



## **A CONTROLLER DESIGN BY QFT METHODOLOGY FOR DYNAMIC POSITIONING OF A MOORED PLATFORM**

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### **ABSTRACT**

An analysis and design of a dynamic positioning system for a moored floating platform has been carried out. The model of the platform is a single degree of freedom SIMO system. The goal is to minimize the drift resulting from the wave action by appropriate thrusters control. An interesting question is that the plant has less degree of freedom for actuation and is more difficult to control. The multivariable QFT robust control technique for underactuated systems is employed. The problem is solved by an iterative multi-stage sequential procedure. It is shown that the control achieves the positioning system.

**Key words:** positioning system, robust control, multivariable control, stability, Nichols chart.

### **INTRODUCTION**

Control of underactuated systems is a typical problem in marine systems. So, problems of tracking, point stabilization or path following for some kind of marine vehicles or dynamic positioning for offshore systems are examples of these types of problems.

In this work we analyse the use of a multivariable QFT design method for underactuated systems. The method is applied to a moored floating platform model. The model is a single-input/multi-output (SIMO) linear time invariant (LTI) system with a single degree of freedom.

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This system has been examined and analysed in the specialized literature, in which is possible to find several robust control methods. For example, (Scherer, *et al.*, 1997) presents an overview of a linear matrix inequality (LMI) approach to the multiobjective synthesis of linear output-feedback controllers. A multiobjective  $H_2/H_\infty$  is proposed to specify the closed-loop objectives in terms of a common Lyapunov function.

In Revilla (2005), this system is used to validate the results obtained in the study about synthesis of reduced-order controllers based on LMI optimization.

In Nakamura, *et al.* (2001) the problem was formulated in the framework of a multimodel-based design of the  $H_\infty$  control law with pole region constraints. Methodology based on LMI was used to solve the problem.

QFT (Quantitative Feedback Theory) (Horowitz, 1992, Yaniv, 1999, Borguesani, *et al.*, 1995, Houppis *et al.*, 2006) is a frequency domain design methodology that has not been very common in naval systems. Our group has applied this technique in the fast ferries stabilization problem (Aranda, *et al.*, 2005) with successful experimental results, and in addition it has been developed an interactive computer-aided control tool (Díaz, *et al.*, 2005a, b).

The objective of this work is to verify that this technique can be extended to underactuated systems, and to show the results obtained in the application of a typical dynamic positioning problem.

## MODEL OF THE PLATFORM SYSTEM

The system consists of a floating platform that is anchored to the bottom of the ocean and equipped with two thrusters, as it is showed in Fig. 1 (the model of a replica of this system and previous control is described in Kajiwara, *et al.* (1995)). The objective is achieving an appropriate thrusters control in order to minimize the drift  $Y$  resulting from the wave action.

The action from the wave is considered as a force  $F$  and a torque  $M$ . The force  $F$  consists of two components  $F = F_1 + F_2$  with the following characteristics:

- $F_1$  is a high-frequency high-amplitude excitation with small drifting effects. Due to its large magnitude, it can not be countered by the thrusters. The spectral energy of  $F_1$  is beyond 5 rad/s.

- $F_2$  is a low-frequency low-amplitude excitation that can cause a large drift. This drifting action has to be eliminated by thrusters control. The spectral energy of  $F_2$  is concentrated between 0 and 1 rad/s.

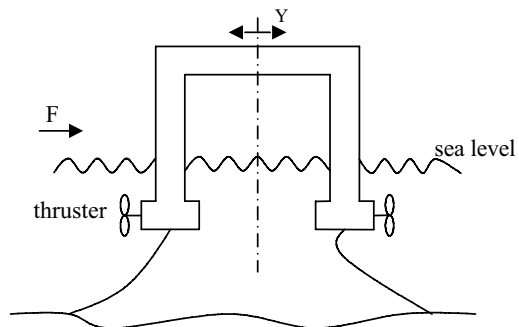


Fig.1. Moored floating platform.



