{2}$ *deg.* to the *West* of *Paris*; it follows therefore that the *Cape* cannot be much more than 14 or 15 *deg.* to the *East* of *Paris*; and undoubtedly it must be less than 18°; for 3 *deg.* is much too great an Error to be committed in so short a Distance of Sailing.

Long of St. Helena. Ib.

The Long. of Madagascar; by Mr. Flamsteed. n. 143. p. 15. *vid. sup. Cap.* IV. § XLVIII.

XXVII. Mr. *Thomas Heathcot* was Chirurgeon to a Ship, which, *Aug.* 19. 1681. lay at the bottom of a deep Bay on the Western Shore of *Madagascar*, and that part which the *Portuguese* and our *Maps* call the *Terra del Gada*; He had then with him on Shore, a Quadrant of two Foot Radius, and a Telescope of 9 Foot, but no Clock; to supply which Defect, he made a *Pendulum* of a String and a Bullet 39 Inches long, that each single Vibration might answer a Second of Time. Waiting the *Beginning* of the *Eclipse* with his Glass, as soon as he saw the true Shadow enter on the *Moon*'s Limb, he caused his Friends who assisted him, to make the Pendulum vibrate, and count its Vibrations; of which they had numbred 140 = 2′ 20″ of Time, when he took the height of *Procyon* (then *East* of the Meridian) 25° 39′. The next day he observed the *Sun*'s Meridional Height with the same Quadrant, whence he found the Latitude of the Place 19° 29′ *South*: hence the time when he took the height of *Procyon* is found 4^h 51′ *mane*, and subtracting the 2′ 20″, past since the observed *Beginning* of the *Eclipse*, its

true *Beginning* was at ————————————	4^h	48′	40″
Which at the *Observatory* here I noted at ———————	1	50	40
Therefore this Part of *Madagascar* is more *Easterly* ———	2	58	00

or 44° 30′, which our *Maps* make 52°; that is 7 $\frac{1}{2}$ *deg.* more remote from it than it really is.

The Long. and Lat. of Balla.

XXVIII. Taking the Observations of the *Occultation* of the *Bull's Eye*, *Oct.* 28. 1680. under the Examination of a *Calculus*, I find that at 8 6′ or or the *Immersion* at *London*, the true Place of the *Moon* correct by *Parallex* was ♊ 4° 32′ 24″; but at 16^h 00′, at *Ballasore Road* (in the *Lat.* of 21° 23′, N. and about 20 Miles E. S. E. from the Town) the true place of the *Moon* was ♊ 5° 54′, that is 1° 21′ 36″, more than at 8^h 6′, at *London*: Now according to the Moon's Velocity at that Time, she passed an Arch of 1° 21′ 36″, in 2^h 8′ 40″, of time; so then at 10^h 14′ 40″, at *London*, the *Moon* was in the same Place as at 16^h 00′, at *Ballasore Road*; whence the Difference of Longitude will be 5^h 45′ 20″, or 86° 20′. *Ballasore* being so much to the *Eastward* of *London*.

fore in India; *by Mr.* Edm. Halley, *Ph. Col.* n. 5. p. 124. *vid. sup. Cap.* IV. § LXVI. Feb. An. 1682.

2. By the Calculation of the *Immersion* of the *Bull's Eye*, *Dec.* 1680. I find that at the 14^h 49′, at *Ballasore* the *Moon*'s true Place was ♊ 6° 30′ 30″, and at 7^h 46′ 12″, the correct Time of the *Immersion* at *Dantzick*, the true Place was ♊ 4° 55′ 11″; that is 1° 35′ 20″, short of the Place deduced from the Observation at *Ballasore Road*; which make in Time 2^h 32″ 40″. Whence it follows, that 10^h 18′ 52″, at *Dantzick* make 14^h 49′, at *Ballasore Road*, and the Difference of Longitude 4^h 30′ 8″; and *Dantzick* being 1^h 15′ 30″, more *Easterly* than *London*, *Ballasore Road* will be from *London* 5^h 45′ 38″, or 86° 24′. and the same Difference of *Meridians* will be found 86° 14′, if you make use of the *Immersion* at *Dantzick*.

Vid. sup. Cap. IV. § LXVII.

3. For farther Confirmation hereof, Mr. *Benj. Harry*, being ashore at *Ballasore* Town, observed with very great Care and Exactness, *Nov.* 18, 1680. that at 9^h 13′, the Star, which *Tycho* calls, *in Cotyla dextra* Aquarii *duarum præcedens*, (and which was then in *Aquarius* 28° 52′, and *Lat.* 2° 46′, *N.*) was in a right Line with the *Cusps* of the *Moon*, then near the first Quarter. The Star's place is confirmed by the Agreement of *Hevelius*'s Observations with those of *Tycho*; and the *Theory* of the *Moon* cannot be considerably faulty in that Part of the *Orb*, it falling precisely on her greatest *Equation*: wherefore by the Theory and Numbers of *Horrox*, the true Place of the *Moon* at 2^h 53′ at *London*, is found ♒ 29° 22′ 10″. but at 9^h 13′, at *Ballasore*, her Place was in ♒ 29° 41′ 17″; that is, 19′ 7″ more than at *London*, which in Time gives 36′; so that 3^h 29′ at *London*, was 9^h 13′, at *Ballasore*, and the Difference of Long. 5^h 44′ or 86° 00′ precisely; which the *Dutch Maps* make full out 99°: And the *French Maps* of *Sanson*, pretending to correct them, have made them 5° worse, and the Error 18° completely. What then is to be thought of the Descriptions of those Places which have been but seldom visited?

XXIX. I have deduced 7^h 23′. for the Difference of Longitudes between *Canton* and *Paris*, from the Exit of *Mercury* out of the *Sun*'s Disk observed at *Canton* and *Nurimburg*; and from the *Eclipses* of the *Moon* observed at *Nurimburg* and *Paris*.

Long. of Canton; *by M.* Cassini. n. 5. p. 371. *Vid. sup. Cap.* IV. § XCVIII.

XXX. From the greatest Meridian Altitude of the Polar Star, observed by the Fathers of the Society of *Jesus*, *Dec.* 31. 1694. with a correct Instrument, which was 42° 16′ 50″. the Refraction being supposed to be 1′ 17″, and the Distance of the Pole *Star* from the *Pole* at that time was 2° 19′ 57″, whence the *Altitude* of the *Pole* is 39° 54′ 56″.

Lat. and Long. of Pekin; *by M.* Ja. Cassini. n. 237. p. 53.

From the leaſt Meridian Altitude of the ſaid *Polar Star*, obſerved on the 7, 8, and 13 Days of *May*, 1695. with a correct Inſtrument to be 37° 36′ 40″. ſuppoſing the Refraction to be 1′ 28″. and the Diſtance of the *Polar Star* from the *Pole* to be 2° 19″ 50″. The *Altitude* of the *Pole* is found 39° 55′ 2″.

	°	′	″
Neglecting the Refraction, the greateſt Altitude of the *Pole-Star* derived from the Obſervation of *Dec.* 31. aforegoing, was at the Beginning of *May*	42	16	43
And the leaſt Altitude of the ſame was then	37	36	40
Therefore the Difference of Altitudes	04	40	03
And the Diſtance of the *Polar Star* from the *Pole*	02	20	01½
And the apparent Altitude of the *Pole*	39	56	41½
To this apparent Altitude the Refraction by my Table is	00	01	10
Wherefore the Altitude of the *Pole* at *Peking* is	39	55	31½

For the Longitude of the City of *Peking*, the Immerſion of the firſt of *Jupiter*'s *Satellites* into the Shadow of *Jupiter* was obſerved *Jan.* 18. 1695. at 12^h 51′ 14″.

Our Tables on that Day repreſent this *Immerſion* to have been at 5^h 18′ 49″. Now the Obſervations made in the ſame Month in the *Royal Obſervatory* at *Paris* ſhew that the Tables were retarded at that time 2′ 30″.

Therefore that *Immerſion* was at *Paris* at 5^h 16′ 19″. Therefore the Difference of Meridians between the Cities of *Paris* and *Peking* will be 7^h 34′ 55″.

But as from other Obſervations the ſame Difference of Meridians has been formerly found to be 7^h 36′. we may aſſume it 7^h 35′½.

Lat. of St. Salvadore. n. 105. p. 91.

XXXI. St. *Salvadore* in *Braſil* is in the *Southern Latitude* 12° 47′.

Lat. of Bridge-Town. n. 189. p. 370.

XXXII. *Bridge-Town* in *Barbadoes* is in the *Northern Latitude* of 12° 58′.

A Deſcription of Nova Zembla; *by* M. Nich. Witſen. n. 101. p. 3. *Fig.* 205. Mar. An. 1674

XXXIII. 1. I herewith ſend you what I have received out of *Muſcovy*, which is a *New Map* of *Nova Zembla* and *Weigats*, as it hath been diſcover'd by the expreſs Order of the *Czar*; and drawn by a Painter, call'd *Panelapoetſki*, who ſent it me from *Moſco* for a Preſent; by which it appears, that *Nova Zembla* is not an *Iſland*, as hitherto it hath been believed to be; and that the *Mare Glaciale* is not a Sea, but a *Sinus*, or Bay, the Waters whereof are freſh: Which is the ſame with what the *Tartars* do alſo aſſure us, who have taſted theſe very Waters in the midſt of the *Sinus*. The *Samojeds*, as well as the *Tartars*, do unanimouſly affirm, that paſſing on the back of *Nova Zembla*, at a conſiderable Diſtance from the Shore, *Navigators* may well paſs as far as *Japan*. And 'tis a great Fault in the *Engliſh* and *Dutch*, that, ſeeking to get to *Japan* on the South-ſide of *Nova Zembla*, they have almoſt always paſs'd the *Weigats*.

The Letter O in the great River *Oby* marks the Place of a Cataract or Fall of Waters. The Letter K denotes the Conjunction of *Zembla* with the *Continent*. The River marked L runs from *China*, called *Kitaie*; which is not every where Navigable, by reaſon of the *Rocks*, and other Inconveniences that obſtruct the paſſing of Veſſels. *Weigats* itſelf is very difficult to paſs, becauſe of the great Quantity of *Ice* continually falling into it out of the River *Oby*, whereby that ſtreight Paſſage is ſtopped up. The *Sa-*

mojeds go every year a Fiſhing upon the ſaid freſh Sea, and that on *Nova Zembla*'s ſide.

2. I formerly thought *Nova Zembla* had been a *Continent:* But I have ſince been better informed, and retracted that Error. And, whereas the late M. *Voſſius* would needs perſuade himſelf, as well as he did others, to their Ruin, that there was a Paſſage to *Japan* by the *North*, and that the *Tartarian* Countries behind *Nova Zembla* did decline immediately towards the South; I did always oppoſe it, and think I can even demonſtrate the Impoſſibility thereof. So that what he wrote, to encourage Mariners to that Attempt, was even directing them to the point of Death; as it afterwards enſued. 193. p. 494.

XXXIV. What is noted with a ſingle Line, is exactly copied from the *Map*, which *M. Sanſon*, one of the moſt illuſtrious *Geographers* of this Age preſented to the *Dauphin*, *An.* 1679. The Names of Cities, whoſe Situation is alſo taken from this *Map*, are written in *Italian* Characters; the Correction of the Poſition of Coaſts (which is deduced from the Obſervations which were made to that End) is marked with a Stroke a little ſhadow'd towards the *Sea*, as is commonly done; and the Names of Cities, whoſe Situation is corrected, are ſet down in *Roman* Characters.

A Map of France; *by M.* Picard; *and M.* De La Hire. n. 226. p. 443. *Fig.* 205. Mar. An. 1697.

The Degrees of *Latitude* are marked on both ſides of the Border, and the Degrees of *Longitude* in the ſame Border, above and below; but the Diviſion of them begins at the *Meridian* that paſſes thro' the *Obſervatory* at *Paris*, by going to *Eaſt* and *Weſt*, and not at the *Meridian* of the Iſle of *Fer*, as hath been eſtabliſhed, becauſe we do not exactly know the Situation of this Iſland in reſpect of the *Obſervatory*.

XXXV. 1. What *Arithmetick*, in whole Numbers and Fractions, as alſo in Decimals and Logarithms, is neceſſary for the ſame: And what Books are beſt for Teaching ſo much thereof. 2. What Vulgar, Practical, Mechanical *Geometry*, performable by the Scale and Compaſs, is ſufficient. 3. What *Trigonometry*, right Lined and Spherical, will ſuffice. 4. How many *Stars* are to be known. .5 What *Inſtruments* are beſt for Uſe at *Sea*, with the Conſtruction of them, and the Manner of uſing them. 6. The whole Skill of the *Magnet*, as to the directive Virtues thereof, and all the Accidents that may befall it. 7. The *Hydrography* of the *Globe* of the Earth, the *Perſpective* of the Coaſts, and the Deſcription of the under-water-bottom of the *Sea*. 8. The Knowledge of Winds and *Meteors*, ſo far as the ſame is attainable. 9. The Hiſtory and Skill of all Sorts of *Fiſhings*. 10. The Art of *Medicine* and *Chirurgery*, peculiar for the *Sea*. 11. The *Common Laws* of the *Admiralty*, and *Juriſdiction* of the *Sea*. 12. The ſeveral *Victuallings* and *Cloathings* fit for Seamen. 13. The whole Science of *Ebbing* and *Flowing*, as alſo of Currents and Eddies at Sea. 14. *Dromometry*, and the Meaſures of a Ship's Motions at Sea. 15. *The Building* of *Ships* of all Sorts, with the ſeveral Riggings and Sails for each Species, and the Uſe of all the Parts and Motions of a Ship. 16. *Naval Oeconomy*, according to the ſeveral

What a Compleat Treatiſe of Navigation *ſhould contain; by* Sir W. Petty. n. 198, p. 657. Mar. An. 1693.

Voyages and Countries. 17. The Art of *Conning*, *Rowing* and *Sailing*, of all the ſeveral ſorts of Veſſels. 18. The *Gunnery*, Fireworks and other Armatures peculiar to Sea and to Sea-Fights. 19. The Art of *Loading* and *Unloading* the Chief Commodities to the beſt Advantage. 20. The Art of *weighing ſunk Ships* and Goods, as alſo of *Diving* for ſunk Goods in deep Water. 21. The General *Philoſophy* of the *Motion* and Figures of the *Air*, the *Sea*, and of Seaſons; of *Timber*, *Iron*, *Hemp*, *Brimſtone*, *Tallow*, &c. And of their ſeveral Uſes in *Naval* Affairs. 22. An Account of 5 or 6 of the beſt *Navies* of *Europe*, with that of the *Arſenals*, *Magazines*, *Yards*, *Docks*, &c. 23. An account of all the *Shipping* able to croſs the *Seas* belonging to each Kingdom and State of *Europe*. 24. An account of all the chief *Commercial* Parts of the World; with mention of what *Commodities* are originally carried from, and ultimately to, any of them. 25. An account of the chief *Sea-Fights*, and all other *Naval Expeditions* and Exploits, relating to *War*, *Trade*, or *Diſcovery*, which have happened in this laſt Century. 26. Of the moſt advantageous Uſe of *Teleſcopes* for ſeveral Purpoſes at *Sea*. 27. Of the ſeveral *Depths* of the *Sea*, and *Heights* of the *Atmoſphere*. 28. The Art of making *Sea-water freſh* and *potable*, and fit for all Uſes in Food and Phyſick at Sea.

The Collection of Secants, and the true Diviſion of the Meridian *in the* Sea-Chart; *by Dr.* Wallis. n. 176. p. 1193. Fig. 206. Nov. An. 1685.

XXXVI. 1. Tho' it be well known, that, in the *Terreſtrial Globe*, all the *Meridians* meet at the *Pole*, (as E P, E P,) whereby the Parallels to the Equator, as they be near to the Pole, do continually decreaſe.

2. And hereby a degree of *Longitude* in ſuch Parallels is leſs than a degree of *Longitude* in the Equator, or a degree of *Latitude*:

3. And that in ſuch Proportion, as is the Co-Sine of *Latitude* (which is the Semidiameter of ſuch Parallel) to the Radius of the *Globe*, or of the Equator.

4. Yet hath it been thought fit (for ſome Reaſons) to repreſent theſe *Meridians*, in the *Sea-Chart*, by parallel ſtraight Lines; as E p, E p.

5. Whereby, each Parallel to the Equator (as L A) was repreſented in the *Sea-Chart*, (as l a) as equal to the Equator E E; and a Deg. of *Longitude* therein, as large as in the Equator.

6. By this means, each Degree of *Longitude* in ſuch *Parallels* was increaſed, beyond its juſt proportion, at ſuch rate as the Equator (or its Radius is greater than ſuch *Parallel*, or the Radius thereof.)

7. But, in the Old *Sea-Charts*, the degrees of *Latitude* were yet repreſented (as they are in themſelves) equal to each other, and to thoſe of the Equator.

8. Hereby, amongſt many other inconveniencies, (as Mr. *Ed. Wright* obſerves, in his *Correction of Errors* of *Navigation*, firſt publiſhed in the Year 1599) the Repreſentation of the Places remote from the Equator was ſo diſtorted in thoſe Charts, as that (for Inſtance) an *Iſland* in the *Latitude* of 60 degrees, (where the Radius of the *Parallel* is but half ſo great as that of the Equator) would have its Length (from *Eaſt* to *Weſt*) in Compariſon of Breadth (from *North* to *South*) repreſented in a double Proportion of what indeed it is.

9. For


Plate VI. Vol. 1. pag. 572.
ENGLAND
Dover
Calais
Dunkyrk
FLANDERS
PICARDY
Dieppe
Amiens
Cherbourg
Haure
Rouen
Caen
NORMANDY
PARIS
THE PARRALEL OF PARIS
S. Malo
M S^t. Michael
Brest
BRITANY
Nantes
POITOU
Rochelle
THE
GASCOGNE
SEA
Tour de Cordouan
Bordeaux
GASCOGNE
Bayonne
SPAIN
OF PARIS
THE MERIDIEN
Lion
LANGUEDOC
Avignon
Montpellier
Narbonne
PROVENCE
Aix
Marseille
Toulon
Antibe
Nice
ITALY
Nova Zembla
Sinus Dulcis
Waigats
Oby flu.
Fig. 205.
Fig. 204.

9. For rectifying this in some Measure, (and of some other Inconveniencies) Mr. *Wright* adviseth, that (the *Meridians* remaining Parallel, as before) the degrees of the *Latitude* remote from the Equator, should at each Parallel be protracted in like Proportion with those of *Longitude*.

10. That is; as the Co-Sine of *Latitude*, (which is the Semidiameter of the Parallel) to the Radius of the *Globe*, (which is that of the Equator:) So should be a degree of *Latitude* (which is every where equal to a degree of *Longitude* in the Equator,) to such degree of *Latitude* so protracted (at such Distance from the Equator;) and so to be represented in the Chart.

11. That is, every where, in such Proportion, as is the respective Secant (for such *Latitude*) to the Radius. For, as the Co-Sine, to the Radius; so is the Radius, to the Secant of the same Arch or Angle; or $\Sigma : R :: R : s$. Fig. 207.

12. So that (by this means) the Position of each Parallel in the *Chart* should be at such Distance from the Equator, compared with so many Equinoctial Degrees or Minutes, (as are those of *Latitude*) as are all the Secants (taken at equal Distances in the Arch) to so many times the Radius.

13. Which is equivalent (as Mr. *Wright* there notes) to a Projection of the Spherical Surface (supposing the Eye at the Centre) on the Concave Surface of a *Cylinder* erected at right Angles to the Plane of the Equator.

14. And the Division of *Meridians*, represented by the Surface of a *Cylinder* erected (on the Arch of *Latitude*) at Right Angles to the Plane of the *Meridian* (or a Portion thereof:) The Altitude of such Projection (or Portion of such Cylindrick Surface) being (at each Point of such Circular Base) equal to the Secant (of *Latitude*) answering to such Point. Fig. 208.

15. This Projection (or Portion of the Cylindrick Surface) if expanded into a Plane, will be the same with a plain Figure, whose Base is equal to a Quadrantal Arch extended (or a Portion thereof) on which (as Ordinates) are erected Perpendiculars equal to the Secants, answering to the respective Points of the Arch so extended: The least of which (answering to the Equinoctial) is equal to the Radius; and the rest continually increasing, till (at the *Pole*) it be infinite. Fig. 209.

16. So that, as E R S L, (a Figure of Secants erected at right Angles on E L, the Arch of *Latitude* extended) to E R R L, (a Rectangle on the same Base, whose Altitude E R is equal to the Radius;) so is E L (an Arch of the Equator equal to that of *Latitude*,) to the Distance of such Parallel, (in the *Chart*) from the Equator.

17. For finding this Distance, answering to each Degree and *Minute* of *Latitude*, Mr. *Wright* (as the most obvious way) adds all the Secants (as they are found calculated in the *Trigonometrical Canon*) from the Beginning to the Deg. or Min. of Latitude proposed.

18. The Sum of all which, except the greatest, (answering to the Figure inscribed) is too little: The Sum of all, except the least, (answering to the circumscribed) is too great; (which is that he follows:) And it would be nearer to the Truth than either, if (omitting all these) we take the Intermediates; for *Min.* $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, *&c.* or (the double of these) *Min.* 1, 3, 5, 7, *&c.* Which yet (because on the Convex-side of the Curve) would be somewhat too little.

19. But

19. But any of these ways are exact enough for the Use intended, as creating no sensible Difference in the *Chart*.

20. If we would be more exact; Mr. *Oughtred* directs, (and so had Mr. *Wright* done before him) to divide the Arch into Parts yet smaller than Minutes, and calculate Secants suiting thereunto.

21. Since the *Arithmetick* of *Infinites* introduced, and (in pursuance thereof) the Doctrine of *Infinite Series*, (for such Cases as would not, without them, come to a determinate Proportion;) Methods have been found for squaring some such Figures.

Fig. 207. 22. In order to a *Quadrature* for this Figure of Secants (by an infinite Series fitted thereunto) put we, for the *Radius* of a Circle, R; the Right Sine of an Arch or Angle, S; the versed Sine, V; the Co-Sine, (or Sine of the Complement) $\Sigma = R - V = \sqrt{Rq - Sq}$; the Secant, $\int$; the Tangent, T.

