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Session on Intelligence

Chair: Dr. H. Laskov

Co-Chair: Dr. M. El-Araby

# Program

"A Concept of a Simulator for Robotics Operations in Space" by  
Dr. M. Laskov, Digital Equip. Corp., Colorado Springs, CO, USA

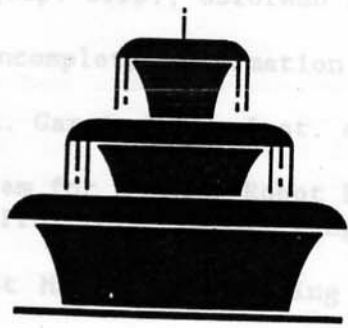
"A Task Planning With Incomplete Information for an Industrial Robot End-effector  
with Sensors" by

Dr. D.H. Kumpul and Dr. E. Garcia, de Auto. Industrial, Madrid, Spain

"A Novel Supervision System for a Dynamic Environment" by  
Prof. S.P. Lancaster and Dr. J. H. M. Lee, King's College, Univ London, London, UK

"V.S.B. Control for Robot Manipulators Using Acceleration Signals Instead of  
Dynamics Model" by

Prof. Tetsu Watanahe, Ritsumeikan Univ., Kita-ku, Kyoto, Japan and Prof. Masao  
Nakajima, Kyoto Univ., Kyoto, Japan



"The Use of Properties of Objects for Its Visual Orientation in Robotics" and  
"Fast Methods for Reliable Thresholding in Robotic Vision" by

Dr. J. M. Garcia, Univ. de Zaragoza, Zaragoza, Spain

## 4th INTERNATIONAL CONFERENCE

on

## SYSTEMS RESEARCH INFORMATICS AND CYBERNETICS

"A Systematic Study of PID Controller Tuning Methods" by  
Dr. F. Morillo, S. Dorado, J.L. Ferrás, and J. Aranda, UNED, Madrid, Spain

"A Robust Sampled-Data Control System for Step Responses" by  
Dr. J. M. Garcia, Univ. de Zaragoza, Zaragoza, Spain

"A Theory of Adaptive Control" by  
Dr. S. Dorado, J.M. de la Cruz, UNED, Dr. J.R. Velasco, UNED, Madrid, Spain

Dr. E. Kabotsho, Univ. del País Vasco, Vizcaya, Spain

"On the Hierarchy of Complex Systems" by  
Dr. G.V. Todorov, Sofia Univ., Sofia, Bulgaria

"Recent Trends for the Solution of Optimal Control Problems" by  
Dr. M. El-Araby, Military Academy, Cairo, Egypt

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# A SYSTEMATIC STUDY OF PID CONTROLLERS TUNING METHODS.

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**Abstract.** This paper presents the results of a comparative study - among five popular tuning methods. To make this study; four processes, - which characteristics vary from a simple lag to pure dead time, has been selected, four approximation techniques to determine the model parameters has been employed, different structures of PI and PID controllers has - been used and closed-loop response to set point and load changes has been simulated. From these results, heuristic rules, that can be used in the - implementation of an expert system for auto-tuning PID controllers, has been obtained.

**Keywords.** PID process control; process parameter estimation; tuning techniques, expert system.

## 1. INTRODUCTION

From the practical standpoint, the adjusting of the PID parameters to obtain a specified closed-loop response is an usual task, known as "Tuning", in process control. There are many tuning methods, some of them based upon parameters determined from the closed-loop response and others based upon parameters determined from the open-loop response. The pioneer method was - proposed by Ziegler and Nichols in 1942. In contrast to the closed-loop - methods, the open-loop technique necessitates only one upset to be imposed on the process.

A well tuned PID controller is shown to provide a level of performance that is equal to or better than more advanced control algorithms. For this reason, in the last years, a lot of research works has been made to ease the tuning by computer and mainly to implement industrial auto-tuning PID regulators.

In this paper we present the results of a comparative study among five open-loop tuning methods. The work is similar to the one made by - Miller, Lopez, Smith and Murril (1967), with the following differences:

- We employ two additional techniques to determine the model parameters, and we do not reject none in the study of the closed-loop response.
- The closed-loop simulation is made with the real process and with the model process.
- In addition to load change, we simulate the closed-loop response to set point, with different structures of PI and PID controllers.

We have followed a systematic procedure to make this work:

- 1.- To select a real process described by its transfer function.
- 2.- To determine the parameters of a first order model with pure - time delay for the process. These parameters vary according to the technique used.