The model of the system has two outputs  $y$  (the horizontal drift  $Y$  and angular deviation from the vertical axis  $\phi$ ), one control input  $u$  (the force delivered by the thrusters  $F_u$ ) and two disturbance inputs  $d$  (the force  $F$  and the torque  $M$  from the wave action). Therefore a single degree of freedom (DOF) SIMO system is presented, with one single input  $F_u$  and two outputs  $(Y, \phi)$ .

The platform dynamics are described by state-space equations (1).

$$\dot{x} = Ax + B \begin{pmatrix} F \\ M \\ F_u \end{pmatrix} \begin{pmatrix} Y \\ \phi \end{pmatrix} = Cx \quad (1)$$

where  $A$ ,  $B$ ,  $C$  are:

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -0.1010 & -0.1681 & -0.04564 & -0.01075 \\ 0.06082 & -2.1407 & -0.05578 & -0.1273 \end{pmatrix}; \quad (2)$$

$$B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.1179 & 0.1441 & 0.1476 \\ 0.1441 & 1.7057 & -0.7557 \end{pmatrix}; \quad C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

In addition, the dynamic of the thrusters are modelled by the first-order transfer function  $G_{act}(s)$ :

$$F_u = G_{act}(s)u \quad G_{act}(s) = \frac{1}{0.7s + 1} \quad (3)$$

where  $u$  is the control input and  $F_u$  the actual force delivered by the thrusters.

## ROBUST CONTROL PROBLEM FORMULATION

The control objectives are:

- Reduce the drifting action  $F_2$  by using the actuators control.
- Maintain the horizontal drift  $|Y| < 0.025\text{m}$ .
- Maintain the angular deviation  $|\phi| < 3$  degrees.
- Keep  $|F_u| < 0.25$  N.
- Make sure that the thrusters have no response to the high-frequency component  $F_1$ .

For design purposes, the system transfer function can be described as:

$$\begin{aligned} y &= P_{\text{plant}}(s) \cdot u + P_d(s) \cdot d \\ u &= -G_{\text{control}}(s) \cdot y \end{aligned} \quad (4)$$

where  $P_{\text{plant}}(s)$  is a transfer functions matrix (2x1) that connects the input  $u$  with the output  $y$ , and  $P_d(s)$  is a transfer functions matrix (2x2) that connects the disturbance  $d$  with the output  $y$ . The control structure is displayed schematically in Fig. 2.

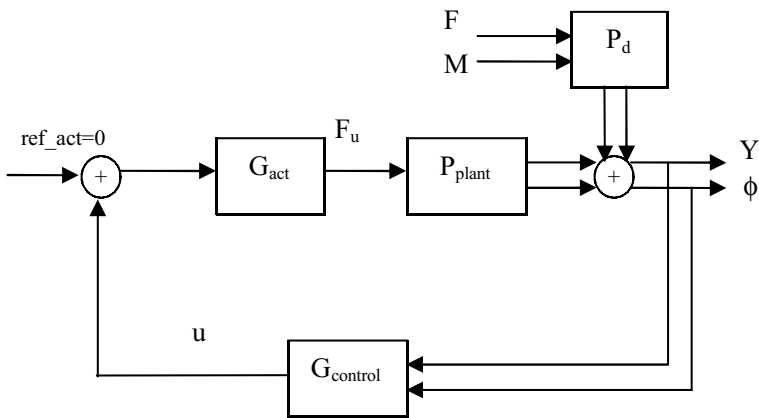


Fig. 2. Single DOF SIMO system with disturbances at the plant's output.

