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Optimal Parameters Selection in Advanced Multi-Metallic Co-Extrusion Based on Independent MCDM Analytical Approaches and Numerical Simulation

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Abstract: Multi-material co-extrusion is a complex thermo-mechanical forming process used to obtain bimetallic billets. Its complexity is due to the combination of diffusion phenomena in the interface of both materials together with the high temperature and pressure generated and the different flow stress characteristics created by the joining of dissimilar materials. Accordingly, the selection of optimal process parameters becomes key to ensure process feasibility. In this work, a comparison among different multi-criteria decision making (MCDM) methodologies, together with different weighting methods, were applied to the simulation results by using DEFORM3D© software to select the optimal combination of process parameters to fulfil the criteria of minimum damage, extrusion force, and tool wear, together with the maximum reduction in the average grain size.

Keywords: multi-material; co-extrusion; MCDM; titanium; magnesium; FEM

MSC: 90B50

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1. Introduction

One of the main problems that arises during part design processes involves balancing weight reduction and the fulfillment of in-service requirements. This problem is especially critical in industries such as aerospace, where the parts work under severe conditions and the reduction of weight is key to obtain the desired performance of the vehicle. Multi-material forming allows designers to combine the mechanical properties of dissimilar materials being the co-extrusion process one of the most highlighted.

Two of the most widely used alloys in the aerospace and automotive industries are titanium alloys, which present excellent mechanical and physical–chemical properties, as well as a good relationship between strength and weight and high corrosion resistance [1], and magnesium alloys, which possess low density and good specific strength [2].

Several studies about the application of multi-material forming processes using these two alloys have been performed over the years. Among those which can be highlighted are the study by Gall et al. [3], which performed Finite Element Method (FEM) simulation together with experiments on Al–Mg billets into hollow profiles during a bimetallic co-extrusion process. Negendanka et al. [4] studied the effect of the die angle on the formation of the diffusion layer during co-extrusion of a bimetallic billet composed by an Mg-core and Al-sleeve. Lehmann et al. [5], by experimenting with hydrostatic, coextruded Al–Mg compounds. However, there are very few studies that explore the combination of titanium and magnesium alloys together. Some of them are, for example, those carried out by Fernández et al. [6,7], who applied ANOVA to determine the influence of the different process parameters together with the effect of the selection of die material on extrusion force and damage during co-extrusion of a Ti_6Al_4V-AZ31B bimetallic billet.

In order to achieve the maximum productivity and the highest performance, proper selection of the process parameters becomes a significant task. Due to the complexity and the number of variables involved, multi objective optimization presents the best approach to obtain a compromise solution for this problem. However, the wide range of Multi-Criteria Decision Making (MCDM) methods, each one with its own pros and cons, makes its choice the first obstacle to overcome, even more when different results can be obtained when applied to the same problem because of the different methods used to determine the weights, scale the objectives, and so on.

The first MCDM method was applied by Pareto in 1896 [8] with his famous 80/20 principle. Another example is Saaty in 1977 [9], who used multi-criteria models to solve problems with conflicting goals. Since 1980, several MCDM methods have been developed and applied to support decision-making in different areas such as supply chain managing contract selection [10], manufacturing process selection [11,12], and material selection [13,14].

From a literature review, is possible to find several examples of applications and even comparisons among MCDM methods regarding the optimization of manufacturing process parameters [15–17], but there is a lack of studies which examine the step before and compare the different weighting methods [18–21] with the MCMD methods, and their effects on the results obtained, being that most of these studies focused on the TOPSIS method [22–25].

This study develops a methodology of comparison between three weighting methods (AHP, Entropy, and Standard Deviation) and their influence on four different MCDM methods (ARAS, TOPSIS, VIKOR, and COPRAS) when applied to a multi-metallic co-extrusion manufacturing process to obtain the optimal parameters under the principle of minimizing the extrusion force, damage, die wear, and grain size. The results of this paper determine the best combination between weighting methods and MCDM, additionally proving that a compromise solution which brings together criteria as disparate as extrusion force, damage, tool wear, and grain size can be reached.

2. Materials and Methods

2.1. Materials, Geometrical Dimensions and Process Parameters

The materials used in this study are a cylindrical sleeve and core made of a titanium alloy UNS R56400 (Ti_6Al_4V) and magnesium alloy UNS M11211 (AZ31B), respectively.

Figure 1 shows the bimetallic cylinder co-extrusion set up with the initial dimensions.

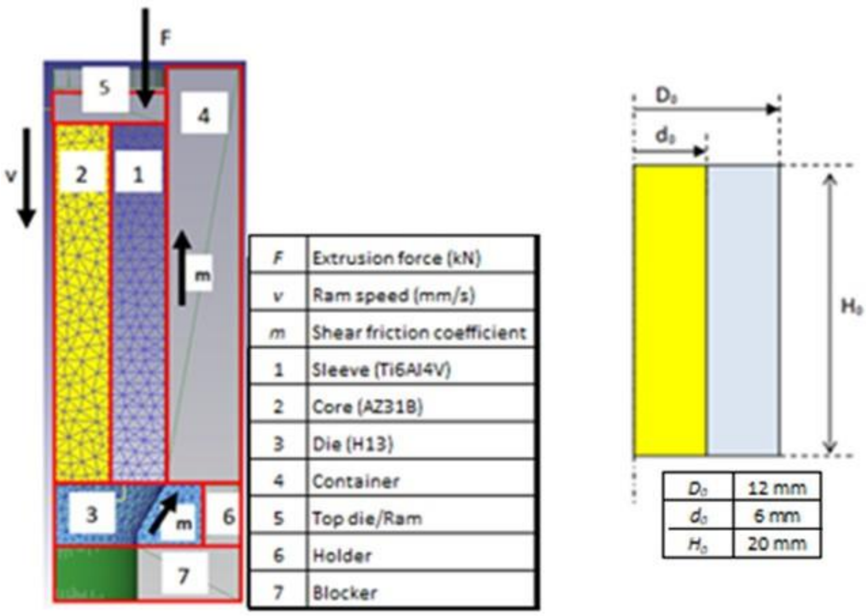

Figure 1. Bimetallic co-extrusion set up and initial billet dimensions.

Chemical compositions, along with the physical and mechanical properties, are shown in Tables 1–4 for the aforementioned materials:

Table 1. Chemical composition of titanium alloy Ti_6Al_4V [26].

Ti (wt.%)	Al (wt.%)	V (wt.%)	Fe (wt.%)	C (wt.%)	O (wt.%)	N (wt.%)	H (wt.%)
Bal.	5.5–6.5	3.5–4.5	0.25	0.08	0.13	0.040	0.012

Table 2. Chemical composition of magnesium alloy AZ31B [27].

Mg (wt.%)	Al (wt.%)	Zn (wt.%)	Mn (wt.%)	Si (wt.%)	Cu (wt.%)	Ca (wt.%)	Fe (wt.%)	Ni (wt.%)
97	2.5–3.5	0.6–1.4	0.20	0.1	0.05	0.04	0.005	0.005

Table 3. Chemical composition of H13 steel [28].

C (wt.%)	Mn (wt.%)	Si (wt.%)	Cr (wt.%)	Mo (wt.%)	Ni (wt.%)
0.32–0.45	0.2–0.5	0.80–1.20	4.75–5.50	1.10–1.75	0.30 max

Table 4. Physical and mechanical properties of the titanium alloy Ti_6Al_4V and magnesium alloy AZ31B [26–29].

Property	AZ31B	Ti_6Al_4V	H13
Density (g/cm^3)	1.74	4.46	7.78
Tensile strength (MPa)	260	895	1990
Yield strength (MPa)	200	828	1650
Elastic modulus (GPa)	44.80	110	210
Poisson's ratio	0.35	0.31	0.3

The parameters affecting the extrusion process considered for this study were the following:

- Ram speed (mm/s) and temperature (°C) as process parameters.
- Die semi-angle (°), shear friction factor, and extrusion ratio (A_0/A_f) as tool parameters.
- Shape factor (H_0/D_0) and diameter ratio (D_0/d_0) as geometric parameters.

Where, A_0 and A_f are the initial and final areas of the cross-section of the billet, D_0 and d_0 are the initial external diameter and internal diameter of the sleeve, and H_0 is the initial billet height.

2.2. Finite Element Modeling and Simulation Preparation

Commercial software DEFORM3D© (v11.2) [30] was used to perform the finite element simulations.

The ram, container, holder, and blocker (extrusion tooling) were modeled as rigid objects. The bimetallic cylinders were modeled as an assembly between two plastic objects (sleeve and core). The die was modeled as an elastic object. All parts were meshed with 7000 tetrahedral elements.

In order to reduce the computation time and the size of the database files, and considering the axial symmetry of the co-extrusion process, only one quarter of the problem was modeled.