- 3.- To apply the five PID tuning methods for each of the foremen--  
tioned models.
- 4.- To simulate the closed-loop response to set point and load chan--  
ges.
- 5.- To measure the characteristics of the different responses for  
subsequent comparisons.

The paper is organized as follows. Section 2 describes the proce--  
sses selected and the four techniques used to model them. Section 3 pre--  
sents the five tuning methods under our study. Section 4 describes how -  
has been simulated the process control system. Section 5, the principal  
part of the paper, shows the results obtained. Some concluding remarks -  
are given in Section 6, the last section.

## 2. PROCESSES AND MODELS.

In general terms, process characteristics vary from a simple lag -  
to pure dead time. So, according to Higham (1985), the transfer functions  
 $G_1(s)$  and  $G_4(s)$  are representatives of extremes, processes with dominant  
lag and processes with dominant dead time:

$$G_1(s) = \frac{1}{(0.5 s + 1)^4 (20 s + 1)} \quad (1)$$

$$G_4(s) = \frac{\exp(-2 s)}{(1.2 s + 1)^5} \quad (2)$$

while the processes described by  $G_2(s)$  and  $G_3(s)$ , mentioned in the paper  
of Krauss and Myron (1984), are representatives of two intermediate cases.

$$G_2(s) = \frac{1}{(2 s + 1) (3 s + 1) (5 s + 1) (30 s + 1)} \quad (3)$$

$$G_3(s) = \frac{1}{(2 s + 1) (3 s + 1) (5 s + 1) (6 s + 1)} \quad (4)$$

selecting these four processes, that will be called Process 1,2,3 and 4,  
described by  $G_1(s)$ ,  $G_2(s)$ ,  $G_3(s)$  and  $G_4(s)$  respectively, we have tried -  
to cover one wide range of industrial processes.

In general, it is not possible to completely characterize a process.  
approximation techniques are employed. Most of these techniques are based  
on the process reaction curve, which is the response of the process to a  
step change in the manipulated variable. By far the most common approxi--  
mation is that of a first order model plus pure time delay. The process  
model transfer function is then:

$$G_m(s) = K \frac{\exp(-T_o s)}{T s + 1} \quad (5)$$

where;  $K$  is the steady state gain,  $T_p$  is the time constant and  $T_o$  is the dead time. There are several procedures to determine the model parameters  $T_p$  and  $T_o$ . In this work we use four of them, that will be called Fit 1, 2, 3 and 4.

**Fit 1:** Is the most old and popular procedure. It makes use of the tangent to the process reaction curve at its point of inflection.

**Fit 2:**  $T_o$  is determined in the same manner as in Fit 1, but  $T_p$  is the one that forces the model response to coincide with the process response at 63.2 percent of the final value.

**Fit 3:** To eliminate the dependence on the tangent line, this method makes coincide the model and process responses at 28.3 and 63.2 percent of final value. Studies, made by Smith (1972), indicate that the model - open-loop response based on Fit 3 always provides an approximation to the process response that is a good or better than the Fit 2 and the Fit 1 in this order.

**Fit 4:** To reduce the estimation error in somewhat noisy processes, this method, proposed by Nishikawa and co-workers (1984), makes use of characteristic areas. Studies, made by us, indicate that Fit 3 and Fit 4 provide very similar approximations in open-loop.

We have applied these four fits to each one of the forementioned processes. The table 1 shows the results obtained, being the model gain  $K = 1$  in all the cases. Looking at this table, it can say that: given a process, the ratio  $T_o/T_p$  is very sensitive to the fit used, as much as its value, and this ratio increases with the fit number.

For the most of the processes encountered in practice, the ratio  $T_o/T_p$  ranges from a small positive value (corresponding to lag dominated processes, for example, liquid level) to about 2 which corresponds to dead time dominant processes, such as those encountered in a paper mill). At a glance of Table 1, it can see that selecting the process 1, 2, 3 and 4 - we have covered one wide range of  $T_o/T_p$  values, increasing this ratio with the process number.

### 3. OPEN-LOOP TUNING METHODS.

The open-loop tuning methods provide a set of formulas, to determine the PID parameters ( $K_p$ ,  $T_i$  and  $T_d$ ), based on the model process parameters ( $K$ ,  $T_p$  and  $T_o$ ). These tuning formulas are empirical equations that predict controller settings for an optimal control criterion. In this work we compare five popular tuning methods, that will be referred as Z-N, 3 C, IAE, ISE, and ITAE, in the same way that Miller and co-workers (1967). The difference among them is the criterion used to predict the PID parameters.