The equation (4) is described in explicit form in (5).

$$\begin{aligned} \begin{pmatrix} Y \\ \phi \end{pmatrix} &= \begin{pmatrix} p_{13}(s) \cdot G_{act}(s) \\ p_{23}(s) \cdot G_{act}(s) \end{pmatrix} \cdot u + \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} \begin{pmatrix} F \\ M \end{pmatrix} \\ u &= -G_{control} \cdot \begin{pmatrix} Y \\ \phi \end{pmatrix} \end{aligned} \quad (5)$$

Once the equations of the system are formulated the control problem is how to design the controller,  $G_{\text{control}}$  such that it simultaneously

- Stabilizes the plant  $P_{\text{plant}}$  and
- Decreases the plant output due to disturbances, i.e, for all  $d$ , the plant output  $y=[Y,\phi]^T$  is bounded by

$$\begin{aligned} |Y(t)| &\leq e_1(t) = 0.025m \\ |\phi(t)| &\leq e_2(t) = 3^\circ \end{aligned} \quad (6)$$



An interesting question is added to the position control design because the plant has less degree of freedom for actuation, that is, it is an underactuated system, and is more difficult to control.

Taking into account all this, the challenge is to study the effectiveness of the QFT technique to accomplish the dynamic positioning of an underactuated system.

## DEVELOPMENT OF THE QFT DESIGN PROBLEM

QFT is a frequency domain design methodology introduced by Horowitz (1992). The foundation of QFT is the fact that feedback is principally needed when the plant is uncertain and/or there are disturbances acting on the plant. Therefore, the feedback control of the platform is a good example for using QFT technique, because the system presents output disturbances (the seaway).

The QFT design procedure involves three basic steps:

- Computation of QFT bounds,
- Design of the controller (loop shaping), and
- Analysis of the design.

QFT converts close-loop magnitude specifications into magnitude constraints on a nominal open-loop function (QFT bounds). A nominal open-loop function is then designed to simultaneously satisfy its constraints as well as to achieve nominal closed-loop stability (loop shaping). It is defined the open-loop function  $L(j\omega)$  as the product of the controller transfer function and the plant transfer function.

$$L(j\omega) = G_{control}(j\omega) \cdot P_{plant}(j\omega) \quad (7)$$

In any QFT design, it is necessary to select a frequency array for computing bounds. In the case of the platform plant, the range of frequencies that belongs to the seaway spectrum will be  $\omega \in [0.1, 10]$ .

When the QFT designed is completed, it is necessary an analysis of the closed-loop response at frequencies other than those used for computing bounds.

### Formulating frequency domain specifications. QFT bounds.

The specifications must be given in terms of frequency response. For the particular case of the design of the dynamic positioning system for the moored platform model, the specifications (6) are given in temporal domain. Therefore, it is necessary to translate these constraints into frequency domain specifications. The QFT specifications used are: the gain and phase margins stability (8), the output disturbance rejection (9) and the control effort (10):

— Gain and phase margins stability

$$\left| \frac{G_{plant} G_{control}}{1 + G_{plant} G_{control}} \right| \leq W_{s1} \quad (8)$$

— Sensitivity reduction

$$\left| \frac{1}{1 + G_{plant} G_{control}} \right| \leq W_{s2} \quad (9)$$

— Control effort

$$\left| \frac{G_{control}}{1 + G_{plant} G_{control}} \right| \leq W_{s3} \quad (10)$$

In Nichols chart, the stability type problem results in bounds about the critical point where the loop response must remain outside the bounds. Sensivity reduction type problems result in bounds about the origin, where the loop response must remain outside the bounds. Control effort type problems result in bounds about the origin where the loop response must remain inside the bounds.

#### Design of the controller. Loop shaping.

The control law of the system in Fig. 2 is:

$$u = -G_{control} \cdot \begin{pmatrix} Y \\ \phi \end{pmatrix} = -(k_1 \quad k_2) \cdot \begin{pmatrix} Y \\ \phi \end{pmatrix} \quad (11)$$

Resolving equation (4)

$$(P_{plant}^{-1} + G_{control}) \cdot y = P_{plant}^{-1} \cdot P_d \cdot d \quad (12)$$

Using the notation  $P_{plant}^{-1} = (\pi_{13} \quad \pi_{23})$ , equation (12) is solved as

$$\begin{aligned} & (\pi_{13} + k_1) \cdot Y + (\pi_{23} + k_2) \cdot \phi = \\ & = (\pi_{13} p_{11} + \pi_{23} p_{21}) \cdot F + (\pi_{13} p_{12} + \pi_{23} p_{22}) \cdot M \end{aligned} \quad (13)$$