Ti_6Al_4V was modeled by using Johnson–Cook constitutive equations [31], and for modeling AZ31B, the exponential model defined by Wen-juan et al. (2012) [32] was used.

The normalized Cockcroft and Latham criterion [33], together with the hydrostatic stress criterion (HSC) [34–36], are used to evaluate the damage factor on the extrudate.

Finally, in order to evaluate the wear of the die and the dynamic recrystallization the Archad's model [37–39] and the Johnson–Melh–Avrami–Kolmogorov (JMAK) model [40,41], respectively, were implemented in the simulations.

2.3. Weighting Methods

The weights of the criteria show their importance. These methods are clustered in three categories. Subjective weighting methods are when the criteria weights are determined dependent of the preferences of decision makers, stakeholders, customer requirements, etc. Objective weighting methods are based on initial data or decision matrix with no involvement of the actors mentioned before. Finally, hybrid weighting methods are a combination of subjective and objective methods, taking features of both methods. Figure 2 shows the weighting methods classification, as well as some example of each.

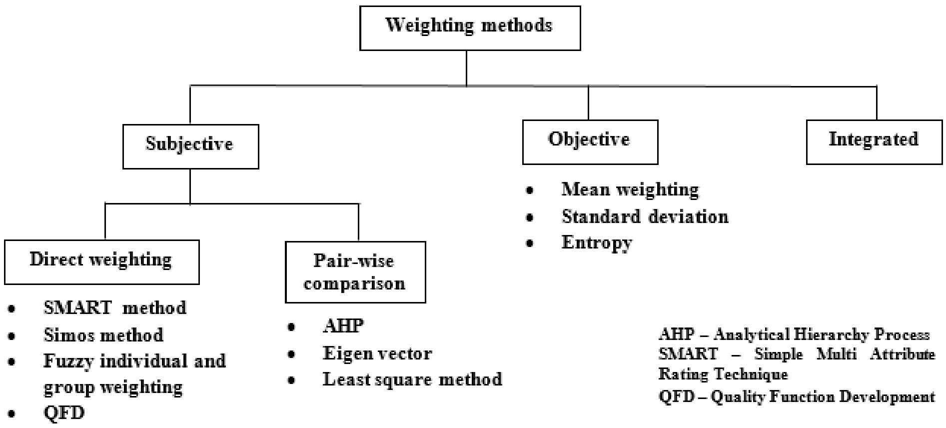

Figure 2. Weighting methods classification.

2.3.1. AHP Method

Analytic Hierarchy Process (AHP) was produced by Thomas L. Saaty in the 1970s [42,43]. It is a structured technique for organizing and analysing complex decisions, based on mathematics and psychology. In this study, AHP is applied to assign weights to the different criteria (extrusion force, damage, tool wear, Ti6Al4V grain size, and AZ31B grain size).

A paired comparison matrix has to be generated by assigning values based on the 9-point Saaty rating scale to the different criteria, as shown in Table 5:

Table 5. The Saaty rating scale [9].

Scale	Numerical Rating	Reciprocal
Extremely preferred	9	1/9
Very strong to extreme	8	1/8
Very strongly preferred	7	1/7
Strongly to very strongly	6	1/6
Strongly preferred	5	1/5
Moderately to strongly	4	1/4
Moderately preferred	3	1/3
Equally to moderately	2	1/2
Equally preferred	1	1

With these values, the $n*n$ pairwise matrix A is generated, as follows:

$$A = \begin{bmatrix} 1 & a_{12} & \ldots & a_{1n} \\ a_{21} & 1 & \ldots & a_{2n} \\ \vdots & \ldots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & 1 \end{bmatrix}$$

The values a_{ij} represent the strength of agreement of ith element respect to jth element. Being a condition that all the values in the diagonal takes value 1 and $a_{ij} = 1/a_{ji}$, where $i, j = 1, 2 \ldots n$.

The next step is to obtain the normalized matrix. The Equation (1) is applied:

$$N^{ij} = \frac{a_{ij}}{\sum_{i=1}^{n} a_{ij}} \; for\ i\ and\ j = 1\ \ldots n. \quad (1)$$

Obtaining this matrix:

$$A^* = \begin{bmatrix} \sum_{i=1}^{n} a_{i1} & \cdots & \sum_{i=1}^{n} a_{in} \end{bmatrix} = [A_1^* \ldots A_n^*]$$

$$N = \begin{bmatrix} N_{12} & N_{12} & \cdots & N_{1n} \\ \vdots & & & \vdots \\ N_{n1} & N_{n2} & \cdots & N_{nn} \end{bmatrix}$$

A first check can be done at this point to ensure that the method is well applied. If the summation of all the elements of each column is equal to 1 then the normalized matrix is correct.

Then, a column matrix composed by the summation of the elements of each row of the normalized matrix can be obtained:

$$N^* = \begin{bmatrix} \sum_{j=1}^{j} N_{1j} \\ \vdots \\ \sum_{j=1}^{n} N_{nj} \end{bmatrix} = \begin{bmatrix} N_1^* \\ \vdots \\ N_n^* \end{bmatrix}$$

Finally, the weights for each criteria are obtained by using Equation (2):

$$W_j = \frac{N_j^*}{\sum N_j^*} \quad (2)$$

A final check is needed to validate the consistency of the measurement scales during the assessment process used to produce matrix A. The recommendation is to calculate the maximal eigenvalue λ_{max}, as shown in Equation (3):

$$\lambda_{max} = \sum (A_i * W_j) \; for\ i\ and\ j = 1\ \ldots n. \quad (3)$$

The consistency index (CI) can be calculated accordingly with Equation (4):

$$CI = \frac{\lambda_{max} - n}{n - 1} \quad (4)$$

where, n is the dimension of pairwise matrix.

The consistency ratio (CR) is used as a guidance value to check for conformity. Equation (5) shows how it can be obtained:

$$CR = \frac{CI}{RI} \quad (5)$$

where, RI is the random index, which is obtained from Table 6, depending on the dimension (n) of our pairwise matrix:

Table 6. Random index values as function of dimension of our pairwise matrix [9].

n	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
CI	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41

The threshold for the *CR* value is 0.1. If this condition is fulfilled, then the importance degree evaluation criteria is assumed to be rational.

2.3.2. Entropy Method

The entropy method [44] is classified within the category of objective weighting methods. It was first proposed by C.E. Shannon in 1948 and is applicable when the data of the decision matrix are known. Entropy is a measure of randomness and disorder in the universe.

First of all, it is necessary to perform the normalization of the arrays of the decision matrix (performance indices) to obtain the project outcomes p_{ij} using Equation (6). Being the decision matrix *D*:

$$D = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix}$$

$$p_{ij} = \frac{x_{ij}}{\sum_{i=0}^{m} x_{ij}} \quad (6)$$

where, *n* is the number of criteria and *m* corresponds with the number of alternatives.

Starting from this normalized matrix, the entropy measure of project outcomes is obtained by means of Equation (7).

$$E_j = -k \sum_{i=1}^{m} p_{ij} \ln p_{ij} \quad (7)$$

with $k = 1/ln\,(m)$.

The objective weight-based definition is given by Equation (8).

$$w_j = \frac{1 - E_j}{\sum_{j=1}^{n} 1 - E_j} \quad (8)$$

2.3.3. Standard Deviation (SD) Method

The SD method [45] is grouped as an objective weighting method and consists of establishing weights based on the standard deviations of the different alternatives from the target.

In order to do that, a normalized matrix is created from the decision matrix *D*, taking into account the beneficial and non-beneficial criteria in accordance with Equation (9) for beneficial and (10) for non-beneficial.

$$D = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix}$$

$$F_{ij} = \frac{x_{ij} - \min x_{ij}}{\max x_{ij} - \min x_{ij}} \quad (9)$$

$$F_{ij} = \frac{\max x_{ij} - x_{ij}}{\max x_{ij} - \min x_{ij}} \quad (10)$$

where, *n* is the number of criteria and *m* the number of alternatives.

Then SD is calculated, as shown in Equation (11):

$$\sigma_j = \sqrt{\frac{\sum_{i=1}^{m} \left(F_{ij} - \overline{F}_j\right)^2}{m}} \tag{11}$$

where, $\overline{F}_j$ is the mean value of each column.

Finally, weights are calculated for each criterion using Equation (12):

$$W_i = \frac{\sigma_j}{\sum_{j=1}^{j} \sigma_j} \tag{12}$$

2.4. MCDM Methods

MCDM methods can be classified in two main groups, according to Hwang and Yoon (1981) [46]: Multi-attribute decision making (MADM) and Multi-objective decision making (MODM).

MADM methods are used to solve discrete problems while MODM are applied towards the resolution of continuous problems. This study is focused on MADM.