The Ziegler and Nichols (Z-N) criterion was that the response of the controlled process to a step change in set point should have a 1/4 decay ratio. The 3 C method adds two constraints to the Z-N criterion, - one requiring that the integral of the error be a minimum and other making  $K_p K_d/T_p = 0.5$ . The IAE, ISE and ITAE methods are based on a criterion of minimizing the integral of absolute error, the integral of error squared and the integral of absolute error multiplied by time to a load change, respectively.

These five approaches to tuning controllers yield tuning relations of the form:

$$Y = A (T / \pi) B$$



where;  $Y = K K_p$  for proportional mode,  $T_p/T_i$  for integral mode,  $T_d/T_p$  for derivative mode,  $A, B = \text{constants}$  for given controller and mode. The Table 2 shows a comparison of the five methods for PI and PID controllers. Plotting these equations is not sufficient for positive indications of which tuning relation is best. For this reason, the comparison will be made on the basis of closed-loop response.

#### 4. CLOSED-LOOP RESPONSE

Computer simulation has been used to calculate closed-loop responses to step changes, in the set point or in the load on the signal control. This simulation has been made for both the real process and the process model acting as controlled process. In this simulation, we have considered the zero-order hold equivalent of the continuous time process or model and we have approximated the pure dead time of Process 4 by a fourth-order Padé approximation. It has been used the following digital PID control algorithm, written in its position form as :

$$u(k) = K_p r(k) a_1 - K_p y(k) + K_i \sum_{i=0}^k (r(i) - y(i)) + K_d (r(k) - r(k-1)) a_2 - K_d (y(k) - y(k-1))$$

$$\text{where; } K_p, K_i = K_p T/T_i, K_d = K_p T_d/T$$

are the proportional, integral and derivative gains,  $T_i, T_d$  are the integral and derivative constants,  $T$  is the sampling period, and  $a_1, a_2$  are two constants that characterize the PID structure. There are three different structures of PID controller that will be called PID ( $a_1 = a_2 = 1$ ), PI-D ( $a_1 = 1, a_2 = 0$ ) and I-PD ( $a_1 = a_2 = 0$ ).

As proposed Moore and co-workers (1969), the sampling period is chosen from one-tenth to one-twentieth of the effective process time constant, and its effect is taken into consideration in the tuning relations by adding one-half the sampling period to the process model dead time. In this work we have chosen  $T = 2$  sec. for the Processes 1,2,3 and  $T = 0,5$  sec. for the Process 4.

#### 5. COMPARATIVE STUDY OF TUNING METHODS

Two reasons have caused us to make this comparative study of tuning methods. The main reason is that: in previous comparisons, the closed-loop system has been simulated with the process model as the controlled process. However, the ratio  $T_o/T_p$  is very sensitive to the fit used and also is very important to the tuning relations, equation (6). Therefore for a process, the results of tuning will depend on the fit and tuning methods. The other reason, less important, is that: always has been considered in tuning methods the classical structures of PI and PID controllers, and there is not information about what can happen when another structure (I-P, PI-D or I-PD), that have a decisive influence on the system response to set point change, is chosen.

We have followed the systematic procedure, described in the introduction, to compare the different combinations of real process, process model, fit technique, tuning method, control structure and perturbations. The comparative study has been divided in two levels.

First level : Given a process (1,2,3 or 4), given a controller, PI or PID, and a control structure. What couple or fit and tuning techniques makes optimum the control?

Second level : Given a controller and a control structure. What fit and tuning method make the control good enough for all the processes ?

The most used criteria of optimal control, based on the closed-loop response to set point or load changes, are the decay ratio and the minimum error integral. As the measure of the decay ratio is not possible in almost all the cases, we have used only the error integral criteria. So, at the first level we consider an optimal combination of fit and tuning techniques to the one that minimizes the corresponding integral, iae (integral of absolute error), ise (integral of error squared) or itae (integral of absolute error multiplied by time), to set point or load change. At the second level we consider a good combination of fit and tuning methods to the one that minimizing the error integral for one or several processes, makes small enough the error integral for the others processes.

In the comparative study we have not rejected none of the four fit techniques for the following reason : If we take into account only the open-loop simulations, the Fit 3 and 4 give the best approximations to real processes. However, the closed-loop simulations show that not always the best approximation in open-loop is also the best in closed-loop. For example, Figure 1 presents the responses to a step change in the manipulated variable of the Process 4, the process model approximated by Fit 1 and by Fit 4. As evident in the diagram, Fit 4 is better approximation to Process 4 than Fit 1. Figure 2 shows the closed-loop responses to a step change in the set point with PID control ( $K_p = 0.65$ ,  $T_i = 10.72$ ,  $T_d = 2.68$  and  $T = 0.5$ ) of Process 4, model 1 and model 4. As evident, the process model obtained by Fit 1, give now the best approximation to the real process.