The design process is based on this equation which depicts the SIMO system of Fig. 2 with one input and two outputs. The equation presents two unknown



quantities (the controllers  $k_1$  and  $k_2$ ). In this way, the problem of the controllers design is solved by a multi-stage procedure by transforming the problem into the design of two sequential SISO systems, as follows:

*First stage.* State initially  $k_2=0$ . From equation (13) and specification  $e_1$  (6),  $k_1$  must be designed such that

$$|Y| = \frac{|\pi_{13}p_{11}F + \pi_{23}p_{12}M| + |\pi_{23}|e_2}{|\pi_{13} + k_1|} \leq e_1, \tag{14}$$

$$\forall d = [F, M]^T \in \{d\}$$

Hence, a first SISO problem is set out. The single DOF SISO system is shown in Fig. 3. The transfer function  $k_1$  is designed such that:

- The system must be stable, and
- For all  $d=[F,M]^T$  the plant output is bounded by  $|Y(\omega)| \leq |e_1(\omega)|$ , where  $|d| \leq |\pi_{13}p_{11}F + \pi_{23}p_{12}M| + |\pi_{23}|e_2$ .

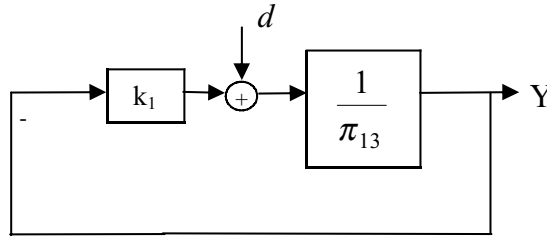


Fig. 3. The SISO feedback system to be solved in the design of  $k_1$ .

*Second stage.* Once  $k_1$  is designed, again from the equation (13) and specification  $e_2$  (6),  $k_2$  is design to satisfy

$$|\phi| = \frac{|\pi_{13}p_{11}F + \pi_{23}p_{12}M| + |(\pi_{13} + k_1)e_1|}{|\pi_{23} + k_2|} \leq e_2 \tag{15}$$

$$\forall d = [F, M]^T \in \{d\}$$

The second single DOF SISO system is set out (Fig. 4). The transfer function  $k_2$  is designed such that:

- the system must be stable and
- for all  $d=[F,M]^T$  the plant output is bounded by  $|\phi(\omega)| \leq |e_2(\omega)|$ , where  $|d| \leq |\pi_{13}p_{11}F + \pi_{23}p_{12}M| + |\pi_{13}+k_1|e_1$ .

Third stage. From the result of  $k_2$ , a new control  $k_1$  is re-designed for the first SISO system, such that it satisfies

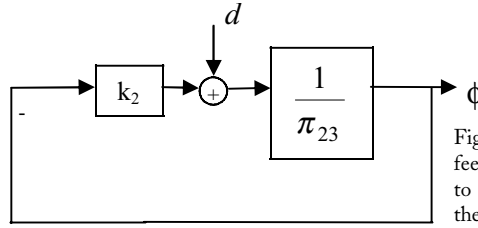


Fig. 4. The SISO feedback system to be solved in the design of  $k_2$ .

$$|Y| = \frac{|\pi_{13}p_{11}F + \pi_{23}p_{12}M| + |\pi_{23} + k_2|e_2}{|\pi_{13} + k_1|} \leq e_1, \tag{16}$$

$$\forall d = [F, M]^T \in \{d\}$$

*i-stage.* The second and third stages repeat successively up to the controllers  $k_1$  and  $k_2$  meet the objectives for the SIMO system in Fig. 2 and the specifications (6).

Thus, the problem is solved by an iterative multi-stage sequential procedure. The idea is schematically depicted in Fig. 6.

For each stage and consequently, for each SISO system, once stability and performance bounds have been computed, the next step in a QFT procedure involves the design (loop shaping) of a nominal function that meets its bounds. The nominal loop  $L(j\omega)$  has to satisfy the worst case of all bounds. The MATLAB toolbox includes an interactive design environment.