MADM can be also clustered depending on the initial information (determinist, stochastic, or uncertain) or depending on the groups of decision makers (single or several groups), but the most common classifications are the ones proposed by Hajkwociz–Collins (2007) [47] and De Brito–Evers (2016) [48]:

- Scoring Methods (COPRAS).
- Distance-based methods (VIKOR and TOPSIS).
- Pair wise comparison methods (AHP).
- Utility/Value methods (ARAS)

2.4.1. ARAS

The Additive Ratio Assessment (ARAS) [17,49] is a method used to select the best alternatives among those given by considering quantitative measurements and utility theory, which determines the relative efficiency. The weight criteria will be those obtained by the methods mentioned before (AHP, Standard Deviation, and Entropy).

In the first step of this method, the definition of the beneficial and non-beneficial criteria of the objective functions is required. After this, the decision matrix can be produced using the following equation, where each column of the matrix represents one of the criteria to be evaluated:

$$D = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix}$$

where, m is the number of alternatives and n is the number of criteria. Typically, this decision matrix is not symmetrical because the number of criteria is less than the number of experimental cases performed.

At this point, normalization is needed to continue, since the variety and unit of the output value differs from the others. By applying a normalization process, the original score is converted into a comparable score by means of Equation (13) for beneficial criteria, and by means of Equations (14) and (15) for non-beneficial criteria:

$$N^{ij} = \frac{x_{ij}}{\sum_{i=0}^{m} x_{ij}} \tag{13}$$

$$x_{ij}^{*} = \frac{1}{x_{ij}} \tag{14}$$

$$N_{ij} = \frac{x^*_{ij}}{\sum_{i=0}^{m} x^*_{ij}} \quad (15)$$

The weight factor matrix (W) is obtained from the multiplication of the normalized value of N and its respective weight factor (previously obtained with AHP, Standard Deviation, and Entropy methods), as Equation (16) shows:

$$W_{ij} = N_{ij} * W_j \quad (16)$$

In order to calculate the degree of utility, first it is necessary to get the optimality function (S_i) for the i^{th} alternative, according to Equation (17):

$$S_i = \sum_{j=1}^{n} W_i\ ;\ i = 0 \ldots m. \quad (17)$$

Finally, the degree of utility (K_i) is determined by the comparison made between each S_i, with the most efficient one (S_0) obtained in the previous step, as can be seen in Equation (18):

$$K_i = \frac{S_i}{S_0} \quad (18)$$

The alternatives are ranked by their value of K_i in an increasing sequence, with the highest value being the best alternative.

2.4.2. TOPSIS

TOPSIS [50,51] is the acronym for Technique for Order Preference by Similarity to Ideal Solution and is a MCDM method initially proposed by Hwang and Yoon in 1981. The concept behind this method is that the best option would be the one closest to the ideal solution, and at the same time, the most remote to the anti-ideal solution.

The first step is to determine the objectives to identify the pertinent evaluation criteria and to define if the objective for each criterion is maximized or minimized. Then, a decision matrix (D) is formulated (same as in the ARAS method).

$$D = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix}$$

The method used to obtain the normalized matrix is slightly different from ARAS, as shown in Equation (19):

$$R_{ij} = \frac{x_{ij}}{\sqrt{\sum x_{ij}^2}};\ i = 1 \ldots m; j = 1 \ldots n. \quad (19)$$

To build the weight-normalized matrix, it is necessary to assign the weights previously calculated by the AHP, SDM, and Entropy methods to the different criteria and then multiply each element of the normalized matrix, as shown in Equation (20):

$$V_{ij} = w_j * R_{ij} \quad (20)$$

Before obtaining the Euclidian distance with the ideal (A^+) and anti-ideal (A^-) solutions, it is required to determine which are the elements of these A^+ and A^-, depending on whether the criteria is to maximize (J) or minimize (J^*), according to Equation (21):

$$A^+ = \{V^+_1 \ldots V^+_n\}\ ;\ Where\ V^+_j = \{max\ V_{ij}\ \ if \in J; min\ \ V_{ij}\ \ if \in J^*\} \quad (21)$$

$$A^- = \{V_1^- \ldots V_n^-\}; \ Where\ V_j^- = \{min\ V_{ij}\ if \in J; max_i\ V_{ij}\ if \in J^*\}$$

Euclidian distance is obtained by Equations (22) and (23):

$$S_i^+ = \sqrt{\sum_{j=1}^{n} \left(V_j^+ - V_{ij}\right)^2} \quad (22)$$

$$S_i^- = \sqrt{\sum_{j=1}^{n} \left(V_j^- - V_{ij}\right)^2} \quad (23)$$

with $i = 1 \ldots m$.

Finally, to settle the relative closeness to the ideal solution, Equation (24) is used:

$$C_i^+ = \frac{S_i^-}{S_i^+ + S_i^-} \quad (24)$$

Now, based on the values obtained, sort the criteria from the highest C_i^+ for the best solution to the lowest C_i^+ for the worst.

2.4.3. VIKOR

The VIKOR method [52,53] is a MCDM originally developed by Serafim Opricovic in 1980 and is an acronym for Serbian VIseKriterijumska Optimizacija I Kompromisno Resenje, which means Multi-criteria Optimization and Compromise Solution.

This methodology is based on the same concept as TOPSIS, which assumes that a compromise solution is acceptable for conflict resolution. The difference of VIKOR in respect to TOPSIS is the addition of a validation step before the compromise solution is declared feasible.

The method begins with the definition of the criteria to be evaluated and the determination of whether the objective is to maximize or minimize each criterion. With this information, the decision matrix (D) is built.

$$D = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix}$$

At this point, the best $f_b{}^*$ and worst $f_b{}^-$ for each criterion is rated according to the values of the decision matrix.

$f_b{}^* = max(x_{ib})$ $f_b{}^- = min(x_{ib})$ Whether the objective is to maximize the criteria.

$f_b{}^* = min(x_{ib})$ $f_b{}^- = max(x_{ib})$ Whether the objective is to minimize the criteria.

Where, $b = 1 \ldots m$, with m being the number of criteria taken into account and $i = 1 \ldots n$, where n is the number of the alternatives considered.

Equations (25) and (26) are used to calculate the Utility measure (S_j) and Regret measure (R_j):

$$S_j = \sum_{b=1}^{m} W\left[\frac{f_b{}^* - f_{ij}}{f_b{}^* - f_b{}^-}\right] \quad (25)$$

$$R_j = max\left[W_b * \left[\frac{f_b{}^* - f_{ij}}{f_b{}^* - f_b{}^-}\right]\right] \quad (26)$$

where, W_b are the weight values obtained by the AHP, Entropy, and Standard Deviation methods explained before.

With these data, the index Q can be obtained by means of Equation (27):

$$Q_a = \upsilon \frac{S_j - S^*}{S^- - S^*} + (1 - \upsilon)\frac{R_j - R^*}{R^- - R^*} \quad (27)$$

where:

$$S^- = max\, S_j$$
$$S^* = min\, S_j$$
$$R^- = max\, S_j$$
$$R^* = min\, S_j$$

υ is a parameter that represents the type of voting used during the process. The rule states that $\upsilon > 0.5$ means "vote by majority rule", $\upsilon = 0.5$ "vote by consensus", and $\upsilon < 0.5$ "with vote".

The best alternative solution is the one with the lowest Q_a value, and it can be recommended if the following conditions are satisfied:

The "acceptable advantage" condition means that $Q(a'') - Q(a') \geq DQ$. With a'' being the alternative with eth second position in the ranking list by Q_a, and a' the first one. DQ is defined by Equation (28):

$$DQ = \frac{1}{(n-1)} \quad (28)$$

where, n is the number of alternatives.

Finally, the "Acceptable stability in decision making" condition implies that the a' alternative must also be the best ranked in S_j and/or R_j. If one of these conditions is not fulfilled, then a set of compromise solutions is proposed.

2.4.4. COPRAS

The COmplex PRoportional ASsessment [54,55] is a MCDM method developed by Zavadskas in 1994, which assumes direct and proportional dependences of the significance and utility degree of the available alternatives under the presence of mutually conflicting criteria.

It is a compensatory method, and as with TOPSIS and VIKOR, it also considers both the ideal and the ideal-worst solutions to solve the problem.

COPRAS begins with the definition of the decision matrix (D), and the normalization of this D, according to Equation (29):

$$D = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix}$$

$$N^{ij} = \frac{x_{ij}}{\sum_{i=0}^{m} x_{ij}} \quad (29)$$

where, m is the number of alternatives.

The weighted normalized decision matrix is obtained by multiplying N_{ij} by the weights from the AHP, Entropy, and Standard Deviation methods, as Equation (30) shows:

$$y_{ij} = N_{ij} * W_j \quad (30)$$

At this point, the beneficial and non-beneficial criteria need to be separated, and then they are summed, as indicated by Equations (31) and (32):

$$S_{+i} = \sum_{j=1}^{n} y_{+ij} \quad (31)$$

$$S_{-i} = \sum_{j=1}^{n} y_{-ij} \quad (32)$$

With n being the number of criteria taken into account.