### 5.1 First level results.

The first level results of the comparative study are shown in the tables 3, 4 and 5. From these tables we can observe :

- a) The Z-N method give the best results in process 1 and 2, dominant lag processes. It is not recommended for dominant dead time processes
- b) The 3 C method is not suitable for integral criteria. Only in the I-P structure give good results.
- c) The IAE, ISE and ITAE methods do not guarantee the minimum in the corresponding integrals.
- d) The Fits 3 and 4 offer better results, in general, than the Fits 1 and 2.
- e) The table 3 and 5 for criteria iae and itae are very similar.

### 5.2 Second level results.

The second level results are shown in the Table 6. In order to obtain this table we have used the four combinations of fit and tuning methods showed in the Tables 3, 4 and 5. Every quartet is applied to the four processes and the corresponding error integrals are calculated and represented. As "recommended" combination we have chosen the one with integral values small enough for all processes. Figure 3 show the case for the first row of Table 3 (iae with set point changes and PI controller). The values do not represented are error integrals too larges (os-



From this table we can observe :

a) The Fits 3 and 4 are the "recomended" approximations in all cases except for I-P structure.

b) The tuning method "recomended" depends on the control structure.

1-. For set point changes with PI or I-P controller can be ITAE.

2-. For set point changes with PID or PI-D controller is Z-N.

3-. For set point changes with I-P controller can be IAE.

4-. For load changes is IAE.

## 6. CONCLUSION

A comparative study among five common tuning methods has been made using four processes with different lag to dead time ratio. Certain rules for fit and tuning recommendations has been obtained for these processes. These rules are purely heuristic.

A general process can be related to one of the forementioned processes by its lag to dead time ratio. The heuristic rules can also be applied to it to determine a robust combination of fit and tuning methods.

Also a certain heuristic rules has been proposed to select the fit and tuning methods, when exists a lack of previous knowledge of the lag to dead time ratio.

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TABLE 1. KEEL PARAMETERS  $T_p$  AND  $T_o$

Process	Process 1			Process 2			Process 3			Process 4		
	$T_p$	$T_o$	$T_o/T_p$	$T_p$	$T_o$	$T_o/T_p$	$T_p$	$T_o$	$T_o/T_p$	$T_p$	$T_o$	$T_o/T_p$
1	23.17	1.79	0.08	45.40	7.22	0.16	18.19	5.21	0.29	6.14	4.52	0.74
2	20.21	1.79	0.09	33.42	7.22	0.22	11.94	5.21	0.44	3.99	4.52	1.13
3	30.06	1.94	0.10	11.20	9.44	0.30	10.37	6.78	0.65	3.44	5.07	1.47
4	19.57	2.04	0.11	30.43	9.52	0.31	9.13	6.93	0.76	2.92	5.11	1.75

TABLE 2. TUNING RELATIONS FOR PI AND PID CONTROLLERS

Method	Mode	A	B
Z - N	P	0.900	-1.000
	I	0.300	-1.000
3 C	P	0.928	-0.946
	I	1.078	-0.583
IAE	P	0.984	-0.986
	I	0.608	-0.707
ISE	P	1.305	-0.959
	I	0.492	-0.739
ITAE	P	0.859	-0.977
	I	0.674	-0.680
Z - N	P	1.200	-1.000
	I	0.500	-1.000
	D	0.500	1.000
3 C	P	1.370	-0.950
	I	1.351	-0.738
	D	0.365	0.950
IAE	P	1.435	-0.921
	I	0.878	-0.749
	D	0.482	1.137
ISE	P	1.495	-0.945
	I	1.101	-0.771
	D	0.560	1.006
ITAE	P	1.357	-0.947
	I	0.842	-0.738
	D	0.381	0.995