23. Then is $\Sigma : R :: R : \int$. That is, Σ) R^2 ($\int = \frac{R^2}{\Sigma}$; the Secant.

24. And $\Sigma : S :: R : T$. That is, Σ) SR ($T = \frac{SR}{\Sigma}$; the Tangent.

Fig. 210. 25. Now, if we suppose the Radius CP, divided into equal Parts, and each of them $= \frac{1}{00} R$;) and on these, to be erected the Co-Sines of Latitude L A.

26. Then are the Sines of Latitude in Arithmetical Progression.

27. And the Secants answering thereunto, $L\int = \frac{R^2}{\Sigma}$.

28. But these Secants, (answering to the Right Sines in Arthemetical Progression,) are not those that stand at equal Distances on the Quadrantal Arch extended. *Fig.* 209.

29. But, standing at unequal Distances (on the same extended Arch;) Namely, on those Points thereof, whose right Sines (whilst it was a Curve) are in Arithmetical Progression. As *Fig.* 211.

Fig. 211. 30. To find therefore the Magnitude of R E L $\int$, *Fig.* 209. which is the same with that of *Fig.* 211, (supposing E L of the same Length in both; however the Number of Secants therein may be unequal) we are to consider the Secants, tho' at unequal Distances, *Fig.* 211, to be the same with those of equal Distances, in *Fig.* 210. answering to Sines in Arithmetical Progression.

31. Now, these Intervals, (or Portions of the Base) in *Fig.* 211. are the same with the intercepted Arches, (or Portions of the Arch) in *Fig.* 210. For this Base is but that Arch extended.

32. And these Arches, (in Parts Infinitely small) are to be reputed equivalent to the Portions of their respective Tangents intercepted between the same Ordinates. As in *Fig.* 210, 212.

Fig. 212. 33. That is equivalent to the Portions of the Tangents of Latitude.

34. And these Portions of Tangents are to the equal Intervals in the Base, as the Tangent (of Latitude) to its Sine.

35. To

35. To find therefore the true Magnitude of the Parallelograms, or Segments of the Figure; we muſt either protract the Equal Segments of the Baſe, *Fig.* 210. (in ſuch Proportion as are the reſpective Tangents to the Sine) to make them Equal to thoſe of *Fig.* 211.

36. Or elſe, (which is equivalent) retaining the equal Intervals of *Fig.* 210, protract the Secants in the ſame Proportion. (For either way the Intercepted Rectangle, or Parallelograms will be equally increaſed) as L M. *Fig.* 212.

37. Namely, as the Sine (of *Latitude*) to its Tangents; ſo is the Secant, *Fig.* 212.
to a Fourth; which is to ſtand (on the Radius equally divided) inſtead of that Secant.

$$S : \frac{SR}{\Sigma} \; (: : \Sigma : R.) : : \frac{R^2 \, R^3}{\Sigma \; : \; \Sigma^2 = R^2 - S^2} = LM.$$

38. Which therefore are as the Ordinates in (what I call *Arith. Infin. Prop.* 104.) *Reciproca Secundanorum:* Suppoſing Σ^2 to be Squares in the Order of Secundans.

39. This (becauſe of $\Sigma^2 = R^2 - S^2$; and the Sines S, in Arithmetical Progreſſion) is Reduced (by Diviſion) into this *Infinite Series:*

$$R : + \frac{S^2}{R} + \frac{S^4}{R^3} + \frac{S^6}{R^5} \; \&c.$$

40. That is, (putting $R = 1$,) $1 + S^2 + S^4 + S^6$, &c.

41. Then, according to the *Arithmetick* of *Infinites*, we are to interpret S, ſucceſſively by 1 S, 2 S, 3 S, &c. till we come to S, the greateſt. Which therefore repreſents the Number of all.

And, becauſe the firſt Member doth repreſent a Series of Equals, the ſecond of Secundans; the third of Quartans, &c. Therefore the firſt Member is to be multiplied by S; the ſecond by $\frac{1}{2}$ S; the third by $\frac{1}{3}$ S; the fourth by $\frac{1}{7}$ S; &c.

43. Which makes the Aggregate, $S + \frac{1}{3} S^3 + \frac{1}{5} S^5 + \frac{1}{7} S^7 + \frac{1}{9} S^9$, &c. $=$ E C L M.

44. This, (becauſe S is always leſs than $R = 1$) may be ſo far continued, till ſome *Power* of S become ſo ſmall, as that it (and all which follow it) may be ſafely neglected.

45. Now, (to fit this to the *Sea-Chart*, according to Mr. *Wright*'s Deſign) having the propoſed Parallel (of *Latitude*) given; we are to find (by the *Trigonometrical Canon*) the Sine of ſuch *Latitude*; and take Equal to it, C L $=$ S. And by this find the Magnitude of E C L M, *Fig.* 212. that is, of REL *ſ*, *Fig.* 211, that is, of REL *ſ*, *Fig.* 209. And then, as RRLE (or ſo many times the Radius) to REL *ſ*, (the Aggregate of all the Secants;) ſo muſt be a like Arch of the Equator (equal to the Latitude propoſed,) to the Diſtance of ſuch Parallel, (repreſenting the Latitude in the *Chart*) from the Equator: Which is the Thing required.

46. The ſame may be obtained in like manner, by taking the verſed Sines in Arithmetical Progreſſion. For, if the right Sines (as here) beginning at the Equator, be in Arithmetical Progreſſion, as, 1, 2, 3, &c. Then will the

the verſed Sines, beginning at the *Pole*, (as being their Complements to the Radius) be ſo alſo.

47. The ſame may be applied in like Manner, (tho' that be not the preſent Buſineſs) to the Aggregate of *Tangents*, anſwering to the Arch divided into equal Parts.

48. For thoſe anſwering to the Radius ſo divided are $\frac{SR}{\Sigma}$; (taking S in Arithmetical Progreſſion.)

49. And then inlarging the Baſe (as in *Fig.* 211.) or the Tangent (as in *Fig.* 212.) in Proportion of the Tangent to the Sine;

$$S : \frac{SR}{\Sigma} \; (: : \Sigma : R) : : \frac{SR}{\Sigma} : \frac{SR^2}{\Sigma^2} = \frac{SR^2}{R^2 - S^2}.$$

50. We have by Diviſion this Series, $S + \frac{S^3}{R^2} + \frac{S^5}{R^4} + \frac{S^7}{R^6} + \frac{S^9}{R^8}$, &c.

51. That is, (putting $R=1$) $S+S^3+S^5+S^7+S^9$, &c.

52. Which (multiplying the reſpective Members by $\frac{1}{2}S$, $\frac{1}{4}S$, $\frac{1}{6}$, $\frac{1}{8}S$, $\frac{1}{10}S$, &c.) becomes $\frac{1}{2}S^2 + \frac{1}{4}S^4 + \frac{1}{6}S^6 + \frac{1}{8}S^8 + \frac{1}{10}S^{10}$, &c.

Which is the Aggregate of Tangents to the Arch whoſe right Sine is S.

53. And this Method may be a Pattern for the like *Proceſs* in other Caſes of like Nature.

Two Problems in Navigation, *propos'd; by Mr.* Nich. Mercator. n. 13. *p.* 215. Jun. An. 1666.

XXXVII. The Line of *Artificial Tangents*, or the *Logarithmical Tangent Line*, beginning at 45°, and taking every half Deg. for a whole one, is found to agree pretty near with the *Meridian-Line* of the *Sea-Chart*, they both growing, as it were, after the ſame proportion. But the Table of *Meridional* Degrees being calculated only to every ſexageſimal Minute of a Degree, ſhews ſome ſmall Difference from the *ſaid Logarithmical Tangent-line*. Hence it may be doubted, whether the Difference do not ariſe from that little Error which is committed by calculating the Table of *Meridional Degrees* only to every Minute.

But, if a certain Rule could be produced, by which the Agreement or Diſagreement of the ſaid two Lines might be ſhewn, the *Helix* or *Spiral* Line of the *Ship's Courſe* would be reduced to a more preciſe Exactneſs than ever was pretended by any.

The ſame Rule would alſo diſcover a far eaſier way of making *Logarithms*, than ever was practiſed or known; and therefore might ſerve, whenever there ſhould be Occaſion, to extend the *Logarithms* beyond the number of Places that are yet extant.

Moreover, ſuch a Rule would enable Men to draw the *Meridian Line* Geometrically, that is, without Tables or Scales; which indeed might alſo be done by ſetting of the *Secants* of every whole or half Degree, if there were not this Inconveniency in it, which is not in my Rule: That a Line compoſed of ſo many ſmall Parts, would be ſubject to many Errors, eſpecially in a ſmall Compaſs.

 The

The ſame Rule will alſo ſerve to find the Courſe and Diſtance between two Places aſſigned, as far as Practice ſhall require it; and that without any Table of *Meridional Parts*, and yet with as much Eaſe and Exactneſs.

And, ſeeing all theſe things do depend on the Solution of this Queſtion, *Whether the Artificial Tangent Line be the true Meridian Line*; it is therefore that I undertake, by God's Aſſiſtance, to reſolve the ſaid Queſtion. And, to let the World know the Readineſs and Confidence I have to make good this Undertaking, I am willing to lay a Wager againſt any one or more Perſons that have a Mind to engage, for ſo much as another Invention of mine (which is of leſs Subtilty, but of a far greater Benefit to the Publick) may be worth to the *Inventor*.

As for the great Advantage, that all *Merchants*, *Mariners*, and conſequently the *Common-Wealth*, may receive from this other *Invention*, it is, in my Judgment, highly valuable, ſeeing it will oftentimes make a *Ship* ſail, tho', according to the common way of *Sailing*, the Wind be quite contrary, and yet as near to the Place intended, as if the Wind had been favourable: Or, if you will, it will enable one to gain ſomething in the intended Way, whether the Wind be good or no, (except only when you go directly *South* or *North*) but the Advantage will be moſt where there is moſt need of it, that is, when the Wind is contrary: ſo that one may very often gain a fifth, fourth, third Part, or more of the intended Voyage, according as it is longer or ſhorter, *viz.* always more in a longer Voyage, where the Gain is more conſiderable, and more welcome, not only by ſaving Time, but alſo Victuals, Water, Fuel, Mens Health, and ſo much Room in the *Ship*.

The Analogy of Logarithmick Tangents to the Meridian Line *Demonſtrated; by Mr.* Edm. Halley. n. 219. *p.* 202. Feb. An. 1696.

XXXVIII. It was firſt diſcovered by chance, and, as far as I can learn, firſt publiſhed by Mr. *Henry Bond* as an Addition to *Norwood*'s *Epitome of Navigation*, about 50 Years ſince, that the *Meridian Line was analogous to a Scale of Logarithmick Tangents of half the Complements of the Latitudes.*

For the Demonſtration of that *Propoſition*, it is requiſite to premiſe theſe four *Lemmata*.

Lemma 1. *In the Stereographick Projection of the Sphere upon the Plane of the Æquinoctial, the Diſtances from the Centre*, which in this Caſe is the Pole, *are laid down by the Tangents of half thoſe Diſtances*; that is, of half the Complements of the Latitudes. This is evident from *Eucl.* 3. 20.

Lemma 2. *In the Stereographick Projection, the Angles, under which the Circles interſect each other, are in all Caſes equal to the Spherical Angles they repreſent*; which is a very valuable *Property* of this Projection.

Demonſt. Let EPBL be any great Circle of the *Sphere*, E the Eye placed in its Circumference; C its Center, P any Point thereof; and let FCO be ſuppoſed a Plane erected at Right Angles to the Circle EPBL, on which FCO we deſign the Sphere to be projected. Draw EP croſſing the Plane FCO in *p*, and *p* ſhall be the Point P projected. To the Point P draw the Tangent APG, and on any Point thereof, as A, erect a Perpendicular AD, at right Angles to the Plane EPBL, and draw the Lines PD, AC, DC; and the Angle APD ſhall be equal to the Spherical Angle contained between the *Fig.* 215.

Planes

Planes APC, DPC. Draw alſo AE, DE, interſecting the Plane FCO in the Points *a* and *d*; and join *a d*, *p d*: I ſay, the Triangle *a d p*, is ſimilar to the Triangle ADP; and the Angle *a p d* equal to the Angle APD. Draw PL, AK, Parallel to F O; and, by reaſon of the Parallels, *a p* will be to *a d*, as A K, to A D: But, (by *Eucl.* 3. 32.) in the Triangle AKP, the Angle AKP=LPE, is alſo equal to APK=EPG: wherefore the Sides AK, AP, are equal; and 'twill be, as *a p* to *a d*, ſo AP to AD. Whence the Angles DAP, *d a p*, being right, the Angle APD will be equal to the Angle *a p d*; that is, the Spherical Angle is equal to that on the *Projection*, and that in all Caſes. Q. E. D.

This *Lemma* I lately received from Mr. *Ab. de Moivre*, tho' I ſince underſtand from Dr. *Hook*, that he long ago produced the ſame thing before the *Society*. However, the *Demonſtration*, and the reſt of the Diſcourſe is my own.

Lemma. 3. *On the Globe, the Rhumb Lines make equal Angles with every Meridian*, and, by the foregoing Lemma, they muſt likewiſe make *equal Angles with the Meridians in the Stereographick Projection on the Plane of the Equator:* They are therefore in that *Projection*, *Proportional Spirals* about the Pole Point.

Fig. 214. Lemma 4. *In the Porportional Spiral it is a known Property, that the Angles* BPC, *or the Arches* BD, *are Exponents of the Rationes of* BP *to* PC: For, if the Arch BD be divided into innumerable equal parts, Right Lines drawn from them to the Center P, ſhall divide the Curve B *c c* C into an Infinity of *Proportionals* between PD and PC, whoſe number is equal to all the Points *d*, *d*, in the Arch BD: Whence, and by what I have delivered concerning the *Conſtruction* of *Logarithms*, it follows, that, as BD to B*d*, or, as the Angle BPC, to the Angle BP *c*, ſo is the *Logarithm* of the *Ratio* of PB to PC, to the *Logarithm* of the *Ratio* of PB to P *c*.

vid. ſup. Cap. 1. § XXVIII.

From theſe *Lemmata* our Propoſition is very clearly *Demonſtrated:* For, by the *Firſt*, PB, P*c*, PC, are the Tangents of half the Complements of the Latitudes in the *Stereographick Projection:* And, by the *Laſt* of them, the Differences of Longitude, or Angles at the Pole between them, are *Logarithms* of the *Rationes* of thoſe Tangents one to the other. But the *Nautical Meridian Line* is no more than a Table of the *Longitudes*, anſwering to each Minute of *Latitude* on the *Rhumb Line*, making an Angle of 45 Degrees with the *Meridian*. Wherefore, the *Meridian Line* is no other than a *Scale of Logarithmick Tangents of the Half Complements of the Latitudes.* Q. E. D.

Corol. 1. Becauſe that in every Point of any *Rhumb Line*, the Difference of *Latitudes* is to the *Departure*, as the Radius to the Tangent of the Angle that *Rhumb* makes with the Meridian; and thoſe equal *Departures* are every where to the *Differences* of *Longitude*, as the Radius to the Secant of the *Latitude*; it follows, that the *Differences* of *Longitude* are, on any *Rhumb*, *Logarithms* of the ſame Tangents, but of differing Species; being proportioned to one another, as are the Tangents of the Angles made with the *Meridian*.

Corol. 2.

Corol. 2. Hence any Scale of Logarithm Tangents (as thoſe of the vulgar Tables made after *Briggs*'s Form; or thoſe made to *Napier*'s, or any other Form whatſoever) is a Table of the Differences of Longitude, to the ſeveral Latitudes, upon ſome determinate *Rhumb* or other: And therefore, As the Tangent of the Angle of ſuch Rhumb, to the Tangent of any other Rhumb; ſo the difference of the *Logarithms* of any two Tangents, to the difference of Longitude on the propoſed Rhumb, intercepted between the two Latitudes, of whoſe half Complements you took the Logarithm Tangents.

Now the *momentary Augment*, or *Fluxion* of the *Tangent-Line* at 45 Degrees, is exactly double to the *Fluxion* of the Arch of the Circle, (as may eaſily be proved) and the Tangent of 45° being equal to the Radius, the *Fluxion* alſo of the *Logarithm Tangent* will be double to that of the Arch, if the *Logarithm* be of *Napier*'s Form: But, for *Briggs*'s Form, it will be as the ſame doubled Arch multiplied into 0,53429, &c. or divided by 2,30258, &c. yet this muſt be underſtood only of the Addition of an indiviſible Arch, for it ceaſes to be true, if the Arch have any determinate Magnitude.

Hence it appears, that, if one Minute be ſuppoſed Unity, the Length of the Arch of one Minute being 0,000290888208665721596154, &c. in parts of the Radius, the Proportion will be as Unity to 2,908882, &c. So Radius to the Tangent of 71° 1′ 42″, whoſe *Logarithm* is 10,46372611720718-325204, &c. and under that Angle is the *Meridian interſected* by that *Rhumb-line*, on which the Differences of *Napier*'s Logarithm Tangents of the Half Complements of the Latitudes are the true Differences of Longitude, eſtimated in Minutes and Parts, taking the firſt 4 Figures for Integers. But for *Vlacq*'s Table, we muſt ſay,

As 2302585, &c. to 2908882, &c. ſo Radius to 1,2633114387244-569212, &c. which is the Tangent of 51° 38″ 9″, and its Logarithm 10,10-1510428507720941162, &c. Wherefore in the *Rhumb-line*, which makes an Angle of 51° 38′ 9″ with the Meridian, *Vlacq*'s *Logarithm Tangents* are the true Differences of Longitude. And this, compared with our *ſecond Corollary*, may ſuffice for the Uſe of the Tables already computed.

But, if a Table of *Logarithm Tangents* be made by Extraction of the Root of the *infiniteth Power*, whoſe Index is the Length of the *Arch* you put for Unity, (as for the Minutes the 0,0002908882*th*, &c. *Power*) which we will call a; ſuch a *Scale* of Tangents ſhall be the true *Meridian Line*, or Sum of all the Secants taken infinitely many. Here the Reader is deſired to have Recourſe to my little Treatiſe of *Logarithms*, that I may not need to repeat it. By what is there delivered it will follow, that putting t for the Exceſs or Defect of any Tangent above or under the Radius or Tangent of 45°: the *Logarithm* of the *Ratio* of Radius to ſuch Tangent will be *vid ſup. Cap. I. § XXVIII.*

$$\frac{1}{m} \text{ into } t - \tfrac{1}{2}t^2 + \tfrac{1}{3}t^3 - \tfrac{1}{4}t^4 + \tfrac{1}{5}t^5, \text{ \&c.}$$

when the *Arch* is greater than 45°, or

$$\frac{1}{m} \text{ into } t + \tfrac{1}{2}t^2 + \tfrac{1}{3}t^3 + \tfrac{1}{4}t^4 + \tfrac{1}{5}{}^5, \text{ \&c.}$$

 when

when it is less than 45°. And by the same Doctrine putting T for the Tangent of any Arch, and t for the Difference thereof from the Tangent of another Arch, the Logarithm of their *Ratio* will be $\frac{1}{m}$ into $\frac{t}{\text{T}} + \frac{tt}{2\text{TT}} + \frac{t^3}{3\text{T}^3} + \frac{t^4}{4\text{T}^4} + \frac{t^5}{5\text{T}^5}$, &c. when T is the greater Term; Or, $\frac{1}{m}$ into $\frac{t}{\text{T}} - \frac{t^2}{2\text{T}^2} + \frac{t^3}{3\text{T}^3} - \frac{t^4}{4\text{T}^4} + \frac{t^5}{5\text{T}^5}$, &c. when T is the lesser Term.

And, if m be supposed, 0,0002908882, &c. $=a$, its reciprocal $\frac{r}{a}$ will be 3437,7467707849392526, &c. which multiplied into the aforesaid *Series* shall give precisely the difference of the Meridional Parts between the two Latitudes, to whose half Complements the assumed Tangents belong.

Nor is it material from whether Pole you estimate the Complements, whether the elevated or depressed; the Tangents being to one another in the same *Ratio*, as their Complements, but inverted.

In the same Discourse I also shewed, that the Series may be made to converge twice as swift, all the even Powers being omitted; and that putting τ for the Sum of the two Tangents, the same *Logarithm* would be $\frac{2}{m}$ or $\frac{2r}{a}$ into $\frac{t}{\tau} + \frac{t^3}{3\tau^3} + \frac{t^5}{5\tau^5} + \frac{t^7}{7\tau^7} + \frac{t^9}{9\tau^9}$, &c. but the Ratio of τ to t, or of the Sum of two Tangents to their Difference, is the same as that of the Sine of the Sum of the Arches, to the Sine of their Difference. Wherefore, if S be put for the *Sine Complement* of the *middle Latitude*, and s for the Sine of half the Difference of *Latitudes*, the same *Series* will be $\frac{2r}{a}$ into $\frac{s}{\text{S}} + \frac{s^3}{3\text{S}^3} + \frac{s^5}{5\text{S}^5} + \frac{s^7}{7\text{S}^7} + \frac{s^9}{9\text{S}^9}$, &c. wherein, as the Differences of *Latitude* are smaller, fewer Steps will suffice. And, if the *Equator* be put for the middle Latitude, and consequently S=R, and s the *Sine* of the *Latitude*, the *Meridional Parts* reckoned from the *Equator* will be $\frac{s}{a} +$
Vid. § XXXVI. $\frac{s^3}{3rra} + \frac{s^5}{5r^4a} + \frac{s^7}{7r^6a}$, &c. which is co-incident with Dr. *Wallis*'s *Solution*. And this same *Series*, being half the *Logarithm* of the *Ratio* of R+s to R—s, that is of the *versed Sines* of the Distances from both Poles, does agree with what Dr. *Barrow* had shewn in his XI. *Lecture*.

The

The ſame *Ratio* of τ to t may be expreſſed alſo by that of the Sum of the Co-ſines of the two *Latitudes*, to the *Sine* of their Difference: As likewiſe by that of the *Sine* of the Sum of the two *Latitudes*, to the Difference of their Co-ſines: Or by that of the *verſed Sine* of the Sum of the *Co-Latitudes*, to the Difference of the *Sines* of the *Latitudes:* Or as the ſame Difference of the *Sines* of the *Latitudes*, to the *verſed Sine* of the Difference of the *Latitudes*; all which are in the ſame *Ratio* of the Co-ſine of the *middle Latitude*, to the *Sine* of half the Difference of the *Latitudes*. As it were eaſy to demonſtrate, if the Reader were not ſuppoſed capable to do it himſelf, upon the bare Inſpection of a Scheme duly repreſenting theſe Lines.