Then, the relative significance of the alternatives is determined by Equation (33):

$$Q_i = S_{+i} + \frac{S_{-min} * \sum_{i=1}^{m} S_{-i}}{S_{-i} * \sum_{i=1}^{m} (S_{-min}/S_{-i})} \tag{33}$$

where, $S_{-min} = min(S_{-i})$.

Finally, the quantitative utility (U_i) is calculated by Equation (34) and the alternatives are sorted by the highest U_i percentage value.

$$U_i = \frac{Q_i}{Q_{max}} * 100\% \tag{34}$$

where, $Q_{max} = max(Q_i)$.

2.5. Methodology

A methodology to compare MCDM methods based on weight assignment to select the optimum process parameters values is presented in this paper. Both weighting and MCDM methods have been selected based on ease of application, results in earlier works, and their popularity. The weighting methods selected are AHP, Entropy, and Standard Deviation, while the MCDM methods are ARAS, TOPSIS, VIKOR, and COPRAS. The methodology steps are shown in Figure 3.

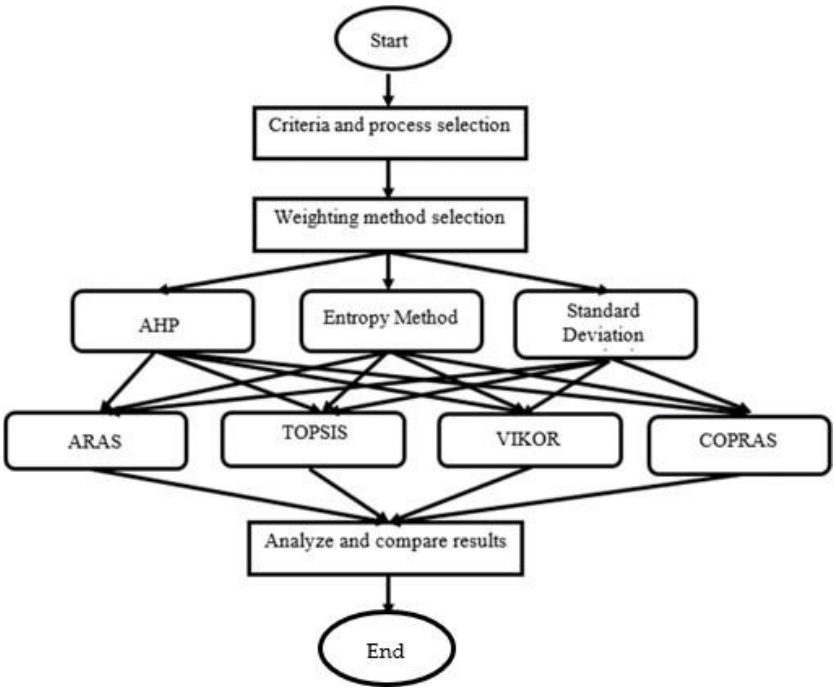

Figure 3. Steps of the methodology.

3. Results

In this paper, a set of simulations of a multi-material co-extrusion process have been performed by using commercial software DEFORM3D© (v11.2), followed by application and comparison of diverse MCDM and weighting methods to establish the optimal process parameters. Table 7 and Figure 4 show the list of simulations carried out in the present work with the process parameters used and the results obtained by each of them.

Table 7. List of simulations with extrusion process parameters.

Number of Simulation	Temperature (°C)	Die Semi-Angle (°)	Ram Speed (mm/s)	Extrusion Ratio (Ao/Af)	Friction	Billet Height (mm)	Core Diameter (mm)
1	350	30	2	1.78	0.10	20	6
2	300	30	2	1.78	0.10	20	6
3	400	30	2	1.78	0.10	20	6
4	450	30	2	1.78	0.10	20	6
5	350	15	2	1.78	0.10	20	6
6	350	45	2	1.78	0.10	20	6
7	350	60	2	1.78	0.10	20	6
8	350	30	1	1.78	0.10	20	6
9	350	30	3	1.78	0.10	20	6
10	350	30	4	1.78	0.10	20	6
11	350	30	2	1.44	0.10	20	6
12	350	30	2	2.25	0.10	20	6
13	350	30	2	1.78	0.05	20	6
14	350	30	2	1.78	0.30	20	6
15	350	30	2	1.78	0.50	20	6
16	350	30	2	1.78	0.10	15	6
17	350	30	2	1.78	0.10	25	6
18	350	30	2	1.78	0.10	30	6
19	350	30	2	1.78	0.10	20	4
20	350	30	2	1.78	0.10	20	8

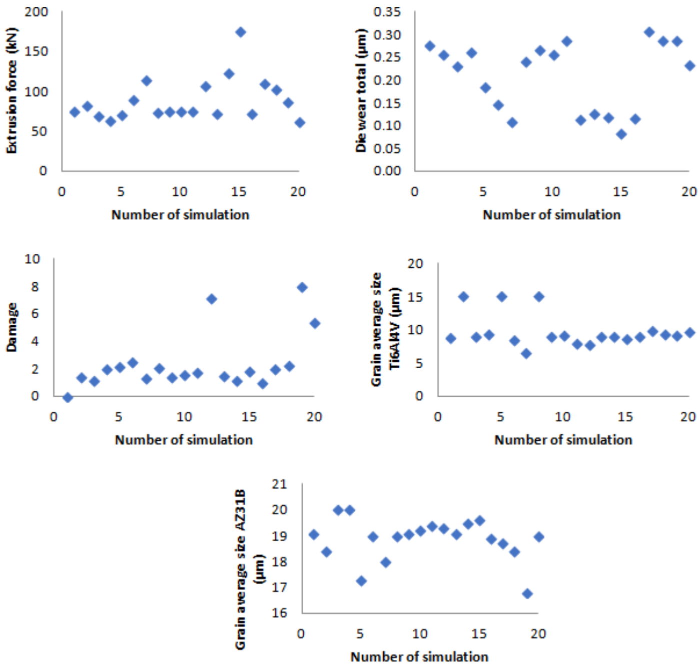

Figure 4. Plot charts of the different co-extrusion criteria obtained during simulations.

3.1. Weighting Methods

In this section, the different weighting methods explained before are applied and compared. For the AHP method, the pairwise matrix is built, as shown in Table 8:

Table 8. AHP pairwise matrix.

Criteria	Extrusion Force	Damage	Tool Wear	Grain Average Size Ti	Grain Average Size Mg
Extrusion Force	1.00	0.33	5.00	3.00	3.00
Damage	3.00	1.00	7.00	5.00	5.00
Tool wear	0.20	0.14	1.00	0.33	0.33
Grain average size Ti	0.33	0.20	3.00	1.00	3.00
Grain average size Mg	0.33	0.20	3.00	0.33	1.00
Total	4.86	1.87	19.00	9.66	12.33

In this pairwise matrix, the order of significance is found to be Damage > Extrusion force > Grain average size Ti > Grain average size Mg > Tool wear. The reasons are as follows:

- Damage is considered the most important factor due to the fact that a high value could mean a central burst or chevron cracking occurrence. In this case, a deeper analysis of the hydrostatic stress using the HSC has to be performed.
- Extrusion force is a factor that can limit the characteristics of the machine to be used. If a high value is required, a more complex machine is needed to perform the process.
- Grain average size is recommended to be as small as possible to enhance the mechanical properties of the final part. The results of simulation AZ31B show a smaller range of variation in the grain size, which is why Ti_6Al_4V is considered moderately preferred.
- Tool wear is an economic factor. The less wear in each extrusion, the longer the change time of the die.

Then, the normalized matrix is obtained:

$$\begin{vmatrix} 0.20547945 & 0.17766497 & 0.26315789 & 0.31034483 & 0.24324324 \\ 0.61643836 & 0.53299492 & 0.36842105 & 0.51724138 & 0.40540541 \\ 0.04109589 & 0.07614213 & 0.05263158 & 0.03448276 & 0.02702703 \\ 0.06849315 & 0.10659898 & 0.15789474 & 0.10344828 & 0.24324324 \\ 0.06849315 & 0.10659898 & 0.15789474 & 0.03448276 & 0.08108108 \end{vmatrix}$$

The weights are calculated from the normalized matrix and shown in Table 9:

Table 9. AHP weights.

	Extrusion Force (kN)	Die Wear Total (μm)	Damage	Grain Average Size Ti6Al4V (μm)	Grain Average Size AZ31B (μm)
In %	0.24	0.04	0.48	0.13	0.08
	24.00%	4.63%	48.81%	13.59%	8.97%

Finally, the consistency ratio is calculated to verify the assumptions taken in the pairwise matrix. In order to do that, the following parameters are calculated:

$\lambda\ max$ = 5.38

CI = 0.09

RI = 1.12

CR = 0.08

The condition $CR < 0.1$ is fulfilled.