### Analysis of the design

Once the controller parameters are designed by using QFT design, the system in closed loop dynamic (Fig. 2) is simulated in order to prove if the control meets the specifications.

### RESULTS

According to the methodology explained, finally the control design procedure was completed in five stages.

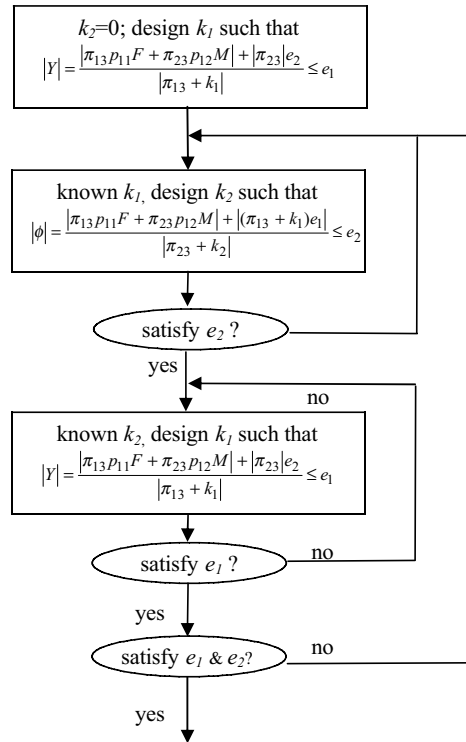


Fig. 6. Scheme of design.





In this work it is shown the results of the QFT design of the two last stages in which the definitive controller  $(k_1, k_2)$  is determined. Specifically, the fourth stage corresponds to the  $k_2$  design, and the fifth stage to  $k_1$  design. Finally, simulations of the system in closed loop are tried in order to examine if the positioning system is achieved. Thus, temporal responses are shown.

#### Design of the controller $k_2$ . Fourth stage.

*Robust Stability and Performance Bounds.* The specifications fixed for QFT design for the second SISO system in Fig. 4 are:

- Gain and phase margins  $W_{s1} = 2.8$ ,
- Sensivity reduction  $W_{s2} = 3$ ,
- Control effort  $W_{s3} = 15$ .

These specifications guarantee adequate gain margins, sensitivity and control effort.

*Control design.* The control  $k_2$  must be designed such that open loop function  $L_2(j\omega)$ :

$$L_2 = G_{control} \cdot P_{plant} = k_2 \cdot 1 / \pi_{23} \quad (17)$$

satisfies the worst case of all bounds (intersection). The controller designed (18) is a second order filter, with 2 poles and 2 zeros.

$$k_2(s) = \frac{\left( \frac{1}{0.26^2} s^2 + \frac{2 \cdot 0.02}{0.26} s + 1 \right)}{\left( \frac{1}{2.4^2} s^2 + \frac{2 \cdot 0.7}{2.4} s + 1 \right)} \quad (18)$$

Figure 7 shows the Nichols chart of the open-loop function  $L_2(j\omega)$  with the given specifications. It is shown that this controller accomplishes the specifications.

#### Design of the controller $k_1$ . Fifth stage

*Robust Stability and Performance Bounds.* The specifications fixed for QFT design for the first SISO system in Fig. 3 are:

- Gain and phase margins  $W_{s1} = 3$ ,
- Sensivity reduction  $W_{s2} = 3$ ,
- Control effort  $W_{s3} = 15$ .

These specifications guarantee adequate gain margins, sensitivity and control effort.

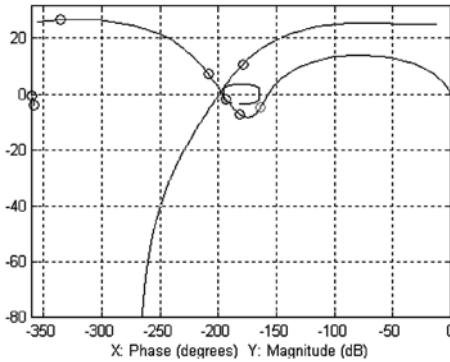


Fig. 7. Nominal open loop function  $L_2(jw)$  and intersection of all bounds, for second SISO system.