This variety of *Expreſſion* of the ſame *Ratio* I thought not fit to be omitted, becauſe by help of the *Rationality* of the *Sines* of 30°, in all Caſes where the Sum or Difference of the *Latitudes* is 30°, 60°, 90°, 120°, or 150°; ſome one of them will exhibit a ſimple *Series*, wherein great Part of the Labour will be ſaved. But the former ſeems for all Uſes the moſt convenient, whether we deſign to make the whole *Meridian-Lines*, or any Part thereof, *viz.* $\frac{2r}{a}$ into $\frac{s}{S}+\frac{s^3}{3S^3}+\frac{s^5}{5S^5}+\frac{s^7}{7S^7}+\frac{s^9}{9S^9}$, *&c.* Wherein a is the Length of any Arch, which you deſign ſhall be the Integer or *Unity* in your *Meridional Parts* (whether it be a Minute, League or a Degree, or any other) S the Co-ſine of the *Middle Latitude*, and s the *Sine* of half the Difference of *Latitudes*; but, the *Secants* being the Reciprocals of the Co-ſine $\frac{s}{S}$ will be equal to $\frac{\int s}{rr}$ putting $\int$ for the *Secant* of the *Middle Latitude*; and $\frac{2r}{a}$ into $\frac{s}{S}$ will be $=\frac{2\int s}{ra}$. This multiplied by $\frac{\int s}{3SS}$ that is by $\frac{\int\int ss}{3rrrr}$, will give the ſecond Step; and that again by $\frac{3\int\int ss}{5rrrr}$, the third Step; and ſo forward till you have compleated as many Places as you deſire. But, the *Squares* of the *Sines* being in the ſame *Ratio* with the *verſed Sines* of the double Arches, we may inſtead of $\frac{ss}{3SS}$ aſſume for our Multiplicator $\frac{v}{3V}$, or the verſed Sine of the Difference of the *Latitudes* divided by thrice the verſed Sine of the Sum of the *Co-Latitudes*, &c. which is the utmoſt *Compendium* I can think of for this Purpoſe; and the ſame *Series* will become $\frac{2sr}{aS}$ into 1 $+\frac{v}{3V}+\frac{v^2}{5V^2}+\frac{v^3}{7V^3}+\frac{v^4}{9V^4}$. Hereby we are enabled to eſtimate the Default of the Method of making the *Meridional Line*, by the continual Addition of the Secants of equi-different Arches, which, as the Differences of thoſe Arches are ſmaller, does ſtill nearer and nearer approach

proach the Truth. If we aſſume, as Mr. *Wright* did, the Arch of one Minute to be Unity, and one Minute to be the common Difference of a Rank of *Arches*; it will be in all Caſes, as the Arch of one Minute to its Chord, ſo the Secant of the middle Latitude to the firſt Step of our Series. This by Reaſon of the near Equality between a and $2s$, which are to one another in the *Ratio* of Unity to 1—0,00000000352566457713, &c. will not differ from the Secant $ſ$, but in the *9th* Figure; being leſs than it in that Proportion. The next Step being $+\frac{2ſ^3s^3}{3arr}$ will be equal to the Cube of the Secant of the *middle Latitude* multiplied into $\frac{2sss}{3arr}$=0,00000000705132908715; which therefore, unleſs the Secant exceed ten Times Radius, can never amount to 1 in the fifth Place. Theſe two Steps ſuffice to make the *Meridian Line*, or *Logarithm Tangent*, to far more Places than any of natural Secants yet extant, are computed to; but, if the third Step be required, it will be found to be $+ſs$ into $\frac{2s^5}{5ar^4}$=0,00000000000000089498; by all which it appears that Mr. *Wright*'s Table does no where exceed the true *Meridian Parts* by fully half a Minute; which ſmall Difference ariſes by his having added continually the Secants 1′, 2′, 3′, &c. inſtead of $0\frac{1}{2}'$, $1\frac{1}{2}'$, $2\frac{1}{2}'$, $3\frac{1}{2}'$, &c. But, as it is, it is abundantly ſufficient for *Nautical* Uſes. That in Sir *Jonas Moor*'s *New Syſtem of the Mathematicks* is much nearer the Truth; but the Difference from *Wright* is ſcarce ſenſible, till you exceed thoſe Latitudes where *Navigation* ceaſes to be practicable; the one exceeding the Truth about half a Minute, the other being a very ſmall Matter deficient therefrom.

For an Example eaſy to be imitated by whoſo pleaſes, I have added the true *Meridional Parts* to the firſt and laſt Minutes of the *Quadrant*.

The *Firſt Minute*, 1,00000001410265862178.

The *Second*,——2,00000005641063806707.

The *Laſt*, or 89° 59′. 30374,9634311414228643, and not 32348,5279, as Mr. *Wright* has it, by the Addition of the Secants of every whole Minute: Nor 30249,8, as Mr. *Oughtred*'s Rule makes it, by adding the Secants of every half Minute. Nor 30364,3, as Sir *Jonas Moor* had concluded it by I know not what Method, tho' in the reſt of his Table he follows *Oughtred*.

The ſame may be deduced independently from the *Arch* itſelf. For if the Latitude from the Equator be eſtimated by the Length of its *Arch* A, Radius being Unity, and the *Arch* put for an Integer be a, as before; the *Meridional Parts* anſwering to that *Latitude* will be $\frac{1}{a}$ into A, $+\frac{1}{6}A^3+\frac{1}{24}A^5+$

$\frac{1}{84}$

60

$\frac{\frac{1}{84}}{60}A^7$ or $\frac{61}{5040}A^7 + \frac{\frac{11}{2880}}{1386}A^9$ or $\frac{1385}{362880}A^9$, &c. which converges much ſwifter than any of the former *Series*, and beſides has the Advantage of A increaſing in Arithmetical Progreſſion; which would be of great Eaſe if any ſhould undertake *de novo* to make the Logarithm Tangents, or the *Meridian-line* to any more Places than now we have them; the Logarithm Tangent to the Arch of $45° + \frac{1}{2}A$ being no other than the aforeſaid Series $A + \frac{1}{6}A^3 + \frac{1}{24}A^5$, &c. in *Napier*'s Form, or the ſame multiplied into 0,43429, &c. for *Briggs*'s.

But, becauſe all theſe *Series* towards the latter End of the Quadrant do *converge* exceeding ſlowly, ſo as to render this Method almoſt uſeleſs, or at leaſt very tedious: It will be convenient to apply ſome other Arts, by aſſuming the Secants of ſome intermediate Latitudes; and you may for *s*, or the *Sine* of *n*, the Arch of half the Difference of Latitudes, ſubſtitute $a - \frac{1}{6}a^3 + \frac{1}{120}a^5 - \frac{1}{5040}a^7 + \frac{1}{362880}a^9$, &c. according to Mr. *Newton*'s Rule for giving the Sine from the *Arch*; and, if *a* be no more than a Degree, a very few Steps will ſuffice for all the Accuracy that can be deſired.

And, if *a* be commenſurable to *a*, that is, if it be a certain Number of thoſe Arches which you make your Integer, then will $\frac{a}{a}$ be that Number, which if we call *n*, the Parts of the Meridional Line will be found to be

$$\frac{sn}{r} \text{ into } \left\{ \begin{array}{l} 1 + \frac{ssaa}{3r^4} + \frac{s^4a^4}{5r^8} + \frac{s^6a^6}{7r^{12}}, \text{ \&c.} \\ -\frac{aa}{6rr} - \frac{ssa^4}{6r^6} - \frac{s^4a^6}{6r^{10}}, \text{ \&c.} \\ +\frac{1a^4}{120r^4} + \frac{13s^2a^6}{360r^8}, \text{ \&c.} \\ -\frac{1a^6}{5040r^6}, \text{ \&c.} \end{array} \right.$$

In this the firſt two Steps are generally ſufficient for nautical Uſes, eſpecially when neither of the Latitudes exceed 60 Degrees, and the Differences of Latitudes do not paſs 30 Degrees.

To conclude, I ſhall only add, that Unity being *Radius*, the Co-ſine of the Arch A, according to the ſame Rules of Mr. *Newton*, will be $1 - \frac{1}{2}A^2 + \frac{1}{24}A^4 - \frac{1}{720}A^6 + \frac{1}{40320}A^8 - \frac{1}{3628800}A^{10}$, &c. from which and the former

former *Series* exhibiting the *Sine* by the Arch, by Diviſion it is eaſy to conclude, that the *natural Tangent* to the Arch A is $A + \frac{1}{3} A^3 + \frac{2}{15} A^5 + \frac{17}{315} A^7 + \frac{62}{2835} A^9$, &c. and the *natural Secant* to the ſame Arch $1 + \frac{1}{2} A^2 + \frac{5}{54} A^4 + \frac{61}{720} A^6 + \frac{227}{8064} A^8$, &c. And from the *Arithmetick of Infinites*, the Number of theſe *Secants* being the Arch A, it follows, that the Sum total of all the *Infinite Secants* on that Arch is $A + \frac{1}{6} A^3 + \frac{1}{24} A^5 + \frac{61}{5040} A^7 + \frac{277}{72576} A^9$, &c. the which, by what foregoes, is the Logarithm Tangent of *Napier*'s Form, for the Arch of $45° + \frac{1}{2} A$, before.

And collecting the infinite Sum of all the *natural Tangents* on the ſaid Arch A, there will ariſe $\frac{1}{2} A A + \frac{1}{12} A^4 + \frac{1}{45} A^6 + \frac{17}{2520} A^8 + \frac{31}{14175}, A^{10}$, &c. which will be found to be the Logarithm of the Secant of the ſame Arch A.

To find the Variation of the Compaſs at Sea; by .... n. 24. p. 435. Apr. An. 1667.

XXXIX. The Height of the Pole, and the *Sun*'s Declination being known, a large Ring-Dial, truly wrought, having a Box with a Compaſs or Needle fixt to its Meridian below, may go as near as any other Inſtrument, to ſhew the Variation of the *Needle* at *Sea*. For, when it is ſet to the juſt Hour and Minute of the Day, the Meridian of it ſtands juſt in its due Place; and ſo ſhews how far the *Needle* varies from it, as exactly as the Largeneſs of the Card will permit.

But, becauſe theſe Dials are ſo rarely juſt, &c. tho' they may be uſed and taken notice of, yet they are not to be relied on. The Thing therefore is to be performed, as followeth:

Find out the *Sun's Azimuthal* diſtance from the Meridian ſome Hours before or after Noon, and then its *magnetical Azimuth*, or diſtance from the Meridian pointed at by the *Needle*; and the difference of thoſe two diſtances is the variation of the *Needle*.

To find the *Sun*'s true *Azimuth*, or by how many Degrees, &c of the Horizon, it is diſtant from the Meridian: Its Declination, its Altitude, and the Elevation of the Pole, muſt all three be known; and thence the true *Azimuth* may be eaſily calculated. The true *Azimuth* of the *Sun* being thus found, and the *magnetical Azimuth* of it, according to your *Needle*, being obſerved, ſubtract the leſſer number from the greater, and the Remander is the Variation of the *Needle*. If the *Magnetical Azimuth* be leſs than the other, then the Variation is towards the ſame ſide of the Meridian, where the *Sun* is; if greater, on the other.

To obſerve the *Sun*'s *Azimuth* by the *Needle*, and the *Needle*'s Variation, to Degrees, any *Needle* long enough to afford upon a Card under it a Circle divided into Degrees, put into a ſquare Box, after the ordinary manner of *Clinatories*, will ſerve turn; by placing the Box ſo, as the Sun may ſhine upon any two oppoſite ſides of it, at the ſame time when the Sun's Height, *&c.* are taken. For then the *Needle*'s Diſtance from the Diameter of the Circle on the Card, that is parallel to thoſe Sides, is the *Magnetick Azimuth* required.

The ſame may be done with an ordinary *Sea-Compaſs*, ſo it have a Circle towards the Limb of the Card, divided into degrees, by faſtning a ſmall Thread, Lute-ſtring or Wire (not of Iron) ſo upon it, as to paſs juſt over the Centre of that Circle; and placing a ſtrait Piece of Wood or Braſs-wire perpendicular on the Edge of the Box at the end of the Thread, and turning it to the Sun till the Shadow of it fall juſt upon the Thread: Then obſerve what Degrees of the Circle on the Card the Thread cuts, by looking plum upon it; and that is the Sun's *Magnetical Azimuth*.

But to have the *Variation* to *Degrees* and *Minutes*, (which is moſt deſireable) then the Obſervation laſt mentioned muſt be made with a *Quadrant*, *Sextant*, or ſome ſuch other Inſtrument, ſo large as to admit of the Diviſion of a *Degree* into *Minutes*: which will require the Radius to be about 3 Foot; the larger the better. If a *Quadrant*; then, it being laid flat, and the ſquare Box with the *Needle* placed upon it, move the *Quadrant* to and again, till that ſide of it, on which the Box is placed, lie parallel to the *Needle* when at quiet: Then the Sight of the *Quadrant* being ſlid along the Limb of it, till the Sun ſhine on both its ſides at the ſame time, the Mid-line, that divides equally the Sight, when the Sun ſhines upon it through the Slit, will mark the *Degree* and *Minute* of the *Sun*'s *Magnetical Azimuth*. All which is eaſy to be put in Practice.

To find this Variation by the Stars, is ſo eaſy, that every Maſter can do it.

A Caution for Obſerving the Variation at Sea; *by Mr.* Edm. Halley, n. 195. p. 571. Oct. An. 1692.

XL. It is a received Error, in the Practice of obſerving the Variation at *Sea*, to take it by the Amplitude of the *Riſing* and *Setting Sun*, when his Centre appears in the Viſible Horizon, whereas he ought to be obſerved when his Under-Limb is ſtill above the Horizon about $\frac{2}{3}$ of his Diameter, or 20 Minutes, upon the Score of the *Refraction*, and the Height of the Eye of the *Obſerver* above the Surface of the *Sea*: Or elſe they are to work the *Amplitude* as they do the *Azimuth*, reckoning the *Sun*'s Diſt. from the *Zenith* 90° 36′.

This, though it be of little Conſequence near the Equinoctial, will make a great Error in *High Latitudes*, where the *Sun* riſes and ſets obliquely.

A Caution to Seamen *bound up the* Engliſh Channel; *by Mr.* Halley, n. 267. p. 725. Nov. Ann. 1700.

XLI. The *Latitudes* of the *Lizard* and *Scilly*, are laid down too far Northerly by near 5 Leagues: For, from undoubted Obſervation, the *Lizard* lies in 49° 55′, the middle of *Scilly* due *Weſt* therefrom, and the South Part thereof neareſt 49° 50′. Whereas in moſt Charts and Books of *Na-*

Navigation they are laid down to the Northward of 50°, and in ſome full 50° 10′. Nor was this without a good Effect, as long as the *Variation* continued *Eaſterly*, as it was when the *Charts* were made. But ſince it is become conſiderably *Weſterly*, (as it has been ever ſince the Year 1657.) and is at preſent about 7½ *Deg.* all Ships ſtanding in, out of the Ocean, *Eaſt* by the *Compaſs*, go two thirds of a Point to the *Northward* of their true Courſe, and in every 80 Miles they ſail, alter their *Latitude* about 10′. So that if they miſs an Obſervation for two or three Days, and do not allow for this *Variation*, they fail not to fall to the *Northward* of their Expectation, eſpecially if they reckon *Scilly* in above 50°, and to run up the *Briſtol Channel*, not without great Danger of all, and the Loſs of many of them. This has been by ſome attributed to the *Indraught* of St. *George*'s *Channel:* But, the *Variation* being allowed, it hath been found, that the ſaid *Indraught* is not ſenſible. It is therefore recommended to all Maſters of Ships, that they ſteer two Watches E. by S. for one E. which will exactly keep their Parallel; as alſo, that they come in, out of the Sea, on a Parallel not more *Northerly*, than 49° 40′, which will bring them fair by the *Lizard*.

XLII. Papers of Leſs General Uſe, omitted.

Pendulum Watches. *n.* 118. *p.* 440. *vid. ſup.* *Cap.* V. *Sect.* V. Mr. *Oldenburg* having publiſhed from *Journal de Scavans*, an Account of M. *Huygen*'s Portable *Watches*, Dr. *Hook*, in the Poſtſcript to his *Deſcription of Helioſcopes*, complains of it, for not having taken Notice, *That this Invention was firſt found out by an* Engliſhman, *and long ſince publiſhed to the World.* To
n. 128. *p.* 710. this Mr. *Oldenburg* anſwers, by relating the *Plain Truth of the Matter:* Where-
n. 129. *p.* 749. upon Dr. *Hook*, in a Poſtſcript to his *Lampus*, further complains, and reflects on Mr. *Oldenburg*'s Integrity and Faithfulneſs in his *Management of the Intelligence of the Royal Society*. This gave Occaſion to the *Council* of that Society to declare, *That Mr.* Oldenburg *had carried himſelf Faithfully and Honeſtly, and given no juſt Cauſe of ſuch Reflections:* To which Mr. *Oldenburg* adds Part of a Letter from Mr. *Huygens* to him, *Offering* (*if Mr.* Oldenburg *believes a Patent in* England *might be worth ſomething*) *all he might there pretend to.* So that if Mr. *Oldenburgh* had a Deſire to take out a Patent, it was for no other Contrivance than Mr. *Huygen*'s.

XLIII. Account of Books and Emendations omitted.

n. 231. *p.* 670. 1. Volumen Primum *Geographorum Gr.* Minorum. *Oxon* in 8*vo.*

n. 231. *p.* 671. 2. *Dionyſii* Periegeſes, Græce & Latine, cum Scholiis Gr. tam Editis quam Ineditis. Cura *Edw. Thwaits*, M. A. *Oxon.* in 8*vo.*

n. 92. *p.* 5172. 3. *Bernhardi Vareni*, M. D. *Geographia* Generalis; aucta & illuſtrata ab *Iſaaco Newtono*, R. S. S. *Cantab.* 1672. in 8*vo.*

n. 231. *p.* 665. 4. *Philippi Cluverii* Introductio in Univerſam *Geographiam*, tam Veterem quam Novam: Tabulis *Geographicis* 46. ac Notis olim ornata, à *Joanne Bunone*; jam vero locupletata Additamentis & Annotationibus *Jo. Frid. Hekelii* & *Jo. Reiſkii*. *Amſt.* 1697. in 4*to.*

5. *Geography Anatomiz'd*; or a Compleat *Geographical Grammar.* Being a short and exact *Analysis* of the whole Body of *Modern Geography*, after a New and Curious Method; by *Pat. Gordon*, M. A. F. R. S. The *Second Edition.* n. 256. p. 335.

6. An Account of the Measure of a *Degree* of a *Great Circle* of the *Earth*; by M. *Picart. Paris.* 1671. *Fol.* Translated into *English*; by Mr. *Waller*, *R. S. Sec. Lond.* 1687. *This Book is here abridg'd, and the Sum of the whole amounts in short to this,* M. Picart *measured on a Plain and straight Ground a space of* 5663 Toises, *to serve for the first* Basis *to divers Triangles*; *by which he hath concluded the* Length *of a* Meridian Line *equivalent to a* Degree *of* Latitude, *to be* 57060 Toises *or* Fathoms, *that is*, 28½ Leagues *and* 60 Toises. n. 112. p. 261. n. 124. p. 596. n. 189. p. 376. n. 126. p. 635.

7. The Seaman's Practice; by Mr. *Richard Norwood*, *Lond.* 1636. in 4*to.* *The Measure of a* Degree *is here extracted from that Book. Mr.* Norwood, *An.* 1635. *having actually measured, for the most Part, the Way from* York *to* London, *and having observed the Meridian Altitudes of the* Sun *in both Places, he found the* Difference *of* Latitude *to be* 20° 28′, *and the* Distance *of their* Parallels 905751 English *Feet*; *and therefore one* Degree *of a* Great Circle *is* 367196 *Feet*, *or* Numero Rotundo 367200 *Feet*, *which is equal to* 69½ English Miles, and 14 Poles; *Whereas the* French *make it no more than* 365000 *such Feet.*

8. Longitude found; by *Hen. Bond*, Sen. *Lond.* 1676. *in* 4*to.* *A Mistake in that Book is here corrected.* n. 95. p. 6065. n. 130. p. 774.

9. A Book published by Mr. *Jo. Moxon*, describing a new sort of Terrestrial Globes, invented by the E. of *Castlemain.* n. 139. p. 9883. *Ph. Col.* n. 1. p. 43.

10. The *English Atlas. Oxford*, for *Moses Pitt.* 1680. *Fol.* *Ph. Col.* n. [illegible]8. p. 39.

11. A New Map of *England*, full 6 Foot Square, wherein *Computed* and *Measured* Miles are entered in Figures; by Mr. *Jo. Adams.* n. 135. p. 886.

12. A large and curious Map of *Great Tartary*; by M. *Nich. Witson.* n. 193. p. 492.

CHAP. VIII.

Architecture. Ship-Building.

Stones fit for Building; by . . . . n. 93. p. 6010. Apr. An. 1673.

I. THERE is a sort of grey *Freestone* at *Paris*, every where on the South side of the River *Sein*, which is of a reasonable coarse Greet, and so soft when first taken out of the *Quarry*, that 'tis drest and hewn with broad sharp Axes, almost as easily as dried Clay, but grows harder and harder in the Air; 'tis very durable, and exceeding fit for *Building*. The *Portland* Stone is of a fine Chalky Greet, fit for all curious hewn and carved Work, tho' not fit for Water or Fire. On the contrary, the *Freestone* in *Kent*, of a whitish grey Colour, lasts well in Air and Water; the Greet thereof less fine and chalky than that of *Portland*. The *Derbyshire Freestone*, tho' it endure the fiercest Fire, is yet brittle, and so unfit for fine and curious Workmanship.

The Choice and Charges of Slate for covering Houses; by Mr. Sam. Colepress. n. 50. p. 1009. Aug. An. 1669.