For the Entropy method, the normalized matrix is:

$$\begin{vmatrix} 0.04238881 & 0.06597512 & 0.02842942 & 0.04546156 & 0.05055585 \\ 0.04663699 & 0.06099129 & 0.02385686 & 0.07714065 & 0.04870302 \\ 0.03916851 & 0.05496847 & 0.03936382 & 0.04628439 & 0.05293806 \\ 0.03568236 & 0.06225345 & 0.04274354 & 0.04834148 & 0.05293806 \\ 0.03976735 & 0.04418737 & 0.05109344 & 0.07714065 & 0.04579142 \\ 0.05053058 & 0.03514114 & 0.02624254 & 0.04309591 & 0.05029116 \\ 0.06410506 & 0.02586695 & 0.04234592 & 0.03342762 & 0.04764426 \\ 0.04159751 & 0.05770154 & 0.02862823 & 0.07714065 & 0.05029116 \\ 0.04230461 & 0.06371716 & 0.03240557 & 0.04628439 & 0.05055585 \\ 0.04217691 & 0.06142081 & 0.03518887 & 0.04679866 & 0.05082054 \\ 0.0419384 & 0.06847687 & 0.14274354 & 0.04062741 & 0.05134992 \\ 0.06025291 & 0.02717757 & 0.03081511 & 0.04011314 & 0.05108523 \\ 0.04062679 & 0.03013044 & 0.02326044 & 0.04628439 & 0.05055585 \\ 0.06965068 & 0.02848820 & 0.03677932 & 0.04649010 & 0.05161461 \\ 0.09860600 & 0.01949619 & 0.02067594 & 0.04443302 & 0.0518793 \\ 0.04078153 & 0.02798574 & 0.03956262 & 0.04587298 & 0.05002647 \\ 0.06235858 & 0.07319051 & 0.04512922 & 0.05044999 & 0.04949709 \\ 0.05784159 & 0.06833866 & 0.1584493 & 0.04787863 & 0.04870302 \\ 0.04885389 & 0.06853062 & 0.10735586 & 0.04690152 & 0.04446797 \\ 0.03473095 & 0.05596188 & 0.04493042 & 0.04983286 & 0.05029116 \end{vmatrix}$$

Then, the entropy array (E_j) is calculated:

E_j = [0.98691712 0.97765503 0.92705489 0.99129917 0.99970911]

The weights are presented in Table 10:

Table 10. Entropy method weights.

	Extrusion Force (kN)	Die Wear Total (μm)	Damage	Grain Average Size Ti6Al4V (μm)	Grain Average Size AZ31B (μm)
	0.11	0.19	0.62	0.07	2.47×10^{-3}
In %	11.15%	19.04%	62.15%	7.41%	0.25%

For the SD method, the decision matrix is:

	74.99224	0.00027495	1.43	8.84	19.10
	82.50792	0.00025418	1.20	15.00	18.40
	69.29504	0.00022908	1.98	9.00	20.00
	63.12752	0.00025944	2.15	9.40	20.00
	70.35448	0.00018415	2.57	15.00	17.30
	89.39628	0.00014645	1.32	8.38	19.00
	113.4116	0.0001078	2.13	6.50	18.00
	73.59232	0.00024047	1.44	15.00	19.00
	74.84328	0.00026554	1.63	9.00	19.10
	74.61736	0.00025597	1.77	9.10	19.20
	74.19540	0.00028538	7.18	7.90	19.40
	106.59656	0.00011326	1.55	7.80	19.30
	71.874960	0.00012557	1.17	9.00	19.10
	123.22264	0.00011872	1.85	9.04	19.50
	174.44900	0.00008125	1.04	8.64	19.60
	72.14872	0.00011663	1.99	8.92	18.90
	110.32180	0.00030502	2.27	9.81	18.70
	102.33056	0.0002848	7.97	9.31	18.40
	86.42996	0.00028560	5.40	9.12	16.80
	61.44432	0.00023322	2.26	9.69	19.00
Max	174.4490	0.00030502	7.97	15.00	20.00
Min	61.44432	0.00008125	1.04	6.50	16.80

The normalized matrix is obtained considering that all criteria are clustered as non-beneficial, because the objective is to minimize all these outcomes from the co-extrusion process.

0.88011187	0.13437905	0.94372294	0.72470588	0.28125
0.81360418	0.22719757	0.97691198	0.00	0.50
0.93052748	0.33936631	0.86435786	0.70588235	0.00
0.98510504	0.20369129	0.83982684	0.65882353	0.00
0.92115229	0.54015284	0.77922078	0.00	0.84375
0.75264777	0.70862940	0.95959596	0.77882353	0.3125
0.54013161	0.88135139	0.84271284	1.00	0.625
0.89250003	0.28846584	0.94227994	0.00	0.3125
0.88143004	0.17643116	0.91486291	0.70588235	0.28125
0.88342925	0.21919828	0.89466089	0.69411765	0.25
0.88716326	0.08778657	0.11399711	0.83529412	0.1875
0.6004392	0.8569424	0.92640693	0.84705882	0.21875
0.90769727	0.80194843	0.98124098	0.70588235	0.28125
0.45331185	0.8325334	0.88311688	0.70117647	0.15625
0.00	1.00	1.00	0.74823529	0.125
0.90527472	0.84189123	0.86291486	0.71529412	0.34375
0.56747384	0.00	0.82251082	0.61058824	0.40625
0.63818985	0.09036064	0.00	0.66941176	0.50
0.7788973	0.08678554	0.37085137	0.69176471	1.00
1.00	0.32086517	0.82395382	0.62470588	0.3125

After calculating the standard deviation for each column, the following array is obtained:
σ = [0.23896758 0.33516811 0.28284746 0.2810343 0.25080053]
The resultant weights are shown in Table 11:

Table 11. Standard Deviation method weights.

	Extrusion Force (kN)	Die Wear Total (μm)	Damage	Grain Average Size Ti6Al4V (μm)	Grain Average Size AZ31B (μm)
	0.17	0.24	0.20	0.20	0.18
In %	17.21%	24.13%	20.37%	20.24%	18.06%

The weighting process results comparison among these three methods is shown in Table 12:

Table 12. Standard Deviation method weights.

	Extrusion Force (KN)	Die Wear Total (μm)	Damage	Grain Average Size Ti6Al4V (μm)	Grain Average Size AZ31B (μm)
AHP	24.00%	4.63%	48.81%	13.59%	8.97%
Entropy	11.15%	19.04%	62.15%	7.41%	0.25%
Standard Variation	17.21%	20.37%	24.13%	20.24%	18.06%

3.2. MCDM Methods

This section presents the results obtained after applying the MCDM methods explained in this paper. To avoid being redundant, only the numbers for weighting values obtained by AHP methods will be shown. For the Entropy and SD methods, only the final results will be presented.

All the methods share the same Decision matrix (D):

74.99224	0.000274950	1.43	8.84	19.10
82.50792	0.000254180	1.20	15.00	18.40
69.29504	0.000229080	1.98	9.00	20.00
63.12752	0.000259440	2.15	9.40	20.00
70.35448	0.000184150	2.57	15.00	17.30
89.39628	0.000146450	1.32	8.38	19.00
113.4116	0.000107800	2.13	6.50	18.00
73.59232	0.000240470	1.44	15.00	19.00
74.84328	0.000265540	1.63	9.00	19.10
74.61736	0.000255970	1.77	9.10	19.20
74.1954	0.000285376	7.18	7.90	19.40
106.59656	0.000113262	1.55	7.80	19.30
71.87496	0.000125568	1.17	9.00	19.10
123.22264	0.000118724	1.85	9.04	19.50
174.449	0.000081250	1.04	8.64	19.60
72.14872	0.000116630	1.99	8.92	18.90
110.3218	0.000305020	2.27	9.81	18.70
102.33056	0.000284800	7.97	9.31	18.40
86.42996	0.000285600	5.40	9.12	16.80
61.44432	0.000233220	2.26	9.69	19.00

The first MCDM method to be applied is ARAS.