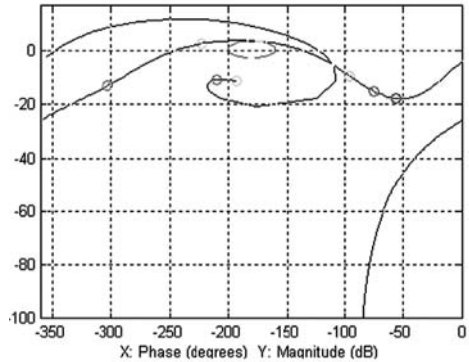


Fig. 8. Nominal open loop function  $L_1(jw)$  and intersection of all bounds, for first SISO system.

*Control design.* The control  $k_1$  must be designed such that open loop function  $L_1(j\omega)$  satisfies the worst case of all bounds (intersection).

$$L_1 = G_{control} \cdot P_{plant} = k_1 \cdot 1 / \pi_{13} \tag{19}$$

The controller designed (20) is a first order filter, with 1 pole and 1 zero. Figure 8 shows the Nichols chart of the open-loop function  $L_1(j\omega)$  with the given specifications. It is shown that this controller meets the specifications.

$$k_1(s) = -0.28 \cdot \frac{\left(\frac{1}{0.52}s + 1\right)}{\left(\frac{1}{1.95}s + 1\right)} \tag{20}$$

### Analysis of the design

Temporal responses of the SIMO system (Fig. 2) in closed loop dynamic are shown. Figures 9 and 10 compare both outputs  $Y$  and  $\phi$  with and without control respectively. It is shown that the control achieves the positioning system.

### CONCLUSIONS

In this work an analysis and design of a dynamic positioning system for a moored floating platform has been carried out. The plant model is a single degree of freedom SIMO system, therefore a notable question is that the plant is underactuated. Thus, it has been used a quantitative design technique (QFT) for synthesizing

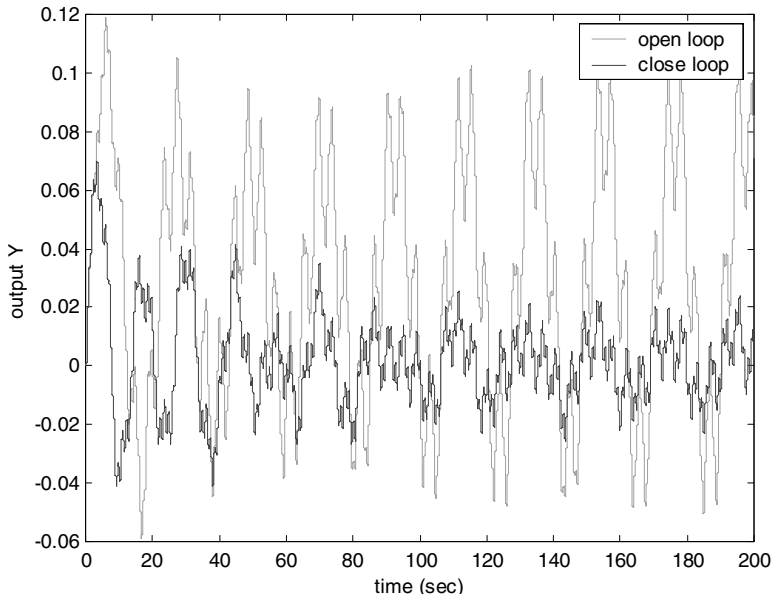


Fig. 9 Comparison of temporal response  $Y$  in open loop (dashed line) and closed loop (solid line).