II. 1. Take the thin cleft Stone, Slat or Shindle, and so knock it against any hard Matter, as to make it yield a Sound; if the Sound be good and clear, that sort of Stone is not crazy, but firm and good. Or,

2. If in hewing it does not break before the Edge of the *Setts*, (the Hewing Instrument of the *Slatters*) you may not much doubt of the Firmness of the Slate. But,

3. If after it hath been exactly weighed (and the Accompt thereof laid by) it be put, and for 2, 4, or 8 Hours left to remain all under Water in a Vessel; and afterward taken up and wiped very clean with Cloths, if then it weigh more than before, 'tis of that kind which imbibes Water, and therefore not so fit to endure any considerable time without rotting the Lathes and Timber.

4. These Stones may be pretty well guessed at, whether they be of a close or loose Texture, by their Colour: For the over blackish Blue is aptest to take in Water; but the lighter Blue is always the firmest and closest. To which may be added the Touch; for a good Stone feels somewhat hard and rough; whereas an open Stone feels very smooth, and as it were Oily.

5. Place your Stone long-ways perpendicular in the midst of a Vessel of Water, (no matter how shallow the Water be, so it exceed half a Foot depth;) and be sure, the upper un-immersed Part of the Stone be not accidentally wetted by the Hand, or otherwise; and so let it remain a Day, or half a Day, or less. If it be a good firm Stone, it will not draw (as they

speak)

ſpeak) Water more than half an Inch above the Level of the Water, and that perhaps but at the Edges only, the Parts of which might be ſomewhat looſened in the Hewing. But a bad Stone will draw Water up to the very top, be the Stone as long as it will, all over.

As for the Charge of covering Houſes with Slate, they may be thus computed.

	s.	d.
1000 of *Efford Small Blue* at the Ship's ſide in *Plymouth* Harbour	5	6
1000 of *Efford Large Blue*	9	9
1000 of *Can Pelmel*	7	0
1000 of *Small Blue* of other Quarries	4	0
1000 of *Large Blue*	8	0

3000 of *Small Blue*, accompted two Tuns in Carriage by Water.

1000 of *Large Blue*, 1 Tun.

3000 of *Small* will cover 1 *Pole* of Work at the 5 Pin Plain.

Every *Pole* of Work is either 6 Foot broad and 14 up, on both ſides, or 168 Foot in Length, and one in Breadth.

3000 of *Large*, will cover two *Poles* of Plain Work.

	s.	d.
Hewing of all ſorts of plain *Pelmel per* 1000	1	6
Pinning per 1000, 8 *d.* *Pins per* 1000. 8 *d.*	1	4

Three Buſhels (*Wincheſter* Meaſure) of good *Lime*, will take 6 Buſhels of freſh Water Sand, and ſerve to lay on one *Pole* of work; tho' much leſs may ſerve the Turn.

300 of Lathes to every *Pole* of Work.

1000 of Lathe Nail to every 300 of Lathes.

An Able Workman may { Lath one *Pole* of Work / Lay on 2000 or more of *Slate* / Hew 1500 plain / Pin 4000 } by the Day.

Chequer-work conſiſts in Angles, Circles, and Semicircles, *&c.* which require no common Skill and Time in hewing and laying.

It is worthy Obſervation, That, if a ſide Wall happen to take Wet by the beating of the Weather, or the like, when nothing elſe will cure it, our *Kerſeying* with *Slate* (which is much uſed in the curious Fronts of Houſes, eſpecially in Towns) will quickly remedy it.

We have ſome ſorts, which by the Conjectures of the more experienced *Helliers*, (or Coverers with Slate), have continued on Houſes ſeveral Hundreds of Years, and yet as firm as when firſt put up.

The beſt Time of Felling of Timber; by Dr. Rob. Plot. n. 192. p. 455. Jan. An. 1691.

III. The *Cuſtom* of *Felling Timber* here in the *South* of *England*, differs from that of *Staffordſhire* in the Time of *Felling*, and Manner of *Barking*. It is *Felled* here in the *Spring*, as ſoon as the *Sap* is found to be fully up, by the Trees putting out, and then *Barked* after the Trees are proſtrate, the *Sap* yet remaining in the Bodies of them: Whereas there it is firſt *Bark'd*, (in the Spring as *here*) but before it is *Felled*, the Trees yet living and ſtanding all the *Summer*, and not *Felled* till the following *Winter*, when the *Sap* is fully in Repoſe.

In

In the *Spring* Seaſon, and ſome time after, all Trees are pregnant and ſpend themſelves (as Animals do in their reſpective Offſprings) in the Production of Leaves and Fruits, and ſo become weaker than at other times in the Year; their Cavities and Pores being then turgid with Juices, or Sap, which (the Trees being felled at that time) ſtill remain in the Pores, having no manner of Means of being otherwiſe ſpent, and there putrefy; not only leaving the Tree full of thoſe Cavities which render the *Timber* weak; but, 2*dly*, breeding a Worm, as both *Pliny* and Mr. *Evelyn* teſtify, that will ſo exceedingly prejudice it, that it becomes altogether unfit for ſtrong Incumbencies, or other robuſt Uſes. 3*dly*, All Timber fell'd at this Time of the Year, whether the Juices putrify, or otherwiſe ſweat forth, or dry away, is not only ſubject to rift and gape, but will ſhrink ſo conſiderably, that a Piece of ſuch *Timber* of a Foot ſquare will uſually ſhrink in the Breadth ¼ of an Inch; than which, ſays *Vegetius*, nothing is more pernicious, if us'd for the *Building* of *Ships*. To which, 4*thly*, the firſt and greateſt *Roman* Emperor, *Julius Cæſar*, adds, that tho' *Ships* may be made of ſuch moiſt Timber, felled in the Spring, yet they will certainly be Sluggs, not near ſo good Sailers as Ships made of Timber felled later in the Year.

In all which Circumſtances, I find, moſt of the *Antients* ſo very well agree, that none of them adviſe the felling of *Timber*, for any ſort of Uſe, before *Autumn*, at ſooneſt; others, not till the Trees have born their Fruit; which, ſays *Theophraſtus*, muſt always be proportionably later, as their Fruits are ripe later in the Year. A third ſort, not till *Mid-winter*; not till *November*, ſays *Palladius*, nay, not till the *Winter Solſtice*, ſays the wiſe *Cato*; and then too in the Decreaſe or *Wane* of the *Moon*, between the 15*th* and 23*d* Day of her Age, ſays *Vegetius*, or rather, according to *Columella*, between the 20*th* and the *New-Moon*. In general, ſays *Theophraſtus*, the *Oak* muſt be fell'd very late in the *Winter*, not till *December*, as the Emperor *Conſtantine Pogonatus* poſitively aſſerts, the *Moon* too being then under the Earth, as 'tis for the moſt part in the Day-time in the firſt Part of its Decreaſe. And the felling of *Oak* within thoſe Limits they call *Tempeſtiva Cæſura*, *Felling Timber* in Seaſon, which they all unanimouſly pronounce (if thus felled) will neither ſhrink, warp, nor cleave, nor admit of Decay, in many Years; it being tough as *Horn*, and the whole Tree in a manner (as *Theophraſtus* aſſerts) as hard and firm as the Heart: with whom alſo agrees our Countryman Mr. *Evelyn*; if you fell not *Oak* (ſays he) till the Sap is in Repoſe, as 'tis commonly about *November* and *December*, after the Froſt has well nipped them, the very *Saplings* thus cut will continue without Decay, as long as the Heart of the Tree.

And the Reaſon of this is given in ſhort by *Vitruvius*, *quia Aeris* Hyberni *vis comprimit & conſolidat Arbores*, becauſe the *Winter* Air doth cloſe the Pores, and ſo conſequently conſolidates all Trees, by which means, the *Oak*, (as he and *Pliny* both expreſs it) will acquire a ſort of *Eternity* in its Duration; and much more will it ſo, if it be Bark'd in the Spring, and left ſtanding all the Summer, expoſed to the Sun and Wind, as is uſual in *Staffordſhire*, and the adjacent Countries; whereby they find, by long Experience, the Trunks of their

their Trees ſo dried and harden'd, that the ſappy Part in a manner, becomes as firm and durable as the Heart itſelf.

Which way of *Barking* and *Felling* of *Timber*, tho' it were unknown to the Ancients (as perhaps it is to all the world beſides theſe few Counties) yet they ſeem not unacquainted with the Rationality of the Practice, The great *Vitruvius* prefers the *Timber* on the *South-ſide* the *Apennine* (where it winds about, and incloſes *Tuſcany* and *Campania*, and ſtrongly reflects the conſtant *Heat* of the *Sun* upon it as it were from a Concave) incomparably before that which grows upon the *North-ſide* of the ſame Hill, in the ſhady moiſt Grounds: Of which his Opinion he renders us this Reaſon, for that the Sun does not only lick up the ſuperfluous Moiſture of the *Earth*, whence the *Trees* are ſupplied in ſuch ſhady Places with too great a Quantity, but in great meaſure exhales the remaining Juices (after the Production of Leaves and Fruits) out of the *Trees* themſelves, rendering the Timber of them the more cloſe, ſubſtantial and durable; which certainly it would do alſo much more effectually, if the *Bark* were taken off in the Spring of the Year, as is accuſtomed in *Staffordſhire*, where the People are content to uſe this Method in their Proviſion of Timber, tho' but for private Uſes.

Much rather then ſhould it be done in ſo publick a Concern as the *Building* of *Ships*, were tough and ſolid Timber is much more neceſſary than in ordinary *Buildings*. There is indeed an *Act* of *Parliament*, 1 *Jac.* I. *Chap.* 22. which forbids the *Felling* of *Timber* for ordinary Uſes (in conſideration of the *Tan*) at any other time but between the 1ſt of *April* and the laſt of *June*, when the Sap is up, and the *Bark* will run; made on Suppoſition, (I gueſs) that ſhould they have admitted *Felling Timber* in any other Seaſon, the *Tanners* would have wanted a Supply of *Bark*. To which I readily anſwer, that I fear the *Legiſlators* that preſſed the making that Act were ignorant that the *Bark* might be taken off in the Spring, and that the Tree would notwithſtanding live and flouriſh till the *Winter* following, as I have ſeen many in *Staffordſhire*: So that tho' the Tree be not *Fell'd* till the *Winter Solſtice*, or *January* following, yet the *Tanner* is not at all defeated of his *Tan*, but has it here in as due Seaſon, as in any of the Southern Counties. The *Legiſlators*, I ſay, were ignorant of this, otherwiſe they would never have made an Act ſo pernicious to the whole Kingdom, as *Felling Timber* at this Seaſon is, for the ſake of a few *Tanners*.

But notwithſtanding this Ignorance, yet then they were ſo wiſe as to except in that Act the *Timber* to be uſed in building of Ships, which may be *Fell'd* in *Winter*, or any other Time; as I am told all the ancient Timber remaining in the *Royal Sovereign* was, it being ſtill ſo hard, that 'tis no eaſy matter to drive a Nail into it.

'Tis true indeed, that the *barking* and *peeling* the Trees ſtanding is ſomewhat more troubleſome, and therefore ſomewhat more chargeable, than when they are proſtrate; and that 'tis likely, People therefore have uſually fell'd their Timber, as well for *Shipping* as other uſes, in the Spring of the Year, for the ſake of the more eaſy and cheap *Barking* it only, rather than any thing elſe. 'Tis too true, that *Timber* is harder to *Fell* in the *Winter*, it being now ſo com-

pact

pact and firm, that the Ax will not make ſo great Impreſſion as it doth in the *Spring*, which will alſo increaſe the Price of the *Felling* ſome ſmall Matter and its *Sawing* afterwards; but how inconſiderable theſe things are in compariſon of the great Good of this manner of *Felling*, I think is ſelf-evident.

The greateſt Objection, that I can foreſee will be urged here in the *South* againſt this Practice, is, that if the *Timber* be not *fell'd* till *Mid-winter*, or *January*, where it grows in *Copſes* and *Woods*, they cannot perhaps incloſe their young *Sprigs* ſo ſoon as ſome may imagine needful, and therefore will be backward to fell their Timber (ſo growing) at that Seaſon. To which I anſwer, That the *Timber* ſo fell'd in *Woods* or *Copſes* may be eaſily carried off before the ſecond Spring, and ſo the Prejudice ſmall, and the firſt it muſt be there, wherever it is fell'd. But *Secondly*, That which will quite remove this inconſiderable Difficulty, is, that perhaps it may be expedient that no Timber whatſoever growing in *Woods* or *Copſes*, be at all bought in the *King*'s *Yards*, for that *Timber* growing in ſuch ſhady Places, and ſo fenced from the *Sun* and *Wind*, as *Timber* in *Woods* for the moſt part is, cannot be ſo good as that which comes from an expoſed Situation, ſuch as it uſually has in *Foreſts*, *Parks*, *Hedge-rows*, and open Fields; whereto it is indifferent at leaſt if not better for the Proprietor, that it be fell'd in *Winter*, (when the Graſs and Corn is gone) than in the *Spring* it ſelf: and the Officers deſigned for that purpoſe may buy all their *Timber* under ſuch Conditions as to be fell'd in *Winter*, enjoining the Proprietor, to take off the Bark in the Spring in due Time, making him ſome ſmall Allowance for the Trouble he will have in peeling it ſtanding.

The Difference of Timber *in different* Countries, *and Fell'd at* different Seaſons; *by M.* Ant. Van. Leuwenhoek. n. 213. *p.* 224. Sep. An. 1694.

IV. It is the common Opinion, That *Timber* which is fell'd in *Winter*, is ſtronger and more laſting, as being more cloſe and firm than that which is fell'd in *Summer*: But, M. *Leuwenhoek*'s Sentiment is, that there is no Difference, except in the *Bark*, and outermoſt Ring of the Wood, which in the *Summer* are ſofter, and ſo more eaſily pierced by the *Worm*; *Wood* conſiſting of hollow Pipes, which in the *Summer* and *Winter* both, are full of Moiſture, they do not ſhrink in the *Winter*, and therefore the *Wood* cannot be cloſer at one time than another, for otherwiſe it would be full of Cracks and Clefts. The ſudden and unexpected rotten of ſome Timber, he conceives to proceed from ſome inward Decay in the Tree before it was fell'd; having obſerved all Trees to begin to decay at firſt in the Midſt, or Heart of the Tree, tho' poſſibly the Tree may ſtand and grow for near an Hundred Years afterwards, and increaſe in Bigneſs all along.

2. He ſays, he was once of Opinion, that Trees growing in good Ground, but increaſing ſlowly, were the beſt and ſtrongeſt Timber; and that thoſe Trees, which in few Years grew large were the ſofteſt and brittleſt; the contrary to which, upon Enquiry of experienced Workmen, he found to be true, and inſtances of an *Elm* of 80 Years Growth, which was 11 Foot in Circumference, and proved excellent tough Timber.

3. The *Age* of *Trees* is to be known by the Number of Rings to be seen when the *Tree* is cut athwart, in each of which Rings is one Circle of large open *Pipes*; now the fewer of these large *Pipes* the stronger the *Timber* is: wherefore by Consequence these *Trees* that make the largest Growth, in a Year, must be the Closer and stronger, and therefore those *Trees* that grow in warm Countries grow fastest, and are the best and toughest Timber; which he confirms by *Riga* and *Dantzick* Oak, which is of slow Growth, and proves spongy and brittle *Timber*, whereas the contrary is observeable in *English* and *French* Oak, which grows faster, and is excellent *Timber*.

V. 1. This Famous *Roman Bridge* at *Pont St. Esprit*, is very crooked, bowing in many Places, and making several unequal Angles, especially in those Places where the Torrent runs strongest, as where the *Turret* stands, 4. In which Place the Angle is most unequal, and the greatest; the *Arches* are very wide, and have their Feet secured by two *Pedestals* that encompass them. Both the *Pedestals* have their several Degrees or Ranks of *Jettings* out, like so many Rows of Stairs or Steps, the lowermost Order pushing out most, the others being less, and going gradually more in; the Second or uppermost *Pedestal* is much less than the first or lowermost, being built a little within its Lines of *Circumference*; 1, 2. Between the great *Arches* there are *Windows*, or, (as it were) small *Arches*; 3. that come down to the very Plane of the second, or uppermost *Pedestal*, dividing the Feet of the great Arches. From this my rude Description it appears to me, that the *Romans* have here contrived all possible ways to break gradually the mighty Force of the *Rhosne*, and to render its Passage easy, and inoffensive to the Feet of the great *Arches*; for here we see so many several *Palisadoes* and *Sluices*, as may be sufficient to defend this wonderful Fabrick against all Storms of the *Torrent*; the several Ranks of *Stairs* jetting from the *Pedestals* (for the most part Triangularly built, and faced well with Free-stone) opposing and breaking the Stream severally, I mean, not all together, or at the same time, by reason of their various Inequalities in standing out: in case the Flood should swell so high (as it frequently does) as to cover both the *Pedestals*, then the small Arches, dividing the Feet of the great ones, help to convey the Water thro', which otherwise might endanger the great *Arches*.

The Bridge at St. Esprit in France; by Dr. Tankred Robinson, n. 160. *p.* 584. *Fig.* 215.

2. That which seems the Foot of the *Arch* is an *Horizontal Arch* gradually contracted, every Stone being of vast Length and Wedge-like, laid level with the Water. This I speak by Memory.

By Dr. Lister, *ib.* 585.

3. The stately *Modern Bridge* at *Avignon* hath yielded in many Places to the extreme Rapidity and Violence of the *Rhosne*. Its Fall, in my Opinion, may be ascribed to three Defects, First, It was not so multangular, as that at St. *Esprit*: Secondly, it wanted in three or four Places, the little *Arches* dividing the Feet of the great ones, and in those Parts it hath suffered most; for where those useful Sluices are, there I observed the Bridge to stand still the most intire. Thirdly, The *Pedestals* (or as you very properly call them *Horizontal Arches*) were not so Geometrically and exactly laid, as those of *Pont St. Esprit*; their Jettings out were few, and they not gradually contracted; so that the Force of the Stream must be greater upon the Fabrick.

Compared with some other Bridges; by Dr. Tank. Robinson. n. 163. *p.* 712.

Tho' the *Tiber* be not ſo ſwift as the *Rhoſne*, yet it is ſubject to greater Inundations, as many Inſcriptions aſſure us. No River ever had ſo many Bridges built with that Magnificence and Art, as this; and tho' they were more pompous and rich in rare Stones, in Sculpture, *&c.* than that I formerly ſent you a Draught of from *Montpelier*; yet they had the like Proviſion for their Security and Preſervation, and their Deſign was much the ſame; which may be ſeen at *Rome* this very Day at the old *Pons Mellius* (now *Ponte Nolle*) near the *Via Flaminia*, in the Marble Remains of the *Pons Æmilius*, repaired with rich Materials by *Antoninus Pius*, on the Side of the *Ripa*, or *Traſtavere*, near the Root of the *Aventine Hill*, where firſt the *Pons Sublicius* ſtood; as alſo in the *Pons Fabricius* and the *Ceſtius*, that leads over to the *Inſula Tiberina*; in all which there are ſtill very fair Marks of the old *Roman* Structure and *Deſign*; and if that prodigious City had not been knocked ſo oft to pieces by *Barbarous Sackers*, we might have had ſtill as clear Proofs from the other Bridges, *viz.* the *Pons Triumphalis*, the *Senaeotius*, &c. But *Gothiſh* and *Northern Torrents* broke all before them.

A Bridge without any Pillar under it; from The Journal of the Phil. Society *of* Oxford. ib. *p.* 714. *Fig.* 216.

VI. A *Timber Bridge* may be built 70 Foot long, or ſomewhat more, without any *Pillar* under it, which may be uſeful in ſome Places where *Pillars* cannot be conveniently built, after this manner; A C, and B O are Beams 28 Foot long, and A B is 32 Foot long. Under the Angles are ſet two large *Braces*, E L, and S R. At each end is a Wall, on which are laid two *Beams* BH, and AD, each 20 Foot long; under theſe two are two *Braces* DE, and RH. There may alſo be Braces at the End of the Arches, that may lie obliquely croſs the Bridge. It may be laid with Planks and Railed. Behind the Walls are *Cauſeys* FD, and AN. The Length of the Bridge CMO, is 70 Foot; in the Height KM is 19 Foot.

An Aqueduct *near* Verſailles n. 171. *p.* 1016. May, An. 1685.

VII. 1. The *Aqueduct* which is to be made near *Maintenon*, for the carrying the River *Bure* to *Verſailles*, will have in Length 7000 Fathom; 462 whereof will be 35 Fathom and 4 Foot high, the reſt will be lower, according to the Difference of the Ground, but no leſs than 5 Foot and 6 Inches high. There will be to the ſaid *Aqueduct* 861 *Arches*, which, where they are higheſt, will have 12 Fathom in Breadth, and 8 Fathom in Thickneſs, diminiſhing to 14 Foot at the Top. The other *Arches* will be leſſer in Breadth as well as Thickneſs, according to the Nature of the Ground. The ſaid *Aqueduct* will have 15 Inches Fall to every Thouſand Fathom in Length; ſo that for the 7000 Fathom, there will be 8 Foot 8 Inches Fall. The River is to paſs by *Maintenon*, *le Parc Eſpernon*, *Gageran*, *Rambouillet*, *les Eſſars*, *le Perrey*, *Cognieres*, and from thence to *Verſailles*. There are 14000 Soldiers that work there, under the Command of the Marqueſs *d'Uxelles*, with three Commiſſaries of War for their Conduct.

n. 176. *p.* 1206. 2. A *Magazine* for the Waters upon the Mountain *Manthoron*, is already cut, which will have 2200 Perches of *Surface* (each Perch being 8 *French* Foot) and 12 Foot in Depth. In another Place, much lower, will be another *Magazine*, to receive the Waters of many *Pools*, the moſt part of which,

as

as yet have no Water in them. In the Valley of *Buc* will be an *Aqueduct*, the Middle whereof will be raiſed 22 Fathom high, for conveying the Pools of *Sarle*, which, it's ſaid, contain much Water, tho' there be nothing but Rain to fill them. This *Aqueduct* is 300 Fathom long, and paſſes thro' two Mountains which have been cut thro' upon that Account. The Valley alſo on both ſides of the *Aqueduct* is raiſed 11 Fathom high, to make Paſſages.