As explained before, all the criteria are considered non-beneficial; therefore, before calculating, the normalized matrix needs to be obtained x_{ij}^{*}:

	0.01333471	3637.02491	0.6993007	0.11312217	0.05235602
	0.01212005	3934.21984	0.83333333	0.06666667	0.05434783
	0.01443105	4365.28724	0.50505051	0.11111111	0.05
	0.01584095	3854.45575	0.46511628	0.10638298	0.05
	0.01421374	5430.35569	0.38910506	0.06666667	0.05780347
	0.01118615	6828.26903	0.75757576	0.11933174	0.05263158
	0.00881744	9276.43785	0.46948357	0.15384615	0.05555556
	0.01358837	4158.52289	0.69444444	0.06666667	0.05263158
	0.01336125	3765.91097	0.61349693	0.11111111	0.05235602
	0.01340171	3906.70782	0.56497175	0.10989011	0.05208333
	0.01347792	3504.14891	0.13927577	0.12658228	0.05154639
	0.00938117	8829.08654	0.64516129	0.12820513	0.05181347
	0.01391305	7963.81244	0.85470085	0.11111111	0.05235602
	0.00811539	8422.8968	0.54054054	0.11061947	0.05128205
	0.00573233	12307.6923	0.96153846	0.11574074	0.05102041
	0.01386026	8574.1233	0.50251256	0.11210762	0.05291005
	0.00906439	3278.47354	0.44052863	0.10193680	0.05347594
	0.00977225	3511.23596	0.12547051	0.10741139	0.05434783
	0.01157006	3501.40056	0.18518519	0.10964912	0.05952381
	0.01627490	4287.79693	0.44247788	0.10319917	0.05263158
Optimal value (OV)	0.0162749	12307.6923	0.96153846	0.15384615	0.05952381

With these values, the normalized matrix is obtained:

	0.05173867	32.78369954	0.71314958	0.28840608	0.21459583
	0.04702577	35.46257844	0.84983659	0.16996732	0.2227598
	0.05599244	39.3481674	0.51505248	0.28327886	0.20493902
	0.06146287	34.74359462	0.47432740	0.27122444	0.20493902
	0.05514928	48.9485647	0.39681086	0.16996732	0.23692372
	0.04340224	61.54918531	0.77257871	0.30423744	0.21572528
	0.03421166	83.61668079	0.47878117	0.39223227	0.22771002
	0.05272287	37.4844188	0.70819715	0.16996732	0.21572528
	0.05184164	33.94546279	0.62564657	0.28327886	0.21459583
	0.05199861	35.21458838	0.57616040	0.28016591	0.21347814
	0.05229433	31.58597145	0.14203397	0.32272275	0.21127734
	0.03639891	79.58431061	0.65793800	0.32686023	0.21237204
	0.05398262	71.78483522	0.87162727	0.28327886	0.21459583
	0.03148771	75.92296577	0.55124535	0.28202542	0.21019386
	0.02224145	110.9400392	0.98058068	0.29508215	0.20912144
	0.05377779	77.28610296	0.51246427	0.28581948	0.21686668
	0.03516983	29.55176116	0.44925282	0.25988886	0.21918611
	0.03791632	31.64985319	0.12795532	0.27384638	0.2227598
	0.04489183	31.56119814	0.18885257	0.27955151	0.24397502
	0.06314658	38.64967922	0.45124066	0.26310730	0.21572528
OV	0.06314658	110.9400392	0.98058068	0.39223227	0.24397502

The normalized weighted matrix is calculated by multiplying the weights obtained before by the normalized matrix.

	0.01241615	16.00173107	0.03300162	0.03920468	0.01925142
	0.01128515	17.30929246	0.03932693	0.02310462	0.01998381
	0.01343696	19.20584930	0.02383451	0.0385077	0.01838511
	0.01474974	16.95835630	0.02194992	0.03686908	0.01838511
	0.01323462	23.89180537	0.01836277	0.02310462	0.02125446
	0.01041559	30.04217110	0.03575176	0.04135672	0.01935275
	0.00821005	40.81332058	0.02215602	0.05331836	0.0204279
	0.01265233	18.29615319	0.03277244	0.02310462	0.01935275
	0.01244086	16.56878797	0.02895234	0.03850770	0.01925142
	0.01247853	17.18824846	0.02666233	0.03808454	0.01915115
	0.01254949	15.41711972	0.00657275	0.04386954	0.01895372
	0.00873494	38.8451198	0.03044666	0.04443197	0.01905193
	0.01295465	35.03819411	0.04033532	0.0385077	0.01925142
	0.00755636	37.05801656	0.02550936	0.03833732	0.01885652
	0.00533746	54.14985795	0.04537723	0.04011219	0.01876031
	0.01290549	37.72336413	0.02371473	0.03885306	0.01945514
	0.00843999	14.42422123	0.02078957	0.03532817	0.01966322
	0.00909909	15.44830042	0.00592124	0.03722549	0.01998381
	0.01077305	15.40502787	0.00873932	0.03800102	0.02188703
	0.01515379	18.86491707	0.02088156	0.03576567	0.01935275
OV	0.01515379	54.14985795	0.04537723	0.05331836	0.02188703

To calculate the degree of utility (K_i), it is necessary to obtain the optimality function (S_i):

	16.1056049
	17.402993
	19.3000136
	17.0503101
	23.9677618
	30.1490479
	40.9174329
	18.3840353
	16.6679403
	17.284625
	15.4990652
	38.9477853
	35.1492432
	37.1482761
	54.2594451
	37.8182926
	14.5084422
	15.5205301
	15.4844283
	18.9560708
OV	54.2855944

Finally, K_i is obtained and the results are sorted by highest value:

0.29668285
0.32058216
0.35552735
0.31408535
0.44151238
0.55537843
0.75374385
0.33865403
0.30704168
0.31840169
0.28550973
0.71746079
0.64748749
0.68431186
0.99951830
0.69665430
0.26726137
0.28590513
0.28524010
0.34919155

For AHP weighted values, the best alternative is number 15. In Tables 13 and 14 below, there is a comparison with the results obtained by using the weight values of the other methods.

Table 13. Best alternative comparison among weighting methods for process parameters using ARAS.

	Temperature (°C)	Die Semi-Angle (°)	Ram Speed (mm/s)	Extrusion Ratio (Ao/Af)	Friction	Billet Height (mm)	Core Diameter (mm)
AHP	350.00	30.00	2.00	1.78	0.50	20.00	6.00
Entropy	350.00	30.00	2.00	1.78	0.50	20.00	6.00
SD	350.00	30.00	2.00	1.78	0.50	20.00	6.00

Table 14. Best alternative comparison among weighting methods for process criteria using ARAS.

	Extrusion Force (kN)	Die Wear Total (μm)	Damage	Grain Average Size Ti6Al4V (μm)	Grain Average Size AZ31B (μm)
AHP	174.45	8.12×10^{-5}	1.04	8.64	19.60
Entropy	174.45	8.12×10^{-5}	1.04	8.64	19.60
SD	174.45	8.12×10^{-5}	1.04	8.64	19.60

In the case of the ARAS method, it can be verified that independent of the weight criterion, the best extrusion parameter combination is number 15.

The next MCDM method to be studied is TOPSIS. As with the ARAS method, the starting point is the decision matrix *D*.

The first difference is the method used to calculate the normalized matrix. With the TOPSIS method, Equation (19) is used:

$$R_{ij} = \frac{x_{ij}}{\sqrt{\sum x_{ij}^2}}$$

Consequently, the normalized matrix obtained is different from the one obtained by using ARAS.

0.18169569	0.27841399	0.10124168	0.19771981	0.22589903
0.19990512	0.25738231	0.08495806	0.33549741	0.21762001
0.16789217	0.23196609	0.14018079	0.20129845	0.23654349
0.15294913	0.26270858	0.15221652	0.21024504	0.23654349
0.17045904	0.18647003	0.18195184	0.33549741	0.20461012
0.21659466	0.14829507	0.09345386	0.18743122	0.22471631
0.27478041	0.10915813	0.15080055	0.14538221	0.21288914
0.17830388	0.24349959	0.10194967	0.33549741	0.22471631
0.18133478	0.26888543	0.11540136	0.20129845	0.22589903
0.18078740	0.25919486	0.12531313	0.2035351	0.22708175
0.17976505	0.28897134	0.50833237	0.1766953	0.22944718
0.25826852	0.11468894	0.10973749	0.17445865	0.22826446
0.17414295	0.12714998	0.08283411	0.20129845	0.22589903
0.29855118	0.12021976	0.13097700	0.20219311	0.23062990
0.42266547	0.08227364	0.07363032	0.19324651	0.23181262
0.17480623	0.11809938	0.14088878	0.19950913	0.22353359
0.26729426	0.30886283	0.16071232	0.21941531	0.22116816
0.24793260	0.28838808	0.56426310	0.20823206	0.21762001
0.20940768	0.28919816	0.38231126	0.20398243	0.19869653
0.14887097	0.23615825	0.16000434	0.21673133	0.22471631

From normalized decision matrix, the weighted one is obtained by multiplying each column by the weight associated to the correspondent criteria.