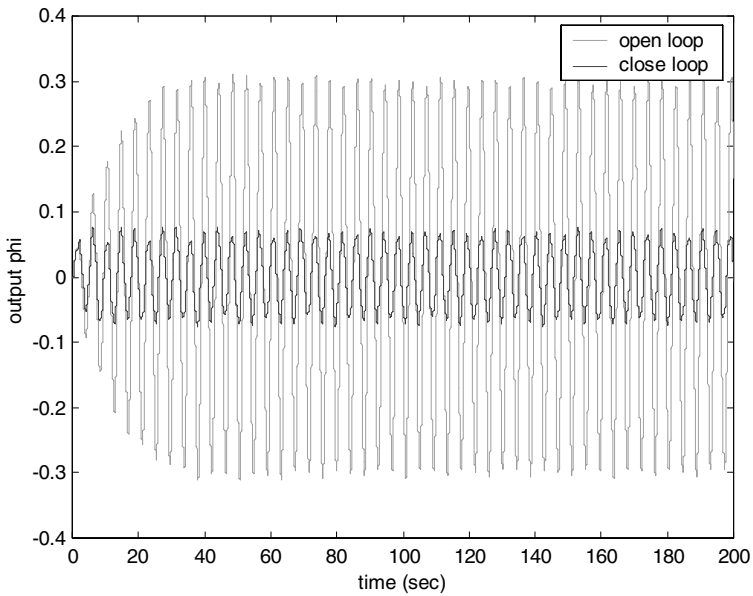


Fig. 10 Comparison of temporal response  $\phi$  in open loop (dashed line) and closed loop (solid line).

the controller for the SIMO plant. The idea consists of transforming the problem into two sequential SISO systems and breaking the design process into a series of iterative stages, in such a way that the solution in the first system is used in the design in the second system, and vice versa. Finally it is shown that QFT design is a robust method very suitable for the implementation, and that accomplishes the objectives efficiently. We have verified that this method is an attractive alternative for robust design of these kinds of systems.

#### ACKNOWLEDGEMENT

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## DISEÑO DE UN CONTROLADOR MEDIANTE LA TÉCNICA QFT PARA POSICIONAMIENTO DINÁMICO DE UNA PLATAFORMA AMARRADA.

### RESUMEN

En este trabajo se ha llevado a cabo el análisis y diseño de un sistema de posicionamiento dinámico para una plataforma amarrada flotante. El modelo de la plataforma consiste en una planta con una entrada y dos salidas (sistema SIMO) de un grado de libertad. La finalidad del controlador es minimizar el desplazamiento provocado por el oleaje mediante un control adecuado de los propulsores. Una cuestión interesante que se plantea es que la planta tiene menos grados de libertad para la actuación y es más difícil de control. El método empleado para el control es la técnica de control robusto QFT para sistemas subactuados. El problema se resuelve mediante un proceso secuencial iterativo multi-etapa. Finalmente se comprueba que el control consigue el posicionamiento dinámico.

### INTRODUCCIÓN

El control de sistemas subactuados es un problema típico en los sistemas navales. Ejemplos de este tipo de problemas son el diseño de controladores para seguimiento de trayectoria, estabilización, y posicionamiento dinámico.

En este trabajo se analiza el uso del método de diseño QFT (del inglés Quantitative Feedback Theory) multivariable para sistemas subactuados, aplicado al modelo de una plataforma amarrada flotante. El modelo es un sistema lineal invariante en el tiempo (LTI) con una entrada y dos salidas (SIMO) con un grado de libertad.

Este tipo de sistemas ha sido examinado y resuelto mediante diferentes métodos de control robusto (Scherer, *et al.*, 1997; Revilla, 2005; Nakamura, *et al.*, 2001).

La técnica QFT (Horowitz, 1992; Yaniv, 1999; Borguesani, *et al.*, 1995; Houpis, *et al.*, 2006) es una metodología de diseño en el dominio de la frecuencia que no es muy común en los sistemas navales. Nuestro grupo de trabajo ha aplicada esta técnica en el problema de estabilización de barcos de alta velocidad (Aranda, *et al.*, 2005) con excelentes resultados.

El objetivo de este trabajo es verificar que esta técnica puede extenderse para sistemas subactuados y mostrar los resultados obtenidos en la aplicación de un problema de posicionamiento dinámico típico.