An *Aqueduct* is alſo making near the Tower of *Stone*, where the Mills raiſe the Water, which will now paſs without Force to the Top of the Mountain: and there be part of it diſtributed into ſeveral great *Ciſterns*, which are making about *Marli* for that Place.

The Elevation of the *Aqueduct* of *Maintenon* is now ſet forth at but 2560 Fathom; whereas it was deſigned to be carried on more than 8000 Fathom, and the Remainder will be made of Earth, which muſt be brought thither; this Opinion prevails, in regard it gives a quicker Diſpatch, tho' it may be doubted, it will not be for the better.

Theſe 2560 Fathoms contain 242 *Arcades*, whoſe *Aperture* is 6 Fathom and $\frac{1}{2}$, and the *Face* of each *Pillar* ſuſtaining the *Arches*, 4 Fathom; there will be then on the ſide of *Maintenon* 33 *Single Arches*, afterwards 71 *Double* ones; (as having one *Arch* upon another) then 46 *Treble* ones; which will generally be 216 Foot 6 Inches high, (*viz.* up to the Floor of the *Channel*) afterwards 72 *Double* ones; then 20 *Single*, which will reach to the Mount of *Earth*, that is to be 50 Foot high.

From the Ground up to the *Second Arcade*, are 16 Fathom; from the *Second* to the *Third*, or upper *Arcade*, are 14 Fathom, (which *Arcades* are Double in Number to thoſe they ſtand upon) and 6 Fathom 6 Inches more to the Floor of the *Channel*, which will at leaſt be 7 Foot high, beſides the *Parapet*.

The *Pillars* by the Ground are 8 Fathom thick; but, what with the *Slopes* and *Shortnings*, which are made in every Story, the Top, where the *Channel* goes, will be but 20 Foot broad. There will likewiſe be at each Pillar a *Buttreſs* jetting out one Fathom, and two Fathom wide.

The intelligent Obſerver, tho' well ſkill'd in things of this Nature, as being no Stranger to the *Writings* of the Ancients, or the famous Ruins and Remainders of their *Fabricks* in *Italy*, and other Places, profeſſes himſelf ſurpriz'd with the Greatneſs of this Undertaking at *Verſailles*, and *Maintenon*; for the Magnificence of the Deſign, the Number of Labourers, the Exceſſiveneſs of the Expence, and the admirable Beauty of the Work.

A very large Stone Chimney, with a peculiar ſort of Arch-Work; by Dr. J. Wallis. n. 166. p. 800. *Fig.* 217.

VIII. Having been lately at *Edgecot* in *Northamptonſhire*, at the Houſe of *Tobias Chancy*, Eſq; he ſhewed me in an Antient Kitchen (now diſuſed) two *Chimneys*, vaſtly large, of *Stone-Work*: Which I took the more Notice of, becauſe of a peculiar Way of *Arch-Work* in the Front of them; whereby, without the Advantage of a *Diſcharge* of *Timber* (which is uſual, in ſuch Caſes, to defend the *Arch-Work* from being overburden'd) an *Arch* of maſſy Stone (in each of them) ſuſtains itſelf at a great Length, tho' almoſt upon a Flat, being very little rais'd in the middle. Over this Arch (after fome walling interpofed) there is another Arch (to defend the former) more raifed from the Flat. The Dimenfions of all, I have thought fit here to fubjoin.

AB, The Breadth between the *Jambs*, from Infide to Infide, 18 Foot.

CD, The Depth of the *Stones* in the Lower Arch, 22 Inches; locked one into another, with a crooked Joint.

DE, The Diftance in Walling, between the Arches, 2 Foot and 7 Inches.

EF, The Depth of the *Stones* for the Upper Arch, 15 Inches: With a ftreight Joint.

GH, The Place of two vaft *Tunnels* of Stone.

K, a *Window* between them.

A new Kind of Stairs; by M. Weighelius. n. 74. p. 2212.

IX. M. *Weighelius* hath lately invented an odd *Bridge*, or kind of Stairs, by which a Man fhall defcend, and yet really be raifed upward; and going as 'twere upon a Plain fhall from a lower, by gently fubfiding, arrive to an upper Story.

Preferving of Ships from being Worm-eaten; by . . . . n. 11. p. 190. Apr. An. 1666.

X. In the *Indian Seas*, there is a kind of fmall *Worms* that faften themfelves to the *Timber* of the Ships, and fo pierce them that they take Water every where; or if they do not altogether pierce them thro', they fo weaken the Wood, that it is almoft impoffible to repair them. Some have employed *Deal*, *Hair* and *Lime*, &c. and therewith Lined their Ships; but befides that, this does not altogether affright the *Worms*, it retards much the Ship's Courfe. The *Portugufe* fcorch their Ships, infomuch that in the *Quick Works* there is made a Coaly Cruft of about an Inch thick. But as this is dangerous, it happening not feldom that the whole Veffel is burnt; fo the Reafon why the *Worms* eat not thro' *Portugal* Ships, is conceived to be the exceeding Hardnefs of the *Timber* employed by them. There is in *Holland* a Man that pretends to have found an admirable Secret to remedy this Evil. And a very worthy Perfon in *London* fuggefts the *Pitch*, drawn out of *Sea-Coals*, for a good Remedy to fcare away thofe noifome *Infects*.

An Account of Lead Sheathing; by Mr. J. Bulteel. n. 100. p. 6192. Jan. An. 1674.

XI. Some few Years fince, Sir *Phil. Howard* and Major *Watfon*, with great Charge and Induftry found out a new Way, by a Manufacture of our own, to preferve the Hulls of Ships from the Worms, *&c.* which is much fmoother and confequently better for Sailing, and more cheap and durable than the Way of *Boards*, *Pitch*, *Tar*, *Rofin*, *Brimftone*, or any *Sheathing* or *Graving* hitherto ufed. The *King* and *Parliament* being fatisfied, upon Examination, of the great Benefit that might redound thereby to his *Majefty* and Subjects in general, for the Inventors Encouragement to make the fame publick, were pleafed, almoft 4 Years fince, to grant them an *Act* of *Parliament* for the fole Ufe of this their Invention, with Penalty and Prohibition to all others. In Profecution whereof, Experiments have been made upon feveral of his *Majefty*'s

Majesty's Ships, *viz.* the *Phœnix*, done three Years ago, has made two Voyages into the *Streights*, &c. and when ſhe was lately taken into the *Dock* at *Woolwick* to be repaired, upon View of the *Maſter Shipwright* and others, her Sheathing was found to be in as good Condition, as at the firſt doing; and the Ship ſo tight during the whole time, that they were forced to heave in Water to keep her ſweet. The *Dreadnought*, a Third Rate, done in *June* 1671; the *Henrietta*, *Lyon*, and *Mary*, all three of the Third Rate, and done a Year and half ſince, being lately laid on Ground at *Sheerneſs* and *Portſmouth*, are found to be all in as good Condition, and the Sheathing to continue as firm and as well as at the firſt doing; as the *Maſter-Builder* and *Aſſiſtant* at *Portſmouth*, and others, have certified.

The *Bread Rooms* alſo of ſome of theſe, and many others of his *Majeſty*'s Ships, have been lined within, almoſt in the ſame manner the *Sheathing* is without; which has prov'd a great Preſervation of the Bread, as ſeveral of the *Purſers* and *Officers* of the ſaid Ships have certified; and by Reaſon of its Duration muſt be much cheaper and better than *Tin*, which is ſo liable to ruſt, or any Way yet uſed.

Alſo the Lead itſelf (which is the principal thing uſed herein) they make ſo cloſe preſſed, ſmooth, and equal, or of what Thickneſs or Thinneſs deſired, that great Uſe may be made thereof about ſeveral other things relating to *Shipping*.

XII. *A Paper of leſs General Uſe* Omitted, *viz.*

Directions for Inquiries concerning *Stones* and other *Materials* for the Uſe of *Building*. n. 93. p. 6010.

XIII. Accounts of *Books* Omitted.

1. *Vitruvius* done into *Engliſh*; by Mr. *Chr. Waſe.* n. 72. p. 2190.

Les dix Livres d' *Architecture* de *Vitruve*, corrigez, & traduits nouvellement en *Francois*, avec des Notes & des Figures; par *Claude Perrault*. *Paris* 1637. in Folio. n. 112. p. 279.

2. Cours d' *Architecture*, enſeigne dans l' *Academy Royale* d' *Architecture*, premiere Partie; par M. *Francois Blondel*, à *Paris*, 1675, in Fol. n. 112. p. 549.

3. *Raphaelis Fabretti Urbinatis* de Aquis & Aquæductibus Veteris *Romæ*, Diſſertationes tres, *Romæ* 1680. in 4*to*. n. 155. p. 466.
n. 95. p. 6071.

4. Modern Fortification, *&c.* by Sir *Jonas Moor*, 1673. in 8*vo*. n. 158. p. 586.

5. Nouvelle Maniere de *Fortifier* les Places; par M. *Blondel*. *Hague*, 1684.

6. *Marci Meibomii* de Fabrica *Triremium* Liber. *Amſtelodami* 1671. in 4*to*. n. 79. p. 3071.

7. *Scheeps-Boow en Beſtier*, that is, *Naval Architecture* and *Conduct*; by *N. Witſen*. *Amſterdam* 1671 in *Folio*. n. 77. p. 3006.

8. L' *Architecture Navale*, avec le *Routier* des *Indes Orientales* & *Occidentales*: par le Sieur *Daſſié*, à *Paris*. 1677. in 4*to*. n. 135. p. 879.

CHAP.

CHAP. IX.

Perspective. Sculpture. Painting.

A Perspective Instrument; by Sir Chr. Wren. n 45 p 898. *Fig.* 218. Mar. An. 1669.

I. A Is a small Sight with a short Arm *B*, which may be turned round about, and moved up and down the small *Cylinder* C D, which is screwed into the Piece E D, at D: This Piece E D moving round about the *Center* E; by which means the Sight may be removed either towards R, or F.

EF is a Ruler fastened on the two *Rulers* GG, which *Rulers* serve both to keep the Square *Frame* S S S S perpendicular, and, by their sliding thro' the square Holes T T, they serve to stay the Sight, either farther from, or nearer to the said Frame; on which Frame is stuck on with a little Wax the Paper OOOO, whereon the *Picture* is to be *drawn* by the *Pen* I. This *Pen* I is by a small Brass Handle V so fix'd to the *Ruler* H H, that the Point I may be kept very firm, so as always to touch the Paper. HH is a *Ruler*, that is always, by means of the small Strings *aaa*, *bbb*, moved *Horizontally*, or Parallel to itself; at the End of which is stuck a small Pin, whose Head P is the Sight, which is to be moved up and down on the Out-Lines of any *Object*.

The Contrivance of the *Strings* is this. The two *Strings a a a*, *b b b*, are exactly of an equal Length. Two Ends of them are fasten'd into a small Leaden Weight Q Q, which is moved in a Socket on the backside of the Frame, and serves exactly to counterpoise the *Ruler* H H, being of equal Weight with it. The other two Ends of them are fasten'd to two small Pins H H, after they have rolled about the small Pulleys N, M M, L L, K K; by means of which Pulleys, if the Pen I be taken hold of, and moved up and down the Paper, the Strings moving very easily, the Ruler will always remain in an Horizontal Position.

The Manner of using it is this: Set the *Instrument* upon a Table, and fix the Sight A, at what Height above the Table, and at what Distance from the Frame S S S S, you please. Then, looking thro' the Sight A, and holding the *Pen* I in your Hand, move the Head of the *Pin* P up and down the Out-Lines of the Object, and the Point I will describe on the Paper OOOO, the Shape of the Object so traced.

A new way of Delineating by Parallel Visual Rays, exactly observing the Symmetry; by Mr. St. Clare, n. *96. p.* 6080. *Fig.* 219. July, An. 1673.

II. ABCD is the *Prosopographick Parallelogram*, H F the Central Style, L C the describing Quill, K A the Index, or an oblong Ruler adapted at right Angles to the Plain of the *Parallelogram*, by means of the winding brass Pin according to the Pattern E. To this Ruler are fixt two Sights P R, S V; in the Middle of P R is a Hole bored at O, and in the Middle of S V a Thread is erected perpendicular to the Ruler R A, in the Middle of which is a certain little Globe or Bead, through which and the Hole O a

Ray

Ray is extended from the Object to the Eye, which while you delineate ſhould not be fixt, but free and at liberty.

1. It is to be obſerved, that a Ray extended through the Hole O and the Bead, will always be perpendicular to the Plain of the *Parallelogram*, or to its Diameter, which is a right Line extended through the deſcribing Style L C, and the fixt Center F, and the ſaid little Globe or Bead, in which Line the Bead will always be found, however the *Parallelogram* may be moved.

2. The ſenſible Delineatory Plain, upon which the Point L moves about, which is the Nib of the drawing Style L C, deſcribing the Image exactly according to the Motion of the Index K A, and upon which the Central Style H F is fixt, is the Plain Q Y X T; but the mere Rational or Mathematical Plain, being a Continuation of the former, is $\varepsilon\delta\beta\gamma$.

3. That all the Rays extended from the Object through the Bead and the Hole O to the Eye, (which is placed in ſo many Points of a tranſparent Medium, by means of the Index K A, as there are Points in the viſible Superficies of the Object to be deſcribed, which are infinite,) will always be parallel to one another.

Perhaps ſome may object, that in Objects at a great Diſtance there can be no uſe of the Sights. But which way will that concern us, ſince this Method of ours is deſign'd only for removing thoſe Difficulties in Delineation, which hitherto have attended the *Parallelogram* of *Scheiner*. For I have often found by experience, (tho' for this the Artiſt does not think the worſe of his Inſtrument,) that by no means there is that Proportion between the Parts of the Copy in the Plain, as there is in the Parts of the Original at a Diſtance.

A Method of caſting Statues of an extraordinary Thinneſs; by M. John Weichard Valvaſor. n. 186. *p.* 259. *Fig.* 220. Jan. An. 1687.

III. I here ſend you a Method of *Caſting Statues* in *Metal*, in Obedience to the Commands of the *Royal Society*; it is as follows. Firſt, I form out of good Clay, that will endure the Fire, and not crack either in drying or burning, ſuch a Figure or *Statue* as I deſire to caſt; when this is well dry, I make, all over the Figure, little Holes of no great Depth (but both Size and Depth proportionate to the Bigneſs of the *Statue*) into which I let ſmall Pieces of *Metal*, and with ſome of the ſame *Clay* fix them firmly in the *Holes*; the Uſe of theſe Bits of *Metal*, *a a a a a*, is to keep the *Core* and *Mould* from touching one another, or falling together when the *Wax* runs out; and that they may remain conſtantly in the ſame fixt Poſture. This done, I ſcrape away with ſome proper Inſtrument, as much of the Clay in thickneſs as I deſign for the thickneſs of my *Statue*; and then laying it in a *Furnace*, I burn the *Core* till it be red hot: (by the *Core* is meant always the *Statue* firſt made in *Clay*,) when it is cold, I rub the *Core* all over with that ſort of *Earth* or Colour, which our *German Potters* uſe to colour the Joints of the Tiles, when they ſet *Stoves* of Tiles or (*Kachel-Ofens*:) this Colour much reſembles *Black Lead* which is uſed to *Deſign* on Paper, and eaſily wipes out with Bread, but it is not the ſame. This Colour I mix with Water, and daub all over the *Core*, becauſe the Metal is found to run freely upon it. There are other Subſtances proper for this Uſe, but I have always made uſe of this, eſpecially for thin *Statues*. This done, I lay upon the *Core* as much yellow Wax mix-

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ed with Pitch or Rofin, as will make the Thicknefs of the intended Statue, which I form in the Wax, with all the Exactnefs poffible.

Here note, that the Particles of *Metal* mentioned to be fet into the *Core*, to keep it a Diftance from the *Mould*, muft be fo fet as to fall in with the Surface of the Wax exactly; and that the Reafon of mixing Pitch or Rofin with the Wax is, becaufe that when it is burnt out, it makes a great Smoke, and that Smoke adhering to the *Mould*, occafions the *Metal* to run more freely; as I have experienced it. Next I put all over upon the Surface of this *Statue* of Wax, little Pieces of Wax, which I call the little *Channels c c c c c*, (all which muft be contrived fo as to enter the great *Channels d d d.*) This done, I cover the *Core* and Wax all over with the fame fort of Clay, that will endure the Fire without cracking; and fo I have my *Concave Statue* or *Mould* made. Upon this I lay the great *Channels* marked *d d d d*, both upright and tranfverfe, formed likewife in Wax, and placed according to Judgment, fo as beft to receive the Ends of the little *Channels c c c c c*, for the more eafy diftribution of the *Metal*. The great *Channels* muft all meet at the top of the *Statue*, fo as to come out by one Hole, as at E, where the Metal is to be poured in: It is alfo neceffary to have a *Channel* or two to let out the Air as the *Metal* enters, as thofe marked *f f*, and there muft be a Hole or two left at the Foot, as *g g*, where the great *Channels* and *waxen Statue* join; and whereat, when the Mould is burnt, the Wax as well of the *Statue* as of the *Channels* may run out. The great *Channels* being thus placed, the Mould muft be again laid over with the fame fort of Clay; (I ufe conftantly to bind about the Mould with Iron Wire, and then lay on more Clay) and when this Mould is well dry, then I heat it red-hot; as I did before the *Core*, fo now both together.

I burn the *Core* firft, that there may not need fo ftrong a Fire to burn the Mould as will melt the fmall bits of Metal: but for fmall manageable *Statues*, of not above a Foot or two high, they may be both burnt together, and there is no need of the Holes *g g*, but the Mould may be inverted, and the Wax run out by the *Channels f f*, and E.

The Mould being thus burnt, I ftop with the fame Clay the two Holes *g g*, and then I bury it in a Pit, and proceed as is ufual in *Cafting* of *Bells* and the like; but care muft be taken, that the *Metal* be very well in Fufion.

If it be a fmall *Statue*, not above a Foot or two high, whofe *Mould* may be managed in one's Hands; then I make me a *Concave Statue* of Wax, of the Thicknefs I defire, and then place upon it all thofe great and leffer Channels, as afore: Which done, I put it all together, into a liquid Subftance made of Plaifter and Tile, or Brick Duft tempered with Water.

If the *Statue* be intended very thin, then I take Copper, and when it is well in Fufion, I mix with it a good quantity of *Zinc*, without obferving any certain Proportion of Weight; the more *Zinc* the better the *Metal* runs. I have fometimes for fmall and thin *Statues* put in above a third part of *Zinc*. I have found by Experience, that this Mineral makes the *Metal* run moft freely, and gives it a fair Golden Colour.

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The Statue being caſt, I take off the *Mould*, and cut off all the little *Channels*; all which, both great and ſmall, are filled with *Metal*, which may be kept for farther Uſe: In theſe there is much more *Metal* than in the whole Statue; for, if the Statue be very thin, there muſt be more and bigger *Channels*, and ſo, the cheaper the Statue, the more weighty the *Channels*, and the more *Metal* remaining.

To know the Quantity of *Metal* requiſite for any intended Work, I take a Lump of the ſame mixture of Wax and Pitch, with which I make the *Mould* of my Statue; and having weighed it, I make a *Mould* upon it, and caſt in the ſame a Lump of *Metal* of the ſame Size; which I weigh, and thereby compute the Proportion of the Weight of the *Metal* and *Wax*; and then, obſerving how many Pounds of Wax I uſe about the *Figure* and Channels, I can calculate to a ſmall matter how much *Metal* I need to melt.

Hitherto I have caſt no Statue above 9 Foot high, but I doubt not but I could, by the ſame Methods, caſt one of any Bigneſs deſired.

A Deſcription of ſome Simple Colours; Mr. Rich. Waller. n. 179 p. 26,30. Jan. An. 1686.

IV. 1. *Spaniſh White* is made of Chalk and Allum burnt together.

Simple Blues.

2. I take the *Lapis Armenius* to be the *Blue Bice* ſold in the Shops, for it is light and friable; formerly brought out of *Armenia*, now from the Silver *Mines* of *Germany*, called *Melochites*, in *High-Dutch*, *Berghblaw*.

3. *Ultramarine* is made of the blueſt *Lapis Lazuli*, which is freeſt from Gold Veins, by Calcination.

4. *Smalt* is made of *Zaffer* and *Pot Aſhes* calcined together in a Glaſs Furnace.

5. *Litmaſe* or *Litmoſe*, I ſuppoſe the Juice of a Plant.

6. *Indigo*, ſaid by *Pliny* to be brought from *India*; a kind of *Mud* adhering to the *Froth* about *Reeds*, and that when tried with a Coal, the true burns with a purple *Flame*, and ſmells of the *Sea*. *Linſchoten* ſays it is called *Anil*, that it grows in *Cabaia*, and is a Plant like *Roſemary*, which is gathered and dried, then wetted with fair Water, and beaten to a Mud.

7. *India Ink*; its Uſe is known to *Pliny*, tho' not its Compoſition; which is yet undiſcovered, except it ſhould be burnt *Rice*, as hath been thought.

Simple Yellows and Reds.

1. *Ceruſe* is the *Ruſt* of *Lead*, made by a vaporous Calcination. *Pliny* writes thus of it in *Cap.* 34. *Lib.* 18. *Ceruſe Pſimithium* is made in the Plumbers Shops, of ſmall Plates of *Lead* laid upon a Veſſel of ſtrong *Vinegar*; what falls into the *Vinegar* is taken out and dried in the *Sun:* and in *Cap.* 6. *Lib.* 35. he ſays it was made at *Rome* of burnt *Marble Flint* quenched in Vinegar.

2. *Maſticot* is a kind of improper *Calx* or *Tin*.

3. *Gutta Cambæ*, or *Cambodia*, the inſpiſſated Juice of a Plant, not well known; it comes from both the *Indies*. Some think it the Juice of *Euphorbium*; others *Scammony* or *Tithymal*; others *Ricinus*; others refer it to the greater *Cataputia*, *Eſula*, or the *Flowers* of the *Indian Ricinus*, and will have it coloured with *Turmerick*; as *Schroder*.