	0.04360298	0.01288385	0.04941609	0.02687718	0.02026543
	0.04797285	0.01191059	0.04146805	0.04560607	0.01952272
	0.04029044	0.01073443	0.06842228	0.02736364	0.02122035
	0.03670444	0.01215707	0.07429692	0.0285798	0.02122035
	0.04090643	0.00862906	0.08881073	0.04560607	0.0183556
	0.05197797	0.00686248	0.04561485	0.02547859	0.02015933
	0.06594127	0.00505139	0.07360578	0.01976263	0.01909831
	0.04278902	0.01126816	0.04976166	0.04560607	0.02015933
	0.04351637	0.01244291	0.05632743	0.02736364	0.02026543
	0.04338501	0.01199447	0.06116537	0.02766768	0.02037154
	0.04313967	0.01337240	0.24811715	0.02401920	0.02058374
	0.06197878	0.00530733	0.05356289	0.02371516	0.02047764
	0.04179049	0.00588398	0.04043135	0.02736364	0.02026543
	0.07164574	0.00556327	0.06392991	0.02748526	0.02068984
	0.10143045	0.00380728	0.03593897	0.0262691	0.02079594
	0.04194966	0.00546515	0.06876784	0.02712041	0.02005323
	0.06414476	0.0142929	0.07844372	0.02982637	0.01984103
	0.05949839	0.01334541	0.27541694	0.02830617	0.01952272
	0.05025325	0.0133829	0.18660621	0.02772849	0.01782509
	0.03572577	0.01092843	0.07809815	0.02946152	0.02015933
A⁺	0.10143045	0.0142929	0.27541694	0.04560607	0.02122035
A⁻	0.03572577	0.00380728	0.03593897	0.01976263	0.01782509

Then, the distance to the ideal solution (A^{+}) and anti-ideal solution (A^{-}) is calculated:

D_j^{+}	D_j^{-}
0.01956117	0.23403861
0.03028185	0.23999658
0.03454403	0.21663412
0.04038909	0.21197447
0.05927627	0.19627868
0.02012987	0.23604237
0.04832116	0.20674868
0.03114406	0.23317247
0.02479289	0.22735737
0.02882775	0.22271237
0.21258354	0.06789948
0.03201143	0.22657484
0.01118059	0.2432685
0.04631054	0.21451981
0.06609285	0.24048647
0.03432669	0.2160158
0.05319383	0.20109593
0.24100142	0.04540224
0.15187762	0.10410771
0.04390472	0.20862622

The last step is to obtain the closeness to the ideal solution and sort the values from highest to lowest.

0.05935214
0.06486590
0.05441376
0.05349463
0.05015999
0.06046750
0.05273535
0.06163134
0.05732822
0.05602110
0.01904465
0.05858914
0.06189945
0.05595327
0.07372818
0.05407793
0.05113664
0.01300337
0.02665005
0.05268458

Using the AHP weights, the best alternative solution is again number 15. After applying the weights from the Entropy and SD methods, the results obtained are shown in Tables 15 and 16.

Table 15. Best alternative comparison among weighting methods for process parameters using TOPSIS.

	Temperature (°C)	Die Semi-Angle (°)	Ram Speed (mm/s)	Extrusion Ratio (Ao/Af)	Friction	Billet Height (mm)	Core Diameter (mm)
AHP	350.00	30.00	2.00	1.78	0.50	20.00	6.00
Entropy	350.00	30.00	2.00	1.78	0.50	20.00	6.00
SD	350.00	30.00	2.00	1.78	0.50	20.00	6.00

Table 16. Best alternative comparison among weighting methods for process criteria using TOPSIS.

	Extrusion Force (kN)	Die Wear Total (µm)	Damage	Grain Average Size Ti6Al4V (µm)	Grain Average Size AZ31B (µm)
AHP	174.45	8.12×10^{-5}	1.04	8.64	19.60
Entropy	174.45	8.12×10^{-5}	1.04	8.64	19.60
SD	174.45	8.12×10^{-5}	1.04	8.64	19.60

The TOPSIS results are aligned with the ARAS ones. In both cases, the best alternative is number 15, independent of the weighting method used.

After obtaining the same results with the ARAS and TOPSIS methods, it is needed to check what happened with the other two pending MCDM methods. Let us start by VIKOR.

In VIKOR, the best ${f_b}^*$ and worst ${f_b}^-$ values for each criteria are obtained directly from decision matrix D.

	74.99224	0.00027495	1.43	8.84	19.10
	82.50792	0.00025418	1.20	15.00	18.40
	69.29504	0.00022908	1.98	9.00	20.00
	63.12752	0.00025944	2.15	9.40	20.00
	70.35448	0.00018415	2.57	15.00	17.30
	89.39628	0.00014645	1.32	8.38	19.00
	113.4116	0.00010780	2.13	6.50	18.00
	73.59232	0.00024047	1.44	15.00	19.00
	74.84328	0.00026554	1.63	9.00	19.10
	74.61736	0.00025597	1.77	9.10	19.20
	74.1954	0.000285376	7.18	7.90	19.40
	106.59656	0.000113262	1.55	7.80	19.30
	71.87496	0.000125568	1.17	9.00	19.10
	123.22264	0.000118724	1.85	9.04	19.50
	174.449	0.00008125	1.04	8.64	19.60
	72.14872	0.00011663	1.99	8.92	18.90
	110.3218	0.00030502	2.27	9.81	18.70
	102.33056	0.0002848	7.97	9.31	18.40
	86.42996	0.0002856	5.4	9.12	16.80
	61.44432	0.00023322	2.26	9.69	19.00
f_i^*	61.44432	0.00008125	1.04	6.50	16.80
f_i^-	174.449	0.00030502	7.97	15.00	20.00

Utility measure (S_j) and Regret measure (R_j) are obtained:

	S_j		R_i
	0.19819819		0.06447916
	0.27255304		0.13593568
	0.24314147		0.08971014
	0.25469310		0.08971014
	0.29791683		0.13593568
	0.18430526		0.06167572
	0.22626210		0.11035833
	0.28450908		0.13593568
	0.21258124		0.06447916
	0.22438568		0.06728261
	0.59702892		0.43245821
	0.22930296		0.09588583
	0.14493217		0.06447916
	0.31230722		0.13119317
	0.35269826		0.23997808
	0.19453387		0.06691129
	0.34290553		0.10379680
	0.70681489		0.48810022
	0.44430736		0.30708759
	0.23004732		0.08592818
S*	0.14493217	**R***	0.06167572
S⁻	0.70681489	**R⁻**	0.48810022

Using the values S^*, S^-, R^* and R^- together with the assumption of vote by consensus (υ = 0.5), the index Q is calculated:

0.05068674
0.20063820
0.12026455
0.13054395
0.22320856
0.03503675
0.12945502
0.21127747
0.06348572
0.07727723
0.83706275
0.11519138
0.00328715
0.23045330
0.39395059
0.05027775
0.22555831
1.00
0.55415898
0.10417802

In VIKOR, the index Q is ranked from the lowest to the highest value, therefore the best alternative solution should be number 13. However, before recommending this alternative as the best compromise solution, the conditions of “Acceptable advantages” and “Acceptable stability in decision making” have to be fulfilled.

In this case, DQ = 0.05263158 due to the number of alternatives is 20. Then:

$Q(2) - Q(1)$ = 0.0317496

$Q(3) - Q(1)$ = 0.0469906

$Q(4)$ $Q(1)$ = 0.04739959

$Q(5) - Q(1)$ = 0.06019857 > DQ

$Q(1) = S^*$

As only the second condition is fulfilled, a set of compromise solutions is presented. These results were obtained using AHP weights. Next, the same process is performed, but using the weights from the Entropy and SD methods, and the results are compared in Tables 17 and 18:

Table 17. Best alternative comparison among weighting methods for process parameters using VIKOR.

	Temperature (°C)	Die Semi-Angle (°)	Ram Speed (mm/s)	Extrusion Ratio (Ao/Af)	Friction	Billet Height (mm)	Core Diameter (mm)
AHP	350.00	30.00	2.00	1.78	0.05	20.00	6.00
Entropy	350.00	30.00	2.00	1.78	0.05	20.00	6.00
SD	350.00	60.00	2.00	1.78	0.10	20.00	6.00

Table 18. Best alternative comparison among weighting methods for process criteria using VIKOR.

	Extrusion Force (kN)	Die Wear Total (μm)	Damage	Grain Average Size Ti6Al4V (μm)	Grain Average Size AZ31B (μm)
AHP	71.87	1.25×10^{-4}	1.17	9.00	19.10
Entropy	71.87	1.25×10^{-4}	1.17	9.00	19.10
SD	113.41	1.07×10^{-4}	2.13	6.50	18.00

Entropy weights lead to the same results as the AHP ones, stating the best alternative solution is number 13, but only fulfilling the condition of "Acceptable stability in decision making". In the case of the SD method, the results are different, and the best compromise solution is number 7, which, in addition, fulfils the two conditions for Q index; however, it is the one with the highest value of damage, which is the criteria considered the most relevant, followed by extrusion force and Ti_6Al_4V grain size. In the case of alternative 7, a deeper analysis using the HSC method is recommended to verify the integrity of the final part.