Set point change, PI Controller

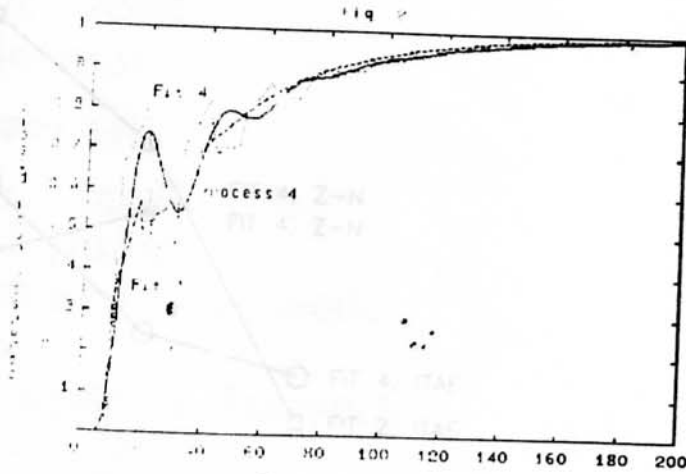
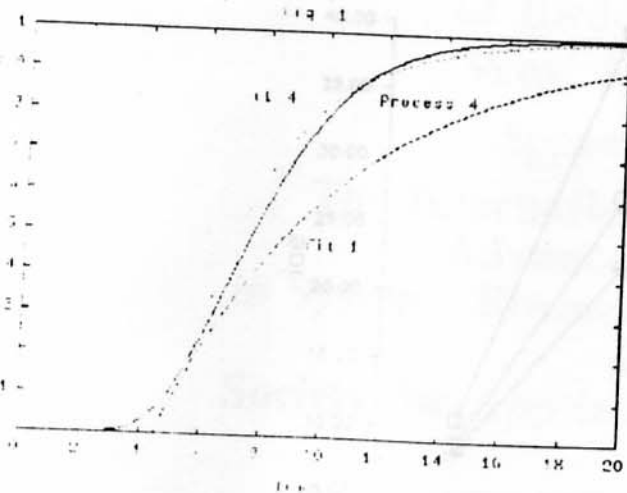




TABLE 3. MINIMUM INTEGRAL OF ABSOLUTE ERROR (IAE)

Change	Control	Process 1		Process 2		Process 3		Process 4	
		Fit	Tuning	Fit	Tuning	Fit	Tuning	Fit	Tuning
Set Point	PI	4	Z-N	4	Z-N	4	Z-N	2	ITAE
	I-P	4	3C	2	ITAE	2	ITAE	4	3C
	PID	4	ITAE	3	Z-N	3	Z-N	4	ITAE
	PI-D	4	Z-N	3	Z-N	3	Z-N	4	ITAE
Load	I-PD	4	IAE	1	Z-N	3	ISE	3	ISE
	PI	2	Z-N	3	ISE	4	ISE	2	ITAE
	PID	4	ITAE	1	Z-N	3	ISE	4	ISE

TABLE 4. MINIMUM INTEGRAL OF ERROR SQUARED (ISE)

Change	Control	Process 1		Process 2		Process 3		Process 4	
		Fit	Tuning	Fit	Tuning	Fit	Tuning	Fit	Tuning
Set Point	PI	4	Z-N	3	Z-N	4	ZE	2	IAE
	I-P	1	3C	1	ITAE	2	ITAE	2	3C
	PID	4	Z-N	3	Z-N	3	Z-N	4	ISE
	PI-D	4	Z-N	3	Z-N	3	Z-N	3	ISE
Load	I-PD	4	3		ISE	3	ZE	2	ISE
	PI	3	ISE	1	Z-N	3	ISE	2	IAE
	PID	3	Z-N	1	ISE	3	ISE	3	ISE

TABLE 5. MINIMUM INTEGRAL OF ABSOLUTE ERROR MULTIPLIED BY TIME (ITAE)

Change	Control	Process 1		Process 2		Process 3		Process 4	
		Fit	Tuning	Fit	Tuning	Fit	Tuning	Fit	Tuning
Set Point	PI	4	Z-N	4	Z-N	4	Z-N	2	ITAE
	I-P	4	3C	4	ITAE	3	ITAE	4	3C
	PID	4	ITAE	3	Z-N	3	Z-N	4	ITAE
	PI-D	4	Z-N	3	Z-N	4	Z-N	4	ITAE
Load	I-PD	4	IAE	2	Z-N	3	IAE	4	ISE
	PI	4	IAE	4	ISE	3	ITAE	2	ITAE
	PID	4	ITAE	1	Z-N	3	ISE	4	ISE

TABLE 6. "RECOMMENDED" FIT AND TUNING METHODS

Change	Control	ise		ise		itae	
		Fit	Tuning	Fit	Tuning	Fit	Tuning
Set Point	PI	4	ITAE	4	ISE	4	ITAE
	I-P	2	ITAE	2	ITAE	3	ITAE
	PID	3	Z-N	4	Z-N	3	Z-N
	PI-D	4	Z-N	3	Z-N	4	Z-N
Load	I-PD	4	IAE	4	3C	3	IAE
	PI	3	ISE	3	ISE	4	IAE
	PID	4	ISE	3	ISE	4	ISE

Set point change, PI Controller