4. *Oker*, a kind of natural Earth. There are two ſorts thereof, the one *Native*, formerly brought out of *Africa*, now from *Dacia* and *Hungary*, and from many Places in *England*, eſpecially in the Foreſt of *Dean:* The other, a factitious Subſtance of *Lead* burnt and quenched in Vinegar. In *Pliny*'s Time it was made of *Rubrica*, or *Ruddle* burnt.

5. *Orpiment*, a fat inflammable Mineral, juſtly ranked amongſt Poiſon for its extreme corroſive Quality. *Pliny* ſays, it was dug up in *Syria* on the Surface of the *Earth*, and that the Emperor *Caligula* had hopes of getting *Gold* out of it; wherefore he cauſed 14 Pounds of it to be tried, which afforded him very good *Gold*, but in ſo ſmall a Proportion, that he loſt by the Trial.

6. *Umber* is a native Earth.

7. *Red-Lead*, a Colour unknown to the Antients, made of *Litharge*, or burnt *Lead*, by a *Reverberatory Calcination*, or of *Ceruſe* put in a Platter over the Fire, which muſt be continually ſtirred till it has acquired a *Red-Lead* Colour. Dr. *Charleton de Foſ.*

8. *Burnt Oker* is the common *Yellow Oker* burnt in the open Fire.

9. *Cinnabar*, or *Vermilion*. There are two ſorts, *Native*, or the *Minium* of the *Antients*, which is the Mineral that yields *Quickſilver*; whereof, and of *Sulphur*, it chiefly conſiſts; it is found in the Mines of *Iſtria*. This Colour was amongſt the *Antient Romans* uſed to ſacred Purpoſes, and on Feſtivals *Jupiter*'s Face was painted therewith, as likewiſe the Bodies of thoſe that entred in Triumph. The *Factitious Cinnabar* is that which we now uſe, and is made by a Sublimation of *Mercury* and *Sulphur*.

10. *Carmin*, made of *Cochineel*.

11. *Lake*, thought to be an *Arabick* Word: It is made of *Flocks* dyed, or ſhavings of ſcarlet Cloth, or of the *Cochineel Inſect*, or elſe of *Kermes-Berries*, their Tincture being extracted with a Lye of *Pot-Aſhes*, and then precipitated with a Solution of *Roch-Allom*. After the ſame manner a *Lake* may be made of any Plant or Flower. There is another ſort of *Lake* made of *Gum-lac*, by extracting its Tincture with Urine.

12 *Sanguis Graconis* is the Gum of a Tree, which looks like dried Blood; 'tis brought out of ſeveral Places in the *Eaſt-Indies*.

13. *Engliſh Reddle*, or *Ruddle*, is found in many Places of *England*; amongſt the reſt, near *Witney* in *Oxfordſhire*.

14. *Lamp-black*, by *Pliny* thus deſcribed: 'Tis made of the Soot of Roſin, or Pitch burnt, Houſes being built on purpoſe for it, that keep in the Smoak.

To make China Varniſhes; *by* *Dr.* Will Sherard. n. 262. *p.* 525. Mar. An. 1700.

V. This way of making ſeveral *China Varniſhes* was firſt ſent from the *Jeſuits* in *China* to the great Duke of *Tuſcany*.

Take of *Crude Varniſh* 60 Ounces, ordinary Water 60 Ounces, mix them well together till the Water diſappears, afterwards put this matter into a wooden Veſſel 5 or 6 Palms long, and 2 or 3 broad, mix them with a wooden *Spatula*, for a whole Day in the *Summer*'s Sun, and for two in the *Winter*. It is afterwards kept in earthen Veſſels with a Bladder over it, and cool. This is the *Varniſh prepared in the Sun*.

Boyling the Oyl of Wood.

Take 20 Ounces of the Oil, called *Oil of Wood*, of that of the *Fruit* 10 Drams; give them 5 or 6 Boils, till it comes to be a little *Yellow*. Let it cool, and put to it 5 Drams of *Quick-lime* powdered.

To give the firſt Grounds called Catniſcia.

Take *Swines Blood* and *Quick-lime* powder'd, mix them well, lay this mixture on the Wood, and, when it is dry, ſmooth it with Pumice-Stones.

Take of the *Varnish* prepared in the Sun, 60 Ounces, *Stone-black Allum*, (suppose to be a sort of *Copperas*) dissolved in a little Water, 3 Drams; 70 Drams of *Lamp-Oil*, call'd by the *Portugueze Azeite de Candea*. It is prepared in a wooden Vessel, as the prepared *Varnish*, observing to put in the *Lamp-Oil* at twice. *To make black Varnish.*

Take of the Oil of *Wood* crude, (called by the *Portugueze Azeite de Pao*) 40 Drams, of the Lamp-Oil, called *de Candea*, Crude, 40 Drams: It is prepared in the Sun in a wooden Vessel, as the prepared Varnish. *Pitch-colour'd Varnish.*

Take 10 Drams of *Cinnabar*, 20 Drams of *Varnish* prepared, a little Oil *de Candea*, or Lamp-Oil; mix them well. *Red Varnish.*

Take of the *Yellow Colour* 10 Drams, 30 Drams of the prepared *Varnish*, with some *Lamp-Oil*. *Yellow Varnish.*

Take of the *Red Varnish* 10 Drams, of the *Black Varnish* 4 Drams; mix them well. *Musk-colour'd Varnish.*

VI. M. *Colbert* being pleased some time since to visit the *Academy Royal* for the Improvement of *Painting* and *Sculpture*, expressed himself to this Effect, That he thought it proper from time to time that the Works of the most *excellent Painters* should be *examined*, and such Observations made thereon as would inform others wherein the *Perfection* of a *Picture* consists. Which hath been ever since practised among them, as the best Means to carry the Art of *Painting* to its highest Perfection; such an *Examen* of the best *Pictures* disclosing many Secrets of that Art, for which there are no Rules, and opening a door to debate many important Questions, not hitherto treated of. *An Examen of Pictures propos'd; by M. Colbert. n. 47. p. 953. May. An. 1669.*

VII. Here is a Man who makes more lively Counterfeits of Nature in *Wax*, than ever I yet saw in *Painting*, having an extraordinary Address in *modelling* the *Figures*, and mixing the *Colours* and *Shadows*; making the Eyes so lively, that they seem to kill, excelling all things of this Art I ever beheld. *Wax Work, and a new kind of Maps in Low Relievo, in France; by .... n. 6. p. 99. Nov. An. 1665.*

I have also seen a new kind of *Maps* in *Low Relievo*, or Sculpture; for Example, the Isle of *Antibe*, upon a Square about 8 Foot, made of Boards, with a Frame like a *Picture*: There is represented the Sea, with Ships, and other Vessels artificially made, with their Cannons, and other Tackle of Wood fix'd upon the Surface, after a new and most admirable manner; the Rocks about the Island exactly form'd, as they are upon the natural Place; and the Island itself with all its Inequalities, and Hills and Dales; the Town, the Forts, the little Houses, Plat-form, and Cannons mounted; and even the Gardens, and Plat-forms of Trees, with their green Leaves standing upright, as if they were growing, in their natural Colours; *in fine*, Men, Beasts, and whatever you may imagine to have any Protuberancy above the Level of the Sea. This new delightful, and most instructive Form of a *Map*, or wooden Country, you are to look upon either Horizontally, or side long, and it affords equally a very pleasant Object.

VIII. Whether the way mentioned by *Kircher* in his *Mundus Subterraneus* will succeed or not, is much doubted by some experienced Men: But 'tis certain, *To Colour Marbles; by .... n. 7. p. 125. Dec. An. 1665.*

tain, that a *Stone-cutter* in *Oxford*, Mr. *Bird*, hath many Years ſince found out a way of doing the ſame thing, in effect, that is there mentioned; and hath practiſed it for many Years: That is, he is able ſo to apply a *Colour* to the Out-ſide of poliſhed *Marble*, as that it ſhall ſink a conſiderable *Depth* into the Body of the Stone, and there repreſent like Figures or Images as thoſe are on the out-ſide; deeper or ſhallower, according as he continues the Application a longer or leſſer while.

An Extraordinary Tincture given to a Stone; by Dr. Salomon Reiſel. n. 179. p. 22.

IX. A Goldſmith at *Stutgard*, a very ingenious Ingraver, by Name *Chriſtopher Muller*, in the Year 1685. while he was rubbing ſome *Aurum fulminans* in a little Scuttle made of *Chalcedony* of the pellucid *Onyx* Colour, or of a horney Caſt, mixed with red Glaſs prepared for Fuſion, and moiſtened with Spring Water, in order to make Enamel, (of which *Antonius Nerius*, as tranſlated by *Andrea Friſio* treats in his Book upon the Art of making Glaſs) found upon repeating it for the third Time, that the Purple Colour of the Powder, which had remained ſome Days dry in the Veſſel, and had ſpread as far as the Lips of it in rubbing, had penetrated this exceeding hard Stone, which the File can make no Impreſſion upon, ſo deep not only in the Scuttle, but in the Piſtle itſelf, and had mark'd it with Spots or pretty regular Circles, that it could neither be taken out by ſimple Water, nor Lixive, or any other aerid Liquor, and all this without hurting the exquiſite Poliſh of the Stone, leaving however ſome Spots here and there ſtill of the *Onyx* Colour. This was repeated ſeveral times in another Veſſel of the ſame Colour, without ever having the ſame Effect. But what is chiefly to be obſerved in the colouring of this Stone, is, that according to the Texture of the Gem, as appeared both to the naked Eye and a Glaſs, in the external Part of the Veſſel, where the Tincture has juſt reached, you can perceive Fibres or circular Layers, according to which it is probable that it encreaſed in its Bulk, freſh Layers of ſtony Matter being laid upon one another; as *Bezoar* and other Stones are encreaſed by new Lamina, and alſo Trees, in whoſe Trunks the Circles or Rings denote the Number of Years that the Tree was a growing. And this purple Colour appears lighter or darker, according as the tinging Stuff met with larger or leſſer Pores, a harder or ſofter Texture of the Stone, and produces circular Streaks ſurrounding a Kind of Vertex, as you ſee round the Pith, or Cor, as it is called, the Grain, or Chaff in other Stones and Wood; intermixed here and there with darker Spots. As the illuſtrious *Boyle* in his *Specim. de Orig. & Virt. Gemmarum.* Sect. 1. p. 22, 23. has obſerved the Points and Joinings of thin Layers or Planes in *Adamant* and *Granates*; which Artiſts call the *Grain*, or ſomething reſembling the Texture of cleft Wood.

Several Writers have mentioned that *Marble*, *Alabaſter* and *Bones* may be tinged by lixivious and acrid Juices: and this may perhaps be expected of Gems, ſince the above mentioned *Boyle*, Sect. 2. p. 123. ſays that a Tincture may evidently be extracted from them; and elſewhere, p. 43 and 190. that *Rock Cryſtal* may be tinged by Mineral Acids, and p. 45. the Sapphire itſelf by ſubterraneous Vapours.

Seeing then it is plain from this Obſervation, that the Gem called the *Chalcedony* was really tinged, tho' it happened accidentally, nor by repeating the Proceſs could the ſame Effect be produced, it deſerves however to be conſidered, whether it was not owing to the Influence of the Stars, or rather ſome other hidden Quality, and a Trial to be made, whether from a Mixture of Salts and acrid Juices there might not be produced a Tincture of that Kind, and even without the Aſſiſtance of Fire, by which the Splendor and Pellucidity of the Gem ſhall not be deſtroyed and its Hardneſs ſtill remain, and therefore its Value be not only preſerved, but even increaſed by its new acquired Colour.

X. *Papers* omitted.

1. A Deſcription of *Scheiner's Stereographick Parallelogram*, and its *Imperfections* conſidered; by Mr. *J. St. Clare*, vid. ſup. Sect. II. n. 96. p. 6085.

2. A Table of *Simple* and *mix'd Colours*, in *Latin*, *Greek*, *French*, and *Engliſh*: with a Specimen of each Colour prefix'd to its proper *Name*; by Mr. *Rich. Waller*. n. 179. p. 24, 29.

XI. *Accounts of Books* omitted.

1. Entretiens ſur les Vies & ſur les Ouvrages des plus excellens *Peintres, Antiens* & *Modernes*; par M. *Felibien*. n. 21. p. 383.

2. An *Idea* of the *Perfection* of *Painting*: Originally written in *French*, by *Roland Freart Sieur de Cambray*; and render'd into *Engliſh* by *J. Evelin*, Eſq; F. R. S. *Lond.* 1668. in 8*vo*. n. 39. p. 784.

3. A General *Idea* of the *Art* of *Painting*, and Relation of ſeven *Conferences* held at *Paris* in the *Academy Royal*, for the Improvement of the *Arts* of *Painting* and *Sculpture*. n. 47. p. 954.

4. Optique de *Portraiture* & *Peinture*, contenant la *Perſpective*, *Speculative*, & *Pratique* Accomplie; &c. Par *Gregoire Huret*, de l' *Academie Royal* de *Peinture* & *Sculpture*. A *Paris*, 1670, *in Fol.* n. 86. p. 5048.

CHAP.

CHAP. X.

Musick.

Of the Trembling of Consonant Strings; by Dr. Wallis. n. 134. p. 839. Mar. An. 1677.

I. IT hath been long since observed, That if a *Viol-string*, or *Lute-string*, be touched with the Bow or Hand, another *String* on the same or another Instrument not far from it, (if an *Unison* to it, or an *Octave*, or the like) will at the same time tremble of its own Accord. But I can now add, That not the whole of that other String doth thus tremble, but the several Parts severally, according as they are *Unisons* to the whole, or the Parts of that String so struck. For Instance, Supposing A C to be an upper *Octave* to $\alpha\,\gamma$, and therefore an *Unison* to each Half of it, stopped at β. If, while $\alpha\,\gamma$ is open, A C be struck; the two Halves of this other, that is, $\alpha\,\beta$, and $\beta\,\gamma$, will both tremble; but not the middle Point at β. Which will easily be observed, if a little Bit of Paper be lightly wrapt about the String $\alpha\,\gamma$, and removed successively from one End of the String to the other.

Fig. 221.

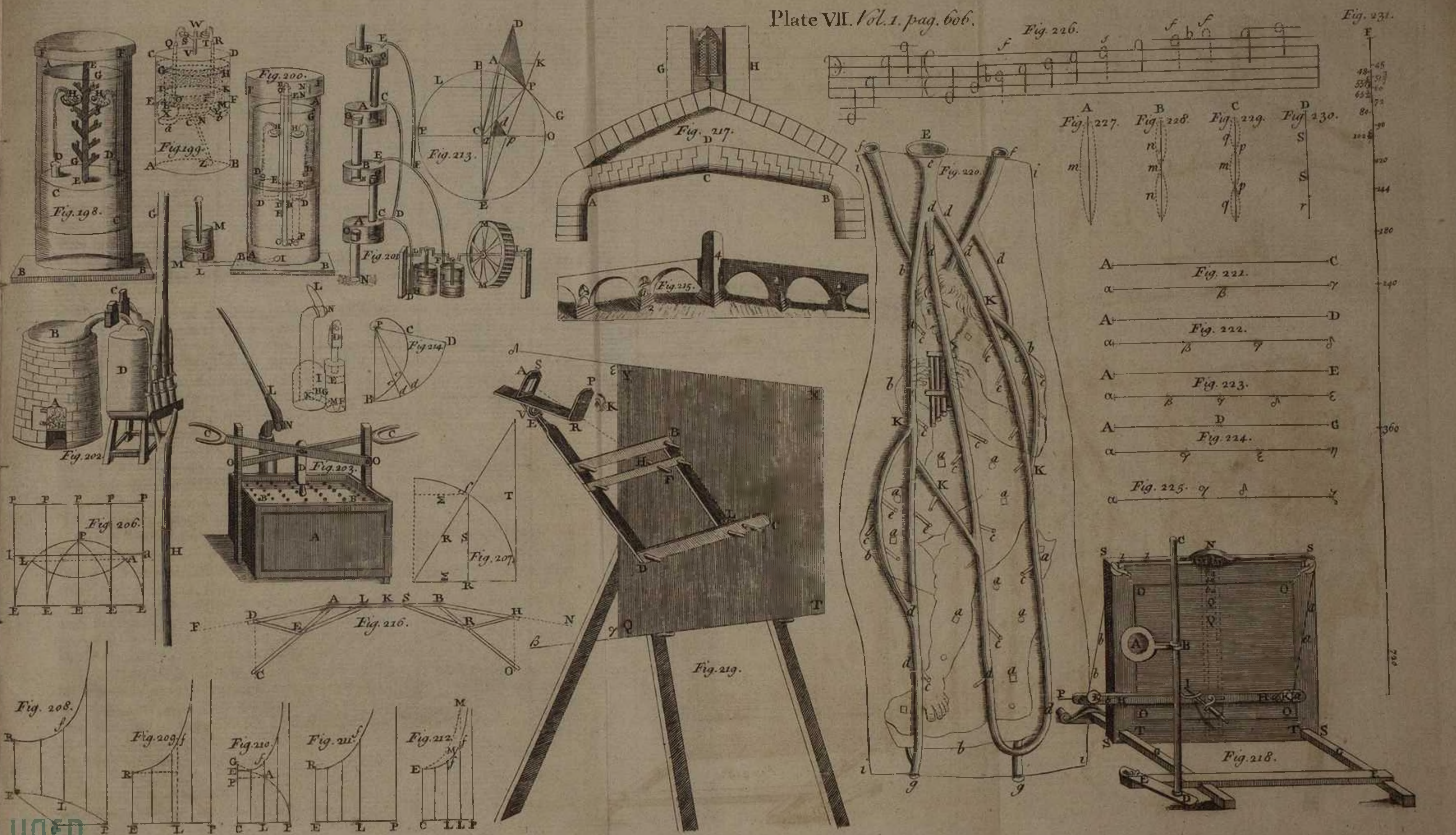
In like manner, If A D be an Upper *Twelfth* to $\alpha\,\delta$, and consequently an *Unison* to its three Parts equally divided in β, γ; if $\alpha\,\delta$, being open, A D be struck, its three Parts $\alpha\,\beta$, $\beta\,\gamma$, $\gamma\,\delta$, will severally tremble, but not the Points $\beta\,\gamma$. In like manner, if A E be a *double Octave* to $\alpha\,\varepsilon$, the four Quarters of this will tremble when that is struck, but not the Points β, γ, δ. So if A G be a *Fifth* to $\alpha\,\eta$; and consequently each Half of that stopped in D, an *Unison* to each third Part of this stopped in $\beta\,\gamma$; while that is struck, each Part of this will tremble severally, but not the Points β, γ; and while this is struck, each of that will tremble, but not the Point D. The like will hold in lesser *Concords*; but the less remarkably, as the Number of Divisions increases.

Fig. 222. *Fig.* 223. *Fig.* 224.

This was first of all, (as I know of) discovered by Mr. *Will. Noble*, M. A. of *Merton College*; and by him shewed to some of our *Musicians*, about three Years since; and after him by Mr. *Tho. Pigot*, A. B. of *Wadham College*, without knowing that Mr. *Noble* had discover'd it before. I add this further, (which I took Notice of upon Occasion of making Trial of the other) that the same *String*, as $\alpha\,\gamma$, being struck in the midst of β, each Part being *Unison* to the other, will give no *clear Sound* at all, but very confused. And not only so (which others have observed, that a *String* doth not sound clear, if struck in the midst) but also, if $\alpha\,\delta$ be struck at β, or γ, where one Part is an *Octave* to the other; and in like manner, if $\alpha\,\varepsilon$ be struck at β or δ; the one Part being a *double Octave* to the other. And so if $\alpha\,\zeta$ be struck in γ or δ; the one Part being a *Fifth* to the other; and so in other like *Consonant* Divisions; but still the less remarkable, as the Number of Divi-

Fig. 225.

Plate VII. Vol. 1. pag. 606.



Diviſions encreaſeth. This and the former I judge to depend upon one and the ſame Cauſe, *viz.* the contemporary Vibrations of the ſeveral *Uniſon* Parts which make the one tremble at the Motion of the other: But, when ſtruck at the reſpective Points of Diviſions, the Sound is incongruous, by Reaſon the Point is diſturbed, which ſhould be at reſt.

A *Lute-ſtring* or *Viol-ſtring* will alſo thus anſwer to a *Conſonant Note* in *Wind Inſtruments*: But not ſo remarkably to the *Wire-ſtrings* of an *Harpſichord*: And we feel the Wainſcot Seats, on which we ſit or lean, to tremble conſtantly at certain Notes on the *Organ*, or other *Wind Inſtruments*; as well as at the ſame Notes on a *Baſs Viol.* I have heard alſo (but cannot aver it) of a thin fine *Venice Glaſs* cracked with the ſtrong and laſting Sound of a *Trumpet*, or *Cornet*, (near it) ſounding an *Uniſon* or a *Conſonant Note* that of the *Tone* or *Ting* of the *Glaſs*.

Concerning theſe *Phænomena*, an Exquiſite Solution is given by Dr. *Narciſſus Marſh*, In Dr. *Plot*'s *Natural Hiſtory* of *Oxfordſhire*. n. 155. p. 872.

The Defects of a Trumpet, and Trumpet-Marine; *by Mr.* Fran. Roberts. n. 195. p. 559. Fig. 226 Oct. An. 1692.

II. The *Extent* of the *Trumpet* cannot be ſtrictly determined; it reaches as *High* as the Strength of the Breadth can force it: But by conſidering its *Notes* within the ordinary Compaſs of the Scale of *Muſick* (from *Double C-fa-ut* to *C-ſol-fa in alt*) the Nature of the higher Notes will plainly appear. Theſe are all ſet down in the Table; only take Notice, that the *Prick'd Notes* are imperfect, not exactly in Tune, but a little *Flatter* or *Sharper* than the Places where they ſtand, according as *f* or *s* is ſet over them.

Here we may make two Enquiries.

1. Whence it comes to paſs, that the *Trumpet* will perform no other *Notes* (in that Compaſs) but only thoſe in the Table, which are uſually called by *Muſicians Trumpet-Notes:*

2. What is the Reaſon that the *7th*, *11th*, *13th* and *14th Notes* are out of Tune, and the others exactly in Tune.

In this Matter we may receive ſome Light from the *Trumpet Marine*, an Inſtrument, tho' as unlike as poſſible to the *Trumpet* in its Frame, one being a *Wind Inſtrument*, the other a *Monochord*, yet has a wonderful Agreement with it in its Effect.