The last method to be analyzed is COPRAS. The normalization matrix is obtained by using Equation (29):

$$N^{ij} = \frac{x_{ij}}{\sum_{i=0}^{m} x_{ij}}$$

0.04238881	0.06597512	0.02842942	0.04546156	0.05055585
0.04663699	0.06099129	0.02385686	0.07714065	0.04870302
0.03916851	0.05496847	0.03936382	0.04628439	0.05293806
0.03568236	0.06225345	0.04274354	0.04834148	0.05293806
0.03976735	0.04418737	0.05109344	0.07714065	0.04579142
0.05053058	0.03514114	0.02624254	0.04309591	0.05029116
0.06410506	0.02586695	0.04234592	0.03342762	0.04764426
0.04159751	0.05770154	0.02862823	0.07714065	0.05029116
0.04230461	0.06371716	0.03240557	0.04628439	0.05055585
0.04217691	0.06142081	0.03518887	0.04679866	0.05082054
0.04193840	0.06847687	0.14274354	0.04062741	0.05134992
0.06025291	0.02717757	0.03081511	0.04011314	0.05108523
0.04062679	0.03013044	0.02326044	0.04628439	0.05055585
0.06965068	0.02848820	0.03677932	0.04649010	0.05161461
0.09860600	0.01949619	0.02067594	0.04443302	0.0518793
0.04078153	0.02798574	0.03956262	0.04587298	0.05002647
0.06235858	0.07319051	0.04512922	0.05044999	0.04949709
0.05784159	0.06833866	0.1584493	0.04787863	0.04870302
0.04885389	0.06853062	0.10735586	0.04690152	0.04446797
0.03473095	0.05596188	0.04493042	0.04983286	0.05029116

Next, the weighted matrix is calculated by multiplying the normalized one by the weights obtained before:

0.01017238	0.00305306	0.01387641	0.00617985	0.00453537
0.01119185	0.00282243	0.01164454	0.01048617	0.00436915
0.00939958	0.00254371	0.01921349	0.00629170	0.00474908
0.00856298	0.00288083	0.02086313	0.00657133	0.00474908
0.00954329	0.00204481	0.02493872	0.01048617	0.00410796
0.01212623	0.00162619	0.01280899	0.00585827	0.00451163
0.01538381	0.00119702	0.02066906	0.00454401	0.00427417
0.00998249	0.00267019	0.01397345	0.01048617	0.00451163
0.01015218	0.00294857	0.01581716	0.00629170	0.00453537
0.01012153	0.0028423	0.01717569	0.00636161	0.00455912
0.01006430	0.00316883	0.06967315	0.00552271	0.00460661
0.01445938	0.00125767	0.01504086	0.00545281	0.00458286
0.00974954	0.00139431	0.01135342	0.00629170	0.00453537
0.01671464	0.00131832	0.01795200	0.00631966	0.00463035
0.02366328	0.0009022	0.01009193	0.00604003	0.0046541
0.00978667	0.00129506	0.01931053	0.00623577	0.00448788
0.01496469	0.00338696	0.02202758	0.00685795	0.00444039
0.01388071	0.00316243	0.07733914	0.00650841	0.00436915
0.01172386	0.00317131	0.05240042	0.00637559	0.00398923
0.00833467	0.00258968	0.02193055	0.00677406	0.00451163

As all the criteria are considered to be non-beneficial. Only $S_{\_i}$, according to Equation (28), has to be calculated:

$$S_{-i} = \sum\nolimits_{j=1}^{n} y_{-ij}$$

Finally, the relative significance of alternatives (Q_i) and quantitative utility (U_i) are determined:

Q_i		U_i
0.06051591		88.1198594
0.05648730		82.2536226
0.05423380		78.9722039
0.05245640		76.3840570
0.04476706		65.1872727
0.06196732		90.2333220
0.04967724		72.3372077
0.05498123		80.0605711
0.05758046		83.8454203
0.05573600		81.1596231
0.02459848		35.8189210
0.05610036		81.6901852
0.06867454		100.00
0.04875969		71.0011115
0.05046210		73.4800694
0.05566054		81.0497473
0.04428486		64.4851229
0.02174176		31.6591243
0.02946848		42.9103388
0.05184648		75.4959312

Ranking U_i from highest to lowest value, the best alternative using COPRAS combined with AHP weighting method is number 13; the same obtained by VIKOR. In Tables 19 and 20, a comparison of combining the Entropy and SD weighting methods with COPRAS is shown:

Table 19. Best alternative comparison among weighting methods for process parameters using COPRAS.

	Temperature (°C)	Die Semi-Angle (°)	Ram Speed (mm/s)	Extrusion Ratio (Ao/Af)	Friction	Billet Height (mm)	Core Diameter (mm)
AHP	350.00	30.00	2.00	1.78	0.05	20.00	6.00
Entropy	350.00	30.00	2.00	1.78	0.05	20.00	6.00
SD	350.00	30.00	2.00	1.78	0.05	20.00	6.00

Table 20. Best alternative comparison among weighting methods for process criteria using COPRAS.

	Extrusion Force (kN)	Die Wear Total (μm)	Damage	Grain Average Size Ti6Al4V (μm)	Grain Average Size AZ31B (μm)
AHP	71.87	1.25×10^{-4}	1.17	9.00	19.10
Entropy	71.87	1.25×10^{-4}	1.17	9.00	19.10
SD	71.87	1.25×10^{-4}	1.17	9.00	19.10

The results after applying the three weighting methods confirm that the best alternative is number 13; no matter which weighting method is used, the best alternative is the same in all cases.

4. Discussion

The main novelty of this paper is the comparison of four different MCDM methods together with three weighting methods to check the grade of influence of choosing a particular weighting method on the final result, and also to evaluate which of the MCDM methods applied fits better in obtaining the optimal process parameters to meet specific criteria to manufacture a bimetallic billet of dissimilar materials by co-extrusion.

The results reveal that in three of the four MCDM methods studied, the best solution obtained is the same, independent of the weighting method used. This becomes especially relevant when observing the great difference in weights obtained for the damage criteria between AHP and SD, the grain size criteria between Entropy and SD, or the total wear among AHP, Entropy, and SD. It must be taken into account that the Entropy and SD methods are independent of previous experience and only consider the current results of the different experiments, while AHP is supported by customer needs or design characteristics. In addition, the AHP method needs a validation to check the consistency of the pairwise comparison matrix.

Regarding the MCDM methods, it has been seen that they are aligned two by two; on one side, ARAS and TOPSIS, and on the other side, VIKOR and COPRAS. It is also relevant to take into account that TOPSIS and VIKOR are both distance-based methods (see Section 2.4) and, therefore, the results they obtain should be similar. Comparing these methods by complexity, TOPSIS can be considered more complex than ARAS because it needs to calculate the Euclidian distances from the ideal and non-ideal solution, and VIKOR is more complex than COPRAS because COPRAS performs a final validation before recommending the best compromise solution.

VIKOR is the only method that obtains different results depending on the weighing method used, leading to the result that alternative 7 is the optimal one when applied together with the SD method.

Table 21 shows a summary of the best alternative for each MCDM method depending on the weighting method applied.

Table 21. Best alternative comparison for each MCDM together with weighting method.

	ARAS	**TOPSIS**	**VIKOR**	**COPRAS**
AHP	Alternative 15	Alternative 15	Alternative 13	Alternative 13
Entropy	Alternative 15	Alternative 15	Alternative 13	Alternative 13
SD	Alternative 15	Alternative 15	Alternative 7	Alternative 13

A comparison among all the alternative solutions has been made in Table 22 to evaluate which can be considered as optimal. After studying the values obtained for each criterion, our conclusion is that the alternative with the best balance among all the criteria considered is number 13.

Table 22. Best alternative comparison for process criteria.

	Extrusion Force (kN)	**Die Wear Total (μm)**	**Damage**	**Grain Average Size Ti6Al4V (μm)**	**Grain Average Size AZ31B (μm)**
Alternative 7	113.41	1.07×10^{-4}	2.13	6.50	18.00
Alternative 13	71.87	1.25×10^{-4}	1.17	9.00	19.10
Alternative 15	174.45	8.12×10^{-5}	1.04	8.64	19.60

In summary, it could be said that both VIKOR and COPRAS are the recommended MCDM methods from the point of view of calculation results. The COPRAS method also obtained the same result independent of the weighting method chosen and it is simpler than VIKOR, considering the computing mechanism. Secondly, the VIKOR method implements two conditions to be fulfilled to confirm the compromise solution or set of compromise solutions; these conditions increase the complexity of application of this method but also provide a security check of the alternative chosen. Finally, the methodology recommended to obtain the optimal manufacturing parameters in a multi-material co-extrusion process is VIKOR together with the Entropy weighting method. Even when considering that is VIKOR a more complex process than COPRAS, the determining factors are the "Acceptable advantages" and "Acceptable stability in decision making" conditions to confirm the compromise solution or set of compromise solutions to be recommended.

In future work, this methodology can be extended to other multi-material manufacturing processes.

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Data Availability Statement: The raw/processed data required to reproduce these findings cannot be shared at this time as the data also form part of an ongoing study.

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