### METODOLOGÍA

El sistema consiste en una plataforma flotante amarrada al fondo del océano, y equipada con dos propulsores. El objetivo es conseguir un control de los propulso-



res adecuado de manera que minimice el desplazamiento derivado de la acción de las olas. El modelo presenta dos salidas  $y$  (el desplazamiento horizontal  $Y$ , y el ángulo de desviación del eje vertical  $\phi$ ), una entrada de control  $u$  (la fuerza  $F_u$  producida por los propulsores), y dos entradas de perturbación (la fuerza  $F$  y el momento  $M$  debido al oleaje). Por tanto se presenta un sistema de un grado de libertad SIMO, con una entrada y dos salidas.

Los objetivos del control son: reducir la fuerza de desplazamiento mediante el control de los actuadores, mantener el desplazamiento horizontal  $Y$  por debajo de los 0.025 m, y la desviación angular  $\phi$  por debajo de los 3 grados, conseguir que la fuerza de los propulsores  $F_u$  no supere los 25 N, y asegurarse que los propulsores no tienen componente en alta frecuencia.

El fundamento de QFT es el hecho de que la realimentación se necesita principalmente cuando la planta tiene incertidumbres y/o hay perturbaciones, por lo que el sistema de la plataforma es un buen ejemplo de aplicación.

El proceso de diseño QFT contempla tres pasos básicos: *i*) cálculo de fronteras, *ii*) diseño del control, *iii*) análisis del diseño. QFT convierte las especificaciones de magnitud en lazo cerrado en restricciones de magnitud de una función en lazo abierto nominal (fronteras). Así, se diseña una función en lazo abierto  $L(j\omega)$ , que se define como el producto de las funciones de transferencia del control y de la planta, de manera que satisfaga simultáneamente tanto sus restricciones como la estabilidad en lazo cerrado (*loop shaping*).

En cualquier diseño QFT, es necesario seleccionar un rango de frecuencias para calcular las fronteras. En el caso de la planta de la plataforma, se emplea un rango de frecuencias que pertenece al espectro del oleaje,  $\omega \in [0.1, 10]$  rad/s.

Finalmente, a partir de la ley de control, se obtiene una ecuación con dos incógnitas  $k_1$ ,  $k_2$ , correspondientes a los dos términos de la matriz de funciones de transferencia del control. El diseño del controlador se basa en esta ecuación, que ayuda a transformar el problema SIMO en el diseño de dos sistemas SISO secuenciales. Así, se resuelve mediante un proceso secuencial iterativo, de tal forma que la solución del control  $k_1$  en el primer sistema SISO se emplea para el diseño de  $k_2$  en el segundo sistema, y viceversa. Las etapas se repiten sucesivamente hasta que los controladores  $k_1$  y  $k_2$  cumplen los objetivos del sistema SIMO original.

## RESULTADOS

Siguiendo el procedimiento planteado, el proceso de diseño del controlador ( $k_1$ ,  $k_2$ ) se completa en cinco etapas. El controlador  $k_1$  final es un filtro de primer orden, con un polo y un cero, y el control  $k_2$  es un filtro de segundo orden, con dos polos y dos ceros. Finalmente, se obtienen simulaciones de las respuestas temporales del sistema en lazo cerrado y se comparan las salidas con y sin control. Se demuestra que el control consigue el sistema de posicionamiento.

## CONCLUSIONES

En este trabajo se ha llevado a cabo el análisis y diseño de un sistema de posicionamiento dinámico para una plataforma amarrada flotante. El modelo de la planta es un sistema SIMO con un grado de libertad, por tanto una notable característica es que la planta es subactuada. Para resolver el problema de posicionamiento, se emplea la teoría cuantitativa realimentada (QFT). Finalmente se demuestra que el diseño QFT es un método robusto que resulta adecuado para la implementación del controlador, y que consigue los objetivos de forma efectiva. Así, hemos verificado que este método es una alternativa atractiva para el diseño de control robusto de sistemas navales.