The Sound is ſo like, as not to be eaſily diſtinguiſhed by the niceſt Ear, and, as it performs the very ſame *Notes*, ſo it has the ſame Defects as a *Trumpet*; for, if the *Strings* be ſtopt in any part but ſuch as produces a *Trumpet-Note*: it yields a harſh and uncouth (not a *Muſical*) *Sound*.

Let us therefore proceed to our firſt Inquiry, and examine what is the Reaſon that the *Trumpet Marine* will perform no other but the *Trumpet-Notes*. It is a known *Experiment* of two *Uniſon Strings*, that ſtriking one of them moves the other; which probably proceeds from hence, that the Impulſes of the Air, which are made by one String, do more eaſily ſet another in Motion, which lies in a Diſpoſition to have its *Vibrations Synchronous* to them, than a Third, whoſe Motion would be croſs.

We may improve this a little farther, by obſerving that a String will move not only at the Striking of an *Uniſon*, but an *8th* or *12th*, tho' after a different Manner.

If

Fig. 227. If an *Unison* be ftruck, it makes one intire Vibration in the whole String, and the Motion is moft fenfibly in the midft at *m*, for there the Vibrations take the greateft Scope.

Fig. 228. If an *8th* is ftruck, it makes two Vibrations; and the Point *m* is in a manner Quiefcent, and the moft fenfible Motion at *n*, *n*.

Fig. 229. If a *12th* be ftruck, then it makes three Vibrations: and the greateft Motion at *q*, *m*, *q*; and hardly to be perceived at *p*, *p*. So that in fhort, this *Experiment* holds when any *Note* is ftruck which is an *Unison* to half the String, and a *12th* to a third Part of it.

Fig. 230. In this Cafe, (the Vibrations of the equal parts of a String being fynchronous) there is no Contrariety in the Motion to hinder each other; whereas it is otherwife, if a *Note* is *Unifon* to S, that does not divide the String into equal Parts; for then the Vibrations of the Remainder *r*, not fuiting with thofe of the other Parts, immediately make a Confufion in the whole.

Now in the *Trumpet-Marine*, you do not ftop clofe, as in other *Inftruments*, but touch the String gently with your Thumb, whereby there is a mutual Concurrence of the upper and lower part of the String to produce the Sound. This is fufficiently evident from that, That if any thing touches the String below the Stop, the Sound will be as effectually fpoiled, as if it were laid upon that Part which is immediately ftruck with the *Bow*. From hence therefore we may collect, that the *Trumpet Marine* yields no *Mufical* Sound, but when the Stop makes the upper part of the *String* an *Aliquot* of the Remainder and confequently of the whole Otherwife, as we juft now remarked, the Vibrations of the Parts will ftop one another, and make a Sound fuitable to their Motion, altogether confufed.

Now that thefe *Aliquot Parts* are the very Stops which produce the *Trumpet-Notes*, fhall be plainly fhewn in the treating of the fecond Inquiry, *viz.* What is the Reafon that the *7th*, *11th*, *13th*, *14th Notes* are out of Tune; and the reft exactly in Tune.

All Writers of the Mathematical Part of *Mufick* agree,

That by Shortning a String	Half a Third Part a Fourth a Fifth a Sixth	the Sound is raifed	an Eighth a Fifth a Fourth a Sharp Third a Flat Third.

From this Foundation all the other *Notes* are derived. The Flat and Sharp Sixth are to be the Flat and fharp *Third* to the *Fourth*, and the *7th* the like to the *5th*: The Second to be a Fifth to the Fourth below, &c. By this Rule let us examine what Notes a *Monochord* fretted in its *Aliquot Parts* will produce.

Fig. 231. Suppofe the *Monochord* F to confift of 720 Parts, and its Tone *Double C-fa-ut*, the firft *Note* in the *Table*; then Half of it will be 360, and a third Part 240, &c.

Now

Now I ſay, *Fretting*, (or *Stopping* with the Thumb) at 360 muſt produce *C-fa-ut*; becauſe 360 being half 720, the Sound will riſe an *Eighth* from *double C-fa-ut*. Again 360 being *C-fa-ut*, 240 muſt make *G-ſol-re-ut*, the third Note in the Table; becauſe 240 being juſt a Third-part leſs than 360, the Sound will riſe a *Fifth* from that Note. After the ſame manner proceeding Step by Step it will be evident that,

180	which is leſs than	240	by juſt	a Fourth	produces	*C-ſol-fa-ut* the	fourth	*Note* in the *Table.*
144		180		a Fifth		*E-la-mi*	fifth	
120		144		a Sixth		*G-ſol-re-ut*	6th	
90		180		Half		*C-ſol-fa*	8th	
80		120		a Third		*D-la-ſol*	9th	
72		90		a Fifth		*E-la*	10th	
60		90		a Third		*G-ſol-re-ut*	12th	
48		60		a Fifth		*B-fa-b-mi*	15th	
45		90		Half		*C-ſol-fa*	16th	

By the ſame Reaſon,

100	which is leſs than	120	by juſt	a 6th	produces	*B-fa-b-mi Flat*,
$67\frac{1}{2}$		90		a 4th		*F-fa-ut*,
54		$67\frac{1}{2}$		a 5th		*A-la-mi-re*,
50		100		Half		*B-fa-b-mi Flat*,

And conſequently,

$102\frac{6}{7}$ the	7th	*Note* in the Table, is a little	*Flatter*	than	*B-fa-b-mi Flat*,
$65\frac{5}{11}$	11th		*Sharper*		*F-fa-ut*,
$55\frac{5}{13}$	13th		*Flatter*		*A-la-mi-re*,
$51\frac{3}{7}$	14th		*Flatter*		*B-fa-b-mi Flat*,

Which anſwers the ſecond Inquiry.

Now to apply this (in a few words) to the *Trumpet*, where the *Notes* are produced only by the different Force of the *Breath*; it is reaſonable to imagine that the ſtrongeſt Blaſt *raiſes* the Sound by breaking the Air within the *Tube* into the ſhorteſt Vibrations, but that no *muſical* Sound will riſe, unleſs they are ſuited to ſome *aliquot Part*, and ſo by Reduplication exactly meaſure out the whole Length of the *Inſtrument*, as in *Fig*. 229. for otherwiſe a Remainder will cauſe the ſame Inconvenience in this Caſe, as in *Fig*. 230. To which if we add, that a *Pipe*, being ſhortened acccording to the Proportions we even now diſcourſed of in a *String*, *raiſes* the Sound in the ſame Degrees, it renders the Caſe of the *Trumpet* juſt the ſame with the *Monochord*.

For a *Corollary* to this Diſcourſe, we may obſerve that the Diſtances of the *Trumpet Notes*, aſcending continually, decreaſed in Proportion of $\frac{1}{1}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, *in infinitum*. For,

The

The	Second	Note in the	First	by	$\frac{1}{2}$	of the *String*, &c.
	Third	Table, differs	Second		$\frac{1}{3}$	
	Fourth, &c.	from the	Third, &c.		$\frac{1}{4}$	

The Division of the Monochord; *by Dr.* J. Wallis. n 238. *p.* 80. Mar. An. 1698.

III. Any *String* or *Chord* of a *Musical Instrument Open* (or at its full length) will ſound (what we call) an *Octave* (or *Diapaſon*) to that of the ſame *String* ſtopt in the Middle, or at half its Length. Hence it is, that we commonly aſſign to an *Octave*, the *Duple* Proportion (or that of 2 to 1) becauſe ſuch is the Proportion of Lengths (taken in the ſame *String*) which give thoſe Sounds. And (upon a like Account) we aſſign to a *Fifth* (or *Diapente*) the *Seſqui-alter* Proportion (or that of 3 to 2.) And to a *Fourth* (or *Diateſſaron*) the *Seſqui-tertian* (or that of 4 to 3.) And to a *Tone* (which is the Difference of a *Fourth* and *Fifth*) the *Seſqui-octave* (or that of 9 to 8 :) Becauſe Lengths (taken in the ſame *String*) in theſe Proportions, do give ſuch Sounds.

And (univerſally) whatever Proportion of Lengths (taken in the ſame *String* equally ſtretched) do give ſuch and ſuch Sounds; ſuch Proportions (of Gravity) we aſſign to the Sounds ſo given.

But, when an *Eighth* (or *Octave*) is ſaid (in common Speech) to conſiſt of 12 *Hemi-tones*, or 6 *Tones*; this is not to be underſtood according to the utmoſt Rigour of *Mathematical* Exactneſs, (of ſuch 6 *Tones*) as what they call the *Diazeutick Tone*, or that of *la*, *mi*, which is the Difference of a *Fourth* and *Fifth*; but, as exact enough for common Uſe. For 6 ſuch *Tones*, (that is, the Proportion of 9 to 8, 6 times repeated) is ſomewhat more than that of an *Octave*, (or the Proportion of 2 to 1.) And conſequently, ſuch an *Hemitone*, is ſomewhat more than the Twelfth-part of an *Eighth* or *Octave*, or *Diapaſon*. But the Difference is ſo little, that the Ear can hardly diſtinguiſh it: And therefore (in common Speech) it is uſual ſo to ſpeak.

And, accordingly, when we are directed to take the Lengths (for what are called the 12 *Hemi-tones*) in *Geometrical* Proportion, it is to be underſtood (not to be ſo in the utmoſt Strictneſs, but) to be accurate enough for common Uſe, for placing the *Frets* on the Neck of a *Viol*, or other *Muſical Inſtrument*, wherein a greater Exactneſs is not thought neceſſary. And this is very convenient, becauſe (thus) the Change of the *Key* (upon altering the Seat of *mi*) gives no new Trouble, for this doth indifferently ſerve any *Key*; and the Difference is ſo ſmall, as not to offend the Ear.

But thoſe who chuſe to treat of it with more Exactneſs, go this way to work.

Preſuppoſing the Proportion for an *Octave* (or *Dia-paſon*) to be that of 2 to 1; they divide this into two Proportions; not juſt equal (for that would fall upon the *Surd* Numbers, as $\sqrt{2}$ to 1;) but near equal (ſo as to be expreſſed in ſmall Numbers.) In order to which, inſtead of taking 2 to 1, they take (the Double of theſe Numbers) 4 to 2; (which is the ſame Proportion as before;) and interpoſe the Middle Number 3. And of theſe three Numbers, 4, 3, 2, that of 4 to 3, is the Proportion of a *Fourth* (or *Dia-teſſaron.*) And that of 3 to 2, the Proportion for a *Fifth* (or *Dia-*

pente.) And theſe two, put together, make up that of an *Octave* (or *Diapaſon*, that of 4 to 2, (or 2 to 1.) And the Difference of thoſe two, that of a *Tone*, or 9 to 8. As will plainly appear in the ordinary Method of *multiplying* and *dividing Fractions*, That is, $\frac{4}{3}\times\frac{3}{2}=\frac{4}{2}=\frac{2}{1}$. And $\frac{4}{3})\ \frac{3}{2}\ (\frac{9}{8}$.

Thus in the common *Scale* (or *Gam-ut*) taking an *Octave*, in theſe Notes, *la*, *fa*, *ſol*, *la*, *mi*, *fa*, *ſol*, *la*; ſuppoſe from E to *e* (placing *mi*, *B-fa-b-mi*, which is called the *natural Scale*;) the Lengths for the Extremes *la*, *la*, an *Octave*, are as 2 to 1, or 12 to 6. Thoſe for *la*, *la*, (in *la*, *fa*, *ſol*, *la*,) or *mi*, *la*, (in *mi*, *fa*, *ſol*, *la*,) a *Fourth*, as 4 to 3, or 12 to 9, or 8 to 6. Thoſe for *la*, *mi*, (in *la*, *fa*, *ſol*, *la*, *mi*,) or *la*, *la*, (in *la*, *mi*, *fa*, *ſol*, *la*,) a *Fifth*, as 3 to 2, or 12 to 8, or 9 to 6. Thoſe for *la*, *mi*, the *Diazeutick-Tone* (or Difference of a *Fourth* and *Fifth*, as 9 to 8. So have we for theſe four Notes *la*, *la*, *mi*, *la*, their proportionate Length in the Numbers 12, 9, 8, 6.

Then, if we proceed in like manner to divide a *Fifth* (or *Dia-pente*,) *la*, *fa*, *ſol*, *la*, *mi*, or *la*, *mi*, *fa*, *ſol*, *la*, or the Proportion of 3 to 2, into near Equals, (taking double Numbers in the ſame Proportion, 6, 4; and interpoſing the middle Number 5;) of theſe three Numbers, 6, 5, 4, that of 6 to 5, is the Proportion of a *leſſer Third*, (called a *Tri-hemitone*, or *Tone* and half,) as *la*, *fa*, (in *la*, *mi*, *fa*.) And that of 5 to 4, is the Proportion of the *greater Third*, (commonly called a *Ditone*, or two *Tones*,) as *fa*, *la*, (in *fa*, *ſol*, *la*,) which two put together make a *Fifth*, as 3 to 2; that is $\frac{6}{5}\times\frac{5}{4}=\frac{6}{4}=\frac{3}{2}$; and their Difference is, as 25 to 24: That is $\frac{6}{5})\ \frac{5}{4}\ (\frac{25}{24}$. So have we for theſe 3 Notes, *la*, *fa*, *la*, their proportionate Lengths in Numbers, as 6, 5, 4.

In like manner, if we divide a *Ditone*, (or *greater Third*,) as *fa*, *la*, (in *fa*, *ſol*, *la*,) whoſe Proportion is as 5 to 4, (or 10 to 8,) into two near Equals (by help of a middle Number 9;) then have we (in theſe three Numbers 10, 9, 8,) that of 10 to 9, for (what they call) the *leſſer Tone*: And that of 9 to 8, for (what they call) the *greater Tone*.

But, whether *fa*, *ſol*, ſhall be made the *Leſſer* (as 10 to 9,) and *ſol*, *la*, the *Greater*, (as 9 to 8;) or, This the *Leſſer*, (as 10 to 9,) and that the *Greater*, (as 9 to 8,) or ſome time This, ſome time That, as there is occaſion, (to avoid what they call a *Schiſm*;) is ſomewhat indifferent: For, either way, the Compound will be as 5 to 4; and the Difference (which they call a *Comma*,) as 81 to 80. This is $\frac{9}{8}\times\frac{10}{9}=\frac{10}{9}\times\frac{9}{8}=\frac{10}{8}=\frac{5}{4}$. And $\frac{10}{9})\ \frac{9}{8}\ (\frac{81}{80}$.

Laſtly, if from that of the *Tri-hemi-tone* (or *Leſſer Third*) *la*, *mi*, *fa*, whoſe Proportion is as 6 to 5; we take that of the *Tone*, *la*, *mi*, (which is the Difference of a *Fourth* and *Fifth*) as 9 to 8; there remains for the *Hemi-tone*, *mi*, *fa*, (or *la*, *fa*,) that of 16 to 15. That is $\frac{9}{8}) \frac{6}{5} (\frac{48}{45} = \frac{16}{15}$.

Or, the *Tri-hemi-tone* (or *leſſer Third*) whoſe Proportion is as 6 to 5, may be divided into three near Equals, (by taking triple Numbers, in the ſame Proportion 18, 15; and interpoſing the two Intermediates 17, 16;) which will therefore be as 18 to 17, and as 17 to 16, and as 16 to 15; That is, $\frac{18}{17} \times \frac{17}{16} \times \frac{16}{15} = \frac{18}{15} = \frac{6}{5}$.

Where alſo the *greater Tone*, whoſe Proportion is as 9 to 8 or 18 to 16, is divided into its two near Equals (commonly called *Hemi-tones*,) that of 18 to 17, and that of 17 to 16: That is, $\frac{18}{17} \times \frac{17}{16} = \frac{18}{16} = \frac{9}{8}$.

And the *leſſer Tone*, that of 10 to 9, or 20 to 18, may be in like manner divided into that of 20 to 19, and that of 19 to 18: That is, $\frac{20}{19} \times \frac{19}{18} = \frac{20}{18} = \frac{10}{9}$.

Which Diviſions of the *Greater* and *Leſſer Tone* anſwer to what is wont to be deſigned by *Flats* and *Sharps*.

So that (by this Compoſition) of theſe Eight Notes, *la*, *fa*, *ſol*, *la*, *mi*, *fa*, *ſol*, *la*, their Proportions ſtand thus; that of *la*, *fa*, (or *mi*, *fa*,) is as 16 to 15. That of *fa*, *ſol*, as 10 to 9, and that of *ſol*, *la*, as 9 to 8; (or elſe that of *fa*, *ſol*, as 9 to 8, and that of *ſol*, *la*, as 10 to 9.) That of *la*, *mi*, as 9 to 8. And, if either of the *Tones* (*Greater* or *Leſſer*) chance to be divided (by *Flats* or *Sharps*) into (what they call) *Hemi-tones*, their Proportions are to be ſuch as is already mentioned.

There may be a like Diviſion of a *Fourth*, (or *Dia-teſſaron*) into two Near Equals: And of ſome others of theſe, into three Near Equals. Which might be of uſe for (what they were wont to call) the *Chromatick* and *Enarmonick Muſick*. But, thoſe ſorts of *Muſick* having been long ſince laid aſide, there is now no need of theſe Diviſions, as to the *Muſick* now in Uſe.

The Imperfection of an Organ; by Dr. J. Wallis. n. 242. p. 249. July, An. 1698.

IV. I think 'tis evident that the *Pipe* in the *Organ* is intended to expreſs a Diſtinct Sound at ſuch a *Pitch*; that is, in ſuch a Determinate Degree of *Gravity* or *Acuteneſs*; or (as it is now called) *Flatneſs* or *Sharpneſs*; And the Relative or Comparative Conſideration of Two (or more) ſuch Sounds or Degrees of *Flatneſs* or *Sharpneſs*, is the Ground of (what we call) *Concord* and *Diſcord*; that is a ſoft, or harſh Coincidence.

Now

Now concerning this, there were amongſt the Antient *Greeks*, Two (the moſt conſiderable) Sects of *Muſicians*: the *Ariſtoxenians*, and *Pythagoreans*.

They both agreed thus far; that *Dia-teſſaron*, and *Dia-pente*, do together make up *Dia-paſon*; That is (as we now ſpeak) a *Fourth* and *Fifth* do together make an *Eighth* or *Octave*: And, the Difference of thoſe two, of a *Fourth* and *Fifth*, they agreed to call a *Tone*, which we now call a *Whole Note*.

Such is that, (in our preſent *Muſick*,) of *la*, *mi*, (or, as it was wont to be called, *re*, *mi*.) For *la*, *fa*, *ſol*, *la*, or *mi*, *fa*, *ſol*, *la*, is a perfect *Fourth*: And *la*, *fa*, *ſol*, *la*, *mi*, or *la*, *mi*, *fa*, *ſol*, *la*,, is a perfect *Fifth*: The Difference of which is *la*, *mi*, which is what the *Greeks* call the *Diazeutick-Tone*; which doth disjoin two *Fourths* (on each ſide of it; and being added to either of them doth make a *Fifth*; Which was, in their *Muſick*, that from *Meſe* to *Parameſe*; that is in our *Muſick*, from A to B: Suppoſing *mi* to ſtand in *B-fa-b-mi*, which is accounted its Natural Poſition.

Now in order to this, *Ariſtoxenus* and his Followers did take that of a *Fourth*, as a known *Interval*, by the Judgment of the Ear; and that of a *Fifth* likewiſe: And conſequently that of an *Octave*, as the Aggregate of both; and that of a *Tone*, as the Difference of thoſe Two.

And this of *Tone* (as a known *Interval*) they took as a common Meaſure, by which they did eſtimate other *Intervals*: And accordingly they accounted a *Fourth* to contain two *Tones* and a Half; a *Fifth* to contain three *Tones* and a Half, and conſequently an *Eighth* to contain ſix *Tones*; or five *Tones* and two *Half Tones*.

And at this Rate our *Practical Muſicians* talk of *Notes* and *Half Notes* at this Day; ſuppoſing an *Octave* to conſiſt of twelve *Hemi-tones*, or *Half Notes*.

But, *Pythagoras* and thoſe who follow him, not taking the Ear alone to be a Competent Judge in a Caſe ſo nice, choſe to diſtinguiſh theſe, not by *Equal Intervals*, but by *Due Proportions*: And this is followed by *Zarline*, *Kepler*, *Cartes*, and others, who treat of *Speculative Muſick* in this and the laſt Age. Accordingly they accounted that of an *Octave*, to be, when the Degree of *Gravity*, or *Acuteneſs* of the one Sound to that of the other, is Double, or as 2 to 1; that of a *Fifth*, when it is *Seſqui-alter*, or as 3 to 2: that of a *Fourth* when *Seſqui-tertian*, or 4 to 3. Accounting that the ſweeteſt Proportion, which is expreſs'd in the ſmalleſt Numbers, and therefore (next to the *Uniſon*) that of, an *Octave*, 2 to 1, then that of a *Fifth*, 3 to 2, and then that of a *Fourth*, 4 to 3.

And thus that of a *Fourth* and *Fifth* do together make an *Eighth*; for $\frac{4}{3}\times\frac{2}{3}=\frac{4}{2}=\frac{2}{1}=2$, or the Proportion of 4 to 3, compounded with that of 3 to 2, is the ſame with that of 4 to 2, or 2 to 1. And conſequently, the Difference of thoſe two, which is that of a *Tone*, or Full Note, is that of 9 to 8. For $\frac{4}{3})\ \frac{3}{2}\ (\frac{9}{8}$; or, if out of the Proportion of 3 to 2, we take that of 4 to 3; the Reſult it that of 9 to 8. Now,

Now according to this Computation, it is manifest, that an *Octave* is somewhat less than Six *Full Notes*. For (as was first demonstrated by *Euclid*, and since by others) the Proportion of 9 to 8, being six Times compounded, is somewhat more than that of 2 to 1. For $\frac{9}{8} \times \frac{9}{8} \times \frac{9}{8} \times \frac{9}{8} \times \frac{9}{8} \times \frac{9}{8} = \frac{531441}{262144}$, is more than $\frac{524288}{262144} = \frac{2}{1}$.

This being the Case; they allowed (indisputably) to that of the *Diazeutick Tone* (*la, mi*,) the full Proportion of 9 to 8, as a thing not to be altered; being the Difference of the *Dia-pente* and *Dia-tessaron*, or the *Fifth* and *Fourth*.

All the Difficulty was, How the remaining *Fourth* (*mi*, *fa*, *sol*, *la*,) should be divided into three parts, so as to answer (pretty near) the *Aristoxenians* Two *Tones* and a half: And might, all together, make up the Proportion of 4 to 3, which is that of a *Fourth* or *Dia-tessaron*,

Many Attemps were made to this purpose: And according to those, they give Names to the Different *Genera* or kinds of *Musick*, (the *Diatonick Chromatick*, and *Enarmonick* Kinds,) with the several *Species*, or lesser Distinctions under those Generals.

The first was that of *Euclid* (which did most generally obtain for many Ages:) Which allows to *fa*, *sol*, and to *sol*, *la*, the full Proportion of 9 to 8; And therefore to *fa*, *sol*, *la*, (which we call the *Greater Third*) that of 81 to 64. (For $\frac{9}{8} \times \frac{9}{8} = \frac{81}{64}$.) And, consequently, to that of *Mi*, *fa*, (which is the Remainder to a *Fourth*) that of 256 to 243. For $\frac{81}{64}) \frac{4}{3} (\frac{256}{243}$; that is, if out of the Proportion of 4 to 3 we take that of 81 to 64, the Result is that of 256 to 243. To this they give the Name of *Limma* (λεῖμμα) that is, the Remainder (to wit, over and above two *Tones*.) But, in common Discourse (when we do not pretend to speak nicely, nor intend to be so understood) it is usual to call it an *Hemi-tone*, or *Half-Note*, (as being very near it) and the other, Two *Whole Notes*. And this is what *Ptolemy* calls *Diatonum Ditonum*, (of the *Diatonick* kind with *Two Full Tones*.)

Against this it is objected (as not the most convenient Division) that the Numbers of 81 to 64, are too great for that of a *Ditone*, or *Greater Third*; which is not harsh to the Ear; but is rather sweeter than that of a single *Tone*, whose Proportion is 9 to 8. And, in that of 256 to 243, the Numbers are yet much greater. Whereas there are many Proportions (as $\frac{5}{4}$, $\frac{6}{5}$, $\frac{7}{6}$, $\frac{8}{7}$, in smaller Numbers than that of 9 to 8; of which, in this Division, there is no Notice taken.

To

To rectify this, there is another Division thought more convenient; which is *Ptolemy*'s *Diatonum Intensum* (of the *Diatonick* Kind, more *Intense* or *Acute* than that other) which instead of two *Full Tones* for *fa*, *sol*, *la*, assigns (what we now call) a *Greater* and a *Lesser Tone*; (which by the more Nice *Musicians* of this and the last Age seems to be more embraced;) Assigning to *fa*, *sol*, that of 9 to 8, (which they call the *Greater Tone*;) and to *sol*, *la*, that of 10 to 9, (which they call the *Lesser Tone*:) And therefore to *fa*, *la*, (the *Ditone* or *Greater Third*) that of 5 to 4. (For $\frac{10}{9} \times \frac{9}{8} = \frac{10}{8} = \frac{5}{4}$.) And consequently to *mi*, *fa*, (which is remaining of the *Fourth*) that of 16 to 15. For $\frac{5}{4}$) $\frac{4}{3}$ ($\frac{16}{15}$. That, if out of that of 4 to 3 we take that of 5 to 4, there remains that of 16 to 15.

Many other ways there are (with which I should not trouble you at present) of dividing the *Fourth* or *Dia-tessaron*, or the proportion of 4 to 3, into three Parts, answering to what (in a looser way of expression) we call an *half Note*, and *two whole Notes*. But this of $\frac{16}{15} \times \frac{9}{8} \times \frac{10}{9} = \frac{4}{3}$, is that which is now received as the most proper.

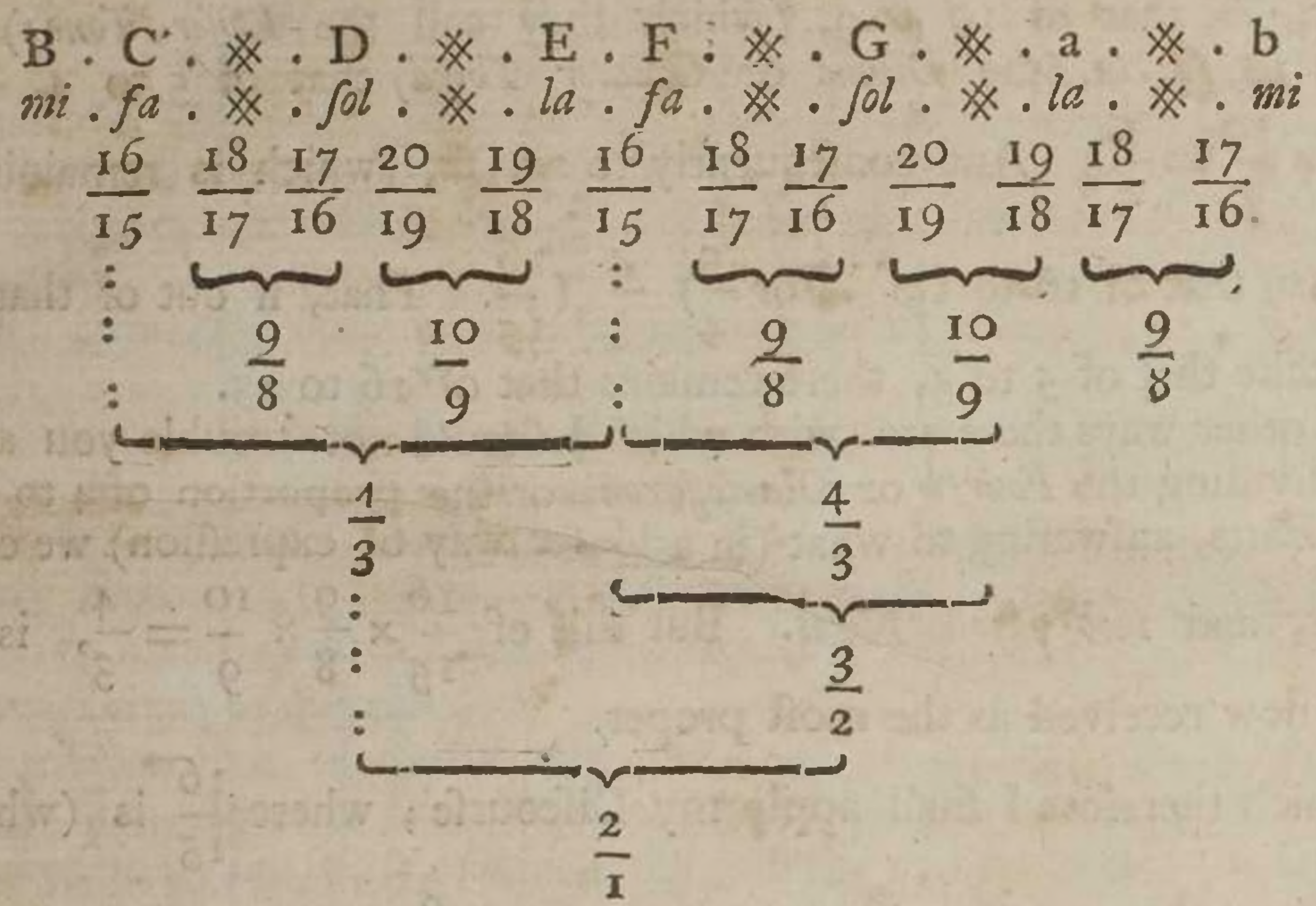
To which therefore I shall apply my Discourse; where $\frac{16}{15}$ is (what we call) the *Hemi-Tone*, or *Half-Note*, in *mi*, *fa*; $\frac{9}{8}$ that of the *greater Tone*, in *fa*, *sol*, and $\frac{10}{9}$ the *lesser Tone*, in *sol*, *la*.

Only with this Addition; that each of those *Tones*, is (upon Occasion) by *Flats* and *Sharps* (as we now speak) divided into two *Hemi-tones*, or *Half Notes*; Which answers to what by the *Greeks* was called *Mutatio quoad Modos* (the Change of *Mood*;) and what is now done by removing *mi* to another *Key*. Namely $\frac{9}{8} = \frac{18}{16} = \frac{18}{17} \times \frac{17}{16}$; and $\frac{10}{9} = \frac{20}{18} = \frac{20}{19} \times \frac{19}{18}$.

Thus by the help of *Flats* and *Sharps* (dividing each *whole Note*, be it the *Greater* or *Lesser*, into two *Half Notes*, or what we call so,) the whole *Octave* is divided into Twelve parts or *Intervals* (contained between Thirteen *Pipes*) which are commonly called *Hemi-tones* or *Half-Notes*; not that each is precisely *Half* a *Note*, but somewhat near it, and so called. And I say, by *Flats* and *Sharps*; for sometime the one, sometime the other is used. As for Instance, a *Flat* in D, or a *Sharp* in C, do either of them denote a Middling Sound (tho' not precisely in the Midst) between C and D; *Sharper* than C, and *Flatter* than D.

Accor-

Accordingly, ſuppoſing *mi* to ſtand in *B fa-b-mi* (which is accounted its *Natural* Seat) the Sounds of each *Pipe* are to bear theſe Proportions to each other, *viz.*



And ſo in each *Octave* ſucceſſively following. And, if the *Pipes* in each *Octave* be fitted to Sounds in theſe Proportions of *Gravity* and *Acuteneſs*, it will be ſuppoſed (according to this *Hypotheſis*) to be perfectly proportioned.

But, inſtead of theſe ſucceſſive Proportions of each *Hemi-tone*; it is found neceſſary (if I do not miſtake the Practice) ſo to order the 13 *Pipes* (containing 12 Intervals which they call *Hemi-tones*) as that their Sounds (as to *Gravity* and *Acuteneſs*) be in continual Proportion, (each to its next following, in *Octave* or *Dia-paſon*, as 2 to 1. Whereby it comes to paſs that each *Pipe* doth not expreſs its proper Sound, but very near it, yet ſomewhat varying from it; which they call *Bearing*; which is ſomewhat of Imperfection in this *Noble Inſtrument*, the Top of all.

It may be aſked, why may not the *Pipes* be ſo ordered, as to have their Sounds in juſt Proportion, as well as thus *Bearing*.

I anſwer, It might very well be ſo, if all *Muſick* were *Compoſed* to the ſame *Key*, or (as the *Greeks* call it) the ſame *Mode*; as for Inſtance, if, in all Compoſitions, *mi*, were always placed in *B-fa-b-mi*. For then the *Pipes* might be ordered in ſuch Proportions as I have now deſigned.

But *Muſical Compoſitions* are made in great Variety of *Modes*, or with great Diverſity in the *Pitch*. *Mi* is not always placed in *B-fa-b-mi*; but ſometimes in *E-la-mi*, ſometimes in *A-la-mi-re*, &c. And (in Sum) there is none of thoſe 12 or 13 *Pipes* but may be made the *Seat* of *mi*. And, if they were exactly

exactly fitted to any one of these Cases, they would be quite out of Order for all the rest.

As for Instance; If *mi* be removed from *B-fa-b-mi*, (by a Flat in B) to *E-la-mi*, instead of the Proportions but now designed, they must be thus ordered,

B	.	※	.	C	.	※	.	D	.	※	.	E	.	F	.	※	.	G	.	※	. a .	b
fa	.	※	.	*sol*	.	※	.	*la*	.	※	.	*mi*	.	*fa*	.	※	.	*sol*	.	※	. *la* .	*fa*.
	$\frac{18}{17}$		$\frac{17}{16}$		$\frac{20}{19}$		$\frac{19}{18}$		$\frac{18}{17}$		$\frac{17}{16}$		$\frac{16}{15}$		$\frac{18}{17}$		$\frac{17}{16}$		$\frac{20}{19}$		$\frac{19}{18}$	$\frac{16}{15}$

where 'tis manifest, that the Removal of *mi* doth quite disorder the whole *Series* of Proportions. And the same would again happen, if *mi* be removed from E to A (by another *Flat* in E,) and again, if removed from A to D. And so perpetually. But the *Hemitones* being made all equal, they do indifferently answer all the Positions of *mi* (tho' not exactly to any;) yet nearer to some than to others. Whence it is, that the same Tune sounds better at one *Key* than at another.

It is asked, whether this may not be remedy'd by interposing more *Pipes*; and thereby dividing a *Note* not only (as now) into *Half Notes*, but into *Quarter-Notes*, or *Half-Quarter-Notes*, &c.

I answer; It may be thus remedy'd in Part; (that is, the Imperfection might thus be somewhat less, and the Sounds somewhat nearer to the just Proportions:) but it can never be exactly true, so long as their Sounds (be they never so many) be in continual Proportion; that is, each to the next subsequent in the same Proportion.

For it hath been long since demonstrated, that there is no such thing as a just *Hemitone* practicable in *Musick* (and the like for the Division of a *Tone* into any Number of equal Parts; three, four or more.) For, supposing the Proportion of a *Tone* or *Full Note*, to be $\frac{9}{8}$ (or, as 9 to 8) that of the *Half-Note* must be as $\sqrt{9}$ to $\sqrt{8}$; that is, as 3 to $\sqrt{8}$. (or 3 to $2\sqrt{2}$) which are *Incommensurable Quantities:* And that of a *Quarter-Note*, as $\sqrt[4]{9}$ to $\sqrt[4]{8}$, which is yet more *Incommensurate*. And the like for any other Number of equal Parts; which will therefore never fall in with the Proportions of Number to Number.

So that this can never be perfectly adjusted for all Keys (without somewhat of *Bearing*) by multiplying Pipes; unless we would for every *Key*, (or every different Seat of *mi*) have a different *Set* of *Pipes*, of which this or that is to be used according as (in the Composition) *mi* is supposed to stand in this or that Seat. Which vast Number of *Pipes*, (for every *Octave*) would vastly increase the Charge; and (when all is done) make the whole impracticable.

A New Tuning of the Lyra Viol; *by S.* Salvetti. n. 87. *p.* 5064. Aug. An. 1672.

V. *S. Salvetti*, about 4 Years ago, invented a *New Tuning* of the *Antient Lyra-Viol* with the usual 13 *Strings*; by Means of which *Tuning*, it is render'd wholly perfect, so that you may express upon it all *Concords*, *Discords*, and also the *Imperfect Concords*, as *sevenths*, *sixths*, &c. as well as upon any *Virginal* that hath the Quarters of Notes upon it. 'Tis true, 'tis only for melancholy and passionate Matter, and not for *Division*, as is the proper Nature of the *Lyra*. I shall only add, that with the abovesaid *Tuning*, it ascends in *Alt* as high as *G-sol-re-ut*; and descends as low as *Double C-fa-ut*; and can make every where the same Concords as above.

The Strange Effects reported of Musick *in Former Times, examined; by Dr.* Wallis n. 243. *p.* 297. Aug. An. 1698

VI. 1. I take it for granted, that much of the Reports concerning the great Effects of *Musick* in former Times, beyond what is to be found in latter Ages, is highly *Hyperbolical*, and next door to Fabulous; and therefore great Abatements must be allowed to the Elogies of their *Musick*.

2. We must consider, That *Musick* (to any tolerable Degree) was then (if not a new, at least) a rare Thing, which the Rusticks, on whom it is reported to have had such Effects, had never heard before; and on such, a little Musick will do great Feats; as we find at this Day a *Fiddle* or a *Bag-pipe* at a Country Morice Dance.

3. We are to consider, that their *Musick* (even after it came to some good Degree of Perfection) was much more plain and simple than ours now-a-days. They had not *Consorts* of two, three, four or more *Parts* or *Voices*: But one single Voice or single Instrument apart which to a rude Ear, is much more taking than more compounded Musick. For that is at a Pitch not above their Capacity; whereas this other confounds it, with a great Noise, but nothing distinguishable to their Capacity.

4. We are to consider, that *Musick* with the *Antients* was of a larger Extent than what we call *Musick* now a days: For *Poetry* and *Dancing* (or comely Motion) were then accounted Parts of *Musick* when *Musick* arrived to some Perfection. Now we know that *Verse* of itself, if in good Measures and affectionate Language, and this set to a *Musical Tune*, and sung by a decent Voice, and accompanied but with *soft Instrumental Musick*, if any, such as not to drown or obscure the *emphatick* Expressions (like what we call *Recitative Musick*) will work strangely upon the Ear, and move all Affections suitable to the *Tune* and *Ditty*; (whether brisk and pleasant, or soft and pitiful, or fierce and angry, or moderate and sedate) especially if attended with a *Gesture* and *Action* suitable. For, 'tis well known, that suitable *Acting* on a *Stage* gives great Life to the Words. Now all this together (which were all Ingredients in what they called *Musick*) must needs operate strongly on the Fancies and Affections of ordinary People unacquainted with such kind of Treatments. For if the deliberate reading of a Romance (when well penn'd) will produce Mirth, Tears, Joy, Grief, Pity, Wrath, or Indignation suitable to the respective Intents of it; much more would it so do, if accompanied with all those Attendants.

5. You

5. You will aſk, perhaps, why may not all this be now done, as well as then? I anſwer, no doubt it may, and with like Effect, if an Addreſs be made in proper Words, with moving Accents in juſt Meaſures, (*Poetical* or *Rhetorical*) with the *emphatick Words*, Words ſet in ſignal Places, pronounced with a good Voice, and a true Accent, and attended with a decent Geſture: all theſe ſuitably adjuſted to the Paſſion, Affection, or Temper of Mind, particularly deſigned to be produced, (be it Joy, Love, Grief, Pity, Courage or Indignation) will certainly *now* as well as *then*, produce great Effects upon the Mind, eſpecially upon a Surprize, and where Perſons are not otherwiſe pre-engaged; and if ſo managed, as that you be (or ſeem to be) in earneſt; and if not over-acted by apparent Affectation.

6. We are to conſider, that the uſual Deſign of what we now call *Muſick*, is very different from that of the *Antients*. What we now call *Muſick* is but what they called *Harmonick*; which was but one Part of their *Muſick*, (conſiſting of Words, Verſe, Voice, Tune, Inſtrument, and Acting;) and we are not to expect the ſame Effect of one Piece, as of the whole.

7. When *Muſick* arrived to great Perfection, it was applied to particular Deſigns of exciting this or that particular Affection, Paſſion, or Temper of Mind; the *Tunes* and *Meaſures* being ſuitably adapted to ſuch Deſigns. But ſuch Deſigns ſeem almoſt quite neglected in our *preſent Muſick.* The chief Deſign now, in our moſt accompliſhed *Muſick*, being to pleaſe the Ear; when by a ſweet Mixture of different *Parts* and *Voices*, with *Cadencies* and *Concord* intermixed, a grateful Sound is produced, which only the judicious *Muſician* can diſcern and diſtinguiſh.

8. 'Tis true, that even this *Compound Muſick* admits of different *Characters:* ſome are more briſk and airy; others more ſedate and grave; others more languid; as the different Subjects do require. But that which is moſt proper to excite particular *Paſſions* or *Diſpoſitions*, is ſuch as is more *Simple*, and *Uncompounded*; ſuch as a Nurſe's *languid Tune*, lulling her Babe to ſleep; or a continual Reading in an *Even Tone*; or even the ſoft Murmur of a little Rivulet, running upon Gravel or Pebbles, inducing a quiet Repoſe of the Spirits: And contrariwiſe, the Briſkneſs of a *Jig*, on a *Kit* or *Violin*, exciting to dance. Which are more operative to ſuch particular Ends, than an *Elaborate Compoſition* of *Full Muſick*.

9. To conclude; If we aim only at pleaſing the *Ear*, by a *ſweet Concert*, I doubt not but our *Modern Compoſitions* may be equal, if not exceed thoſe of the *Antients*; amongſt whom I do not find any Footſteps of what we call ſeveral Parts or Voices, (as *Baſe*, *Treble*, *Mean*, &c. ſung in Concert) anſwering each other, to complete the Muſick. But if we would have our Muſick ſo adjuſted, as to excite particular Paſſions, Affections, or Tempers of Mind, (as that of the *Antients* is ſuppoſed to have done) we muſt apply more ſimple Ingredients, fitted to the Temper we would produce. And this, I doubt not, but a Judicious Compoſer may ſo effect, that (with the Help of ſuch *Hyperboles*, as with which the *Antient Muſick* is wont to be ſet off) our *Muſick* may be ſaid to do as great Feats as any of theirs.

VII. *Ac-*

VII. *Account of Books*, Omitted.

n. 143. p. 20. 1. *Claudii Ptolemæi Harmonicorum* Libri tres, Ex Cod. MSS. undecim, nunc primum *Græce* editi. *Jo. Wallis*, S. S. Th. D. Recensuit, Edidit, Versione & Notis Illustravit, & Auctuarium adjecit. *Oxon* 1682. in 4*to.*

n. 231. p. 668. 2. *Porphyrii* Commentarius in Librum Primum *Harmonicorum Claudii Ptolemæi :* atque *Manuelis Bryennii* Commentarius in tres Libros *Harmonicos* ejusdem *Ptolemæi* (Qui soli restant ex *Græcis Musicæ* Scriptoribus nondum editis.) *Græce* ac *Latine*. Curâ *Jo. Wallisii*, S. Th. D. *Fol.*

n. 80. p. 3095. 3. An Essay to the Advancement of *Musick*; by *Tho. Salmon*, M. A. *Lond.* 1672. in 8*vo.*

n. 90. p. 5153. n. 100. p. 7000. 4. *Syntagma Musicæ*; Treating of *Musick Philosophically*, *Mathematically*, and *Practically*; by *J. Birchensha*, Esq; This Book was preparing for the Press, 1674.

n. 100. p. 6194. 5. *Musica* Speculativa del *Mengoli*. in *Bologna* 1670. *in* 4*to.*

n. 133. p. 835. 6. A *Philosophical Essay of Musick. Lond.* 1677. *in* 4*to.*

n. 208. p. 67. 7. A Treatise of the *Natural Grounds* and *Principles* of *Harmony*; by *Will. Holden*, D. D. *Lond.* 1694. *in* 8*vo.*

FINIS.

PHILOSOPHICAL
TRANSACTIONS

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